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THE POLARIZED TWO-LOOP SPLITTING FUNCTIONS*

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We present a brief description of the light-cone gauge calculation of the spin-dependent next-to-leading order splitting functions.

It has recently become possible to perform analyses of the spin-dependent parton distributions of a longitudinally polarized hadron at next-to-leading order (NLO) accuracy of QCD. A first such phenomenological NLO study, taking into account all available experimental data on polarized deep-inelastic scattering has been presented in [1], followed by the analyses [2]. An indispensable ingredient here are the polarized two-loop splitting functions (or anomalous dimensions) $\Delta P_{ij}^{(1)}$ which appear in the NLO Q^2-evolution equations for the spin-dependent parton densities. Results (in the $\overline{\rm MS}$ scheme) for the $\Delta P_{ij}^{(1)}$ have first been obtained in [3] where the Operator Product Expansion (OPE) formalism was used. The results were afterwards confirmed in [4] using the somewhat more efficient method developed in [5] and employed in the unpolarized case in [6,7,8], which is based on the factorization properties of mass singularities and on the use of the axial gauge. In this paper we give a brief description of our calculation [4].

To begin with, we collect all ingredients for a NLO analysis of longitudinally polarized deep-inelastic scattering in terms of the spin-dependent structure function $g_1(x,Q^2)$. Beyond LO, there are two different short-distance cross sections, ΔC_q and ΔC_g, for scattering off incoming polarized quarks and gluons, respectively. Thus the NLO expression for g_1 reads in general:

$$\begin{aligned} g_1(x,Q^2) &= \frac{1}{2}\sum_{i=1}^{n_f} e_i^2 \left\{ \Delta q_i(x,Q^2) + \Delta\bar{q}_i(x,Q^2) + \right. \\ &+ \left. \frac{\alpha_s(Q^2)}{2\pi}\left[\Delta C_q \otimes (\Delta q_i + \Delta\bar{q}_i) + \frac{1}{n_f}\Delta C_g \otimes \Delta g\right](x,Q^2)\right\}, \quad (1) \end{aligned}$$

where n_f is the number of flavors and $\otimes$ denotes the usual convolution. Here, the polarized parton distributions $\Delta f \equiv f^{\uparrow} - f^{\downarrow}$ $(f = q, \bar{q}, g)$ are to be evolved in Q^2 according to the NLO spin-dependent Altarelli-Parisi [9] evolution equations. We adopt the following perturbative expansion of the evolution kernels:

$$\Delta P_{ij}(x,\alpha_s) = \left(\frac{\alpha_s}{2\pi}\right)\Delta P_{ij}^{(0)}(x) + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{(1)}(x) + \dots \quad (2)$$

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We emphasize that neither of the NLO corrections, ΔC_i and $\Delta P_{ij}^{(1)}$, are physical quantities since they depend on the factorization scheme adopted. Needless to mention that the same scheme has to be chosen in the calculation of both in order to obtain a meaningful result. Conversely, once the ΔC_i and $\Delta P_{ij}^{(1)}$ are known in one scheme it is possible to perform a factorization scheme transformation, i.e., to shift terms between them without changing a physical quantity like g_1, hereby redefining the polarized NLO parton distributions [10].

Defining $\Delta q_i^{\pm} \equiv \Delta q_i \pm \Delta \bar{q}_i$ one finds the following NLO evolution equations for the non-singlets (NS) Δq_i^- and $\Delta q_i^+ - \Delta q_j^+$:

$$\frac{d}{d\ln Q^2}(\Delta q_i^+ - \Delta q_j^+) = \Delta P_{qq}^+(x,\alpha_s(Q^2)) \otimes (\Delta q_i^+ - \Delta q_j^+) \,, \tag{3}$$

$$\frac{d}{d\ln Q^2}\Delta q_i^- = \Delta P_{qq}^-(x,\alpha_s(Q^2)) \otimes \Delta q_i^- \,, \tag{4}$$

where we have suppressed the obvious argument (x, Q^2) in all parton densities and taken into account that there are two different NS splitting functions, $\Delta P_{qq}^{\pm}$, beyond LO (see, e.g., [8]). Defining $\Delta\Sigma \equiv \sum_i (\Delta q_i + \Delta \bar{q}_i)$ one has in the flavor singlet sector:

$$\frac{d}{d\ln Q^2}\begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} = \begin{pmatrix} \Delta P_{qq}(x,\alpha_s(Q^2)) & \Delta P_{qg}(x,\alpha_s(Q^2)) \\ \Delta P_{gq}(x,\alpha_s(Q^2)) & \Delta P_{gg}(x,\alpha_s(Q^2)) \end{pmatrix} \otimes \begin{pmatrix} \Delta\Sigma \\ \Delta g \end{pmatrix} . \tag{5}$$

The qq-entry in the singlet matrix of splitting functions is written as

$$\Delta P_{qq} = \Delta P_{qq}^+ + \Delta P_{qq}^S \,. \tag{6}$$

Thus, at NLO, we will have to derive the splitting functions $\Delta P_{qq}^{\pm,(1)}$, $\Delta P_{qq}^{S,(1)}$, and those involving gluons. The general strategy to do this in the method of [5,6,7] consists of first expanding the squared matrix element ΔM for (polarized) virtual photon–polarized quark (gluon) scattering into a ladder of two-particle irreducible (2PI) kernels [5] C_0, K_0,

$$\Delta M = \Delta\left[C_0(1 + K_0 + K_0^2 + K_0^3 + \ldots)\right] \equiv \Delta\left[\frac{C_0}{1-K_0}\right] . \tag{7}$$

We now choose the light-cone gauge by introducing a light-like vector n ($n^2 = 0$) with $n \cdot A = 0$. In this gauge the 2PI kernels are finite as long as external legs are kept unintegrated, such that collinear singularities only appear when integrating over the lines connecting the rungs of the ladder [5]. This allows for projecting out the singularities by introducing the projector onto polarized physical states, $\Delta\mathcal{P}$. Thus ΔM can be written in the factorized form

$$\Delta M = \Delta C \Delta\Gamma \,, \tag{8}$$

where $\Delta C = \Delta C_0/(1-(1-\Delta\mathcal{P})K_0)$ is the (finite) short-distance cross section, whereas $\Delta\Gamma$ contains all (and only) mass singularities. Working in dimensional regularization ($d = 4 - 2\epsilon$) in the $\overline{\rm MS}$ scheme one has explicitly [6]:

$$\Delta\Gamma_{ij} = Z_j \left[\delta(1-x)\delta_{ij} + x \ {\rm PP} \int \frac{d^dk}{(2\pi)^d} \delta(x - \frac{kn}{pn}) \Delta U_i K \frac{1}{1-\Delta\mathcal{P}K} \Delta L_j \right] , \tag{9}$$

where 'PP' extracts the pole part, Z_j ($j = q(g)$) is the residue of the pole of the full quark (gluon) propagator, and we have defined $K = K_0(1-(1-\Delta\mathcal{P})K_0)^{-1}$. k is the momentum of the parton leaving the uppermost kernel in $\Delta\Gamma$. The spin-dependent projection operators onto physical states are given by

$$\Delta U_q = -\frac{1}{4kn}\gamma_5 \not{n}, \ \ \Delta L_q = -\not{p}\gamma_5 \ ; \ \ \Delta U_g = i\epsilon^{\mu\nu\rho\sigma}\frac{n_\rho k_\sigma}{kn}, \ \ \Delta L_g = i\epsilon^{\mu\nu\rho\sigma}\frac{p_\rho n_\sigma}{2pn} \,. \tag{10}$$

Finally, it can be shown [6] that the coefficient of the $1/\epsilon$ pole of $\Delta\Gamma$ is related to the splitting functions we are looking for:

$$\Delta\Gamma_{qq} = \delta(1-x) - \frac{1}{\epsilon}\left(\frac{\alpha_s}{2\pi}\Delta P_{qq}^{(0)}(x) + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{qq}^{(1)}(x) + \ldots \right) + O\left(\frac{1}{\epsilon^2}\right) \tag{11}$$

and analogously for the flavor singlet case. Explicit examples of the Feynman diagrams contributing to the $\Delta\Gamma_{ij}$ can be found in [6,4,8].

We see from Eq. (10) that there is a new ingredient in the polarized calculation which requires extra attention: The Dirac matrix γ_5 and the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ enter. A prescription for dealing with these (genuinely *four*-dimensional) quantities in $d = 4 - 2\epsilon$ dimensions has to be adopted which must be free of algebraic inconsistencies. Our calculation [4] was performed using the original definitions for γ_5 and $\epsilon_{\mu\nu\rho\sigma}$ of [11] (HVBM scheme) which is usually regarded as the most reliable prescription. Here γ_5 retains its *four*-dimensional definition, $\gamma_5 \equiv i\epsilon^{\mu\nu\rho\sigma}\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma/4!$, with the ϵ-tensor being a genuinely four-dimensional object. As a consequence one finds that

$$\{\gamma^\mu, \gamma_5\} = 0 \ \ \text{for } \mu = 0,1,2,3 \, ; \ \ [\gamma^\mu, \gamma_5] = 0 \ \ \text{otherwise} \,. \tag{12}$$

Thus the matrix element squared of a graph will in general depend on scalar products defined in the '$(d-4)$-dimensional' subspace. Special care has to be taken of such terms in loop and phase space integrals [4].

Another more technical remark concerns the use of the light-cone gauge, which plays a crucial role in the calculation. The light-cone gauge denominator $1/(n \cdot l)$ in the gluon propagator can give rise to additional divergencies in loop and phase space integrals. We follow [6,7,8] to use the principal value (PV) prescription to regulate such poles:

$$\frac{1}{n \cdot l} \rightarrow \frac{1}{2}\left(\frac{1}{n \cdot l + i\delta(pn)} + \frac{1}{n \cdot l - i\delta(pn)}\right) = \frac{n \cdot l}{(n \cdot l)^2 + \delta^2 (pn)^2} \,. \tag{13}$$

The PV prescription appears to be the most convenient choice from a practical point of view; it leads, however, to the feature that the renormalization 'constants' depend [6,8] on the longitudinal momentum fractions x.

We express the $\overline{\mathrm{MS}}$ results of our calculation in the HVBM scheme in terms of the unpolarized NLO NS splitting functions $P_{qq}^{\pm,(1)}$ of [6] and of the recent polarized OPE results $\Delta\tilde{P}_{ij}^{(1)}$ of [3], exploiting the fact that the contributions $\sim \delta(1-x)$ to the diagonal splitting functions are necessarily the same as in the unpolarized case since they are determined by Z_j in (9). One then has:

$$\begin{aligned}
\Delta P_{qq}^{\pm,(1)}(x) &= P_{qq}^{\mp,(1)}(x) - 2\beta_0 C_F (1-x) \;, \\
\Delta \hat{P}^{(1)}(x) &= \Delta \hat{\tilde{P}}^{(1)}(x) - \frac{\beta_0}{2}\hat{A}(x) + \left[\hat{A}(x), \hat{P}^{(0)}(x)\right]_{\otimes} \;, \\
\Delta C_q(x) &= \Delta \tilde{C}_q(x) - 4C_F(1-x) \;, \\
\Delta C_g(x) &= \Delta \tilde{C}_g(x) \;,
\end{aligned} \tag{14}$$

where $\hat{P}^{(0)}$ and $\hat{P}^{(1)}$ denote the LO and NLO parts, respectively, of the singlet evolution matrix, and

$$\hat{A}(x) \equiv 4C_F(1-x)\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \,.$$

In Eq. (14) we have also included the results for the short-distance cross sections ΔC_q, ΔC_g. As indicated in Eq. (14), the '+' and '−' combinations of the NS splitting functions have interchanged [1,12] their roles. Eqs. (3,14) therefore imply that the combination $\Delta P_{qq}^{+,(1)} = P_{qq}^{-,(1)} - 2\beta_0 C_F(1-x)$ would govern the Q^2-evolution of, e.g., the polarized NS quark combination

$$\Delta A_3(x, Q^2) = \left(\Delta u^+ - \Delta d^+\right)(x, Q^2) \,.$$

Since the first moment (i.e., the x-integral) of the latter corresponds to the nucleon matrix element of the NS axial vector current $\bar{q}\gamma^\mu\gamma_5\lambda_3 q$ which is conserved, it has to be Q^2-independent. Keeping in mind that the integral of the unpolarized $P_{qq}^{-,(1)}$ vanishes already due to fermion number conservation [6], it becomes obvious that the additional term $-2\beta_0 C_F(1-x)$ in (14) spoils the Q^2-independence of the first moment of $\Delta A_3(x,Q^2)$. On the other hand, as pointed out earlier, we are free to perform a factorization scheme transformation to the results in (14). It turns out [4] that the scheme transformation that removes the term $-2\beta_0 C_F(1-x)$ from $\Delta P_{qq}^{\pm,(1)}$ in Eq. (14) eliminates *at the same time all* extra $(1-x)$-terms on the r.h.s. of (14), leaving ΔC_g unchanged. Thus our final results after the transformation are in complete agreement with those of [3]. We note that the presence of the $(1-x)$-terms in our original HVBM scheme result (14) can be traced back to the fact that in this scheme the d-dimensional polarized LO quark-to-quark splitting function is no longer equal to its unpolarized counterpart due to the non-anticommutativity of γ_5 (see (12)), artificially violating helicity conservation.

Our complete final results for the $\Delta P_{ij}^{(1)}(x)$ can be found in [4] and need not be repeated here. We mention that compact expressions for the Mellin-moments of the polarized NLO splitting functions, defined by

$$\Delta P_{ij}^{(1),n} \equiv \int_0^1 x^{n-1} \Delta P_{ij}^{(1)}(x) dx \tag{15}$$

as well as their analytic continuations to arbitrary complex n, can be found in [1]. To work in Mellin-n space is very convenient for a numerical analysis of parton distributions since the evolution equations can be solved analytically here. For illustration we show the entries $\Delta P_{ij}^{(1),n}$ of the NLO part of the singlet evolution matrix as a function of real Mellin-n in Fig. 1, comparing them to the unpolarized $P_{ij}^{(1),n}$ as obtained from [13]. One observes, in particular, that $\Delta P_{ij}^{(1),n} \to P_{ij}^{(1),n}$ for $n \to \infty$ (i.e., for $x \to 1$), except [1] for $\Delta P_{gq}^{(1),n}$. We finally note that the values for the first moments of the $\Delta P_{ij}^{(1)}(x)$ turn out to be [3,4,14]

$$\begin{aligned} &\Delta P_{qq}^{(1),n=1} = -3C_F T_f \;, \quad \Delta P_{gq}^{(1),n=1} = -\frac{9}{4}C_F^2 + \frac{71}{12}N_C C_F - \frac{1}{3}C_F T_f \;, \\ &\Delta P_{qg}^{(1),n=1} = 0 \;, \quad \Delta P_{gg}^{(1),n=1} = \frac{17}{6}N_C^2 - C_F T_f - \frac{5}{3}N_C T_f \equiv \frac{\beta_1}{4} \;, \end{aligned} \tag{16}$$

where $C_F = 4/3$, $N_C = 3$, $T_f = n_f/2$.

References

1. M. Glück, E. Reya, M. Stratmann, W. Vogelsang, *Phys. Rev.* D **53**, 4775 (1996).

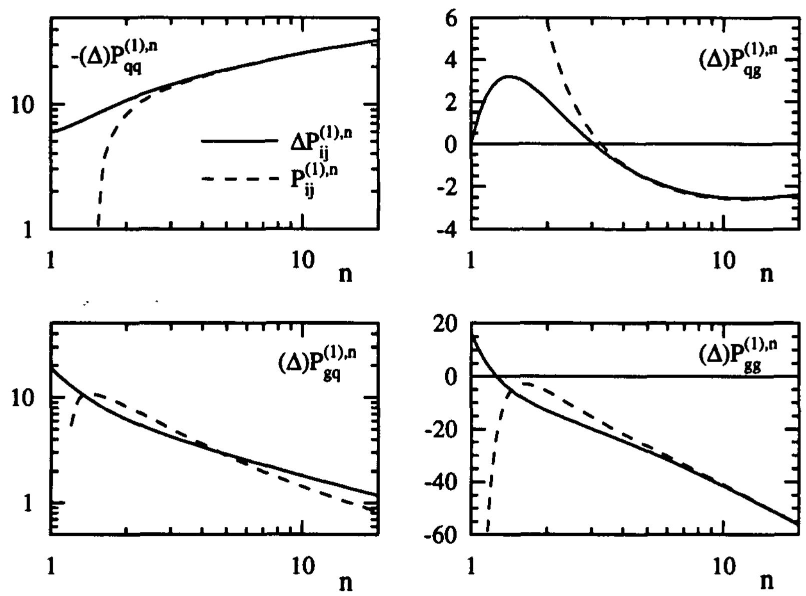

Figure 1: Mellin-moments of the polarized and unpolarized NLO singlet splitting functions.

2. R.D. Ball, S. Forte, G. Ridolfi, *Phys. Lett.* B **378**, 255 (1996); T.K. Gehrmann, W.J. Stirling, *Phys. Rev.* D **53**, 6100 (1996).
3. R. Mertig, W.L. van Neerven, *Z. Phys.* C **70**, 637 (1996).
4. W. Vogelsang, Rutherford RAL-TR-95-071, to appear in *Phys. Rev.* D; RAL-TR-96-020, to appear in *Nucl. Phys.* B.
5. R.K. Ellis *et al.*, *Nucl. Phys.* B **152**, 285 (1979).
6. G. Curci, W. Furmanski, R. Petronzio, *Nucl. Phys.* B **175**, 27 (1980).
7. W. Furmanski, R. Petronzio, *Phys. Lett.* **97B**, 437 (1980).
8. For a detailed description of the calculation in the unpolarized case see: R.K. Ellis, W. Vogelsang, RAL-TR-96-012, hep-ph/9602356.
9. G. Altarelli, G. Parisi, *Nucl. Phys.* B **126**, 298 (1977).
10. see, e.g., M. Glück, E. Reya, *Phys. Rev.* D **25**, 1211 (1982).
11. G. 't Hooft, M. Veltman, *Nucl. Phys.* B **44**, 189 (1972); P. Breitenlohner, D. Maison, *Comm. Math. Phys.* **52**, 11 (1977).
12. M. Stratmann, W. Vogelsang, A. Weber, *Phys. Rev.* D **53**, 138 (1996).
13. E.G. Floratos, C. Kounnas, R. Lacaze, *Nucl. Phys.* B **192**, 417 (1981).
14. J. Kodaira, *Nucl. Phys.* B **165**, 129 (1980); G. Altarelli, B. Lampe, *Z. Phys.* C **47**, 315 (1990); S.A. Larin, *Phys. Lett.* B **303**, 113 (1993).