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A New Method of Determining the Bose Condensate Fraction in Superfluid ^{4}He

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The phenomena of superfluidity in ^{4}He and superconductivity are generally recognised as a direct manifestation of quantum behaviour on a microscopic scale. Since the suggestion of London [1] that the presence of superfluidity in liquid ^{4}He at low temperatures is linked with the phenomenon of Bose-Einstein condensation, it has gradually become accepted that both superconductivity and superfluidity are linked with the presence of a Bose condensate (BC). The presence of a BC is in fact one of the fundamental assumptions of modern field theoretical treatments of superfluidity [2,3,4,5]. In contrast to an ideal Bose gas, where all atoms occupy the zero momentum state at T=0, the condensate is depleted in an interacting system [6]. The first estimate of an 8% BC fraction f in ^{4}He at T=0 was made by Penrose and Onsager [7]. Many calculations of increasing sophistication and using a variety of different methods have been made since this early estimate [8,9] and the most recent calculations give f=9% at T=0 and 1 atm. pressure.

Many experimental attempts have been made to measure the Bose condensate (BC) fraction in superfluid ^{4}He, but the current situation is that no unambiguous experimental evidence exists for the presence of a BC. The situation until 10 years ago was reviewed by Sears [10]. The main advance since then has been the direct measurements of the momentum distribution in liquid helium made by Sokol et al, [11] using deep inelastic neutron scattering. However the condensate fraction extracted from these measurements is very assumption dependent and Sokol [12] has emphasised that his experimental data is consistent with the assumption that superfluid helium contains no BC.

In a recent paper [13] it was shown that in the presence of a Bose condensate fraction f, one would expect that the coherent scattering of X-rays and neutrons from ^{4}He should be reduced in intensity by a factor $(1-f)^2$. This was formally demonstrated by considering the Fourier components of the many particle wavefunction and also by considering the implications of the presence of a BC for the real space behaviour of the wavefunction. This result has the following physical interpretation. In the presence of a BC fraction f, each particle has a probability f of being in the zero momentum state, which is uniformly distributed over the sample volume. Since scattering from this state can only occur for wavevector $q=0$, (i.e. a uniform medium scatters only at zero angle) it follows that scattering from each particle at $q \neq 0$ is reduced in intensity by a factor $(1-f)$. The coherent scattering is the product of amplitudes from two particles and hence is reduced in intensity by a factor $(1-f)^2$.

Experimental measurements of the static structure factor $S(q)$, do in fact show that the coherent structure of $S(q)$ is reduced in intensity in the superfluid, and following a suggestion by Hylands et al [14,15] that the pair correlation function $g(r)$ should scale by a factor $(1-f)^2$, values of f were derived from measurements of $S(q)$. However a number of authors [16,17,18,19,20,21] have cast doubt on this method and the majority opinion in the literature is that the apparent agreement between theory and values of f derived from this procedure is coincidental. This paper presents further experimental evidence that the coherent contribution to $S(q)$ is reduced in intensity by a factor $(1-f)^2$ and suggests that the condensate fraction f is directly related to the velocity of sound in the liquid.

At low q scattering is almost entirely coherent and hence one would expect that $S(q)$ should scale as $(1-f)^2$. It is well known [22] that at low q, $S(q)$ is determined by density fluctuations and that these can be related by thermodynamic arguments to the isothermal compressibility κ_T and hence to the velocity of sound c.

$$S(0) = \rho k_B T \kappa_T = \gamma k_B T / mc^2 \kappa_T \tag{1}$$

Here ρ is the density T the temperature, k_B Boltzmann's constant, γ the ratio of the principle specific heats and m is the mass of a helium atom. Both X-ray and neutron scattering measurements show that this relation is accurately satisfied in the normal and superfluid phases of ^{4}He [23]. If one assumes that the change in $S(0)$ with T is entirely due to the development of a BC, it follows that $\rho k_B = \alpha(1-f)^2$ where α is a constant and hence that f can be directly derived from thermodynamical properties of ^{4}He. Thus the variation with temperature of the BC fraction can be derived from the relationship,

$$f(T) = 1 - \sqrt{\frac{\rho(T)\kappa_T(T)}{\rho(T_\lambda)\kappa_T(T_\lambda)}} = 1 - \frac{c(T_\lambda)}{c(T)}\sqrt{\frac{\gamma(T_\lambda)}{\gamma(T_\lambda)}} \approx 1 - \frac{c(T_\lambda)}{c(T)} \tag{2}$$

where T_λ denotes the superfluid transition temperature. The variation of c with both temperature and pressure was measured many years ago[24,25]. γ can be calculated from measured values of the principle specific heats [26,27] and within experimental error, at ambient pressure is independent of temperature in the superfluid, with a value of 0.96. Taking $c(T_\lambda)$=219 m/sec as the value of c at the transition temperature, we obtain from equation 2 the values for f listed in Table 1 at different temperatures and pressures.

Figure 1 shows values of f derived from equation 2 as a function of temperature at 1 atm. pressure. The measurements of Whitney and Chase [28] were used to calculate f at temperatures below 1.25 K. Previous experimental measurements of f and theoretical calculations are shown for comparison. As the superfluid is cooled there is an anomalous decrease in density from 0.1460 gm/cm^3 at 2.2K to 0.1450 gm/cm^3 at 0.2K [29] and one would expect that c should change purely as a result of this density change. In ^{4}He c increases linearly with increasing density, by 4.9 m/sec for an increase in density of 0.001gm/cm^3. (This behaviour is independent of temperature for $1<T<3$K and pressure for $1<P<25$ atm.) Thus one would expect that c at 0.2K should be lower than c at 2.2K by ~4.9m/sec as a result of the density change. If one corrects for this effect by adding 4.9m/sec to the measured value of c at 0K, one obtains f=9.9% at T=0 rather than the figure of 8.1% derived without correction. The full line shows the values of f derived at different temperatures after this density correction has been applied. Another contribution to c comes from the creation of rotons and phonons and the resulting interaction between first and second sound. However, the analysis of Whitney and Chase [28] suggests that this is a very small effect which changes c by only a few cm/sec and can therefore be neglected. Figure 2 shows f at 1.25 K as a function of pressure. It is clear that both the magnitude and general trends of the behaviour of f with pressure agree well with theoretical calculations.

Since the velocity of sound can be measured relatively easily and accurately, the link between c and f, which has been pointed out in this paper, opens the possibilty of accurate measurements of f in a variety of environments, which have been previously unaccessible to measurement. For example the Bose condensate fraction of helium in confined geometries could be determined from small angle X-ray scattering measurements. The agreement between theory and experiment which has already been obtained provides very strong experimental evidence for the existence of a Bose condensate in superfluid ^{4}He. However further theoretical work on the corrections to values of f derived from this method and further experimental measurements of c and γ as a function of temperature and pressure are clearly desirable.

Table 1 Values of *f* derived from equation 2. Values of *f* (in %) are listed as a function of temperature at different pressures. The pressures are in atm. The values of *c* used in the calculation of *f* were taken from reference 25

T (K)	*P=1*	*P=2.5*	*P=5*	*P=10*	*P=15*	*P=20*	*P=25*
1.25	*7.2*	*7.0*	*5.5*	*4.3*	*4.3*	*3.8*	*4.7*
1.50	*6.4*	*6.6*	*5.1*	*4.0*	*4.0*	*3.5*	*3.9*
1.75	*5.6*	*5.2*	*4.4*	*3.7*	*3.4*	*2.6*	*2.0*
1.80	*5.2*	*4.8*	*4.1*	*2.7*	*2.8*	*0.0*	*0.0*
1.90	*3.9*	*4.0*	*3.4*	*1.7*	*1.9*	*0.0*	*0.0*
2.00	*3.1*	*3.2*	*2.6*	*0.3*	*0.0*	*0.0*	*0.0*
2.10	*0.9*	*0.4*	*0.4*	*0.0*	*0.0*	*0.0*	*0.0*

Figure1. The Bose condensate fraction *f* as a function of temperature at a pressure of 1 atm. The points oo show the values of *f* derived from equation 2. The dotted line shows the values of *f* obtained after correction for the change in density, as described in the text. The squares are values obtained from neutron scattering measurements, the crosses from path integral Monte Carlo (PIMC) calculations [9] and the triangle was obtained from Greens function Monte-Carlo (GFMC) calculations[30].

Figure2 The Bose condensate fraction *f* as a function of density at a temperature of 1.25K. The circles are values of *f* obtained from equation 2. The crosses are values derived from neutron scattering [11], the squares are GFMC calculations [30] and the triangles PIMC calculations [9].

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Figure 1

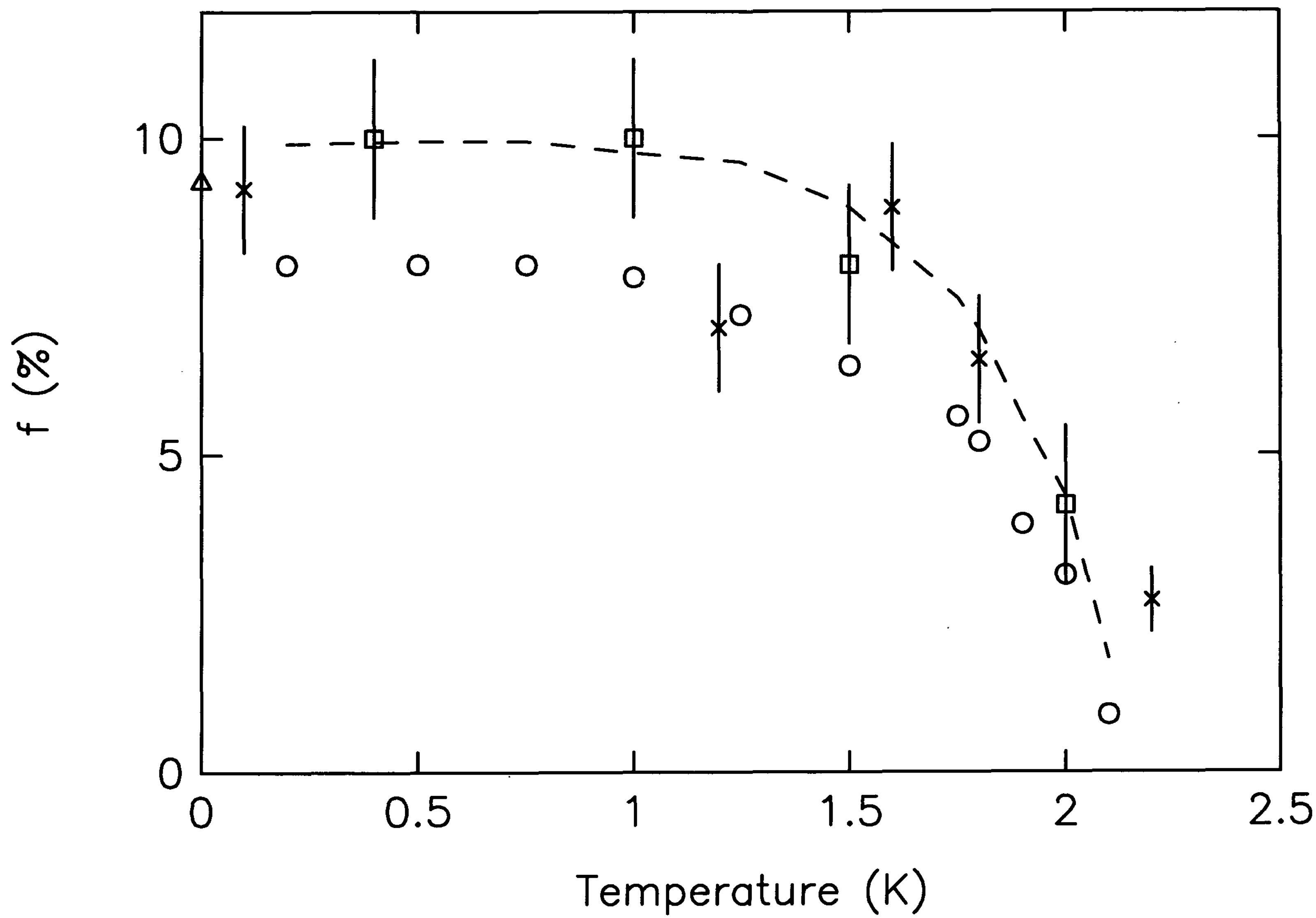

Figure 2