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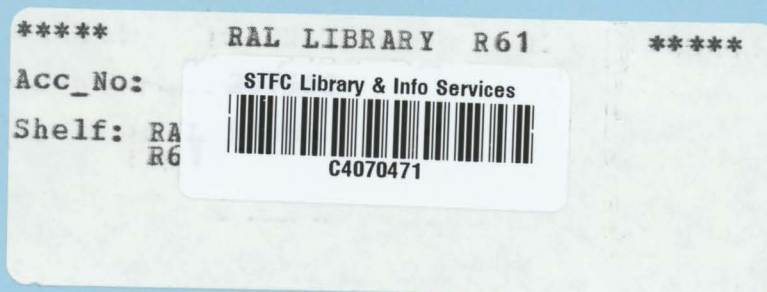
Nonlinear Effects in the Beat Wave Accelerator Scheme

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Abstract

In this article the concept of the beat wave accelerator is studied with emphasis put on the plasma physics. An important effect is the relativistic nature of the electrons oscillating in the electric field of the beat wave. Various instabilities are presented which could limit the overall efficiency of the accelerating process, these include the modulational instabilities of Langmur waves due to both relativistic and ponderomotive nonlinearities. In the later the ions become important for the saturation of the Langmuir wave.

Introduction

The study of generating beat waves in plasmas has been going on for more than ten years, with the first experiments being done in 1971 (Stansfield, Nodwell and Meyer (1971)). Kaufmann et. al., (1972) and Rosenbluth et. al., (1973) considered theoretically the generation of Langmuir waves in Tokomaks by beating two laser beams whose frequency difference matched the plasma frequency. The aim was to heat the plasma with the resultant plasma wave, which decayed by collisional damping or Landau damping. The problem naturally arose in the study of Laser fusion with Stimulated Raman and Brillouin scattering. The production of very high energy electrons was a result. In an article by Lin and Dawson (1974) describing the generation of fast particles in laser plasma experiments the concept of beat wave generation was discussed. Modification experiments in the ionosphere (Wong et. al., 1978) have also used the beat wave concept to create large amplitude plasma waves with a full theory being developed for production of ion sound waves by Fried et. al., (1979). Recently the idea for using such a plasma process as an alternative method of high energy acceleration has been proposed by Tajima and Dawson (1979), Joshi et. al., (1981) and Ruth and Chao (1982) and is now commonly called the laser beat wave accelerator (Lawson (1983)).

The beat wave accelerator depends on the generation of a large amplitude plasma mode with a phase velocity close to the velocity of light. The generation of such a plasma mode is possible by beating together two laser beams of frequencies and wavenumbers $(\omega_1, \underline{k}_1)$ and $(\omega_2, \underline{k}_2)$ such that the beat wave has $w = \omega_1 - \omega_2$ and $\underline{k} - \underline{k}_1 - \underline{k}_2$. The process is related to stimulation forward Raman scattering which can be considered as the single pump treatment. The general equations describing the beat wave mechanism and stimulated Raman scattering are therefore the same.

The plasma mode generated by the beat process grows linearly at first whereas it grows exponentially from noise in the Raman process, these two processes will therefore compete with each other if the Raman process is sufficiently fast. The saturation level of the plasma mode will, however, he determined by nonlinear processes which have still to be fully investigated.

Similar nonlinear saturation processes will operate in both the beat wave process and the Raman process. The Raman process, however, can be detrimental to the operation of the beat wave accelerator since in the Raman process different modes can be excited resulting in laser light being scattered out of the interaction region. The beat wave process is intrinsically non-linear because of the large amplitude waves involved. There are nonlinear problems associated

with the laser beams as well as the large amplitude plasma wave. A study of the beat wave accelerator will therefore involve the development of nonlinear methods in both analytic and computational treatments.

Model and derivation of the nonlinear Equations

The beat wave accelerator concept using laser beams to generate large amplitude Langmuir waves in a plasma is very similar to the four wave Raman forward scattering in an under-dense plasma. Stimulated Raman scattering is normally considered as a three wave process where the incident transverse (laser beam) decays into another transverse wave and a Langmuir wave. The beat wave accelerator relies on two laser beams beating together in a plasma, producing a beat disturbance at the local plasma frequency/ In an under-dense plasma where $\omega_o \gg \omega_{pe}, \omega_o$ is the laser frequency and ω_{pe} is the plasma frequency, the forward Raman scattering becomes important. For phase matching the wavenumber k_ℓ of the Langmuir mode is much smaller than the laser wavenumber $k_\ell \ll k_o$, under these conditions we must consider an up-shifted or anti-Stokes transverse component as well as the down shifted Stokes component, since both can be considered to be resonant with the initial laser wave and the Langmuir wave. The instability then becomes a "four wave" process with the incident laser beam $(\omega_o, \underline{k}_o)$ decaying into a Stokes wave $(\omega_1, \underline{k}_1)$ and an anti-Stokes wave $(\omega_2, \underline{k}_2)$ together with a density disturbance at $(\omega_\ell, \underline{k}_\ell)$. To describe this effect we will consider the coupling process to conserve momentum exactly and energy only approximately with the relations

$$\underline{k}_o = \underline{k}_{1,2} \pm \underline{k}_\ell, \quad \omega_o \approx \omega_{1,2}$$

In writing these relations we assume there is a frequency mismatch in the system, this allows coupling to both the upper and lower sidebands.

The plasma model we use to analyse the problem is the relativistic two fluid equations together with Maxwell's equations and Poisson's equations. The use of a relativistic treatment is necessary when we come to examine the longitudinal beat wave which is driven to very large amplitudes such that the quiver velocity in the longitudinal electric field approaches the velocity of light. We will show later that the relativistic corrections ultimately saturate the growth of the beat wave.

Starting from the equations:

$$\frac{\partial n_j}{\partial t} + \underline{\nabla} \cdot \left(n_j \underline{v}_j \right) - 0 \tag{1}$$

$$\left(\frac{\partial}{\partial t} + \underline{v}_j.\underline{\nabla}\right)\gamma_j\underline{v}_j + \frac{KT_j}{n_jm_j}\underline{\nabla}n_j + \nu_j\underline{v}_j = \frac{q_j}{m_j}\left(\underline{E} + \underline{v}_j \times \underline{B}\right) \quad (2)$$

$$\underline{\nabla}.\underline{E} = \frac{1}{\varepsilon_o}\sum_j n_jq_j \quad (3)$$

$$\underline{\nabla} \times \underline{E} = -\mu_o\frac{\partial\underline{H}}{\partial t} \quad (4)$$

$$\underline{\nabla} \times \underline{H} = \underline{J} + \varepsilon_o\frac{\partial\underline{E}}{\partial t} \quad (5)$$

where

$$J = \sum_j n_jq_j v_j, \; \gamma_j = \left(1 - v_j^2/c^2\right)^{-\frac{1}{2}}$$

and $j == i, e$, we obtain the following equations for a plane polarized electromagnetic wave E_T and a electrostatic density perturbation δn.

$$\left(\frac{\partial^2}{\partial t^2} + \nu_{ei}\frac{\omega_{pe}^2}{\omega_T^2}\frac{\partial}{\partial t} + \omega_{pe}^2 - c^2\underline{\nabla}^2\right)\underline{E}_T = \frac{-en_o}{\varepsilon_o}\left(\underline{v}_e.\underline{\nabla}\right)\underline{v}_e + \frac{e}{\varepsilon_o}\frac{\partial}{\partial t}\left(n_e\underline{v}_e\right) \quad (6)$$

$$\left(\frac{\partial^2}{\partial t^2} + \nu_{ei}\frac{\partial}{\partial t} - \frac{KT_e}{m_e}\nabla^2 + \omega_{pe}^2\right)\delta n - n_o\underline{\nabla}.\left[\left(\underline{v}_e.\nabla\right)\underline{v}_e + \frac{e}{m_e}\left(\underline{v}_e \times \underline{B}\right)\right] \quad (7)$$

where ν_{ei} is the electron ion collision frequency. The left hand side of equation (7) contains the relativistic correction term and is important whenever the quiver velocity in the longitudinal field approaches c, the quiver velocity in the transverse fields is always much less than c for the case considered.

Beat Wave Generation

The equation describing the generation of a longitudinal beat wave produced by two high frequency transverse waves is obtained from equation (7). Assuming that the wave fields (electromagnetic and electrostatic) are given by products of a slowly varying amplitude, (on the time scale of the pump frequency) times the plane wave determined by the linear dispersion relation. The transverse waves E are represented by:

$$\underline{E}_T = r_e \left\{ \underline{E}_j(\underline{x}, t) \exp i \left(\underline{k}_j . \underline{x} - \omega_j t \right) \right\} ; j = 0, 1, 2,$$

and the density perturbation δn by: $\delta n = R_e \left\{ N(\underline{x}, t) \exp i \left(\underline{k}_\ell . \underline{x} - \omega_\ell t \right) \right\}$

Using the small amplitude approximation and neglecting damping, relativistic effects, pump depletion and other non-linear processes the equation for the longitudinal plasma wave becomes

$$\frac{\partial N(t)}{\partial t} = -i \frac{n_o e^2}{4 m_e^2 \omega_o \omega_1} \frac{k_\ell^2}{\omega_\ell^2} E_o E_1 \tag{8}$$

Letting $N/n_o = A(t) E^{i\phi t}$ we find the longitudinal plasma wave grows linearly in time with

$$A(t) = A(o) + \frac{e^2 E_o E_1}{4 m_e^2 \omega_o \omega_1 c^2} \omega_{pe} t \tag{9}$$

In this approximation the wave amplitude would grow until $A(t) = 1$ ie. when the electron quiver velocity in the longitudinal wave equal c. The amplitude, however, will saturate well before reaching this level by pump depletion and non-linear effects such as the relativistic correction to the plasma frequency. If we include the relativistic effects the equation for the density perturbation can be written as:

$$i \frac{\partial N}{\partial t} + \frac{3}{16} \frac{\omega_{pe}}{n_o^2} \mid N \mid^2 N = \frac{n_o e^2 k_\ell^2}{4 m_e^2 \omega_o \omega_1 \omega_{pe}} E_o E_1 * \tag{10}$$

If we include spatial variation as well as temporal variation the equation becomes a driven non-linear Schrodinger equation. From equation (10) we find that the wave amplitude saturates at

$$A = \left(\frac{3}{16} \frac{e^2 E_o E_1}{m_e^2 \omega_o \omega_1 c^2} \right)^{\frac{1}{3}} \tag{11}$$

which was first derived by Rosenbluth and Liu (1972).

For the parameters given in the Ruth and Chao design model and also in the first RAL study report Lawson (1983) (see Table 1) the wave is found to saturate well before the wave breaking limit, at a value given by:

$$\frac{e E_\ell}{m_e \omega_{pe} c} = \frac{\delta n}{n} \simeq 0.15 \tag{12}$$

therefore wave breaking as a thermalization process is not a problem in this model. This saturation level for the Langmuir field sets the upper limit for the effective peak accelerating field to be $1.8GV/m$. Although other processes such as the modulational instability and filamentation of the Langmuir wave due to the relativistic effect must be taken into account. These will produce a broadening in frequency and the creation of spikes in space and the formation of solitons.

To describe the modulational instability we write the density disturbance δn as the sum of a pump wave and two other components the Stokes and anti-Stokes waves $\delta n = R_e \left\{ N_o(x,t) \exp i\left(\underline{k}_o.\underline{x} - \omega_o t\right) + N_{1,2}(\underline{x},t) \exp i\left(\underline{k}_{1,2}.\underline{x} - \omega_{1,2} t\right) \right\}$ where $N_j(\underline{x},t)$ is determined by the non-linear interaction. Using a perturbation procedure on equation (7) and neglecting pondermotive force effects we obtain the following equations for these waves:

$$\left(\frac{\partial}{\partial t} + \gamma_\ell\right) N_o = i\Gamma \left[\mid N_o \mid^2 N_o + \mid N_1 \mid^2 N_o + \mid N_2 \mid^2 N_o + 2N_o * N_1 N_2 e^{i(\delta_1 + \delta_2)t}\right]$$

$$\left(\frac{\partial}{\partial t} + \gamma_\ell\right) N_1 = i\Gamma \left[\mid N_o \mid^2 N_1 + \mid N_1 \mid^2 N_1 + \mid N_2 \mid^2 N_1 + N_o^2 N_2 * e^{-i(\delta_1 + \delta_2)t}\right]$$

$$(13)$$

$$\left(\frac{\partial}{\partial t} + \gamma_\ell\right) N_2 = i\Gamma \left[\mid N_o \mid^2 N_2 + \mid N_1 \mid^2 N_2 + \mid N_2 \mid^2 N_2 + N_o^2 N_1 * e^{-i(\delta_1 + \delta_2)t}\right]$$

where $\gamma_\ell = \nu_{ei}/2$, is the linear damping rate, $\Gamma = \frac{3}{16}\frac{\omega_{pe}}{n_o^2}$ is the coupling coefficient and $\delta_{1,2} = \omega_o - \omega_{1,2}$ is the frequency mismatch. A fuller derivation of these equations will be presented in a future publication. Solving these equations for $N = a$ constant and assuming the amplitudes $N_1 e^{i\delta_1 t}$ and $n_2^* e^{-i\delta_2 t}$ vary ass $\exp(-i\omega t)$ results in the following dispersion relation for $n_o \gg N_1, N_2$

$$(\omega - \delta_1 + i\gamma_\ell)(\omega + \delta_2 + i\gamma_\ell) - (\delta_1 + \delta_2) K = 0 \qquad (14)$$

where $K = \Gamma \mid N_o \mid^2$

Solving this equation we obtain the following threshold for instability:

$$K = -\left[\gamma_\ell^2 + \Delta^2\right]/\Delta$$

where $\Delta = \delta_1 + \delta_2$ ie. we have instability only when $\Delta = 0$. However, from the definitions of δ_1 and δ_2 we find that:

$$\Delta \approx \frac{-\mid k_{MOD} \mid^2 v_{Te}^2}{\omega_o}$$

and so Δ is a negative definite, $\underline{k}_{MOD} = \underline{k}_o \mp \underline{k}_{1,2}$ is the modulation wavenumber. The growth rate resulting from (14) can be expressed as:

$$\frac{\gamma_g}{\omega_o} = -\frac{\gamma_\ell}{\omega_o} + \kappa_s \left(\frac{2K}{\omega_o} - \kappa_s^2 \right)^{\frac{1}{2}} \qquad (15)$$

where $\kappa_s = \frac{1}{\sqrt{2}} \frac{k_{MOD}e}{\omega_o}$, and the threshold can be expressed as

$$\frac{3}{16} W_\ell / n m_e c^2 > \frac{1}{4} \frac{\gamma_\ell}{\omega_o}$$

where $W_\ell = \frac{1}{2} \varepsilon_o \mid E_\ell \mid^2$. This threshold is much less than amplitude levels reached in most laboratory experiments. The Langmuir modulational instability (Bingham and Lashmore-Davies (1979)) due to pondermotive effects becomes important for $\frac{W_\ell}{n m_e c^2} > \frac{\gamma_\ell}{\omega_o} \frac{v_{Te}^2}{c^2}$ which is a much smaller threshold than the relativistic value. However, the pondermotive effects involve the ion motion and so time scales for this process to occur will be the ion time scales $\sim 1/\omega_{pi}$, where ω_{pi} is the ion plasma frequency, which is much longer than the time for the relativistic effects to occur. For short time scales the relativistic term can be the dominant one. Modulational type instabilities indicate the onset of strong Langmuir turbulence with the generations of a broad frequency spectrum and cavity formation. Parametric three wave decay processes are forbidden when the condition $k_e \lambda_{DE} < (m_e/m_i)^{\frac{1}{2}}$ is satisfied (Bingham and Lashmore-Davies (1979)). For the parameters considered in the Ruth and Chao design this condition is satisfied.

So far we have only considered the Langmuir modulational instability, filamentation and self-modulation of electromagnetic waves and the oscillating two stream instability where the nonlinear coupling term is due to low frequency ponderomotive force can also occur. In this section we will consider self-modulation of relativistic Langmuir waves, which arise in the beat wave accelerator scheme, due to weak relativisitic as well as the nonlinear coupling of the wave with ion modes.

The nonlinear frequency shift produced by the relativistic mass correction is well known in the study of large amplitude plasma waves. This term is responsible for the relativistic de-tuning and saturation of the wave in the beat wave

accelerator. Other terms such as coupling to low frequency ion modes could also contribute to de-tuning and thus saturation of the wave. These nonlinear frequency shifts can also be responsible for the modulational instability of the wave which leads eventually to frequency broadening and the development of strong turbulence, and thus limiting the usefulness of the scheme for particle acceleration.

Starting from the fluid equations and Maxwell's equations (1) - (5) the equation describing the 1-D Langmuir wave field E in the weak relativistic limit $v_{osc}^2/c^2 \ll 1$ including low frequency density perturbations is given by

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2\left(1 + \frac{\delta n_e^l}{n_o}\right) + 3v_{Te}^2\frac{\partial^2}{\partial x^2}\right)E = -\frac{3}{8}\omega_{pe}^2\frac{v_{osc}^2}{c^2}E \qquad (16)$$

where δn_e^l is a low frequency density perturbation and $v_{osc}(= e|E|/m_e\omega)$ is the electron quiver velocity in the high Langmuir field E.

Using the equation $E = A_o \exp i(k_o x - \omega_o t)$ to describe the high frequency field and the W.K.B approximation with $\frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial \tau} - i\omega_o, |\frac{\partial}{\partial \tau}| \ll \omega_o$ and $\frac{\partial}{\partial x} \rightarrow \frac{\partial}{\partial x} + ik_o$ we obtain the envelope equation for the slowly varying wave amplitude A_o

$$i\left(\frac{\partial}{\partial \tau} + v_g\frac{\partial}{\partial x}\right)A_o + \frac{v_g'}{2}\frac{\partial^2}{\partial x^2}A_o - \frac{\omega_{pe}^2}{2\omega_o}\left(\frac{\delta n_e^l}{n_o} - \frac{3}{8}\frac{v_{ose}^2}{c^2}\right)A_o = 0 \qquad (17)$$

where $v_g = 3k_o v_{Te}^2/\omega_o$ and $v_g' = 3v_{Te}^2/\omega_o$ are the group velocity and group velocity dispersion of the Langmuir wave respectively.

The equation for the low frequency density perturbation in the fluid approximation is given by

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\frac{\partial^2}{\partial x^2}\right)\delta n_e^l = \frac{\epsilon_o}{2m_i}\frac{\omega_o^2}{\omega_{pe}^2}\frac{\partial^2}{\partial x^2}\mid A_o\mid^2 \qquad (18)$$

where c_s is the ion sound speed. Equations (17) and (18) represent generalised Zakharov (1972) equations. Note that the relativistic nonlinearity operates on time scales of the order of $c/v_{osc}\omega_{pe}$ which can be of the order of electron plasma time scales whereas the low frequency density response operates on ion-plasma periods which is much longer than the electron plasma period. This means that these two effects will set in at different times.

Another regime exists when the ion response is treated using kinetic theory (Bingham et al 1988). This regime is important when $\frac{m_e}{m_i}\frac{T_i}{T_e} \simeq k^2\lambda_{De}^2$ in

this regime the low frequency mode is an ion quasi-mode where the density perturbation is not a normal mode of the system.

Equation (17) can be analysed for stability of a constant amplitude high frequency pump against modulations at (Ω, k_s) in three regimes:

1. The quasi-static or hydrodynamic regime.

2. The relativistic regime.

3. The kinetic regime.

In the quasi-static or hydrodynamic regime we use the approximation $\frac{\partial^2}{\partial t^2} \ll k_s c_s$ to express δn_e^l in terms of E_ℓ which is now given by

$$\delta n_e^l = \frac{\epsilon_o}{2m_i c_s^2} \mid E_\ell \mid^2 \qquad (19)$$

In the kinetic regime the expression for δn_e^l is given in terms of the electron and ion susceptibilities (Bingham et al 1988) by

$$\delta n_e^l = -\frac{\epsilon_o k_s^2}{m_i \omega_{pe}^2} \frac{\chi_e(1+\chi_i)}{1+\chi_e+\chi_i} \mid E_\ell \mid^2 \qquad (20)$$

where $\chi_e(\chi_i)$ are the electron (ion) susceptibility given by

$$\chi_j = \frac{1}{k_s^2 \lambda_{De}^2}[1 + \xi_j Z(\xi_j)], j = e, i \qquad (21)$$

$\xi_j = \frac{\omega_l}{\sqrt{2k_s v_{Tj}}}$, and Z is the plasma dispersion function.

We now consider stability of a constant amplitude high frequency pump against modulations at (Ω, k_s) where $\Omega \ll \omega_o$. Assuming as before the pump wave $A_o = R_e\{A_o + A_1(\psi_1) + A_2(\psi_2)\}$ where A_o is a constant and A_1 and A_2 are the stokes and anti-stokes sidebands which vary as $e^{i\psi}$ with $\psi = (\Omega t \mp k_s x)$ equation (17) can be linearised to obtain the usual dispersion relation.

$$(\Omega - k_s v_g) = \pm i(\frac{1}{2}k_s^2 v_g' C_k \mid A_o \mid^2 - \frac{1}{4}k_s^4 v_g'^2)^{\frac{1}{2}} \qquad (22)$$

where the coefficient C_k for the three cases is given by

$$C_{kHYDRO} = \frac{1}{8}\frac{\epsilon_o \omega_{pe}}{n_o k T_e} \tag{23}$$

$$C_{kKIN} = \frac{\epsilon_o \omega_{pe}}{2n_o k T_e}\frac{1 + \chi_i}{1 + (k_s \lambda_{De})^{-1} + \chi_e} \tag{24}$$

$$C_{kREL} = \frac{3}{16}\frac{e^2}{\omega_{pe} m_e^2 c^2} \tag{25}$$

From equation (22) we find that the real part of Ω is given by

$$R_e \Omega = k_s v_g \tag{26}$$

which is the frequency of the low-frequency quasi-mode.

The threshold for self-modulation is given by

$$\mid A_o \mid^2 = \frac{1}{2}\frac{k_s^2 v_g'}{C_k} \tag{27}$$

and the maximum growth rate occurs for

$$k_s^2 v_g' = C_k \mid A_o \mid^2 \tag{28}$$

and is given by

$$\gamma_{MAX} = \frac{1}{2}C_k \mid A_o \mid^2 \tag{29}$$

For the weakly relativistic case the growth rate is given by

$$\gamma_{REL} = \frac{3}{32}\frac{v_{osc}^2}{c^2}\omega_{pe} \tag{30}$$

for a wavenumber $k_{s(REL)} = \frac{1}{4}\frac{\omega_{pe}}{v_{Te}}\frac{v_{osc}}{c}$ and for the kinetic regime the maximum growth rate

$$\gamma_{KIN} \cong \frac{1.2}{\sqrt{2}}k_s v_{osc} \tag{31}$$

for the wavenumber $k_{s(KIN)} = \frac{1}{6}\frac{\omega_{pe}}{v_{Te}}\frac{v_{osc}}{v_{Te}}$. This is much larger than the modulational wavenumber for relativistic self-modulation by a factor of about c/v_{Te},

it could be even larger than the pump wavenumber $k_o = \omega_{pe}/c$. This would result in both forward and backward propagating waves which would be more easily damped by Landau damping than the original pump wave and would produce strong electron acceleration and heating in both directions. Simulations by Lin and Tsintzadze (1976) of relativistic plasma waves demonstrate such as an effect when ion motion is included. The effect of modulational type instabilities on the saturation of plasma beat waves has been demonstrated by Mora et al (1988). This is easily seen by including the driving term due to two high frequency pump waves on the left hand side of equation (17) resulting in the equation for the beat wave including both relativistic and low frequency density perturbations given by

$$i\left(\frac{\partial}{\partial t} + v_g \frac{\partial}{\partial x}\right)A_o + \frac{v_g'}{2}\frac{\partial^2}{\partial x^2}A_o + \left(C_{kREL} + C_{kHYDRO.})\right)\mid A_o\mid^2 A_o = \frac{n_o}{4}\omega_{pe}\alpha_1\alpha_2 \tag{32}$$

where $\alpha_1\alpha_2$ are the normalised quiver velocities of the high frequency pump fields E_1 and E_2 given by $\alpha_{1,2} = \frac{eE_{1,2}}{me\omega_{1,2}c}$. From the time dependent part of equation (32) we find in the linear approximation the wave amplitude grows according to $A_o(t) = A_o(o) + n_o\alpha_1\alpha_2\omega_{pe}t/4$ (Rosenbluth and Lui 1972). The frequency shift causes the wave to saturate at a value given by

$$A_o = n_o[\omega_{pe}\alpha_1\alpha_2/(C_{kREL} + C_{kHYDRO})]^{\frac{1}{3}} \tag{33}$$

which reduces to the Rosenbluth and Liu (1972) limit when we only consider relativistic nonlinearities. We can see easily from equation (33) that the role of the nonlinear frequency shift due to both relativistic nonlinearities and the low frequency density perturbation is to lower the saturation level for the waves. These extra nonlinearities may also explain why the beat wave experiment at the Rutherford Appleton Laboratory saturated at very low values less than 1%. The ponderomotive force nonlinearity above predicts a saturation level lower than the relativistic saturation level by a factor $\left(\frac{3}{2}\frac{v_{Te}^2}{c^2}\right)^{\frac{1}{3}}$. This, however, is only true if $v_{osc}^2 < v_{Te}^2$ i.e the perturbation expansion breaks down if this is not the case. The modulational instability due to ion density fluctuations in the strong wave limit has been studied by Mora et al (1988).

The self-modulational instability of electromagnetic waves due to relativistic effects has been described by Max, Arons and Langdon (1974). In this paper they described self-modulation due to a single electromagnetic wave. In the beat wave accelerator where there are two electromagnetic waves the theory has been

developed by McKinstrie and Bingham (1989), in fact the theory considers an arbitary number of finite-amplitude waves.

Saturation by particle trapping can also contribute to the final level of the Langmuir wave field. Coffey (1971) has shown that the amplitude of the Langmuir wave saturates at a level given by:

$$\frac{E_\ell}{\sqrt{4\pi n_o m_e v_{ph}^2}} = \left(1 - \frac{1}{3}\beta - \frac{8}{3}\beta^{\frac{1}{4}} + 2\beta^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

where $\beta = 3v_{Te}^2/v_{ph}^2$. For the Ruth and Chao model the wave saturates at the level $\delta n/n \simeq 0.8$, for a plasma temperature of $10eV$, which is much higher than that determined by relativistic effects. In determining the saturation due to particle trapping the plasma temperature must be determined. The plasma temperature can change due to non-linear heating by the large amplitude Langmuir wave.

Stimulated Raman Scattering

One of the drawbacks in the beat wave accelerator scheme is the fact that the high powered laser beams can under-go non-linear scattering processes which result in some of the laser energy being scattered out of the interaction zone. Stimulated Raman scattering is one such non-linear process, as mentioned before the beat wave process is a special case of stimulated Raman forward scattering. To describe stimulated Raman scattering, we can use the same set of equations (6) and (7), however, instead of dealing with two pump waves we will consider only one pump and two high frequency scattered waves. When the frequency of the plasma is much less than the laser frequency we need to consider coupling to both upper and lower sidebands, this inherently produces a frequency mismatch in the system with the result that the low frequency being determined by the laser parameters. As before we assume that the waves can be described by a linear phase times a slowly varying amplitude in space and time,

$$E_{T0,1,2} = R_e\left\{E_{0,1,2}(\underline{x},t)\exp i\left(\underline{k}_{0,1,2}.\underline{x} - \omega_o t\right)\right\}$$

$$\delta n = R_e\{N(\underline{x},t)\exp i(\underline{k}_\ell.\underline{x})\}$$

where E_{T0} represents the pump wave and E_{T1}, E_{T2} are the Stokes and anti-Stokes waves respectively. The density perturbation is assumed to be a driven

response, its frequency of oscillation will be determined from the dispersion relation.

Expanding the distribution function for the transverse waves about their linear values we obtain the following reduced equations for the Stokes and anti-Stokes waves:

$$\left(\frac{\partial}{\partial t} + \underline{v}_1 \cdot \frac{\partial}{\partial \underline{x}} + \gamma_T\right) E_1\left(\underline{x}, t\right) = -ic_T E_o N * e^{-i\delta_1 t} \tag{34}$$

$$\left(\frac{\partial}{\partial t} + \underline{v}_2 \cdot \frac{\partial}{\partial \underline{x}} + \gamma_T\right) E_1\left(\underline{x}, t\right) = -ic_T E_o N * e^{-i\delta_1 t} \tag{35}$$

where v_1, v_2 are the group velocities, $c_T = \frac{e^2}{4m_e \varepsilon_0 \omega_o}$ is the coupling coefficient and $\delta_{1,2} = \omega_o - \omega_{1,2}$ is the frequency mismatch with $\delta_1 \simeq \underline{k}_\ell \cdot \underline{v}_o - \frac{c^2 k^2}{2\omega_o}$

The equation for the density perturbation is given by:

$$\left(\frac{\partial^2}{\partial t^2} - v_{Te}^2 \Delta^2 + \omega_{pe}^2 + \gamma_\ell \frac{\partial}{\partial t}\right) N = -C\left[E_o E_1^* e^{-i\delta_1 t} + E_o^* E_2 e^{i\delta_2 t}\right] \tag{36}$$

where $C = \frac{n_o e^2 k_\ell^2}{2m_e^2 \omega_o}$ is the coupling coefficient and γ_ℓ represents damping collisional or Landau damping. Introducing the new amplitudes $\alpha_1 = E_1^* e^{i\delta_1 t}$ and $\alpha_2 = E_2 e^{i\delta_2 t}$ and assuming α_1, α_2 and N vary as $\exp(i\omega t)$, we obtain the following dispersion relation for $\underline{k}_\ell \ll \underline{k}_o$ from equations (16, (17) and (18).

$$\left[(\omega - \underline{k}_\ell \cdot \underline{v}_o + i\gamma_T)^2 - \delta^2\right]\left[\omega^2 - \omega_{pe}^2 - v_{Te}^2 k_\ell^2 + i\omega \gamma_e\right] - \frac{1}{2}\delta^2 \omega_{pe}^2 \frac{v_{osc}^2}{c^2} = 0 \tag{37}$$

where v_{osc} is the quiver velocity in the pump field, $\delta = \frac{-c^2 k_\ell^2}{2\omega_o}$.

This dispersion relation (37) is well known and has been solved for a number of different cases (Nishikawa (1968), Bingham and Lashmore-Davies (1976)). For $\underline{k}_\ell \| \underline{v}_o$ we have forward scatter, with the short wavelength pump wave being modulated by the long wavelength electrostatic wave resulting in the generation of high frequency transverse sidebands and bunching of the plasma particles. This is the single pump analogue of the beat wave process the growth rate and threshold obtained from equation (37) are given by:

$$\gamma = \frac{1}{\sqrt{2}} \frac{v_{osc}^2}{c^2} \frac{\omega_{pe}^2}{\omega_o^2} \tag{38}$$

$$\frac{v_{osc}^2}{c^2} = 8\gamma_T \frac{\omega_o}{\omega_{pe}^2} \tag{39}$$

The plasma wave frequency is given by the real part of ω ie. $R_e\omega = k_\ell \left(1 - \frac{\omega_{pe}^2}{\omega_o^2}\right)^{\frac{1}{2}} c \approx k_\ell c$ ie. the same as in the beat process.

This "four-wave" modulational instability results in the broadening of the laser frequency and generation of broad-band plasma oscillations.

For the parameters used in the Ruth and Chao model $\gamma \approx 2.5 \times 10^{11} sec^{-1}, I_{LASER} \approx 10 I_{THRESHOLD}$ where I_{LASER} is the laser intensity.

For $\underline{k_\ell} \perp \underline{k_o}$ we have a purely growing instability with a standing longitudinal wave set up whose wavenumber is perpendicular to the laser propagation direction resulting in the break-up of the beam into filaments with a growth rate $\gamma = \frac{\omega_{pe}}{\sqrt{2}} \frac{v_{osc}}{C}$ and threshold $\frac{v_{osc}^2}{2\omega_{pe}}$. For the parameters given in the Ruth and Chao design $\gamma \approx 2.5 \times 10^{11} sec^{-1}, I_{LASER} \approx 9 I_{THRESHOLD}$.

The above processes are "four-wave" modulational type instabilities resulting when $\underline{k_\ell}$ is either parallel or perpendicular to $\underline{k_o}$, for intermediate cases where $\underline{k_\ell}.\underline{k_o} = k_e k_o \cos\theta, \theta \neq 0, \pi/2$, the process becomes a three wave resonant side-scatter instability with the following resonance condition for forward side scattering:

$$k_\ell = k_o \cos\theta - \left(k_o^2 \cos^2\theta - \frac{2\omega_{pe}\omega_o}{c^2}\right)^{\frac{1}{2}}$$

This results in a scattering angle $\theta_s \approx \sin^{-1}\left(\frac{2\omega_{pe}}{\omega_o}\right)^{\frac{1}{2}}$ with a growth rate $\gamma \approx \frac{\omega_{pe}}{\sqrt{2}} \frac{v_{osc}}{c}$ and threshold $\frac{v_{osc}^2}{c^2}$ and threshold $\frac{v_{osc}^2}{c^2} \approx \frac{\gamma_e \gamma_T}{2\omega_{pe}^2}$. This corresponds to a scattering angle $\theta_s \approx 5^o$, growth rate $\gamma \approx 2.5 \times 10^{11} sec^{-1}$ and a threshold level $I_{THRESHOLD} \simeq I_{LASER}/1000$ for the Ruth and Chao model. This instability results in the laser light being scattering out of the interaction column and poses a serious threat to the laser beat-wave scheme. Other instabilities associated with the laser beams are the filamentation and self-focusing instabilities (Bingham and Lashmore-Davies (1976), Max et al (1974)) these cause the incident plane wave to break up into a number of filaments of higher laser intensity. The laser intensity used in the Ruth and Chao design is 16 times the threshold intensity and the growth length is of the order of 5cm. A summary of the different types of instabilities can be found in Table 1. The parameters used to prepare table 1 correspond to those used in the design study (Lawson (1983)).

The three wave forward Raman process is the most serious instability since it has the fastest growth rate. For laser pulse lengths greater than about 50 psec the loss of energy from the laser through this process becomes the dominant loss process, with the light being scattered at an angle of 5° out of the main beam. The four wave processes are not quite so serious, however, over a long propagation path frequency broadening due to these processes could become a serious problem, also the possibility of large radial electric fields being set up by the perpendicularly standing Langmuir wave could disrupt the electron beam. The filamentation process is also seen to be important for propagation lengths greater than 5cm. This process will amplify any spatial non-uniformities on the laser beam.

Conclusion

In this report we have discussed some of the plasma physics problems associated with the beat wave accelerator. We have shown that an important effect is the frequency shift, due to the relativistic mass correction of the Langmuir wave. This relativistic term determines the saturation level of Langmuir wave ($\delta n/n = 0.15$), this limits the effective peak accelerating field E_z to $1.8 GV/m$, it also contributes to frequency broadening and cavity formation (breaking the beam up into smaller wavepackets). The different type of instabilities associated with the large amplitude Langmuir wave and the laser beams have been discussed and are shown to lead to important effects which require further investigation.

Table 1.

Summary of growth rates and thresholds for the various laser-plasma interactions. The plasma and laser parameters used are given in Table 1, they correspond to those used in the design study (Lawson (1983)).

	Threshold W/m	Growth rates s^{-1}
Beat-Wave process	-	4.3×10^9
3-Wave Forward Raman Scattering	3.3×10^{12}	2.5×10^{11}
4-Wave Forward Raman Scattering ($\underline{k}_l \perp \underline{k}_o$	3.6×10^{14}	5×10^8
4-Wave Forward Raman Scattering ($\underline{k}_l \perp \underline{k}_o$	3.6×10^{14}	2.5×10^{11}
Filamentation	2×10^{14}	5 cm

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