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A Wave Model for the Aurora

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ABSTRACT

A wave-particle interaction model is proposed for the electron acceleration that leads to discrete aurora. Measurements of accelerated electrons are used to deduce the wave spectrum and the nature of the waves responsible for acceleration. The waves are identified as lower-hybrid waves. Generation of the waves by various free energy sources are considered and the wave energy required to account for the aurora is calculated and compared with observations.

1. INTRODUCTION

Observations of auroral-electron and ion energy distributions with peaks in the range 1-20 keV suggest that the mechanism which accelerates the particles in the direction of the magnetic field is a velocity-dependent, statistical, process (Hall and Bryant (1974); Bryant et al (1978)), and not one based on a quasi-static potential difference. The need to consider an alternative to the potential difference model has been pointed out also by Whalen and Daly (1979), Sharp et al (1980), and Birn et al (1981).

In this paper we will consider the possibility of electron acceleration by electrostatic plasma turbulence, where the electrons gain energy from those plasma waves which slowly overtake them, the energy gained during each interaction being a small fraction of their final energy. The electrons absorb energy from the waves and diffuse in velocity space. Kaplan and Tsytovich (1973) and have shown that electrostatic wave-particle interactions can be very efficient in accelerating particles to produce non-thermal tails. It is also well known (Sturrock, (1966)) that high energy tails can be produced through stochastic acceleration by waves of appropriate high phase velocity.

2. THE WAVE SPECTRUM

In the upper part of figure 1 velocity space density is plotted versus velocity for electron distributions measured at two locations within the stream producing an auroral arc. These are typical of what is observed (Bryant, (1981)). We propose that various features of the electron distributions can be accounted for by acceleration by waves that have the spectra shown in the lower part of figure 1. In this model the non-thermal tails of the electron distributions are produced by the heating that results from the absorption of waves that have a broadband spectrum, with phase velocities ranging from a cut-off at 2×10^7 ms^{-1} to the speed of light. Also a general and consistent trend which is illustrated in figure 1 is that changes in the electron velocity space density at velocities less than the cut-off are independent of those at the higher velocities (Whalen and Daly (1979); Hall (1980) and Bryant (1981). The invariance of the electron densities at velocities less than the cut-off is attributable to the absence of waves at the lower velocities.

The feature of the wave spectrum that leads to a peak in the electron distribution is the transition section, where the wave energy density increases to a non-thermal plateau.

3. THE NATURE OF THE WAVES

For a plasma with an electron temperature, $Te \sim 1$ keV, which is a typical value for the source region of auroral particles, electrostatic waves, such as lower-hybrid and Langmuir waves, have phase velocities along the field lines ranging from a cut-off at the electron thermal velocity, $v_{Te} = 2 \times 10^7$ ms^{-1}, to greater than the speed of light. The cut-off at low phase velocity is due to Landau damping, which allows only those waves with phase velocity $v_{ph} > v_{Te}$ to exist, where v_{Te} is the electron thermal velocity (see below). Thus lower-hybrid and Langmuir waves have the range of phase velocity that is necessary to account for the acceleration of auroral electrons. We will concentrate on lower hybrid waves because, as is shown in section 5, these are observed to carry most power. In laboratory plasmas lower hybrid waves have been shown to be extremely effective in accelerating electrons along magnetic field lines and in producing high energy tails in electron distribution functions (Boyd et al (1976)). This work has been concerned primarily with lower-hybrid current drive in Tokamaks (Liu et al, (1982)). Numerical simulations by McBride et al (1972) and Tanaka and Papadopoulos (1983) show how large field strengths in the lower hybrid mode are generated by the modified two stream instability and how effective these waves are in forming high energy tails.

The dispersion relation for electrostatic waves in a magnetized plasma is (Stix (1962):

$$1 + \frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} = 0 \qquad (1)$$

for $\omega_{ci}^2 << \omega^2 << \omega_{ce}^2$

where $k_\perp$ and $k_\parallel$ are the perpendicular and parallel components of the wave-number with respect to the ambient magnetic field B_0, $k^2 = k_\perp^2 + k_\parallel^2$,

$\omega_{ce,i} = \dfrac{eB_0}{m_{e,i}}$ are the electron and ion (angular) gyrofrequencies, and

$\omega^2_{pe} = n_0 e^2/m_e\varepsilon_0$ is the plasma (angular) frequency.

For $k_\parallel^2 << k^2$, solutions to (1) are called lower hybrid waves. Their frequency is given by

$$\omega^2 = \omega_{Lh}^2 [1 + (k_\parallel/k)^2 (m_i/m_e)] \tag{2}$$

where $\omega_{Lh}^2 = \omega_{pi}^2/(1 + \omega_{pe}^2/\omega_{ce}^2)$ is called the lower hybrid reasonance frequency. These waves have $\underline{k}$ nearly perpendicular to $\underline{B}_0$, there is however an electric field component, and wave-number $k_\parallel \leqslant (m_e/m_i)^{\frac{1}{2}} k$, parallel to $\underline{B}_0$. The group velocities parallel and perpendicular to the magnetic field can be obtained from (2), and are given by:

$$v_{g\parallel} = \omega_{Lh} \frac{k_\parallel}{k^2} \frac{m_i}{m_e} \left(1 - \frac{k_\parallel^2}{k^2}\right) \Big/ \left(1 + \frac{k_\parallel^2}{k^2} \frac{m_i}{m_e}\right)^{\frac{1}{2}} \hat{\underline{z}}$$

(3)

$$v_{g\perp x,y} = - \frac{k_\parallel k_{x,y}}{k^2} \omega_{Lh} \frac{k_\parallel}{k^2} \left(\frac{m_i}{m_e}\right) \Big/ \left(1 + \frac{k_\parallel^2}{k^2}\left(\frac{m_i}{m_e}\right)\right)^{\frac{1}{2}} \hat{\underline{x}}, \hat{\underline{y}}$$

$\underline{B}_0$ is in the z-direction.

From equation (3) and the condition $k_{\parallel}{}^2 < k^2$ we obtain the ratio $v_{g\parallel}/v_{g\perp x,y} \approx k^2/k_{\parallel}k_{\perp}$ it follows that $v_{g\parallel} > |v_{gx},_y|$, therefore most of the energy flows parallel to the magnetic field. The phase velocity along the field is given by $v_{ph_{\parallel}} = \omega/k_{\parallel}$. Using the relations $k_{\parallel} \leqslant (m_e/m_i)^{\frac{1}{2}}/k$ and $k \approx \omega/c_s$, where $c_s(=[(KT_e + KT_i)/m_i]^{\frac{1}{2}})$ is the ion-acoustic velocity, K is Boltzmann's constant, $T_{e,i}$ are the electron, ion, temperatures respectively we find that $v_{ph\parallel} \geqslant c_s\ (m_i/m_e)^{\frac{1}{2}}$, ie, $v_{ph\parallel} \geqslant v_{Te}$. Thus the phase velocity along the field line is greater than the electron thermal speed.

4. WAVE GENERATION AND ACCELERATION PROCESS

There are a number of free energy sources on auroral field lines which can generate non-thermal levels of lower-hybrid waves. All these sources ultimately derive their power from the solar wind. One model which transfers the energy from the solar wind to the magnetosphere uses magnetic-field re-connection (see review by Galeev (1982)). The interaction between the solar wind and the magnetosphere compresses the plasma sheet transversely, the tail then narrows leading to the tearing instability and the formation of a re-connection point. This in turn leads to enhanced particle flows (De Cöster and Frank (1979); Frank et al (1976); Hones et al (1972)). As the plasma sheet is compressed the magnetic field increases leading to the enhancement of temperature and density gradients between the plasma sheet and tail lobes.

Lower hybrid waves can be driven unstable by several mechanisms. Cross-field currents due to density, temperature and magnetic field gradients, differential electron-ion drift or $\underline{E} \times \underline{B}_0$ drift, these can all act as driving mechanisms (McBride et al (1972); Chen (1974)). Various names are given to the processes which result in the growth of the waves. If gradients drive the instability, the process is known as the lower-hybrid drift instability, if convection is the driver it is called the modified two-stream instability. In both cases there is a threshold requirement of $v_d \geqslant v_{Ti}$, where v_d is the electron-ion relative drift speed and v_{Ti}, $(= (KT_i/m_i)^{\frac{1}{2}})$, is the ion thermal speed. The maximum growth rate γ_{max} $(= \omega_{Lh}/2)$, occurs at wave number $k_{max} = \sqrt{3}\ \omega_{Lh}/v_d$, and at real frequency $\omega = \sqrt{3}\ \omega_{Lh}/2$ (McBride et al (1972). Growth of the lower hybrid waves is insensitive to the electron-ion temperature ratio, T_e/T_i, and can take place even if $T_i >> T_e$, which is a condition on auroral field lines that limits other electrostatic instabilities, such as the current-driven ion-acoustic instability.

McBride et al (1972) have shown that the lower-hybrid wave growth saturates at an energy density given by

$$W_{Lh}/n_o KT_e \approx 0.05 \;/(1 + \omega_{pe}^2/\omega_{ce}^2) \tag{4}$$

where W_{Lh} $(= \frac{1}{2}\,\varepsilon_o|E|^2)$ is the energy density of the lower hybrid wave with an electric field E, and n_o is the plasma density. Particle acceleration is the process which limits the growth and leads to the saturation (McBride et al (1972)).

The statistical acceleration of electrons in the tail of the distribution due to interaction with lower-hybrid turbulence can be described by the quasi-linear diffusion equation (Tsytovich (1970); Davidson (1972))

$$\frac{\partial f_e}{\partial t} = \frac{\partial}{\partial v_\parallel} D_\parallel \frac{\partial f_e}{\partial v_\parallel} \tag{5}$$

where f_e is the electron distribution function, and $D_\parallel$ is the quasi-linear velocity diffusion coefficient, $D_\parallel = |\Delta v|^2/\tau$, where Δv is the change in velocity of the particle due to wave particle interaction in time τ. Bhadra et al, (1983) and Liu et al (1982) have shown that

$$D_\parallel = 16\pi^2 \frac{e^2}{m_e^2} \frac{k_\parallel}{\omega_{pe}} \varepsilon_k \tag{6}$$

where ε_k is the wave energy density per unit wavenumber. For an acceleration region of length L the effective range of $k_\parallel$ is

$$k_m \leq k_\parallel \leq k_o \tag{7}$$

where k_m is determined by L, and k_o is determined by the strong Landau damping that takes place for $k_\parallel \approx \omega_{LH}/2v_{Te}$. The total wave energy density is thus

$$W_{Lh} = (k_o - k_m)\, \varepsilon_k \simeq k_o \varepsilon_k \tag{8}$$

The time for electrons to be accelerated from v_1 to v_2 by lower hybrid waves is given by

$$\tau_{acc} = (v_2 - v_1)^2 / D_\parallel \tag{9}$$

A characteristic acceleration length $L_{acc} = \langle v \rangle \tau_{acc}$, where $\langle v \rangle$ is the average electron velocity, is given by (6), (7) and (8) as

$$L_{acc} = \frac{\omega_{Lh}}{4\pi\omega_{pe}^2} \left(\frac{n_o\, KT_e}{W_{Lh}} \right) \left(\frac{m_i}{m_e} \right) \frac{(v_2 - v_1)^2}{v_{Te}^2} \frac{k_o}{k_\parallel} \langle v \rangle \tag{10}$$

In section 5 we show that using existing lower hybrid wave measurements this length is of the order of $2 \sim 3 \times 10^6$ m which is a suitably short distance for auroral electron acceleration.

5. LOWER-HYBRID TURBULENCE ON AURORAL FIELD LINES

For lower-hybrid turbulence to be responsible for producing the electron distributions shown in Figure 1, there must be sufficient power available in the these modes. Using the theoretical value for the saturated wave energy density given by equation (4) we find that the normalized wave energy density on auroral field lines between 3 Re and 5 Re (where $\omega_{ce} > \omega_{pe}$) would saturate at 5×10^{-2}. Taking this value for the wave energy density with an electron density of 10^{-7} m^{-3} and an electron temperature of 1 keV, the power $(P = \frac{1}{2}\, \varepsilon_o |E|^2 v_g)$ in the lower-hybrid waves between 3

Re and 5 Re in a flux tube which has an area corresponding to 1 degree longitude by 1 degree latitude at 1 Re (measured from the earth's centre) and a latitutde of 70° is 1.5×10^9 W per deg long deg lat. The power observed to be carried by the electrons in the same flux tube at 1 Re is (12×10^7 - 72×10^7) W per deg long deg lat. This is obtained from the curves in Figure 1, which represents energy fluxes corresponding to 3×10^{-2} W m^{-2} for the lower energy curve and 21×10^{-2} W m^{-2} for the higher energy curve. Simulations by Tanaka and Papadopoulos (1983) demonstrate that 60% of an initial ion-stream energy is transferred via lower-hybrid waves to electrons, producing a high-energy tail extending to 7 v_{Te}. This result demonstrates the efficiency of the process in transferring energy from an ion stream to electrons via lower-hybrid waves. In this process the wave power is only of the same order of magnitude as that of the electrons, since the final electron energy is as high as 60% of the initial ion energy. Ion streams observed in the plasma sheet boundary layer carry a power of 3.6×10^{10} W per deg long deg lat (Hall et al (1983), using the energy flux of 10^{-3} W m^{-2} obtained by De Coster and Frank (1979) at 20 Re). This represents of course a power two orders of magnitude greater than that in the auroral electron stream.

Published observations of electrostatic wave enegy densities do not yet constitute a comprehensive survey. Most measurements are in fact taken far from the auroral region, but there are some clear pointers to the level of lower-hybrid turbulence that may be generated on field lines associated with auroral arcs. Scarf et al (1973, 1975), using results obtained from Ogo 5 found that the spectrum peaked near the lower-hybrid frequency with a normalized energy density of 10^{-3} - 10^{-4} corresponding to electric field strengths between 0.2 - 0.5 Vm^{-1}. The most intense measurements were seen in the region of steep gradients. Gurnett and Frank (1977) and Mozer et al (1979) also reported intense electric field strengths ($\simeq$ 50 mVm^{-1}) at the lower hybrid frequency, again with much lower field strengths at the Langmuir frequency. Using the value of 10^{-3} for the normalized energy density in (10) we find that the distance L_{acc} required to accelerate electrons from v to 2v, where $v = 2 \times 10^7$ m sec^{-1} and $k_0/k_\parallel \simeq 2$, is 2×10^6 m , ie of the order of 1 Re.

The observed power in the lower-hybrid waves between 3 Re and 5 Re in a fluid tube which has an area corresponding to 1 degree longitude and 1 degree latitude at 1 Re and a latitude of 70° is 5×10^7 Watts per deg long deg lat, for an normalized energy density of 10^{-3}. This is less than but of the same order as that carried by the electrons shown in Figure 1.

There is nothing inconsistent in this result especially since the rocket measurements were made over an auroral arc selected because of its high intensity, whilst the satellite measurements were not necessarily taken in the region of an auroral event. This is the kind of result expected from the theoretical considerations above in which there will be an equilibium between production and loss of wave power. We expect that the normalized energy wave density in the location of strong auroral events to be as much as 10^{-2}.

CONCLUSION

We have identified, as an alternative to the potential difference model, a process which can accelerate electrons to produce discrete aurora. This alternative process is stochastic acceleration by the lower hybrid turbulence observed on auroral field lines. We have shown that the lower-hybrid waves have the phase velocities (> 2 x 10^7 ms^{-1}) that are necessary to account for the acceleration in the direction of the magnetic field. The observed wave energy density is sufficient to accelerate electrons over a suitably sort distance (of the order of 1 Re). Wave powers (5 x 10^7 Watts/deg long deg lat) have been observed having the same order of magnitude as electron powers. It is suggested that the energy which drives the process could come from the ion streams observed by De Coster and Frank (1979) in the high-latitude plasma sheet boundary layer.

Other aspects of a wave model such as nonlinear wave processes like filamentation of waves may explain certain observed aurora phenomena such as striations or filaments in the auroral arc. The observed levels of wave energy density are strong enough to make nonlinear wave-particle and wave-wave processes possible. These would include nonlinear Landau-damping, particle trapping and three and four wave parametric interactions. Further study on these processes is underway.

Details that need yet to be evaluated are the exact from of the wave spectrum, the way the electron distribution evolves as a result of acceleration by lower-hybrid waves, and the contribution to the wave generation from the various free energy sources.

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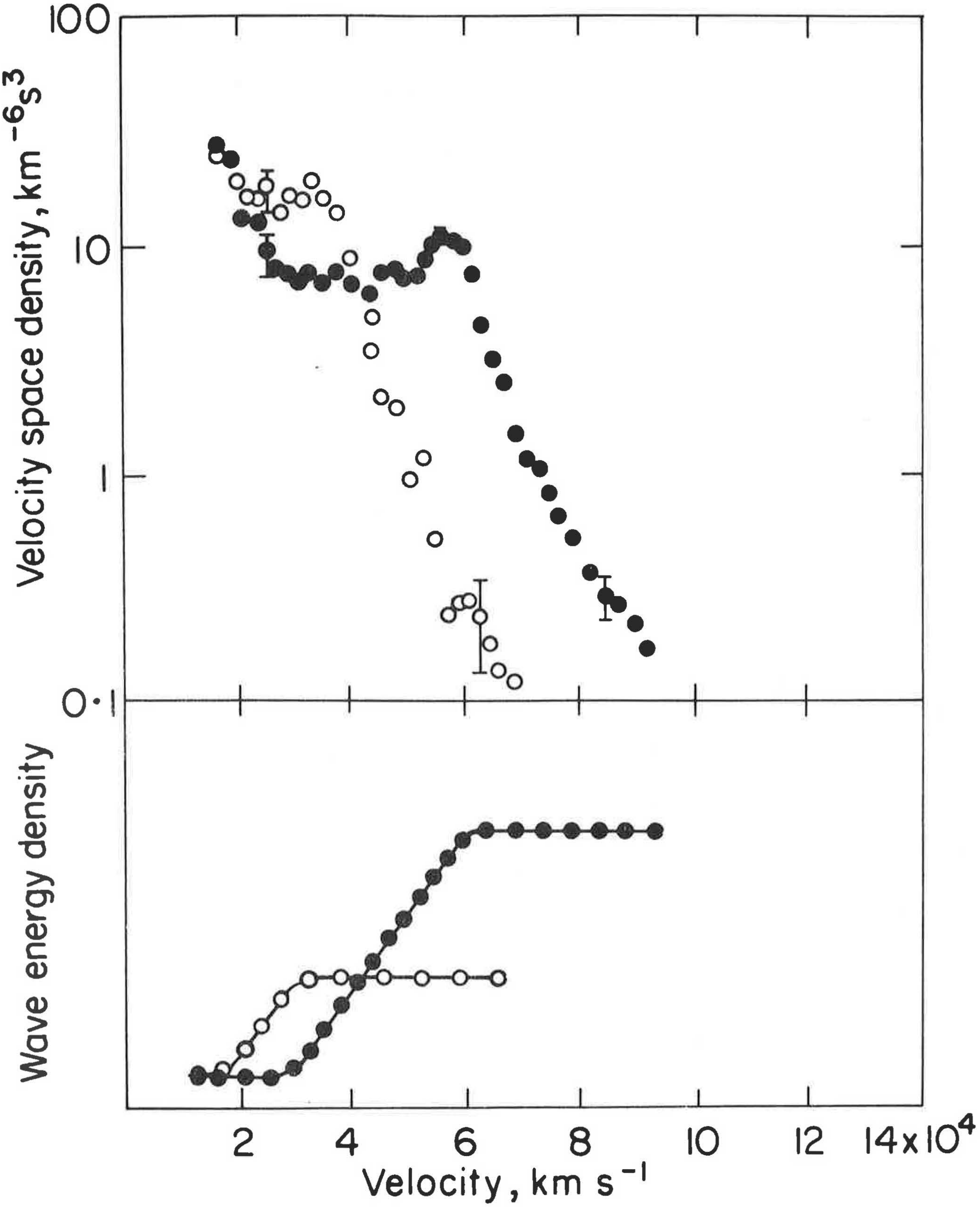
Figure 1



100
10
1
0·1
Velocity space density, km^{-6}s^3
Wave energy density
2
4
6
8
10
12
14x10^4
Velocity, km s^{-1}