

# THE DYNAMICS OF ULTRA-COMPACT AND ISOCHRONOUS GEV ENERGY FFAGS

R.B. Appleby<sup>#</sup>, S. Tygier, J. Garland, H.L. Owen,

The University of Manchester, M13 9PL, UK, Cockcroft Institute, Daresbury Laboratory,  
Warrington, WA4 4AD, UK

C. Johnstone<sup>##</sup>, Particle Accelerator Corp, Batavia, IL 60510, USA

## Abstract

The FFAG (Fixed-Field Alternating-Gradient) accelerator is a class of accelerators that comprises the best features of the cyclotron and the synchrotron, combining fixed magnetic fields with strong focusing gradients for stable, low-loss operation. Here, a new type of medium-energy 1 GeV isochronous (CW) FFAG has been developed in a racetrack layout that supports two opposing synchrotron-like straights, permitting both high-gradient RF modules and efficient injection and extraction in a highly compact footprint. In this paper we present beam dynamic simulations for this compact racetrack FFAG, and compare the differences between an equivalent circular and racetrack configuration. A comparison of the FFAG dynamics with the 800-MeV (Daeδalus) cyclotron is briefly presented.

## INTRODUCTION

Historically, cyclotrons have been the highest current, most compact accelerator technology for protons up to a few hundred MeV. Higher energies require separated sectors in the cyclotron - like the 590-MeV PSI [1] or 500-MeV TRIUMF cyclotrons [2] - in order to insert strong accelerating (RF) systems. Stronger acceleration at relativistic energies is required to overcome beam losses during acceleration and extraction, due to both resonance crossing (machine tunes cannot remain constant without strong focusing) and the ever-reducing orbit separation for isochronous operation. Lower-energy cyclotrons utilize Dee-shaped RF components between poles to achieve compactness; however, the accelerating gradient is insufficient for low-loss operation. Once space is inserted between the magnetic sectors of the cyclotron the footprint of the cyclotron grows rapidly and the cyclotron no longer constitutes a compact solution.

Weak- and edge-focusing becomes problematic for containment of high-intensity beams and tolerance to space charge forces. In order to achieve milliamps of average current, proton accelerators require both strong-focusing and CW operation to mitigate space charge. As relativistic energies are approached, the orbit separation on consecutive turns must decrease to maintain the isochronous condition; i.e. constant revolution frequency for a fixed-frequency acceleration system. High acceleration gradients are therefore required to separate orbits on different acceleration turns ('cross-talk') and to limit losses at extraction (to avoid massive shielding and unmanageable component activation). However, orbit

separation also scales with circumference so compact accelerators must deploy strong RF gradients. The weak-focusing nature of traditional cyclotrons does not permit long (several metre) straight sections without the use of separated sectors and a significant scaling-up of machine radius and size. The addition of strong focusing gradients (and corresponding strong beam envelope control) to conventional cyclotron fields - including reversed gradients to confine the beam envelope in both transverse planes - does allow insertion of long synchrotron-like straight sections and thus the efficient implementation of high-gradient, multiple-cavity RF modules, even for example SCRF cryomodels. Furthermore, recent non-scaling FFAG designs utilise a racetrack shape - essentially a recirculating linear accelerator with FFAG arcs. This new generation of ultra-compact non-scaling FFAGs with constant machine tunes are described in this work, specifically an example 0.2 - 1 GeV proton FFAG with a 4.5 m radius, and a racetrack lattice with a 7m by 11m footprint that could utilise either high-gradient normal-conducting RF or SRF structures [3].

## DESIGN CONCEPTS

The dynamics of FFAGs - both scaling [4] and non-scaling - are dominated by synchrotron-like dynamics [4]. In an FFAG all conventional focusing terms are described by the following thin-lens approximation:

$$1/f_F = k_F l + \frac{\vartheta}{\rho_F} + \frac{\eta}{\rho_F}. \quad (1)$$

where  $f_F$  and  $\rho_F$  are the focal length in the horizontal plane and bend radius respectively,  $l$  is the horizontally-focussing magnet half length,  $k_F$  the "local" horizontally focusing gradient for an arbitrary field order,  $\theta$  the sector bend angle and  $\eta$  the edge crossing angle (the tangent is approximated). In the vertical (non-bending) plane the sector bend term is not available. The synchrotron relies on the first strong-focusing gradient term whilst the cyclotron on the last two terms which are the centripetal (weak focusing) and edge focusing terms, but the FFAG utilizes all three terms for beam envelope control and dynamical stability. Unlike the scaling version, the non-scaling FFAG further optimizes the bend/reverse gradients and edge angles independently; this is critical for compactness and continuous wave (CW) dynamics. The reverse gradient is particularly important for a stronger vertical tune than can be achieved in a cyclotron. In addition, in the non-scaling FFAG (only), the different focusing principles may be combined in different and varying relative strengths through the acceleration cycle -

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#robert.appleby@manchester.ac.uk  
##cjj@fnal.gov

this varying composition can be exploited to control the machine tune through the acceleration cycle without applying the field scaling law. The terms can be inter-played to achieve stable dynamics in highly flexible lattice structures.

The design of the non-scaling FFAG with its flexibility is even more powerful with the use of a nonlinear field expansion. The nonlinear gradient has the advantage of providing increased focusing in both transverse planes (as a function of radius), thereby providing a high constant tune in both planes while preserving considerable freedom in magnet and layout parameters. Furthermore, the orbital path length can be constrained such that the revolution time at each momentum scales with velocity, and simultaneously the machine tune can be controlled through edge- and weak-focusing effects independent of path length - thereby impacting tune but not revolution frequency. Unlike the cyclotron - which relies on a predominately dipole field or fixed B-field scaling with  $\gamma$  and is therefore limited in adapting path length to velocity as the energy becomes relativistic - the non-scaling FFAG can maintain the isochronous condition well into relativistic energy regimes.

## LATTICE DESIGN OF 2M RING

The nonlinear design principles described above have been applied to produce a 200 MeV to 1 GeV quasi-isochronous ( $dt/t \approx \pm 3.5\%$ ) circular FFAG with a small footprint [5] which would utilise magnetic fields of 5T or less. The machine footprint is shown in Fig. 1 (left), with four 2m straights and each cell composed of an FDF triplet.

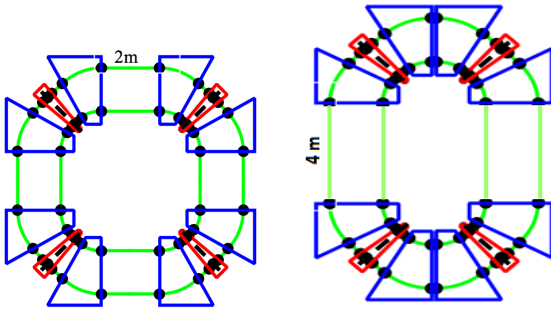


Figure 1: The 2m circular (left) and racetrack FFAG (right) layouts.

As the demands for compactness increase, eventually all inter-component straight sections become too short for effective extraction, or even injection. A racetrack configuration would optimize compactness yet retain the capability to insert high-gradient RF modules in one straight and injection/extraction systems in the opposite straight. To this end we have designed a racetrack variant that eliminates two of the straights and lengthens the remaining two opposing straights shown in Fig 1. We have also constructed an intermediate-stage racetrack

FFAG with two straights of 1.5m and two straights of 2.5m which preserves the circumference and tune (denoted here as a “mini-racetrack”), and which allows a comparison between the circular and racetrack designs. The beam orbits through the common bend structures are shown in Fig. 2 for the range of machine energies.

The ring tunes for the ring and the racetrack version computed with PyZgoubi assuming an Enge fringe field [6] are shown in Fig. 3. Note that the tunes are constant with energy in the horizontal plane but have vertical variations that will be addressed in future work. The design orbits computed with COSY [7] are shown in Fig. 3.

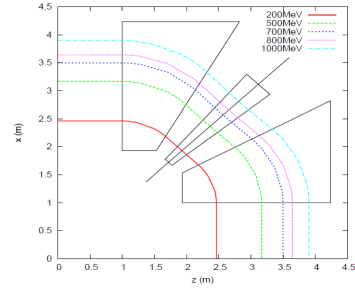


Figure 2: Particle tracks through the FDF bend structure for 200 MeV to 1000 MeV protons.

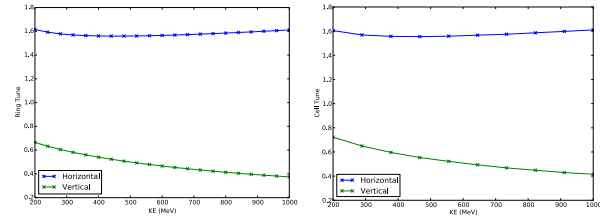


Figure 3: Machine tunes for the 2m ring (left) and the mini-racetrack FFAGs (right) as a function of beam energy.

Interestingly, the lattices are almost identical in machine tunes and dynamics if the circumference is preserved, as can be seen in the machine tunes in Figure 2. The following simulations thus apply to both variants.

## DYNAMIC APERTURE

We now consider the dynamic aperture of these FFAGs and present an algorithm (implemented in PyZgoubi) for its calculation. The period of time over which particles must be followed in simulation to predict the dynamic aperture depends upon the application of that accelerator. For the ultra-compact FFAGs discussed here if implemented with SRF, then the protons remain in the accelerator for only around 40 turns, but with NCRF as in the PSI cyclotron, the number of turns will increase by about an order of magnitude.

Dynamic aperture is expressed here in terms of the limit in transverse particle amplitudes or (equivalently) the limit in transverse single particle actions  $J_{x,y}$  over the number of acceleration turns and is calculated by increasing  $J_x$  and  $J_y$  until the particle is lost in X

acceleration turns (loss defined by the actual physical apertures, by a particle deflected by more than 90 degrees in any magnet or if a particle attempts to make too many steps in a magnet). We therefore consider increments of  $J_x$  and increments of  $J_y$  or some combination of both defined the angle in the  $(J_x, J_y)$  plane. For the large horizontal and small vertical apertures of fixed-field machines, the DA is best described in terms of these two planes.

Table 1: The dynamic aperture for the circular 2m FFAG computed with PyZgoubi

Dynamic aperture	Horizontal $\pi$ .mm.mrad	Vertical $\pi$ .mm.mrad
200 MeV 40 turns	74700	107
200 MeV 200 turns	72600	91
800 MeV 40 turns	156300	364
800 MeV 200 turns	155100	356

Note that the single-particle trajectories are generally elliptical at small amplitudes and but become non-elliptical at larger amplitudes due to the non-linear motion. Hence we compute the dynamic aperture by combinations of particle coordinates based on the particle action e.g. we increase  $J_x$  by increasing the single particle coordinates along the four axes  $(+x, 0)$ ,  $(0, +x')$ ,  $(-x, 0)$  and  $(0, -x')$  and similarly for  $J_y$  although generally we find approximate symmetry about the origin. The DA has been computed in the horizontal and vertical using this method over 40 and 200 turns. The DA is simulated using the PyZgoubi framework, with a series of random horizontal misalignments applied to all magnetic elements to study the DA reduction for a realistic lattice. The misalignments are modelled as a Gaussian distribution truncated at  $3\sigma$ . The DA is shown as the mean DA over the seeds, with an error bar corresponding to the RMS variation over the seeds. Table 1 shows the DA achieved for the 2m circular FFAG in the horizontal and vertical places computed with Pygoubi and an Enge form for the fringe field. Fig. 4 shows the fractional decrease of dynamic aperture with respect to the ideal case for the circular 2m FFAG in terms of normalised stored emittance as a function of misalignment magnitude. The DA shows a slow decline for misalignments up to 600  $\mu$ m. The same calculation for the racetrack is shown in Fig. 5 and shows a faster decline of DA with misalignment width. However both machines are very tolerant to misalignments in the horizontal plane.

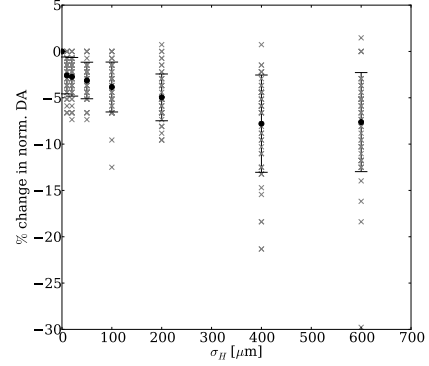


Figure 4: The relative drop in dynamic aperture as a function of misalignment for the circular 2m FFAG computed with PyZgoubi.

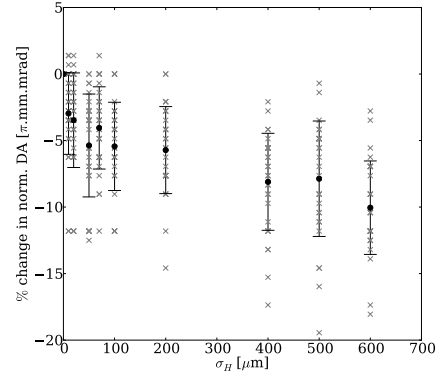


Figure 5: The relative drop in dynamic aperture as a function of misalignment for the racetrack 2m FFAG computed with PyZgoubi.

## SUMMARY

In summary, 0.2 – 1 GeV FFAG designs have been presented which have an approximate size of 9 m (diameter) and where the racetrack variant is about 7 m 11m; the racetrack lattice includes long straights up to 4m and the tunes are insensitive to the length of the straight section. The dynamic aperture for the circular and racetrack layout has been determined, and the racetrack configuration does not significantly impact the dynamic aperture compared to the equivalent circular lattice. Both designs are fairly insensitive to horizontal misalignments. Tracking shows an increase of DA with energy. This is in contrast to the DA predicted for the 8-sector version of the Daedaleus cyclotron [8] which decreases with energy. The tracking indicates a large factor (of at least 4) increase in DA at 800 MeV for the FFAG described in this work with a factor of 4 decrease in footprint, when compared to previous published results [8]. Future work will study smaller configurations with higher fields.

## REFERENCES

- [1] <http://abe.web.psi.ch/accelerators/ringcyc.php>
- [2] see <http://www.triumf.ca/research-program/research-facilities/main-cyclotron-beam-lines> and

<https://cycops.triumf.ca/cycfac.htm>

- [3] S. Kutsaev, et. al., “High Gradient Superconducting Cavity Development for FFAG”, to be published in Proceedings of Cyclotrons’13.
- [4] C. Prior, Editor ICFA Beam Dynamics Newsletter #43, August 2007.
- [5] C. Johnstone et al, The proceedings of Cyclotron 2013
- [6] S. Tygier, R.B. Appleby et al, To be submitted to the journal of Computational Physics
- [7] COSY INFINITY reference
- [8] F. Meot, “MW-class 800 MeV/n  $H_2^+$  SC-Cyclotron for ADS Application Design Study and Goals”, IPAC12, New Orleans, LA, May 2012, THPPR064, p. 412 (2012).