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SCIENCE RESEARCH COUNCIL

COMPUTER SIMULATION OF THE QUENCHING OF A SUPERCONDUCTING MAGNET

by

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When a superconducting magnet reverts to the normal state, the energy stored in the magnet is dissipated in the resistive regions of the winding. This process can give rise to problems of excess voltage, overheating of the winding and violent boil off of the liquid helium.

Various protection schemes have been proposed[1], the only one to be considered here is the use of an external dump resistor; fig. 1 shows the circuit. At the start of the quench, the circuit breaker is opened, and some fraction of the energy is released in the protection resistor, away from the cryogenic environment. The remaining energy is dissipated in the resistive regions of the magnet and in the persistent current switch (when fitted). A short computer programme, 'quench', has been written to calculate how the current decays and where the energy is released.

The increase in magnet resistance with time after the quench is a complicated process, two factors are involved:

(a) the increase in size of the normal region which spreads outwards in three dimensions.

(b) the increase in resistivity of the normal region as it gets hot.

The circuit equation:

$$IR_{mag}\,(I,\,\theta,\,t) + I\,(R_{sw} + R_{prot}) + L\,\frac{di}{dt} = 0$$

is difficult to solve analytically[2] but lends itself very well to a simple stepwise integration technique. Starting at t=0, $\frac{dI}{dt}_{(t=0)}$ is known, from this $I_{(t=\Delta t)}$ may be found and hence $R_{mag}\,(t=\Delta t)$ and $\frac{dI}{dt}\,(t=\Delta t)$ calculated. Proceeding to time $2\Delta t$, $I_{(t=2\Delta t)}$ is found from $I_{(t=\Delta t)}$ and $\frac{dI}{dt}\,(t=\Delta t)$, hence $R_{mag}\,(t=2\Delta t)$ and $\frac{dI}{dt}\,(t=2\Delta t)$ and so on.

The model adopted for the spreading of the normal region is illustrated in fig. 2. The velocity of propagation of normality is given approximately by:

$$U = \frac{I}{A\delta Cp}\left(\frac{k\rho}{\theta_c}\right)^{\frac{1}{2}}$$

Where:

A = Conductor area of cross section

δ = Density

Cp = Specific heat

k = Thermal conductivity

ρ = Electrical resistivity

θ_c = Critical temperature

This may be simplified to $U = U_o I/I_o$ in a direction parallel to the conductor and $U' = \alpha U_o I/I_o$ in a direction perpendicular to the conductor[2] where:

$$U_o = \frac{I_o}{A\delta Cp}\left(\frac{k\rho}{\theta_c}\right)^{\frac{1}{2}}$$

$$\alpha = (k \text{ perpendicular}/k \text{ parallel})^{\frac{1}{2}}$$

There is also a slow dependence of U on the magnetic field through θ_c but this is neglected and it is assumed the quench takes place in the high field region of the coil. The normal region therefore spreads to fill a conical volume, the incremental volume increase during Δt at time $n\,\Delta t$ being given by:

$$\Delta V = 2\pi\alpha^2 \quad \Delta x\ (x^2 + x\Delta x + \Delta x^2/_3)$$

Where:

$$\Delta x = U\ (I)\ \Delta t$$

$$x = \sum_n U(I)\ \Delta t$$

Under certain circumstances the sideways spreading may not take place around a full circle, in a cooled coil for example, normality may only spread upwards. This may be simply included in the expression for ΔV. It is also necessary to terminate the increase in volume when the normal region reaches the coil boundary or, when the two normal fronts, travelling in opposite directions, meet each other at the far side of the magnet.

Upon becoming resistive each incremental volume will immediately start to heat up.[3]

$$\int \frac{I^2\rho(\theta)dt}{A} = \int Cp(\theta)\delta A d\theta$$

$$\int \frac{I^2}{A^2}\, dt \;=\; \int \frac{\delta\, Cp}{\rho(\theta)}\, d\theta \;=\; G(\theta)$$

The function $G(\theta)$ for copper is plotted in fig. 3, it has been approximated, for the calculation, by three straight line segments. At each Δt, the quantity $\sum_n I^2\, \Delta t$ is therefore summed for each incremental volume and, from this the temperature, resistivity (fig. 4) and finally the resistance of the increment is calculated. These resistances are then summed over all the increments to yield R_{mag} which is then substituted in equation 1. The use of $G(\theta)$ and $\rho(\theta)$ for pure copper implies that the magnet winding consists mainly of copper. This is a good approximation for most fully stabilised magnets but it may sometimes be necessary to modify the form of $G(\theta)$, $\rho(\theta)$ and also U_o to take account of the superconductor, insulation, structural materials etc.

The programme has been checked by comparison with the observed current decay of several magnets; fig. 5 shows a typical example(4). It has also been used to predict the behaviour of the superconducting bending magnet presently under construction. For this purpose, the calculated values of U_o and α were checked by direct measurements on a small test solenoid with the same winding construction. Fig. 6 shows the calculated current decay at optimum energy extraction for a constant protection resistor and also for a 'Metrosil' where $R = R_o\, (I_o/I)^{0.75}$. Fig. 7 shows how the fraction of the stored energy dissipated in the cryostat depends on the value of the protection resistor. This fraction first decreases with increasing R_{prot} as less energy is released in the magnet, it then rises again as more energy is dissipated in the persistent current switch. The resistance of the switch when 'off' i.e. in the normal state is 10 ohm. Fig. 8 shows the peak temperature rise in the magnet which occurs at the point where the quench starts. Evidently there is no danger of this magnet burning out, even with zero protection resistance, the peak temperature rise is only 150^{o}K. It is however possible to markedly reduce the helium boil off at quench by a suitable choice of protection resistor. The total stored energy is 1 MJoule i.e. 400 litres of liquid helium but the minimum of fig. 8 is only 40 kJoules or 16 litres of liquid. This represents a very considerable saving in cost and also avoids the danger of a high pressure being developed in the cryostat.

APPENDIX : USE OF PROGRAMME

Five data cards are required:

1) Δt, $N(t)$ (F10.4, I10)
2) U_o, A, ℓ, (3F 10.4)
3) α, β, X_{max}, Y_{max} (4F 10.4)
4) I_o, R_{prot} γ, R_{sw}, L (5F 10.4)

Where:

Δt	=	Time interval (secs)
$n(t)$	=	Total number of intervals
U_o	=	Initial velocity parallel to conductors
A	=	Conductor area of cross section
ℓ	=	Conductor length per unit volume of winding
α	=	Ratio perpendicular to parallel velocity
β	=	Fraction of circle over which perpendicular spreading takes place
X_{max}	=	Limiting distance for parallel spreading
Y_{max}	=	Limiting distance for perpendicular spreading
I_o	=	Initial current
R_{prot}	=	Protection resistor
γ	=	Protection resistor index, set zero for constant resistor
R_{sw}	=	Resistance of persistent current switch when 'off', set to large value if no switch
L	=	Coil Inductance

REFERENCES

1) P.F. Smith — Rev. Sci. Inst. Vol. 34 No. 4 pp. 368.

2) Z.J.J. Stekly — Avoo-Everett Research Report 135 (BSD - TDR - 62 - 116).

3) B.J. Maddock and G.B. James — Proc. I.E.E. Vol. 115 No. 4. pp. 543

4) H. Desportes — Experiences sur Cable Supraconducteur Semi-Stabilisé, presented at the Conference on High Magnetic Fields, Grenoble, 1966.

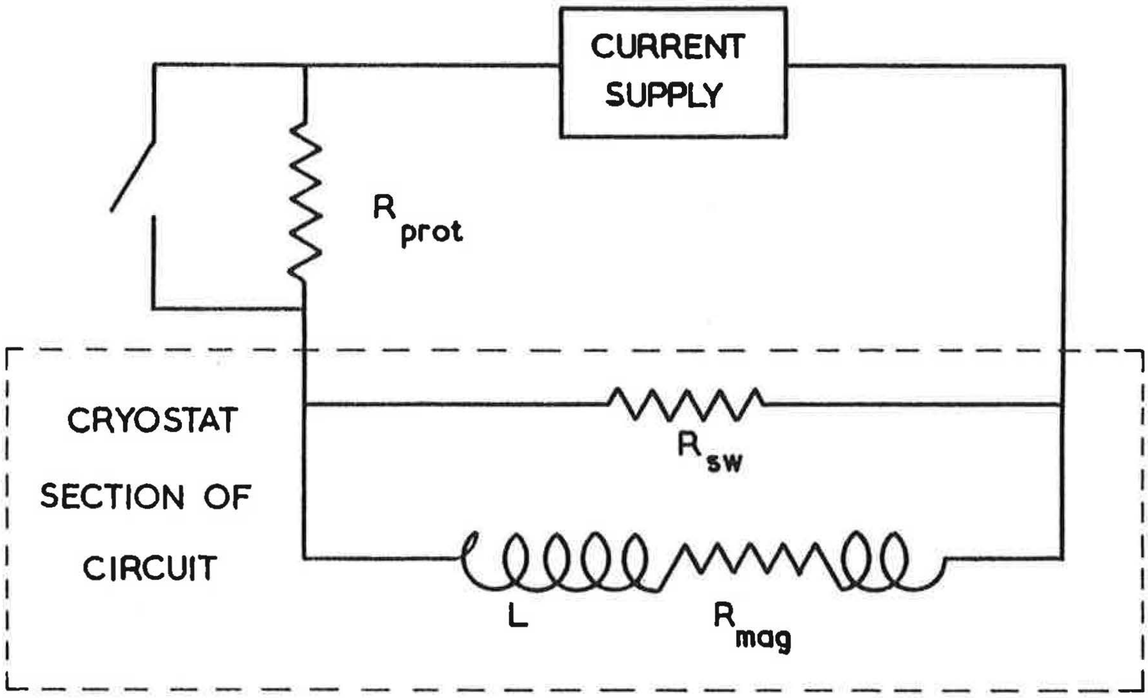

Fig. 1 CIRCUIT DIAGRAM

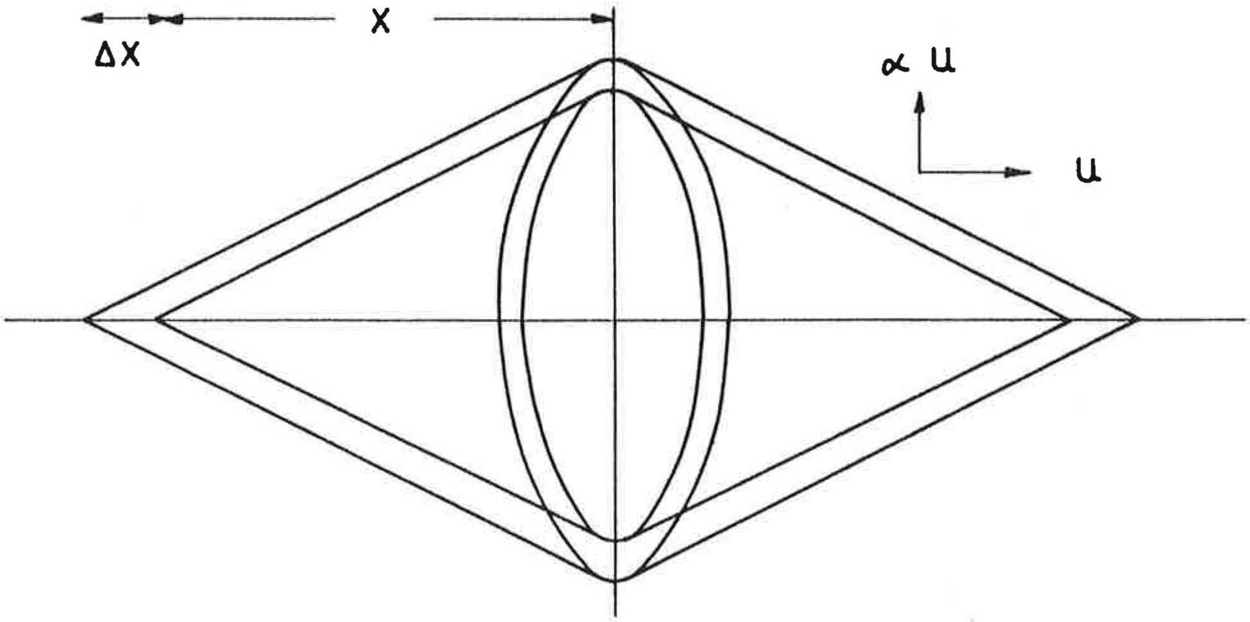

Fig. 2 SPREADING OF THE NORMAL REGION

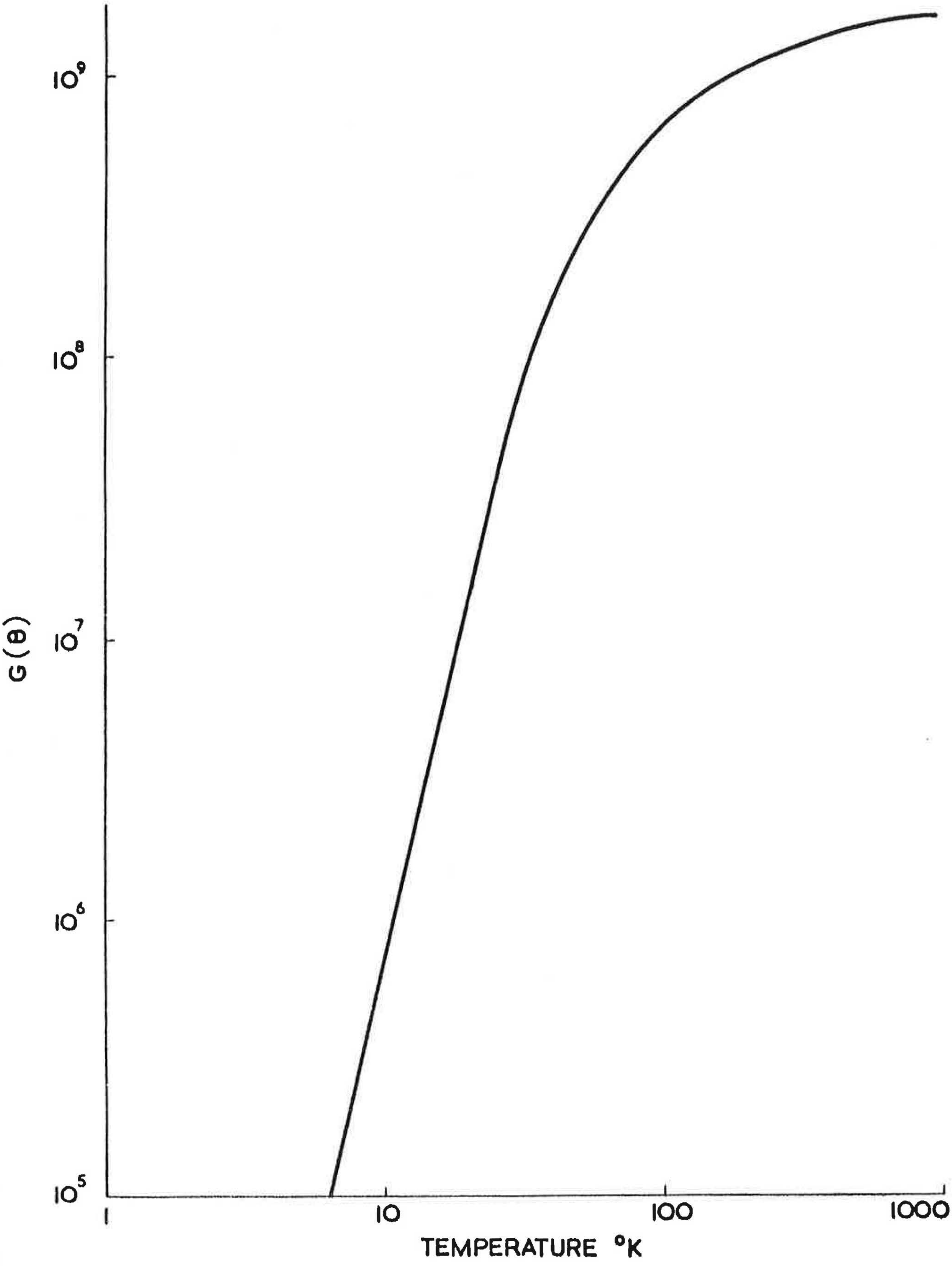

Fig. 3 G (θ) FOR COPPER

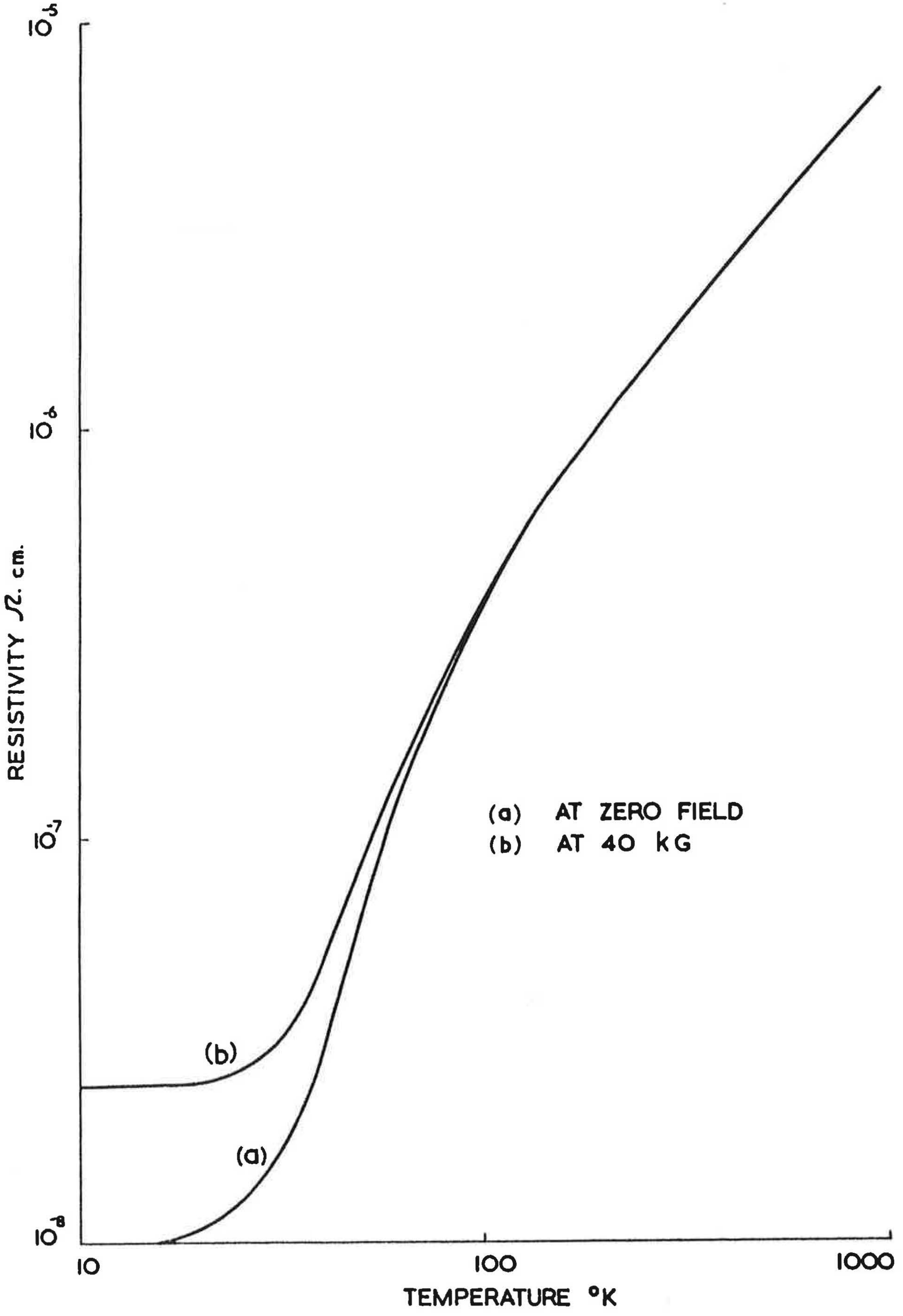

Fig. 4 RESISTIVITY OF COPPER

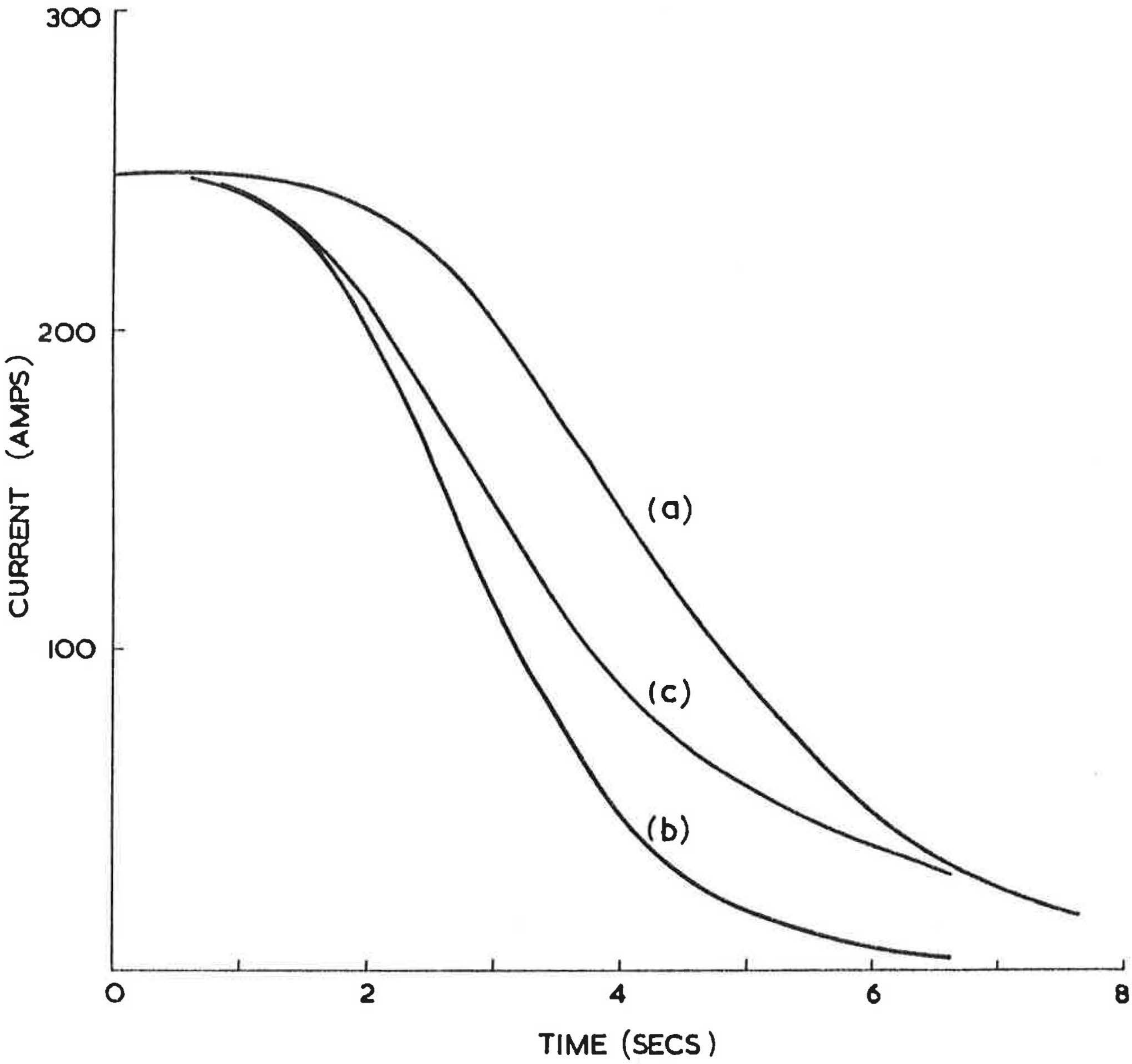

Fig. 5 CURRENT DECAY OF DESPORTES MAGNET B4 (3)

(a) (b) COMPUTER SOLUTIONS WHEN NORMALITY STARTS AT CENTRE AND INNER CORNER OF WINDING RESPECTIVELY

(c) MEASURED CURRENT DELAY

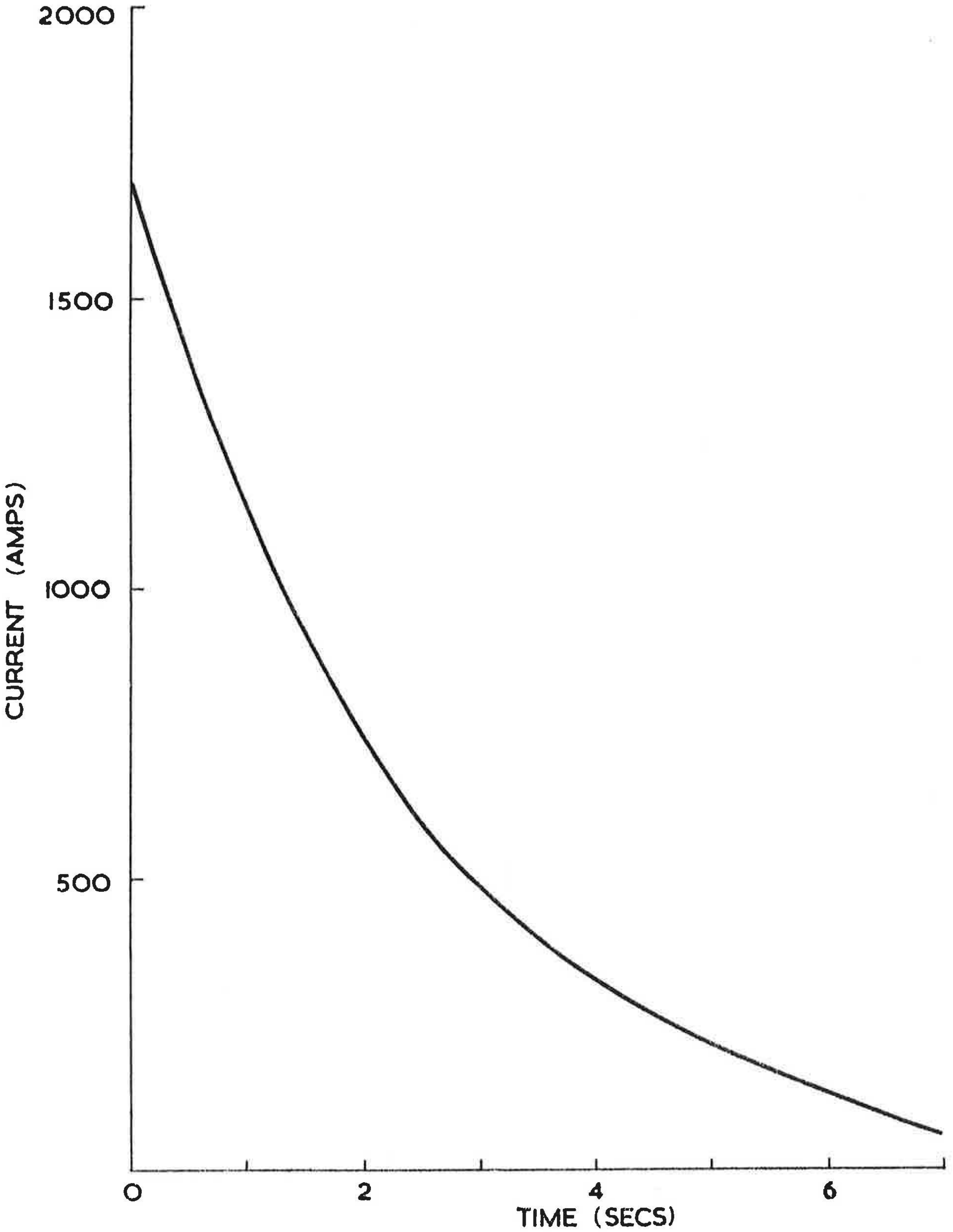

Fig. 6 CURRENT DECAY OF SUPERCONDUCTING BENDING MAGNET

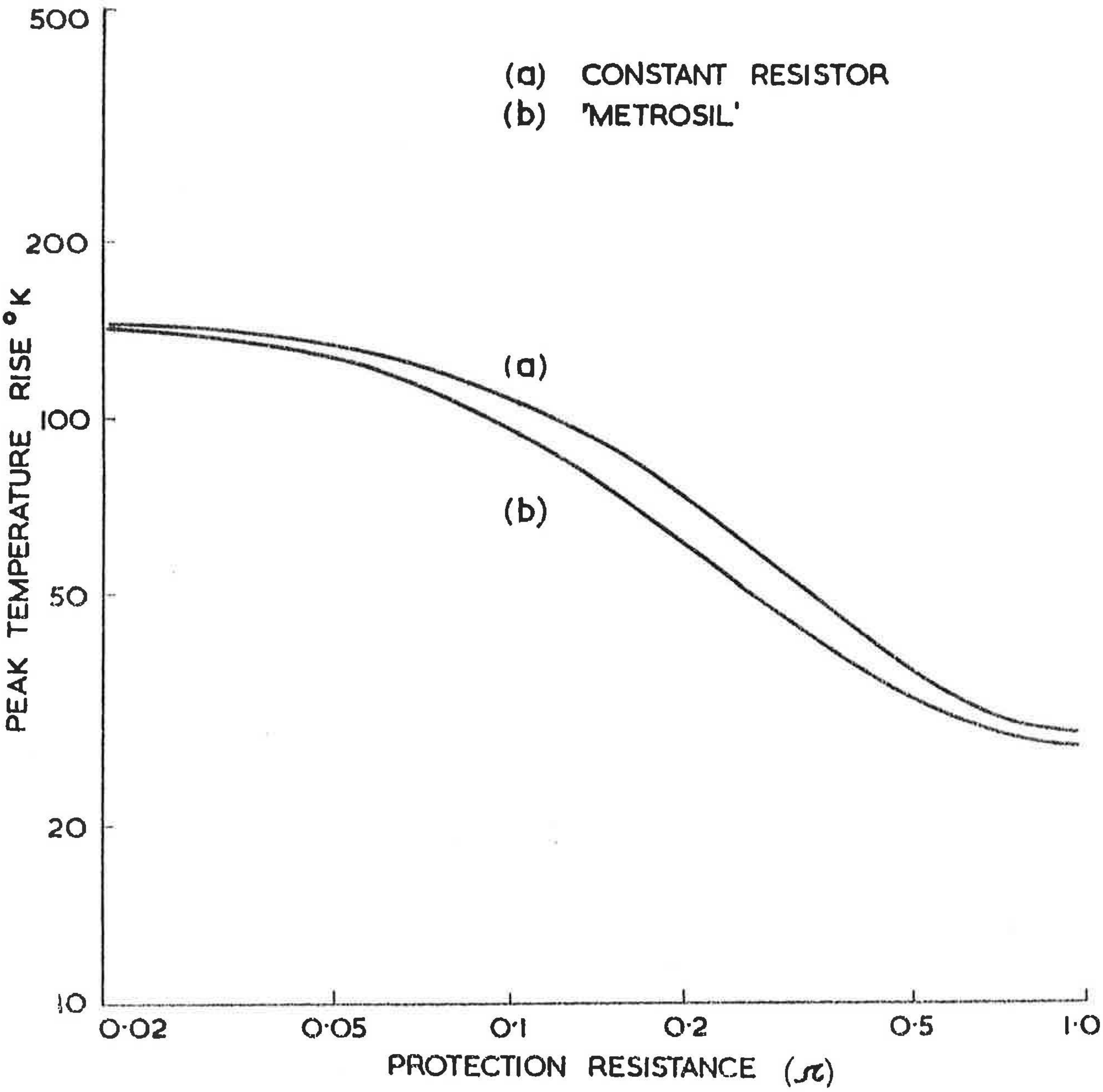

Fig. 8 SUPERCONDUCTING BENDING MAGNET, PEAK TEMPERATURE RISE