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A New Approach to Yukawa Textures in Supersymmetric Unified Models With Gauged Family Symmetries

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Abstract

The origin of texture zeroes in the Yukawa matrices may be accounted for by appealing to a broken gauged family symmetry such as $U(1)_X$, where such symmetries arise naturally from string theories. In order to improve the predictive power of such models we appeal to quark-lepton unification where additional *Clebsch texture zeroes* appear, leading to an entirely new class of models. We illustrate these ideas in the context of the Pati-Salam gauge group $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ supplemented by a $U(1)_X$ gauged family symmetry. The gauge symmetries are broken down to those of the minimal supersymmetric standard model which is the effective theory below 10^{16} GeV. The combination of the $U(1)_X$ family symmetry and the Pati-Salam gauge group leads to a successful and predictive set of Yukawa textures involving both kinds of texture zeroes. We discuss both symmetric and non-symmetric textures in models of this kind, and in the second case perform a detailed numerical fit to the charged fermion mass

and mixing data. Two of the Yukawa textures allow a low energy fit to the data with a total χ^2 of 0.39 and 1.02 respectively, for three degrees of freedom.

1 Introduction

The pattern of quark and lepton masses and quark mixing angles has for a long time been a subject of fascination for particle physicists. In terms of the standard model, this pattern arises from three by three complex Yukawa matrices (54 real parameters) which result in nine real eigenvalues plus four real mixing parameters (13 real quantities) which can be measured experimentally. In recent years the quark and lepton masses and mixing angles have been measured with increasing precision, and this trend is likely to continue in the future as lattice QCD calculations provide increasingly accurate estimates and B-factories come on-line. Theoretical progress is less certain, although there has been a steady input of theoretical ideas over the years and in recent times there is an explosion of activity in the area of supersymmetric unified models. This approach presumes that at very high energies close to the unification scale, the Yukawa matrices exhibit a degree of simplicity, with simple relations at high energy corrected by the effects of renormalisation group (RG) running down to low energy. For example the classic prediction that the bottom and tau Yukawa couplings are equal at the unification scale can give the correct low energy bottom and tau masses, providing that one assumes the RG equations of the minimal supersymmetric standard model (MSSM)[1][1]. In the context of the MSSM it is even possible that the top, bottom and tau Yukawa couplings are all approximately equal near the unification scale [3], since although this results in the top and bottom Yukawa couplings being roughly the same at low energy, one can account for the large top to bottom mass ratio by invoking a large value of $\tan\beta$ defined as the ratio of vacuum expectation values (VEVs) of the two Higgs doublets of the MSSM.

These successes with the third family relations are not immediately generalisable to the lighter families. For the remainder of the Yukawa matrices, additional ideas are required in order to understand the rest of the spectrum. One such idea is that of texture zeroes: the idea that the Yukawa matrices at the unification scale are rather sparse; for example the Fritzsch ansatz [4]. Although the Fritzsch texture does not work for supersymmetric unified models, there are other textures which do, for example the Georgi-Jarlskog (GJ) texture [5] for the down-type quark and lepton matrices:

$$\lambda^E = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & -3\lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & \lambda_{12} & 0 \\ \lambda_{21} & \lambda_{22} & 0 \\ 0 & 0 & \lambda_{33} \end{pmatrix}. \tag{1}$$

[1] The next-to-MSSM (NMSSM) with an additional low energy gauge singlet works just as well [2].

After diagonalisation this leads to $\lambda_\tau = \lambda_b$, $\lambda_\mu = 3\lambda_s$, $\lambda_e = \lambda_d/3$ at the scale M_{GUT} which result in (approximately) successful predictions at low energy. Actually the factor of 3 in the 22 element above arises from group theory: it is a Clebsch factor coming from the choice of Higgs fields coupling to this element.

It is observed that if we choose the upper two by two block of the GJ texture to be symmetric, $\lambda_{12} = \lambda_{21}$, and if we can disregard contributions from the up-type quark matrix, then we also have the successful mixing angle prediction

$$V_{us} = \sqrt{\lambda_d/\lambda_s}. \tag{2}$$

This last relation supports the idea of symmetric matrices, and a texture zero in the 11 position. Motivated by the desire for maximal predictivity, Ramond, Roberts and Ross (RRR) [6] have made a survey of possible symmetric textures which are both consistent with data and involve the maximum number of texture zeroes. Assuming GJ relations for the leptons, RRR tabulated five possible solutions for the up-type and down-type Yukawa matrices. We list them below for completeness:

Solution 1:

$$\lambda^U = \begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix} \tag{3}$$

Solution 2:

$$\lambda^U = \begin{pmatrix} 0 & \lambda^6 & 0 \\ \lambda^6 & 0 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 2\lambda^3 \\ 0 & 2\lambda^3 & 1 \end{pmatrix} \tag{4}$$

Solution 3:

$$\lambda^U = \begin{pmatrix} 0 & 0 & \sqrt{2}\lambda^4 \\ 0 & \lambda^4 & 0 \\ \sqrt{2}\lambda^4 & 0 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 4\lambda^3 \\ 0 & 4\lambda^3 & 1 \end{pmatrix} \tag{5}$$

Solution 4:

$$\lambda^U = \begin{pmatrix} 0 & \sqrt{2}\lambda^6 & 0 \\ \sqrt{2}\lambda^6 & \sqrt{3}\lambda^4 & \lambda^2 \\ 0 & \lambda^2 & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{6}$$

Solution 5:

$$\lambda^U = \begin{pmatrix} 0 & 0 & \lambda^4 \\ 0 & \sqrt{2}\lambda^4 & \lambda^2/\sqrt{2} \\ \lambda^4 & \lambda^2/\sqrt{2} & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 2\lambda^4 & 0 \\ 2\lambda^4 & 2\lambda^3 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{7}$$

where $\lambda = 0.22$, and the top and bottom Yukawa couplings have been factored out for simplicity. These textures are valid at the unification scale. All the solutions involve texture zeroes in the 11 entry. Solutions 1,2, and 4 involve additional texture zeroes in the 13=31 positions which are common to both up-type and down-type matrices. Solutions 3 and 5 have no texture zeroes which are common to both up-type and down-type matrices, apart from the 11 entry. Thus solutions 1,2 and 4 involve rather similar up-type and down-type matrices, while solutions 3 and 5 involve very different textures for the two matrices.

Having identified successful textures[2], the obvious questions are: what is the origin of the texture zeroes? and: what is the origin of the hierarchies (powers of the expansion parameter λ)? A natural answer to both these questions was provided early on by Froggatt and Nielsen (FN) [8]. The basic idea involves a high energy scale M, a family symmetry group G, and some new heavy matter of mass M which transforms under G. The new heavy matter consists of some Higgs fields which are singlets under the vertical gauge symmetry but non-singlets under G. These break the symmetry G by developing VEVs V smaller than the high energy scale, There are also some heavy fields which exist in vector-like representations of the standard gauge group. The vector-like matter couples to ordinary matter (quarks, leptons, Higgs) via the singlet Higgs, leading to "spaghetti-like" tree-level diagrams. Below the scale V the spaghetti diagrams yield effective non-renormalisable operators which take the form of Yukawa couplings suppressed by powers of $\lambda = V/M$. In this way the hierarchies in the Yukawa matrices may be explained, and the texture zeroes correspond to high powers of λ.

A specific realisation of the FN idea was provided by Ibanez and Ross (IR) [9], based on the MSSM extended by a gauged family $U(1)_X$ symmetry with θ and $\bar{\theta}$ singlet fields with opposite X charges, plus new heavy Higgs fields in vector representations[3]. Anomaly cancellation occurs via a Green-Schwarz-Witten (GSW) mechanism, and the $U(1)_X$ symmetry is broken not far below the string scale [9]. By making certain symmetric charge assignments, IR showed that the RRR texture solution 2 could be approximately reproduced. To be specific, for a certain choice of $U(1)_X$ charge assignments,

[2]Over the recent years, there has been an extensive study of fermion mass matrices with zero textures [7].

[3]The generalisation to include neutrino masses is straightforward [10].

IR generated Yukawa matrices of the form:

$$\lambda^U = \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} \bar{\epsilon}^8 & \bar{\epsilon}^3 & \bar{\epsilon}^4 \\ \bar{\epsilon}^3 & \bar{\epsilon}^2 & \bar{\epsilon} \\ \bar{\epsilon}^4 & \bar{\epsilon} & 1 \end{pmatrix}, \quad \lambda^E = \begin{pmatrix} \bar{\epsilon}^5 & \bar{\epsilon}^3 & 0 \\ \bar{\epsilon}^3 & \bar{\epsilon} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{8}$$

These are symmetric in the expansion parameters ϵ and $\bar{\epsilon}$, which are regarded as independent parameters. This provides a neat and predictive framework, however there are some open issues. Although the order of the entries is fixed by the expansion parameters, there are additional parameters of order unity multiplying each entry, making precise predictions difficult. A way to address the problem of the unknown coefficients has been proposed in [11] where it has been shown that the various coefficients may arise as a result of the infra-red fixed-point structure of the theory beyond the Standard Model.

Note that the textures for up-type and down-type matrices are of similar form, although the expansion parameters differ. Also note that there are no true texture zeroes in the quark sector, merely high powers of the expansion parameter. Thus this example most closely resembles RRR solution 2 with approximate texture zeroes in the 11 and 13=31 positions. However, without the inclusion of coefficients, the identification is not exact. The best fit to solution 2 of RRR is obtained for the identification $\epsilon \equiv \lambda^2$, $\bar{\epsilon} \equiv \lambda$ (alternative identifications, like $\epsilon \equiv \lambda^2$, $\bar{\epsilon} \equiv 2\lambda^3$ lead to larger deviations). However even this choice does not exactly correspond to RRR solution 2, as can be shown by taking solution 2 and inserting the numerical values of the entries:

$$\lambda^U = \begin{pmatrix} 0 & 1\times10^{-4} & 0 \\ 1\times10^{-4} & 0 & 5\times10^{-2} \\ 0 & 5\times10^{-2} & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 0 & 5\times10^{-3} & 0 \\ 5\times10^{-3} & 2\times10^{-2} & 2\times10^{-2} \\ 0 & 2\times10^{-2} & 1 \end{pmatrix} \tag{9}$$

We compare these numbers to the order of magnitudes predicted by the symmetry argument, making the identifications $\epsilon \equiv \lambda^2$, $\bar{\epsilon} \equiv \lambda$

$$\lambda^U = \begin{pmatrix} 3\times10^{-11} & 1\times10^{-4} & 5\times10^{-6} \\ 1\times10^{-4} & 2\times10^{-3} & 5\times10^{-2} \\ 5\times10^{-6} & 5\times10^{-2} & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} 5\times10^{-6} & 1\times10^{-2} & 2\times10^{-3} \\ 1\times10^{-2} & 5\times10^{-2} & 2\times10^{-1} \\ 2\times10^{-3} & 2\times10^{-1} & 1 \end{pmatrix} \tag{10}$$

Comparison of Eq.9 to Eq.10 shows that while λ^U is in good agreement, λ^D differs. In Eq.10, the $23 = 32$ element is an order of magnitude too large. When the unknown couplings and phases are inserted the scheme can be made to work. However, some tuning of the unknown parameters is implicit, and to some extent this is in contrast with the object of the symmetries approach, where one had hoped to understand at least the order of magnitude of the entries of the Yukawa matrices using symmetries. We observe that a better fit to λ^D could be obtained by introducing a small parameter δ into all the elements apart from the 33 renormalisable element, so that Eq.8 gets replaced by[4]

$$\lambda^U = \begin{pmatrix} \epsilon^8 & \epsilon^3 & \epsilon^4 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^4 & \epsilon & 1 \end{pmatrix}, \quad \lambda^D = \begin{pmatrix} \delta\bar{\epsilon}^8 & \delta\bar{\epsilon}^3 & \delta\bar{\epsilon}^4 \\ \delta\bar{\epsilon}^3 & \delta\bar{\epsilon}^2 & \delta\bar{\epsilon} \\ \delta\bar{\epsilon}^4 & \delta\bar{\epsilon} & 1 \end{pmatrix} \tag{11}$$

The idea is that the suppression factor δ originates from some flavour independent physics, while the parameters ϵ and $\bar{\epsilon}$ control the flavour structure of the matrices. For example, suppose we take $\bar{\epsilon} \equiv \lambda$ as in the previous example but scale down the entries by a factor of $\delta = 0.2$. Then we would have,

$$\lambda^D = \begin{pmatrix} 1 \times 10^{-6} & 2 \times 10^{-3} & 4 \times 10^{-4} \\ 2 \times 10^{-3} & 1 \times 10^{-2} & 4 \times 10^{-2} \\ 4 \times 10^{-4} & 4 \times 10^{-2} & 1 \end{pmatrix} \tag{12}$$

which provides a better description of the numerical values required by the RRR analysis for solution 2 in Eq.9, at the expense of introducing the parameter δ. This example indicates that if family symmetries are to give the correct order of magnitude understanding of Yukawa textures without any tuning of parameters, then an extra parameter δ needs to be introduced as above.

Another aspect of the fermion mass spectrum not addressed by only $U(1)_X$ flavour symmetries, is that of the mass splitting within a particular family. For example the GJ texture in Eq.1 provides a nice understanding of the relationship between the charged lepton and down-type quark Yukawa couplings within a given family, and in the simplest $U(1)_X$ scheme such relations are either absent or accidental, as seen in Eq.8 where the form of λ^E has been fixed by a parameter choice. Unless such parameters are predicted by the theory, as in the extension of the initial IR scheme that is discussed in [11], the only antidote is extra unification. Then, the leptons share a representation with the quarks, and the magic GJ factors of three originate from the fact that the quarks have three colours. For example the $SO(10)$ model of Anderson et al [12] with both low energy Higgs doublets unified into a single $\underline{10}$ -representation predicts Yukawa unification for the third family, GJ relations for the charged leptons and down-type masses, and other Clebsch relations involving up-type

[4]In our scheme we will have a Unified Yukawa matrix. This, as we are going to see, will imply a common expansion parameter for the up and down-type mass matrices and the presence of a factor δ in the up-quark mass matrix as well.

quarks. As in the IR approach, the approach followed by Anderson et al is based on the FN ideas discussed above. Thus for example, only the third family is allowed to receive mass from the renormalisable operators in the superpotential. The remaining masses and mixings are generated from a minimal set of just three specially chosen non-renormalisable operators whose coefficients are suppressed by a set of large scales. The 12=21 operator of Anderson et al is suppressed by the ratio $(45_1/M)^6$, while the 23=32 and operators are suppressed by $(45_{B-L}/45_1)^2$ and $(45_{B-L}S/45_1^2)$ where the 45's are heavy Higgs representations. In a complicated multi-scale model such as this, the hierarchies between different families are not understood in terms of a family symmetry such as the the $U(1)_X$ of IR. Indeed it is difficult to implement a family symmetry in this particular scheme, and the latest attempts based on global $U(2)$ [13] abandon it. In any case[5], models such as $SO(10)$ are not "string friendly" and simple orbifold compactifications in which candidate gauge $U(1)_X$ family symmetries are present do not easily emerge.

In this paper we shall combine the $U(1)_X$ family symmetry approach of IR with the idea of Clebsch relations to describe the mass relations within a particular family. The combination of the two ideas provides a very attractive framework for describing the fermion mass spectrum. It is clear that the way to obtain Clebsch relations is to unify the quarks with the leptons. It is equally clear that too much unification can lead to too many different scales which negates the idea of the $U(1)_X$ family symmetry, and causes problems with string compatibility. Therefore we shall consider the simplest "string friendly" unified extension of the standard model, which can lead to Clebsch relations of the kind we desire. In this way we are led to the Pati-Salam gauge group [14] which was considered as a unified string model [15],[16] some time ago. This Pati-Salam gauge group has recently been the subject of renewed interest from the point of view of fermion masses [17], and an operator analysis has shown that it is possible to obtain desirable features such as Yukawa unification for the third family, and GJ type relations within this simpler model. A particular feature of the published scheme which we would like to emphasise here is the idea of *Clebsch texture zeroes* which arise from the group theory of the Pati-Salam gauge group. These Clebsch zeroes were used to account for the lightness of the up quark compared to the down quark, for example [17]. However the operator analysis of [17] did not address the question of the hierarchy between families (no family symmetry was introduced for example), nor the question of the origin of the non-renormalisable operators. Here we shall introduce a $U(1)_X$ gauge

[5]Here we restrict our discussion in string constructions based on $k = 1$ level of Kac-Moody algebras.

symmetry into the model and combine it with the Clebsch relations previously used, to provide a predictive scheme of fermion masses and mixing angles. We shall also ensure that we obtain the correct order of magnitude for all the entries of the Yukawa matrices from the symmetry breaking parameter, using structures like that of Eq.11. In our case the quantity δ will be identified with a bilinear of heavy Higgs fields which are responsible for generating the Clebsch structures, while the parameters such as ϵ will have trivial Clebsch structure (singlets under the vertical gauge group) but will generate family hierarchies from the flavour symmetry. This corresponds to there being two types of heavy Higgs fields: Pati-Salam gauge singlets (corresponding to IR θ and $\bar{\theta}$ fields) which break the $U(1)_X$ family gauge group but leave the Pati-Salam group unbroken, and Pati-Salam breaking fields whose bilinear forms are $U(1)_X$ singlets but transform non-trivially under the Pati-Salam gauge group, thereby giving interesting Clebsch structures. The non-renormalisable operators of interest must therefore involve both types of Higgs fields simultaneously.

The layout of the paper is as follows: In section 2 we briefly review the string-inspired Pati-Salam model. In section 3 we introduce our new approach based on the combined operators mentioned above, using the symmetric textures of RRR as an example. In section 4 we review the non-symmetric operator analysis in ref.[17] and then introduce a non-symmetric version of our new approach. In section 5 we perform a full numerical analysis of the non-symmetric models. In section 6 we review the $U(1)_X$ family symmetry approach, and perform an analysis relevant for the full (symmetric and non-symmetric) model. Finally section 7 concludes the paper.

2 The Model

Here we briefly summarise the parts of the model which are relevant for our analysis. For a more complete discussion see [15]. The gauge group is,

$$\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R. \tag{13}$$

The left-handed quarks and leptons are accommodated in the following representations,

$$F^{i\alpha a} = (4,2,1) = \begin{pmatrix} u^R & u^B & u^G & \nu \\ d^R & d^B & d^G & e^- \end{pmatrix}^i \tag{14}$$

$$\bar{F}^i_{x\alpha} = (\bar{4},1,\bar{2}) = \begin{pmatrix} \bar{d}^R & \bar{d}^B & \bar{d}^G & e^+ \\ \bar{u}^R & \bar{u}^B & \bar{u}^G & \bar{\nu} \end{pmatrix}^i \tag{15}$$

where $\alpha = 1, \ldots, 4$ is an SU(4) index, $a, x = 1, 2$ are $\mathrm{SU}(2)_{L,R}$ indices, and $i = 1, 2, 3$ is a family index. The Higgs fields are contained in the following representations,

$$h_a^x = (1, \bar{2}, 2) = \begin{pmatrix} h_2{}^+ & h_1{}^0 \\ h_2{}^0 & h_1{}^- \end{pmatrix} \tag{16}$$

(where h_1 and h_2 are the low energy Higgs superfields associated with the MSSM.) The two heavy Higgs representations are

$$H^{\alpha b} = (4, 1, 2) = \begin{pmatrix} u_H^R & u_H^B & u_H^G & \nu_H \\ d_H^R & d_H^B & d_H^G & e_H^- \end{pmatrix} \tag{17}$$

and

$$\bar{H}_{\alpha x} = (\bar{4}, 1, \bar{2}) = \begin{pmatrix} \bar{d}_H^R & \bar{d}_H^B & \bar{d}_H^G & e_H^+ \\ \bar{u}_H^R & \bar{u}_H^B & \bar{u}_H^G & \bar{\nu}_H \end{pmatrix}. \tag{18}$$

The Higgs fields are assumed to develop VEVs,

$$< H >=< \nu_H >\sim M_X, \quad < \bar{H} >=< \bar{\nu}_H >\sim M_X \tag{19}$$

leading to the symmetry breaking at M_X

$$\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \longrightarrow \mathrm{SU}(3)_C \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \tag{20}$$

in the usual notation. Under the symmetry breaking in Eq.20, the bidoublet Higgs field h in Eq.16 splits into two Higgs doublets h_1, h_2 whose neutral components subsequently develop weak scale VEVs,

$$< h_1^0 >= v_1, \quad < h_2^0 >= v_2 \tag{21}$$

with $\tan\beta \equiv v_2/v_1$.

In addition to the Higgs fields in Eqs. 17,18 the model also involves an SU(4) sextet field $D = (6, 1, 1)$. The superpotential of the model is a simplified version of the one in ref.[15]:

$$W = \lambda_1^{ij} F_i \bar{F}_j h + \lambda_2 H H D + \lambda_3 \bar{H}\bar{H} D + \lambda_4^{ij} H \bar{F}_j \phi_i + \mu h h \tag{22}$$

where ϕ_i, $(i = 1, 2, 3)$ are singlets under the PS-symmetry. The last term generates the higgs mixing between the two SM higgs doublets in order to prevent the appearance of a massless electroweak axion. Note that this is not the most general superpotential that is invariant under the gauge symmetry. Additional terms not included in Eq.22 may be forbidden by imposing suitable discrete or continuous symmetries, the details of which need not concern us here. The D field does not develop a VEV but the terms in

Eq.22 HHD and $\bar{H}\bar{H}D$ combine the colour triplet parts of H, $\bar{H}$ and D into acceptable GUT-scale mass terms [15]. When the H fields attain their VEVs at $M_{GUT} \sim 10^{16}$ GeV, the superpotential of Eq.22 reduces to that of the MSSM augmented by neutrino masses. Note that the last term in Eq.22 is proportional to the dimensionful parameter μ. Below M_X the part of the superpotential involving matter superfields is just

$$W = \lambda_U^{ij} Q_i \bar{U}_j h_2 + \lambda_D^{ij} Q_i \bar{D}_j h_1 + \lambda_E^{ij} L_i \bar{E}_j h_1 + \lambda_N^{ij} L_i N_j h_2 + \ldots \quad (23)$$

where N_i are the superfields associated with the right-handed neutrinos. The Yukawa couplings in Eq.23 satisfy the boundary conditions

$$\lambda_1^{ij}(M_{GUT}) \equiv \lambda_U^{ij}(M_{GUT}) = \lambda_D^{ij}(M_{GUT}) = \lambda_E^{ij}(M_{GUT}) = \lambda_{\nu_D}^{ij}(M_{GUT}). \quad (24)$$

Thus, Eq.(24) retains the successful relation $m_\tau = m_b$ at M_{GUT}. Moreover from the relation $\lambda_U^{ij}(M_{GUT}) = \lambda_{\nu_D}^{ij}(M_{GUT})$, and the fourth term in (22), we obtain through the see-saw mechanism light neutrino masses $\sim \mathcal{O}(m_u^2/M_{GUT})$ which satisfy the experimental limits.

3 The New Approach: Symmetric Textures

In this section we briefly review the results of the operator analysis of ref.[17], then introduce our new approach based on the combined operators discussed in section 1. We discuss the RRR textures as a simple example of the new method.

The boundary conditions listed in Eq.24 lead to unacceptable mass relations for the light two families. Also, the large family hierarchy in the Yukawa couplings appears to be unnatural since one would naively expect the dimensionless couplings all to be of the same order. This leads us to the conclusion that the λ_1^{ij} in Eq.22 may not originate from the usual renormalisable tree level dimensionless coupling. We allow a renormalisable Yukawa coupling in the 33 term only and generate the rest of the effective Yukawa couplings by non-renormalisable operators that are suppressed by some higher mass scale. This suppression provides an explanation for the observed fermion mass hierarchy.

In ref.[17] we restricted ourselves to all possible non-renormalisable operators which can be constructed from different group theoretical contractions of the fields:

$$O_{ij} \sim (F_i \bar{F}_j) h \left(\frac{H\bar{H}}{M^2}\right)^n + \mathrm{H.}c. \quad (25)$$

where we have used the fields $H, \bar{H}$ in Eqs.17,18 and M is the large scale $M > M_X$. The idea is that when $H, \bar{H}$ develop their VEVs, such operators will become effective Yukawa couplings of the form $hF\bar{F}$ with a small coefficient of order M_{GUT}^2/M^2. We considered up to $n = 2$ operators. The motivation for using $n = 2$ operators is simply that such higher dimension operators are generally expected to lead to smaller effective couplings more suited to the 12 and 21 Yukawa entries. According to our present approach we shall restrict ourselves to $n = 1$ operators with the required suppression factors originating from a separate flavour sector. We will leave the question of the definite origin of the operators for now. Instead we merely note that one could construct a FN sector to motivate the operators, or that one might expect such operators to come directly out of a string theory. In section 6 we shall introduce a $U(1)_X$ family symmetry into the model, which is broken at a scale $M_X > M_{GUT}$ by the VEVs of the Pati-Salam singlet fields θ and $\bar{\theta}$. According to the ideas discussed in section 1 we shall henceforth consider operators of the form

$$O_{ij} \sim (F_i\bar{F}_j)h \left(\frac{H\bar{H}}{M^2}\right) \left(\frac{\theta^n\bar{\theta}^m}{M_\theta^n M_{\bar{\theta}}^m}\right) + \text{h.c.} \tag{26}$$

where we have assumed the form of the operators in Eq.25 corresponding to $n = 1$ and glued onto these operators arbitrary powers of the singlet fields $\theta, \bar{\theta}$. Note that the single power of $(H\bar{H})$ is present in every entry of the matrix and plays the role of the factor of δ in Eq.11. However, unlike the previous factor of δ, the factor of $(H\bar{H})$ here carries important group theoretical Clebsch information. In fact Eq.26 amounts to assuming a sort of *factorisation* of the operators with the family hierarchies being completely controlled by the $\theta, \bar{\theta}$ fields as in IR, with m, n being dependent on i, j, and the horizontal splittings being controlled by the Clebsch factors in $(H\bar{H})$. However this factorisation is not complete since we shall assume that the Clebsch factors have a family dependence, i.e. they depend on i, j. We offer no explanation for the family dependence of the Clebsch factors but simply select the Clebsch factor in each entry in an *ad hoc* way.

As a first example of our new approach we shall consider the RRR textures discussed in section 1. Our first observation is that, restricting ourselves to $n = 1$ operators, there are no large Clebsch ratios between the up-type and down-type quarks for any of the operators. This means that it is very difficult to reproduce RRR solutions such as solution 2 where the 12 element of the down-type matrix in Eq.9, for example, is 50 times larger than its up-type counterpart. Of course this can be achieved by requiring an accurate cancellation between two operators, but such a tuning of coefficients looks

	QUh_2	QDh_1	LEh_1	LNh_2
O^A	1	1	1	1
O^B	1	-1	-1	1
O^C	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^D	$\frac{1}{\sqrt{5}}$	$\frac{-1}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^G	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0
O^H	4/5	2/5	4/5	8/5
O^K	8/5	0	0	6/5
O^M	0	$\sqrt{2}$	$\sqrt{2}$	0
O^N	2	0	0	0
O^R	0	$\frac{8}{5}$	$\frac{6}{5}$	0
O^W	0	$\sqrt{\frac{2}{5}}$	$-3\sqrt{\frac{2}{5}}$	0
O^S	$\frac{8}{5\sqrt{5}}$	$\frac{16}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$

Table 1: When the Higgs fields develop their VEVs at M_{GUT}, the $n = 1$ operators utilised lead to the effective Yukawa couplings with Clebsch coefficients as shown. We have included the relative normalisation for each of the operators. The full set of $n = 1$ operators and Clebsch's is given in Appendix 1. These $n = 1$ operators were used in the lower right hand block of the Yukawa matrices in the analysis of ref. [17].

ugly and unnatural, and we reject it. On the other hand the $n = 1$ Clebsch's in Table 1 include examples of *zero Clebsch's*, where the contribution to the up-type matrix, for example, is precisely zero. Similarly there are *zero Clebsch's* for the down-type quarks (and charged leptons). The existence of such *zero Clebsch's* enables us to reproduce the RRR texture solutions 3 and 5 without fine-tuning. Interestingly they are precisely the solutions which are not possible to obtain by the standard IR symmetry approach, which favours solutions 1,2 and 4 and for which the up-type and down-type structures are similar. Thus our approach is capable of describing the RRR solutions which are complementary to those described by the IR symmetry approach[6]. To take a specific example let us begin by ignoring the flavour dependent singlet fields, and consider the symmetric $n = 1$ operator texture,

$$\lambda = \begin{pmatrix} 0 & O^M & O^N \\ O^M & O^W + s.d. & O^N \\ O^N & O^N & O_{33} \end{pmatrix} \tag{27}$$

[6]In [19], two of us used an alternative approach in order to reproduce the structure of solutions 1 and 3 of RRR by the implementation of a symmetry. These solutions were found to lead to the optimal predictions for neutrino masses and mixings. This has been achieved by a proper choice of charges (integer/half-integer) and by imposing residual Z_2 symmetries which forbid different entries in the up and down-quark mass matrices.

where O_{33} is the renormalisable operator, *s.d.* stands for a sub-dominant operator with a suppression factor compared to the other dominant operator in the same entry. Putting in the Clebsch's from Table 1 we arrive at the component Yukawa matrices, at the GUT scale, of

$$\lambda^U = \begin{pmatrix} 0 & 0 & 2\lambda^U_{13} \\ 0 & \lambda^U_{22} & 2\lambda^U_{23} \\ 2\lambda^U_{13} & 2\lambda^U_{23} & 1 \end{pmatrix} \tag{28}$$

$$\lambda^D = \begin{pmatrix} 0 & \sqrt{2}\lambda^D_{12} & 0 \\ \sqrt{2}\lambda^D_{12} & \lambda^D_{22}\sqrt{2}/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{29}$$

$$\lambda^E = \begin{pmatrix} 0 & \sqrt{2}\lambda^D_{12} & 0 \\ \sqrt{2}\lambda^D_{12} & 3\lambda^D_{22}\sqrt{2}/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{30}$$

where λ^D_{22} and λ^E_{22} arise from the dominant O^W_{22} operator and λ^U_{22} comes from a sub-dominant operator that is relevant because of the texture zero Clebsch in the up sector of O^W_{22}. The zeroes in the matrices correspond to those of the RRR solution 5, but of course in our case they arise from the Clebsch zeroes rather than from a family symmetry reason. The numerical values corresponding to RRR solution 5 are,

$$\lambda^U = \begin{pmatrix} 0 & 0 & 2\times10^{-3} \\ 0 & 3\times10^{-3} & 3\times10^{-2} \\ 2\times10^{-3} & 3\times10^{-2} & 1 \end{pmatrix}, \; \lambda^D = \begin{pmatrix} 0 & 5\times10^{-3} & 0 \\ 5\times10^{-3} & 2\times10^{-2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{31}$$

Thus, the hierarchy $\lambda^U_{22} << \lambda^D_{22}$ is explained by a Clebsch zero and a suppression factor of the sub-dominant operator. Using Eq.31 we can read off the values of the couplings which roughly correspond to a unified matrix of dominant couplings

$$\lambda = \begin{pmatrix} 0 & 3\times10^{-3} & 1\times10^{-3} \\ 3\times10^{-3} & 2\times10^{-2} & 2\times10^{-2} \\ 1\times10^{-3} & 2\times10^{-2} & 1 \end{pmatrix} \tag{32}$$

where we have extracted the Clebsch factors. We find it particularly elegant that the whole quark and lepton spectrum is controlled by a unified Yukawa matrix such as in Eq.32 with all the vertical splittings controlled by Clebsch factors.

At this stage we could introduce a $U(1)_X$ symmetry of the IR kind, and the flavour dependent singlet fields in order to account for the horizontal family hierarchy of couplings in Eq.32. In the present case we must remember that there is a small quantity δ multiplying every non-renormalisable entry as in Eq.11, corresponding to the $n = 1$

bilinear $\delta \equiv \frac{v\bar{v}}{M^2}$ which we have required to be present in every non-renormalisable entry. Thus we can understand Eq.32 as resulting from a structure like,

$$\lambda = \begin{pmatrix} \delta\epsilon^8 & \delta\epsilon^3 & \delta\epsilon^4 \\ \delta\epsilon^3 & \delta\epsilon^2 & \delta\epsilon \\ \delta\epsilon^4 & \delta\epsilon & 1 \end{pmatrix} \tag{33}$$

where we identify $\epsilon \equiv \lambda = 0.22$ and set $\delta \approx 0.2$ which gives the correct orders of magnitude for the entries, rather similar to the case we discussed in Eq.12. Here of course the considerations apply to the unified Yukawa matrix, however, not just the down-type quark matrix. The details of the $U(1)_X$ family symmetry analysis are discussed in section 6. Here we simply note that such an analysis can lead to a structure such as the one assumed in Eq.33.

A similar analysis could equally well be applied to RRR solution 3. In both cases we are led to a pleasing scheme which involves no unnatural tuning of elements, and naturally combines the effect of Clebsch's with that of family symmetry suppression, in a simple way. The existence of the Clebsch texture zeroes thus permits RRR solutions 3 and 5 which are impossible to obtain otherwise within the general framework presented here.

4 Non-Symmetric Textures

In this section we up-date the non-symmetric textures based on both $n = 1$ and $n = 2$ operators introduced in ref.[17], then extend the new approach introduced in the previous section to the non-symmetric domain. As in the previous section, we shall begin by ignoring the effect of the singlet fields, which will be discussed in section 6.

As discussed in Appendix 2 we shall modify the analysis of Ref.[17] to only include the lower 2 by 2 block Ansatz:

$$A_1 = \begin{bmatrix} O^W_{22} + s.d. & 0 \\ O^C_{32} & O_{33} \end{bmatrix}. \tag{34}$$

This is then combined with the upper 2 by 2 blocks considered in ref.[17]:

$$B_1 = \begin{bmatrix} 0 & O^1 \\ O^{Ad} & X \end{bmatrix} \tag{35}$$

$$B_2 = \begin{bmatrix} 0 & O^2 \\ O^{Ad} & X \end{bmatrix} \tag{36}$$

	QUh_2	QDh_1	LEh_1	LNh_2
O^{Ad}	$\frac{4\sqrt{2}}{25}$	$\frac{12\sqrt{2}}{25}$	$\frac{9\sqrt{2}}{25}$	$\frac{3\sqrt{2}}{25}$
O^{Dd}	$\frac{1}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$
O^{Md}	$\frac{\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$	$\frac{6\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$
O^1	0	$\sqrt{2}$	$\sqrt{2}$	0
O^2	0	$\frac{8}{5}$	$\frac{6}{5}$	0
O^3	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0

Table 2: Clebsch coefficients of $n = 2$ operators previously utilised.

$$B_3 = \begin{bmatrix} 0 & O^3 \\ O^{Ad} & X \end{bmatrix} \tag{37}$$

$$B_4 = \begin{bmatrix} 0 & O^1 \\ O^{Dd} & X \end{bmatrix} \tag{38}$$

$$B_5 = \begin{bmatrix} 0 & O^2 \\ O^{Dd} & X \end{bmatrix} \tag{39}$$

$$B_6 = \begin{bmatrix} 0 & O^3 \\ O^{Dd} & X \end{bmatrix} \tag{40}$$

$$B_7 = \begin{bmatrix} 0 & O^1 \\ O^{Md} & X \end{bmatrix} \tag{41}$$

$$B_8 = \begin{bmatrix} 0 & O^2 \\ O^{Md} & X \end{bmatrix}, \tag{42}$$

where X stands for whatever is left in the 22 position, after the lower 2 by 2 submatrix has been diagonalised. The Clebsch coefficients of the $n = 2$ operators used in Eqs.35-42 are displayed in Table 2 but we refer the reader to ref.[17] for the explicit realisation of these operators in terms of the component fields for reasons of brevity. The Ansätze listed above present problems because of the breakdown of matrix perturbation theory[7]. For purposes of comparison with the new scheme involving only $n = 1$ operators, we will recalculate the predictions for each of the models from ref.[17] numerically in the next section.

We now turn our attention to the new approach introduced in the previous section, based on $n = 1$ operators together with singlet fields which for the moment we shall ignore. In this case the 21 operator used in ref.[17] which gave an up Clebsch coefficient

[7]When the magnitudes of H_{21}, H_{12} and H_{22} are calculated they are each of the same order in the down Yukawa matrix, thus violating the hierarchy in Eq.88 that was assumed in the calculation of the predictions.

1/3 times smaller than the down Clebsch is not available if we only use $n = 1$ operators. We must therefore use a combination of two operators in the 21 position that allow the up entry to be a bit smaller than the down entry. We require that the combination provide a Clebsch relation between λ_{21}^D and λ_{21}^E for predictivity. The two operators cancel slightly in the up sector, but as shown later this cancellation is $\sim O(1)$ and therefore acceptable. The result of this is that the prediction of V_{ub} is lost; however this prediction was almost excluded by experiment anyway, and a more accurate numerical estimate which does not rely on matrix perturbation theory confirms that V_{ub} in ref.[17] is too large. So the loss of the V_{ub} prediction is to be welcomed! The Clebsch effect of the 12 operator (with a zero Clebsch for the up-type quarks) can easily be reproduced at the $n = 1$ level by the operator O^M for example.

To get some feel for the procedure we will follow, we first discuss a simple example of a non-symmetric texture, ignoring complex phases for illustrative purposes. Restricting ourselves to $n = 1$ operators, we consider the lower block to be A_1 and the upper block to be the modified texture as discussed in the previous paragraph. Thus we have,

$$\lambda = \begin{pmatrix} 0 & O^M & 0 \\ O^M + O^A & O^W + s.d. & 0 \\ 0 & O^C & O_{33} \end{pmatrix} \tag{43}$$

where O_{33} is the renormalisable operator. Putting in the Clebsch's from Table 3 we arrive at the component Yukawa matrices, at the GUT scale, of

$$\lambda^U = \begin{pmatrix} 0 & 0 & 0 \\ \lambda_{21}^U & \lambda_{22}^U & 0 \\ 0 & \sqrt{2}\lambda_{32}^U/\sqrt{5} & 1 \end{pmatrix} \tag{44}$$

$$\lambda^D = \begin{pmatrix} 0 & \sqrt{2}\lambda_{12}^D & 0 \\ \sqrt{2}\lambda_{21}^D & \lambda_{22}^D/\sqrt{5} & 0 \\ 0 & -\sqrt{2}\lambda_{32}^U/\sqrt{5} & 1 \end{pmatrix} \tag{45}$$

$$\lambda^E = \begin{pmatrix} 0 & \sqrt{2}\lambda_{12}^D & 0 \\ \sqrt{2}\lambda_{21}^D & 3\lambda_{22}^D/\sqrt{5} & 0 \\ 0 & -3\sqrt{2}\lambda_{32}^U/\sqrt{5} & 1 \end{pmatrix} \tag{46}$$

where λ_{22}^U and λ_{22}^D arise from the difference and sum of two operators whose normalisation factor of $\sqrt{5}$ has been explicitly inserted, and similarly for λ_{21}^U and λ_{21}^D. To obtain the numerical values of the entries we use some typical GUT-scale values of Yukawa couplings and CKM elements (see ref.[17]) as follows:

$$\lambda_{33} = 1, \lambda_c = 0.002, \lambda_s = 0.013, \lambda_\mu = 0.04, \lambda_u = 10^{-6}, \lambda_d = 0.0006, \lambda_e = 0.0002, \tag{47}$$

$$V_{cb} = 0.05,\ V_{us} = 0.22,\ V_{ub} = 0.004 \tag{48}$$

where we have assumed,

$$\alpha_s = 0.115,\ m_b = 4.25,\ \tan\beta = 55,\ m_t = 180GeV \tag{49}$$

The textures in Eqs.44, 45 and 46 imply that the 22 eigenvalues are just equal to the 22 elements (assuming matrix perturbation theory is valid – see later), and $\lambda^U_{32} = V_{cb}/2 = 0.025$. Thus we have $\lambda^U_{22} = 0.004$,$\lambda^D_{22} = 0.03$. The remaining parameters are determined from the relations,

$$\lambda_u = 0,\ \lambda_d = 3\lambda_e = \lambda^D_{21}\sqrt{2}\lambda^D_{12}/\lambda_s,\ V_{ub} = \lambda^U_{21}V_{cb}/\lambda_c \tag{50}$$

Note that the up quark mass looks like it is zero, but in practice we would expect some higher dimension operator to be present which will give it a small non-zero value. We thus have three equations and three unknowns, and solving we find $\lambda^U_{21} = 2\times10^{-4}$, $\lambda^D_{21} = 2\times10^{-3}$, $\lambda^D_{12} = 3\times10^{-3}$. The difference between λ^U_{21} and λ^D_{21} requires suppression of O^A caused by the Clebsch zero in the dominant operator O^M. Thus the unified Yukawa matrix involves operators with the following approximate numerical coefficients,

$$\lambda = \begin{pmatrix} 0 & 3\times10^{-3} & 0 \\ 3\times10^{-3} & 1.5\times10^{-2} & 0 \\ 0 & 2.5\times10^{-2} & 1 \end{pmatrix} \tag{51}$$

where we have extracted the Clebsch factors, and the 22 and 21 values in Eq.51 refer to each of the two operators in this position separately. The numerical values in Eq.51 are not dissimilar from those in Eq.32, in particular the upper 2 by 2 block is symmetrical with the same values as before. In this case the lower 2 by 2 block has a texture zero in the 23 position, as well as the 31 and 13 positions, but otherwise the numerical values are very similar to those previously obtained in Eq.32. Thus this particular non-symmetric texture can be described by a structure of the kind,

$$\lambda = \begin{pmatrix} \delta\epsilon^{big} & \delta\epsilon^3 & \delta\epsilon^{big} \\ \delta\epsilon^3 & \delta\epsilon^{1or2} & \delta\epsilon^{big} \\ \delta\epsilon^{big} & \delta\epsilon & 1 \end{pmatrix} \tag{52}$$

where we identify $\epsilon \equiv \lambda = 0.22$ and set $\delta \approx 0.1$ as before. Can such a structure for the ϵ's be obtained from the $U(1)_X$ symmetry? This will be discussed in section 6.

There is no reason to restrict ourselves to non-symmetric textures with a zero in the 13 and 31 position, as assumed in ref.[17]. For example the following texture is also viable, amounting to a hybrid of the symmetric case considered in Eq.27 and the non-symmetric lower block just considered.

$$\lambda = \begin{pmatrix} 0 & O^M & O^N \\ O^M & O^W & 0 \\ O^N & O^C & O_{33} \end{pmatrix} \tag{53}$$

Here, O_{33} is the renormalisable operator. We now perform a general operator analysis of the non-symmetric case, assuming $n = 1$ operators for all non-zero entries (apart from the 33 renormalisable entry). In this general analysis there are two classes of texture: those with universal texture zeroes in the 13 and 31 position (essentially $n = 1$ versions of the textures considered in ref.[17]) and new textures with non-zero entries in the 13 and/or 31 position. For now we will not consider the cases with operators in the 13 or 31 positions for reasons of brevity. In the general analysis we repeat the above procedure, being careful about phases, and obtain some numerical estimates of the magnitude of each entry which will be explained in terms of the $U(1)_X$ family symmetry as discussed in the next section.

With the above discussion in mind, we consider the new scheme in which the dominant operators in the Yukawa matrix are O_{33}, O^C_{32}, O^W_{22}, $O_{21}, \tilde{O}_{21}$ and O_{12}, where the last three operators are left general and will be specified later. We are aware from the analysis in ref.[17] that O_{12} must have a zero Clebsch coefficient in the up sector. A combination of two operators must then provide a non-zero O_{21} entry to provide a big enough V_{ub}, an additional much more suppressed operator elsewhere in the Yukawa matrix gives the up quark a small mass. At M_{GUT} therefore, the Yukawa matrices are of the form

$$\lambda^I = \begin{bmatrix} 0 & H_{12}e^{i\phi_{12}}x^I_{12} & 0 \\ H_{21}x^I_{21}e^{i\phi_{21}} + \tilde{H}_{21}\tilde{x}^I_{21}e^{i\tilde{\phi}_{21}} & H_{22}x^I_{22}e^{i\phi_{22}} & 0 \\ 0 & H_{32}x^I_{32}e^{i\phi_{32}} & H_{33}e^{i\phi_{33}} \end{bmatrix}, \tag{54}$$

where only the dominant operators are listed. The I superscript labels the charge sector and x^I_{ij} refers to the Clebsch coefficient relevant to the charge sector I in the ij^{th} position. ϕ_{ij} are unknown phases and H_{ij} is the magnitude of the effective dimensionless Yukawa coupling in the ij^{th} position. Any subdominant operators that we introduce will be denoted below by a prime and it should be borne in mind that these will only affect the up matrix. So far, the known Clebsch coefficients are

$$\begin{aligned} &x^U_{12} = 0 \\ &x^U_{22} = 0 \quad x^D_{22} = 1 \quad x^E_{22} = -3 \\ &x^U_{32} = 1 \quad x^D_{32} = -1 \quad x^E_{32} = -3. \end{aligned} \tag{55}$$

We have just enough freedom in rotating the phases of $F_{1,2,3}$ and $\bar{F}_{1,2,3}$ to get rid of all but one of the phases in Eq.54. When the subdominant operator is added, the Yukawa matrices are

$$
\begin{aligned}
\lambda^U &= \begin{bmatrix} 0 & 0 & 0 \\ H_{21}^U e^{i\phi_{21}^U} & {H_{22}}' e^{i{\phi_{22}}'} & 0 \\ 0 & H_{32}x_{32}^U & H_{33} \end{bmatrix} \\
\lambda^D &= \begin{bmatrix} 0 & H_{12}x_{12}^D & 0 \\ H_{21}^D & H_{22}x_{22}^D & 0 \\ 0 & H_{32}x_{32}^D & H_{33} \end{bmatrix} \\
\lambda^E &= \begin{bmatrix} 0 & H_{12}x_{12}^E & 0 \\ H_{21}^E & H_{22}x_{22}^E & 0 \\ 0 & H_{32}x_{32}^E & H_{33} \end{bmatrix},
\end{aligned}
\tag{56}
$$

where we have defined

$$
\begin{aligned}
H_{21}^U e^{i\phi_{21}^U} &\equiv H_{21}x_{21}^U e^{i\phi_{21}} + \tilde{H}_{21}\tilde{x}_{21}^U e^{i\tilde{\phi}_{21}} \\
H_{21}^{D,E} &\equiv H_{21}x_{21}^{D,E} e^{i\phi_{21}} + \tilde{H}_{21}\tilde{x}_{21}^{D,E} e^{i\tilde{\phi}_{21}}
\end{aligned}
\tag{57}
$$

We may now remove ${\phi_{22}}'$ by phase transformations upon $\bar{F}_{1,2,3}$ but ϕ_{21}^U may only be removed by a phase redefinition of $F_{1,2,3}$, which would alter the prediction of the CKM matrix V_{CKM}. Thus, ϕ_{21}^U is a physical phase, that is it cannot be completely removed by phase rotations upon the fields. Once the operators $O_{21}, \tilde{O}_{21}, O_{12}$ have been chosen, the Yukawa matrices at M_{GUT} including the phase in the CKM matrix are therefore identified with $H_{ij}, {H'}_{22}, \phi_{21}^U$.

5 Numerical Analysis of Masses and Mixing Angles

In this section we discuss the numerical procedure used to analyse the non-symmetric cases introduced in the previous section. We shall perform an analysis on the new approach based on $n = 1$ operators only, and also re-analyse and up-date the original scheme of ref.[17] for comparison.

The basic idea is to do a global fit of each considered Ansatz to m_e, m_μ, m_u, m_c, m_t, m_d, m_s, m_b, $\alpha_S(M_Z)$, $|V_{ub}|$, $|V_{cb}|$ and $|V_{us}|$ using m_τ as a constraint. We assume that the whole SUSY spectrum of the MSSM lies at $M_{SUSY} = m_t$ and that the MSSM remains a valid effective theory until the scale $M_{GUT} = 10^{16}$ GeV. Not wishing to include neutrino masses in this analysis, we simply set the right-handed Majorana neutrino mass of each family to be 10^{16} GeV so that the neutrinos are approximately massless and hence their masses do not affect the RGEs below M_{GUT}. Recall the parameters introduced in Eq.56: $\phi_{21}^U \equiv \phi$, $H_{21}^U \equiv H_{21}'$, $H_{21}^D \equiv H_{21}$, H_{22}', H_{22}, H_{12}, H_{32}, H_{33}. The values of these 8 parameters plus α_S at the GUT scale are determined by the fit.

The matrices λ^I are diagonalised numerically and $|V_{ub}(M_{GUT})|, |V_{us}(M_{GUT})|$ are determined by

$$V_{CKM} = V_{UL} V_{DL}^\dagger, \tag{58}$$

where V_{UL}, V_{DL} are the matrices that act upon the $(u,c,t)_L$ and $(d,s,b)_L$ column vectors respectively to transform from the weak eigenstates to the mass eigenstates of the quarks. We use the boundary conditions $\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = 0.708$, motivated by previous analyses based on gauge unification in SUSY GUT models [18]. $\lambda_{u,c,t,d,s,b,e,\mu,\tau}$, $|V_{us}|$ and $|V_{ub}|$ are then run[8] from M_{GUT} to 170 GeV$\approx m_t$ using the RGEs for the MSSM. Below M_{GUT} the effective field theory of the Standard Model allows the couplings in the different charge sectors to split and run differently. The λ_i are then evolved to their empirically derived running masses using 3 loop QCD$\otimes$1 loop QED [17]. m_τ^e and $\lambda_\tau^p(m_\tau)$ then[9] fix $\tan\beta$ through the relation [12]

$$\cos\beta = \frac{\sqrt{2} m_\tau^e(m_\tau)}{v \lambda_\tau^p(\lambda_\tau)}, \tag{59}$$

where $v = 246.22$ GeV is the VEV of the Standard Model Higgs. Predictions of the other fermion masses then come from

$$\begin{aligned} m_{c,t}^p &\approx \lambda_{c,t}^p(m_{c,t}) \frac{v \sin\beta}{\sqrt{2}}, \\ m_{d,s,b}^p &\approx \lambda_{d,s,b}(m_{1,1,b}^e) \frac{v\cos\beta}{\sqrt{2}} \\ m_{e,\mu}^p &\approx \lambda_{e,\mu}(m_{1,\mu}^e) \frac{v\cos\beta}{\sqrt{2}}, \end{aligned} \tag{60}$$

where $m_1 \equiv 1$ GeV. There are twelve data points and nine parameters so we have three degrees of freedom (dof). The parameters are all varied until the global χ^2/dof is minimised. The data used (with 1σ errors quoted) is [20]

$$m_e = 0.510999 \text{ MeV}$$

[8]All renormalisation running in this paper is one loop and in the $\overline{\text{MS}}$ scheme. The relevant renormalisation group equations (RGEs) are listed in ref.[17].

[9]The superscript e upon masses, mixing angles or diagonal Yukawa couplings denotes an empirically derived value, whereas the superscript p denotes the prediction of the model for the particular fit parameters being tested.

$$
\begin{aligned}
m_\mu &= 105.658 \text{ MeV} \\
m_\tau &= 1.7771 \text{ GeV} \\
m_c &= 1.3 \pm 0.3 \text{ GeV} \\
m_t^{phys} &= 180 \pm 12 \text{ GeV} \\
m_d &= 10 \pm 5 \text{ MeV} \\
m_s &= 200 \pm 100 \text{ MeV} \\
m_b &= 4.25 \pm 0.1 \text{ GeV} \\
|V_{ub}| &= (3.50 \pm 0.91)10^{-3} \\
|V_{us}| &= 0.2215 \pm 0.0030 \\
\alpha_S(M_Z) &= 0.117 \pm 0.005
\end{aligned}
\tag{61}
$$

$|V_{cb}|$ is fixed by H_{32} which does not influence the other predictions to a good approximation and so $|V_{cb}|$ and H_{32} effectively decouple from the fit. We merely note that in all cases, to predict the measured value of $|V_{cb}|$, $H_{32} \sim 0.03$. Note that no errors are quoted upon the lepton masses because m_τ is used as a constraint on the data and because m_e, m_μ were required to be satisfied to 0.1% by the fit. In this way we merely use the lepton masses as 3 constraints, using up 3 dof. We did not perform the fit with smaller empirical errors on the lepton masses because of the numerical roundoff and minimisation errors associated with high χ^2 values generated by them. Also, 0.1% is a possible estimate of higher loop radiative corrections involved in the predictions. Note that no other theoretical errors were taken into account in the fit. The largest ones may occur in derivations of m_b due to the large λ_b coupling [21] and the non-perturbative effects of QCD near 1 GeV. It is not clear how to estimate these errors since the error on m_b depends upon soft parameters which depend on the SUSY breaking mechanism in a very model dependent way and non-perturbative QCD is an unsolved problem. The correlations between the empirical estimations of the current quark masses are also not included. A potentially large error could occur if the ansatze considered are not exact but are subject to corrections by higher dimension operators. We discuss this point further in section 6.

The results obtained from this analysis are given in Table 3. Out of 16 possible models that fit the texture required by Eqs.55,54, 11 models fit the data with χ^2/dof< 3. Out of these 11 models, 5 fit the data with χ^2/dof< 2 and these are displayed in Table 3. The operators listed as $O_{12}, O_{21}, \tilde{O}_{21}$ describe the structure of the models and the entries $H_{22}, H_{12}, H_{21}, \cos\phi, H_{33}, H_{22}', H_{21}'$ are the GUT scale input parameters of

Model	1	2	3	4	5
O_{12} $O_{21}+\tilde{O}_{21}$	O^M O^M+O^A	O^W O^G+O^H	O^R O^M+O^A	O^R O^G+O^H	O^R O^R+O^S
$H_{22}/10^{-2}$	2.88	2.64	2.69	2.67	6.15
$H_{12}/10^{-3}$	2.81	4.41	2.13	0.70	1.21
$H_{21}/10^{-3}$	1.30	5.97	1.76	4.33	1.91
$\cos\phi$	0.87	1.00	0.20	1.00	0.61
H_{33}	1.18	1.05	1.05	1.07	4.6
$H_{22}{}'/10^{-3}$	1.91	1.87	1.87	1.87	2.87
$H_{21}{}'/10^{-3}$	1.94	1.62	1.63	1.66	0.76
$\alpha_S(M_Z)$	0.119	0.118	0.118	0.118	0.126
m_d/MeV	6.25	1.03	8.07	4.14	11.9
m_s/MeV	158	150	154	152	228
m_c/GeV	1.30	1.30	1.30	1.30	1.30
m_b/GeV	4.24	4.25	4.25	4.25	4.13
m_t^{phys}/GeV	182	180	180	180	192
$\lvert V_{us}\rvert$	0.2211	0.2215	0.2215	0.2215	0.2215
$\lvert V_{ub}\rvert/10^{-3}$	3.71	3.51	3.50	3.52	3.50
$\tan\beta$	59.5	58.3	58.3	58.5	65.7
χ^2/dof	0.34	1.16	0.13	0.55	1.84

Table 3: Results of best-fit analysis on models with $n = 1$ operators only. Note that the input parameters $H_{ij}, H_{ij}{}', \cos\phi$ shown are their values at the scale M_{GUT}. All of the mass predictions shown are running masses, apart from the pole mass of the top quark[11], $m_t^{phys} \equiv m_t(1+\frac{4\alpha_S(m_t)}{3\pi})$. The CKM matrix element predictions are at M_Z.

Model	B_1	B_2	B_3	B_4	B_5	B_6	B_7	B_8
$\alpha_S(M_Z)$	0.123	0.123	0.123	0.124	0.123	0.124	0.125	0.124
m_d/MeV	7.58	9.12	4.64	6.18	7.49	3.63	3.53	4.53
m_s/MeV	215	240	179	210	217	179	200	187
m_c/GeV	1.29	1.38	1.35	1.16	1.29	1.32	0.86	1.31
m_b/GeV	4.19	4.17	4.19	4.19	4.19	4.18	4.20	4.19
m_t^{phys}/GeV	188	189	189	189	188	189	190	189
$\|V_{us}\|$	0.2212	0.2213	0.2214	0.2212	0.2212	0.2215	0.2212	0.2214
$\|V_{ub}\|/10^{-3}$	4.52	4.37	4.05	4.22	4.56	3.74	3.85	3.98
$\tan\beta$	63.2	63.6	63.4	63.7	63.2	63.8	64.3	63.6
χ^2/dof	0.95	0.96	1.00	1.05	0.97	1.16	1.87	1.04

Table 4: Predictions of best-fit analysis on models from ref. [17] with $n = 2$ operators included. All of the mass predictions shown are running masses, apart from the pole mass of the top quark. The CKM matrix element predictions are at M_Z.

the best fit values of the model. The estimated 1σ deviation in $\alpha_S(M_Z)$ from the fits is ± 0.003 and the other parameters are constrained to better than 1% apart from $\cos\phi$, whose 1σ fit errors often cover the whole possible range. Out of the predictions shown in Table 3, m_d discriminates between the models the widest. $\alpha_S(M_Z)$ takes roughly central values, apart from model 5 for which the best fit is outside the 1σ errors quoted in Eq.61 on $\alpha_S(M_Z)$. m_s, $|V_{ub}|$ are within 1σ of the data point and m_c, $|V_{us}|$ are approximately on the central value for all 5 models. Models 3,1 and 4 are very satisfactory fits to the data with χ^2/dof< 1. We conclude that the χ^2 test has some discriminatory power in this case since if all of the models were equally good, we would statistically expect to have 11 models with χ^2/dof< 1, 3 models with χ^2/dof$= 1 - 2$ and 2 models with $\chi^2 = 2 - 3$ out of the 16 tested.

We now briefly return to the original models with upper blocks given by B_{1-8} in Eqs.35-42 [17]. After again isolating the only physical phase to λ^U_{21}, a numerical fit analogous to the above was performed using the same data in Eq.61. The main difference in the fit with these models is that there are now 4 degrees of freedom in the fit (since there is one less parameter). All eight models in question fit the data with $\chi^2 < 2$ and these are displayed in Table 4. We do not display the best fit input parameters because they are largely irrelevant for the discussion here. 1σ fit deviations of $\alpha_S(M_Z)$ are again 0.003 for B_{1-8}. Note that whereas these models are able to fit $|V_{us}|, m_s, m_d, m_b, m_c$ fairly well, their predictions of $\alpha_S(M_Z)$ are high and outside the

1σ empirical error bounds. $|V_{ub}|$ is naturally high in these models (as found in ref.[17]) and this forces $\alpha_S(M_Z)$ to be large, where $|V_{ub}|$ may decrease somewhat. To fit m_b with a high $\alpha_S(M_Z)$ requires a large H_{33} element and this is roughly speaking why m_t^{phys} is predicted to be quite high. In each model the high value of $\alpha_S(M_Z)$ required is the dominant source of χ^2 apart from B_7, where m_c is low.

In comparison to the new scheme with $n = 1$ operators only, the old scheme with $n = 2$ operators fits the data pretty well, although not quite as well as models 1,3,4. The old scheme also has one more prediction than the new one. However, the preferred models are the ones incorporating the $U(1)_X$ symmetry since they go deeper into the reasons for the zeroes and hierarchies in the Yukawa matrices.

6 $U(1)_X$ Family Symmetry

In our discussion of the symmetric textures, we assumed that we could obtain the same structure as IR. Of course, as we have already mentioned, the case we are examining is different in two aspects: (a) the fermion mass matrices have the same origin, and thus the same expansion parameter and (b) all differences between the different charge sectors arise from Clebsch factors. As a starting point, we will therefore briefly repeat the IR analysis for **symmetric mass matrices** in our framework; we then go on to consider the non-symmetric case, with the goal of being able to reproduce the numerical values (at least to an order of magnitude) of the successful ansatze given in the previous section.

The structure of the mass matrices is determined by a family symmetry, $U(1)_X$, with the charge assignment of the various states given in Table 5.

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	h_1	h_2	H	$\bar{H}$
$U(1)_X$	α_i	α_i	α_i	α_i	α_i	α_i	$-2\alpha_3$	$-2\alpha_3$	x	-x

Table 5: $U(1)_X$ charges assuming symmetric textures.

The need to preserve $SU(2)_L$ invariance requires left-handed up and down quarks (leptons) to have the same charge. This, plus the additional requirement of symmetric matrices, indicates that all quarks (leptons) of the same i-th generation transform with the same charge α_i. Finally, lepton-quark unification under $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ indicates that quarks and leptons of the same family have the same charge (this is a different feature as compared to IR, where quarks and leptons of the two lower generations have different charges under the flavour symmetry). The full anomaly free Abelian group involves an additional family independent component, $U(1)_{FI}$, and with this freedom $U(1)_X$ is made traceless without any loss of generality[12]. Thus we set $\alpha_1 = -(\alpha_2 + \alpha_3)$. He we consider the simplest case where the combination $H\bar{H}$ is taken to have zero charge. This is consistent with our requirement that it plays no role in the mass hierarchies, other than leading to a common factor δ for all non-renormalisable entries.

If the light Higgs h_2, h_1, responsible for the up and down quark masses respectively, arise from the same bidoublet $h = (1, 2, 2)$, then they have the same $U(1)_X$ charge so that only the 33 renormalisable Yukawa coupling to h_2, h_1 is allowed, and only the 33 element of the associated mass matrix will be non-zero. The remaining entries are generated when the $U(1)_X$ symmetry is broken. This breaking is taken to be spontaneous, via Standard Model singlet fields, which can be either **chiral** or **vector** ones; in the latter case, which is the one studied in IR, two fields θ, $\bar{\theta}$, with $U(1)_X$ charge -1, +1 respectively and equal VEVs are introduced. When these fields get a VEV, the mass matrix acquires its structure. For example, the 32 - entry in the up quark mass matrix appears at $O(\epsilon)$ because U(1) charge conservation only allows the term $c^c t h_2 (\theta/M_2)^{\alpha_2 - \alpha_3}$ for $\alpha_2 > \alpha_3$, or $c^c t h_2 (\bar{\theta}/M_2)^{\alpha_3 - \alpha_2}$, for $\alpha_3 > \alpha_2$. Here $\epsilon = (< \theta > /M_2)^{|\alpha_2 - \alpha_3|}$ where M_2 is the unification mass scale which governs the higher dimension operators. In IR, a different scale, M_1, is expected for the down quark and lepton mass matrices.

In our case however, all charge and mass matrices have the same structure under the $U(1)_X$ symmetry, since all known fermions are accommodated in the same multiplets of the gauge group. The charge matrix is of the form

$$\begin{pmatrix} -2\alpha_2 - 4\alpha_3 & -3\alpha_3 & -\alpha_2 - 2\alpha_3 \\ -3\alpha_3 & 2(\alpha_2 - \alpha_3) & \alpha_2 - \alpha_3 \\ -\alpha_2 - 2\alpha_3 & \alpha_2 - \alpha_3 & 0 \end{pmatrix} \tag{62}$$

Then, including the common factor δ which multiplies all non-renormalisable entries, the following pattern of masses is obtained (for vector-like singlets):

$$\frac{M_u}{m_t} \approx \frac{M_d}{m_b} \approx \frac{M_\ell}{m_\tau} \approx \begin{pmatrix} \delta\epsilon^{|2+6a|} & \delta\epsilon^{|3a|} & \delta\epsilon^{|1+3a|} \\ \delta\epsilon^{|3a|} & \delta\epsilon^2 & \delta\epsilon \\ \delta\epsilon^{|1+3a|} & \delta\epsilon & 1 \end{pmatrix}, \tag{63}$$

[12]Since we assume that the 33 operator is renormalisable, the relaxation of the tracelessness condition does not change the charge matrix since any additional FI charges can always be absorbed into the Higgs h_i charges.

where[13] $a = \alpha_3/(\alpha_2 - \alpha_3)$. We emphasise that the entries in Eq.63 describe the magnitudes of the dominant operators, and do not take the Clebsch zeroes of the different charge sectors into account. Note the existence of a single expansion parameter, for all three matrices. Another interesting point is that a unique charge combination a appears in the exponents of all matrices, as a result of quark-lepton unification. Actually, unlike what appears here, in most schemes the lepton mass matrix is described in the generic case by two parameters (since the charges of quarks and leptons of the lower generations are not related). For $a = 1$, one generates the structure in Eq. 33 for the unified fermion mass matrices.

Before passing to the non-symmetric case, let us make a few comments on the possibility of having chiral or vector singlets, as well as on the charge of the Higgs fields. Suppose first that θ is a chiral field. From the form of the charge matrix, we observe that if the 22 and 23 entries have a positive charge, α_3 is negative (for all these entries to be non-vanishing at the same time). Moreover the hierarchy 1:3 between the 23 and 12 elements indicates that α_2 would have to be zero in the chiral case, and thus the 13 element would tend to be larger than desired. We can say therefore that in the symmetric case with vector fields generates the mass hierarchies in a more natural way.

Concerning the h_1, h_2 higgses, there are two kinds originating from free fermionic string models: those coming from Neveu-Schwarz sector which in general have integer (including zero) $U(1)_X$ charges, and those arising from twisted sectors, which usually carry fractional $U(1)_X$ charges. Which of these cases acquire VEVs, is decided from the phenomenological analysis. For example, to obtain the structure of Eq.33 we see that the charges of $h_{1,2}$ may not be zero, since in such a case the 12 element which is proportional to the Higgs charge would be unacceptably large. For the non-symmetric case of course this feature does not necessarily hold. Finally, the H, $\bar{H}$ fields (the $SU(4)$ higgses) tend to be non-singlets under extra $U(1)_X$ symmetries. What the charge under these symmetries can be, and whether our assumptions are consistent in the framework of realistic superstring models will be discussed in a future publication. We now proceed to discuss the **non-symmetric case**, which in the framework of $U(1)_X$ symmetries has been extensively studied in [22]. Here, we will examine what constraints one may put on the various possibilities for non-symmetric textures, in the model under study.

[13]In this simplest (and more predictive) realisation, $h_b \approx h_t$ therefore we are in the large $\tan\beta$ regime of the parameter space of the MSSM.

The charge assignment for this case appears in Table 6. Fields that belong to the

	Q_i	u_i^c	d_i^c	L_i	e_i^c	ν_i^c	h_1	h_2	H	$\bar{H}$
$U(1)_X$	β_i	α_i	α_i	β_i	α_i	α_i	$-\beta_3-\alpha_3$	$-\beta_3-\alpha_3$	x	-x

Table 6: $U(1)_X$ charges for non-symmetric textures.

same representation of $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ are taken to have the same charge. Again, it is clear that all fermion mass matrices will have the same structure. With this charge assignment we may proceed as in the symmetric case, and calculate the possible mass matrices that may arise. The charge matrix is now

$$\begin{pmatrix} -\alpha_2-2\alpha_3-\beta_2-2\beta_3 & \alpha_2-\alpha_3-\beta_2-2\beta_3 & -\beta_2-2\beta_3 \\ -\alpha_2-2\alpha_3+\beta_2-\beta_3 & \alpha_2-\alpha_3+\beta_2-\beta_3 & \beta_2-\beta_3 \\ -\alpha_2-2\alpha_3 & \alpha_2-\alpha_3 & 0 \end{pmatrix} \tag{64}$$

We now want to find which charge assignments may generate a mass matrix as close as possible to the form in Eq.52, keeping in mind that there is no reason to restrict ourselves to non-symmetric textures with a zero in the 13 and 31 position.

Let us initially check whether it is possible to generate the above structure by chiral singlet fields. We assume for a starting point that for the 32 entry we have $\alpha_2-\alpha_3>0$ (without a loss of generality since we can always choose the sign of one entry in the charge matrix). The 23 entry has to be small, indicating that (a) either $\beta_2-\beta_3<0$ or (b) $\beta_2-\beta_3$ is positive and large (≥ 2). Case (b) is excluded, since it would indicate that the 22 charge, which is always the sum of the 23 and 32 charges, would be unacceptably large as well. What about case (a)? A negative number can not dominate the 22 entry in the chiral case, thus $|\beta_2-\beta_3|$ would have to be smaller than $|\alpha_2-\alpha_3|$. This clearly reverses the hierarchy between the 22 and 32 elements, which indicates that the 22 element may be equal or smaller than the 32.

For this reason we are going to look for solutions in the case of vector singlets, where it is the absolute value of the charges that matters. Here, what changes from the previous case is that a solution with a small and positive $\alpha_2-\alpha_3$ and a large negative $\beta_2-\beta_3$ is allowed. The 23 and 32 elements have the correct hierarchy, while the 22 element can also be sufficiently small, as a result of a cancellation between terms of opposite sign, with the negative contribution being dominant. What can we say about the rest of the structure and how restrictive should we be when looking for solutions? We could allow for a small asymmetry between the 12 and 21 entries. Actually, λ_{12}^D can be slightly larger than λ_{21}^D. This, combined with the fact that there are unknown coefficients of order unity indicates that we can have an asymmetry of order ϵ between the 12 and 21 entries. We will keep solutions with such an asymmetry, even in the case that $\lambda_{12}^D < \lambda_{21}^D$, due to this coefficient ambiguity as well as the ambiguity in the experimental value of the up and down quarks (the lepton masses however are well defined). We also need not drop solutions with a large 13 or 31 entry, if they are compatible with the numerics.

On this basis, we have looked for solutions in the following way: for the charges of the elements 12-21-22-32 we made all possible charge assignments (such that lead up to a 4^{th} power in terms of the expansion parameter for the resulting mass matrices, for the 12 and 21 entries). This, each time fixes all charges α_2, α_3, β_2, β_3. We then looked at what the charges of the other entries are and whether the generated hierarchies are consistent with the phenomenology.

The restrictions we require in order to identify a viable solution, are (besides of course that the only renormalisable term is in the 33 position)

$$\begin{aligned}
|\text{charge}(11)| &> |\text{charge}(12)| \\
|\text{charge}(11)| &> |\text{charge}(21)| \\
|\text{charge}(21)| &> |\text{charge}(22)| \\
|\text{charge}(12)| &> |\text{charge}(22)| \\
|\text{charge}(13)| &> |\text{charge}(22)| \\
|\text{charge}(31)| &> |\text{charge}(22)| \\
|\text{charge}(32)| &\leq |\text{charge}(22)| O(\epsilon) \\
|\text{charge}(12)| &\approx |\text{charge}(21)| O(\epsilon) \\
|\text{charge}(23)| &> |\text{charge}(22)|
\end{aligned} \tag{65}$$

Then, we end up with the following possibilities:

Case 1:

$$\alpha_2 = -2/3, \;\; \alpha_3 = -5/3, \;\; \beta_2 = -2, \;\; \beta_3 = 0, \;\; Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^6 & \delta\epsilon^3 & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^4 & \delta\epsilon & 1 \end{pmatrix} \tag{66}$$

Case 2:

$$\alpha_2 = -1, \;\; \alpha_3 = -2, \;\; \beta_2 = -2, \;\; \beta_3 = 0, \;\; Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^7 & \delta\epsilon^3 & \delta\epsilon^2 \\ \delta\epsilon^3 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^5 & \delta\epsilon & 1 \end{pmatrix} \tag{67}$$

Case 3:

$$\alpha_2 = -4/3, \ \ \alpha_3 = -7/3, \ \ \beta_2 = -2, \ \ \beta_3 = 0, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^8 & \delta\epsilon^3 & \delta\epsilon^2 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^6 & \delta\epsilon & 1 \end{pmatrix} \tag{68}$$

Case 4:

$$\alpha_2 = -4/3, \ \ \alpha_3 = -1/3, \ \ \beta_2 = 0, \ \ \beta_3 = -2, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^6 & \delta\epsilon^3 & \delta\epsilon^4 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & 1 \end{pmatrix} \tag{69}$$

Case 5:

$$\alpha_2 = -4/3, \ \ \alpha_3 = -7/3, \ \ \beta_2 = -3, \ \ \beta_3 = 0, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^9 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^3 & \delta\epsilon^2 & \delta\epsilon^3 \\ \delta\epsilon^6 & \delta\epsilon & 1 \end{pmatrix} \tag{70}$$

Case 6:

$$\alpha_2 = -1, \ \ \alpha_3 = -2, \ \ \beta_2 = -7/3, \ \ \beta_3 = -1/3, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^8 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^3 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^5 & \delta\epsilon & 1 \end{pmatrix} \tag{71}$$

Case 7:

$$\alpha_2 = -5/3, \ \ \alpha_3 = -8/3, \ \ \beta_2 = -3, \ \ \beta_3 = 0, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^{10} & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^4 & \delta\epsilon^2 & \delta\epsilon^3 \\ \delta\epsilon^7 & \delta\epsilon & 1 \end{pmatrix} \tag{72}$$

Case 8:

$$\alpha_2 = -4/3, \ \ \alpha_3 = -7/3, \ \ \beta_2 = -7/3, \ \ \beta_3 = -1/3, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^9 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^6 & \delta\epsilon & 1 \end{pmatrix} \tag{73}$$

Case 9:

$$\alpha_2 = -4/3, \ \ \alpha_3 = -1/3, \ \ \beta_2 = -1/3, \ \ \beta_3 = -7/3, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^7 & \delta\epsilon^4 & \delta\epsilon^5 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & 1 \end{pmatrix} \tag{74}$$

Let us also list for completeness a few cases with a larger splitting between the 21 and 12 entries (up to $O(\epsilon^2)$):

Case 10:

$$\alpha_2 = -4/3, \ \ \alpha_3 = -1/3, \ \ \beta_2 = 1/3, \ \ \beta_3 = -5/3, \ \ Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^5 & \delta\epsilon^2 & \delta\epsilon^3 \\ \delta\epsilon^4 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^2 & \delta\epsilon & 1 \end{pmatrix} \tag{75}$$

Case 11:

$$\alpha_2 = -2/3, \;\; \alpha_3 = -5/3, \;\; \beta_2 = -7/3, \;\; \beta_3 = -1/3, \;\; Y_{u,d,\ell} = \begin{pmatrix} \delta\epsilon^7 & \delta\epsilon^4 & \delta\epsilon^3 \\ \delta\epsilon^2 & \delta\epsilon & \delta\epsilon^2 \\ \delta\epsilon^4 & \delta\epsilon & 1 \end{pmatrix} \tag{76}$$

Of course, here we also have the cases with the opposite charge assignment[14]. Among the various choices, we see that

- The charge of the Higgs fields $h_{1,2}$ is always different from zero.
- There are cases where the 13 and 31 elements are large.

We may now examine the results of Table 3 in the context of the $U(1)_X$ symmetry discussion above. We take all models that fit the data with $\chi^2/\text{dof}< 1$, i.e. models 1,3,4. We define in each of these models, H_{ij}^{emp} as being the dimensionless and dominant effective coupling constants in the $SU(4)\otimes SU(2)_L\otimes SU(2)_R$ unified Yukawa matrix for the best fit parameters.

Then, model 1 has

$$H_{ij}^{emp} \sim \begin{bmatrix} 0 & 0.003 & 0 \\ 0.001 & 0.03 & 0 \\ 0 & 0.03 & 1 \end{bmatrix}. \tag{77}$$

We see that case 1 above does not fit this pattern very well if all dimensionless couplings are $\sim O(1)$ because in case 1, H_{21} is suppressed in comparison to H_{12}. Cases 4,9 do not possess approximate texture zeroes in the 31 position and this would affect $|V_{ub}|$ strongly. Similar objections can be raised about other cases, except for cases 2,7,8. Case 2 with $\epsilon = 0.21, \delta = .14$ yields

$$\begin{bmatrix} 2.10^{-6} & 0.001 & 6.10^{-3} \\ 0.001 & 0.03 & 6.10^{-3} \\ 6.10^{-5} & 0.03 & 1 \end{bmatrix}, \tag{78}$$

which fits Eq.77 well apart from a factor $\sim$3 in the 12 position. The next sub-dominant operator in the 22 position needs to be 2.10^{-3} according to Table 3. The values of ϵ and δ used in Eq. 78 give the subdominant operator in the 22 position to be $\sim 6.10^{-3}$. This is acceptable, but a closer match occurs for the next higher dimension operator, which has magnitude $\sim 10^{-3}$. An ambiguity occurs in that we have not set the normalisation of the sub-dominant operator due to its numerous possibilities and so the original

[14]The presence of fractional charges implies the existence of residual discrete symmetries after the breaking of the abelian symmetry.

discrepancy factor of ~ 3 could easily be explained. Below, we do not consider the numerical size of the sub-dominant operator because it is clear that some operator can be chosen that will fit the required number well. If the charge assignments under the $U(1)_X$ symmetry were the same as in this case, we would have succeeded in explaining why the assumption of texture zeros was valid. For example, the 13 element in Eq.78 being 6×10^{-3} instead of zero only affects mixing angle and mass predictions by a small amount. We have also explained the hierarchies between the elements in terms of the different mass scales involved in the non-renormalisable operators by not having to choose dimensionless parameters of less than 1/3 (or greater than 3). Case 7 with $\epsilon = 0.36, \delta = 0.08$ gives

$$\begin{bmatrix} 2.10^{-6} & 0.001 & 4.10^{-3} \\ 0.001 & 0.01 & 4.10^{-3} \\ 6.10^{-5} & 0.03 & 1 \end{bmatrix}. \tag{79}$$

We should note that at this level, we may naively expect 8% corrections to the constraint in Eq.79 through the next order of δ operators in each element. We could have attempted to include these possible errors in the numerical fits but we did not due to the fact that they are very model dependent. Deeper model building in terms of constructing the non-renormalisable operators out of extra fields or examining underlying string models would be required to explain why this should not be the case. It should also be borne in mind that explanations for exact texture zeroes can be made in this context by setting fractional $U(1)_X$ - charges on the heavy fields in the operators, or by leaving certain heavy fields out of the FN model. Case 8 with $\epsilon = 0.36, \delta = 0.08$ gives the same results as in Eq.79, except with the (22) element as 0.03.

From Table 3 we see that model 3 (the model that fits the data the best) has

$$H_{ij}^{emp} \sim \begin{bmatrix} 0 & 0.002 & 0 \\ 0.002 & 0.03 & 0 \\ 0 & 0.03 & 1 \end{bmatrix}. \tag{80}$$

Choosing $\epsilon = 0.26, \delta = 0.12$ in case 2 gives a good match to Eq.80:

$$\begin{bmatrix} 9.10^{-8} & 0.002 & 8.10^{-3} \\ 0.002 & 0.03 & 8.10^{-3} \\ 6.10^{-5} & 0.03 & 1 \end{bmatrix}. \tag{81}$$

Case 7 with $\epsilon = 0.40, \delta = 0.07$ or case 8 with the same ϵ and δ both give a fairly good match as well.

Model 4 is different in the sense that it possesses a hierarchy between the 12 and

21 entries of the effective Yukawa couplings:

$$H_{ij}^{emp} \sim \begin{bmatrix} 0 & 0.0007 & 0 \\ 0.004 & 0.03 & 0 \\ 0 & 0.03 & 1 \end{bmatrix}. \tag{82}$$

Here, case 1 with $\delta = 0.2, \epsilon = .15$ predicts

$$\begin{bmatrix} 3.10^{-7} & 0.0007 & 4.10^{-3} \\ 0.004 & 0.03 & 4.10^{-3} \\ 10^{-5} & 0.03 & 1 \end{bmatrix}, \tag{83}$$

an extremely good match to Eq.82. Case 6 with $\epsilon = 0.28, \delta = 0.11$ provides a good match also.

Thus we see that we can explain the hierarchies and texture zero structures of the models that fit the data best. In general, it seems likely that we have enough freedom in setting charges to attain the required hierarchies for the Yukawa matrices. Before passing to the conclusions, let us briefly comment on how the basic features of the $U(1)_X$ symmetries that we have discussed arise in string constructions.

In realistic free fermionic string models [23, 16] there are some general features: At a scale $M_{string} \sim 5g_{string} \times 10^{17}$GeV, one obtains an effective $N = 1$ supergravity model with a gauge symmetry structure which is usually a product of non-Abelian groups times several $U(1)$ factors. The non-Abelian symmetry contains an observable and a hidden sector. The massless superfields accommodating the higgs and known chiral fields transform non-trivially under the observable part and usually carry non-zero charges under the surplus $U(1)$-factors. The latter, act as family symmetries in the way described above. Some of them are anomalous, but it turns out that one can usually define new linear $U(1)$ combinations where all but one are anomaly-free. The anomalous $U(1)$ is broken by the Dine Seiberg Witten mechanism [24], in which a potentially large Fayet-Iliopoulos D-term is generated by the VEV of the dilaton field. A D-term however breaks supersymmetry and destabilizes the string vacuum, unless there is a direction in the scalar potential which is D-flat and F-flat with respect to the non-anomalous gauge symmetries. If such a direction exists, some of the singlet fields will acquire a VEV, canceling the anomalous D term, so that supersymmetry is restored. Since the fields corresponding to such a flat direction typically also carry charges for the non-anomalous D-terms, they break all $U(1)$ symmetries spontaneously. For the string model in ref.[16], the expected order of magnitude for the VEV of the singlet fields is $< \Phi_i > \sim (0.1 - 0.3) \times M_{string}$. Thus, their magnitude is of the right order to produce the required mass entries in the mass matrices via non-renormalisable operators.

Finally we mention that the presence of a gauged family symmetry such as $U(1)_X$ is in principle quite dangerous since its presence can lead to large off-diagonal squark and slepton masses which can mediate flavour-changing processes at low energy. In particular the D term associated with $U(1)_X$ is in general only approximately flat due to soft supersymmetry breaking terms, and this can lead to family-dependent squark and slepton masses with unacceptably large mass splittings. This is a generic problem of any model with a gauged family symmetry, however the $U(1)_X$ symmetry here is non-asymptotically free with a large beta function so that its gauge coupling rapidly becomes very small below the string scale, leading to small X gaugino masses. It has been suggested [25] that the possible infra-red structure of the theory could help by relating the soft scalar masses to the small gaugino masses, thereby making them naturally smaller than the squark and slepton masses, or by enforcing $< \theta > = < \bar{\theta} >$ as an infra-red fixed point of the theory. We refer the reader to ref.[25] for more details.

7 Conclusions

We have combined the idea of a gauged $U(1)_X$ family symmetry with that of quark-lepton unification within the framework of a string-inspired Pati-Salam model. Our basic assumption is that the non-renormalisable operators above the unification scale are of the form in Eq.26. These operators factorise into a factor $(H\bar{H})$ and a factor involving the singlet fields $\theta, \bar{\theta}$. The singlet fields $\theta, \bar{\theta}$ break the $U(1)_X$ symmetry and provide the horizontal family hierarchies while the $H, \bar{H}$ fields break the SU(4)$\otimes$SU$(2)_L\otimes$SU$(2)_R$ symmetry and give the vertical splittings arising from group theoretic Clebsch relations between different charge sectors. The factor $(H\bar{H})$ also provides an additional flavour independent suppression factor δ which helps the fit. The quark and lepton masses and quark mixing angles are thus described at high energies by single unified Yukawa matrix whose flavour structure is controlled by a broken $U(1)_X$ family symmetry, and all vertical splittings controlled Clebsch factors.

An important feature of the scheme is the existence of Clebsch zeroes which allow an entirely new class of textures to be obtained. For example the RRR solutions 3 and 5 may be reproduced by this scheme which are complementary to the RRR solution 2 favoured by the IR approach. These Clebsch zeroes were also a feature of the non-symmetric textures discussed in Ref.[17], which were previously analysed in the absence of any family symmetry. We have extended these models to incorporate the $U(1)_X$ family symmetry and found that although one prediction is lost, new models exist which appear to fit the data better than the original models. To be precise, a global fit to the data with 3 dof is described, in which three models are singled out with χ^2/dof< 1. By comparison a recent paper [26] performed a global χ^2 analysis for some $SO(10)$ models, including the mass and mixing data. With 3 dof, they obtain a χ^2/dof$\sim 1/3$ for the best model. While our fit to model 3 has a smaller χ^2/dof than this, it is difficult to make a comparison as in ref.[26] quark mass correlations from data, as well as the effect of large $\tan\beta$ on m_b has been included[15]. Moreover, at this level of difference of χ^2 between models, the χ^2 test is subject to large statistical fluctuations. Similarly, we do not statistically distinguish between the fits in Tables 3,4 since both contain good fits to the data with χ^2/dof< 1.

We find it remarkable that the tight constraints coming from the restricted Clebsch structures of the unified theory, and the non-trivial allowed family patterns dictated by the $U(1)_X$ symmetry can both be satisfied and allow such successful fits to the quark and lepton masses and mixing angles. However our new approach brings with it new unanswered questions which more complete theories should address. The main unanswered question of the present models is that of the flavour dependence of the Clebsch factors: why is one specific group theoretical contraction in a particular entry of the Yukawa matrix singled out to be dominant over the others? Also, why do the dominant non-renormalisable operators always contain the $H\bar{H}$ pair? The answers to these questions must lie in a more fundamental model. If this model were of the FN type with extra heavy fields that produce the spaghetti diagrams that yield the necessary operators, the model would have to include some adjoint representations. This is essentially because all of the models require a Clebsch coefficient $|Y^D_{22}/Y^E_{22}| = 3$ and all of the operators providing this factor (see Table 7) involve C_{15}, the adjoint tensor of SU(4) from Eq.85. This could take some of the motivation for the model away, as one of its benefits is that it fits into string models easily because of its lack of non-fundamental representations of the gauge group.

We could solve this apparent difficulty by adding an extra operator in the 22 position but this would diminish the predictivity of the model. Alternatively, one may re-examine models like those in refs.[15, 16] in which the string construction leaves us

[15]These involve the soft terms, thus a larger number of parameters are involved in the fit.

with a supersymmetric 422 effective field theory below the string scale. One would then have to check whether it is possible to construct a phenomenologically viable model of flavour that gives the correct choice of operators once a particular flat direction is chosen. Our preliminary investigations of these questions show that the gross features of the string construction of refs.[15, 16] that lead to the gauge group of Eq.13 are similar to our model but with some noticeable differences. $U(1)_X$ family symmetries are a consequence of the string construction, but there are four of them with one being anomalous. There are several charged singlets to take the role of the θ fields and the $H\bar{H}$ pair is charged under $U(1)_X$. It will clearly be interesting to examine the consequences of the string construction in detail, and we hope to return to these issues in a future publication [27].

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Appendix 1. $n = 1$ Operators

The $n = 1$ operators are by definition all of those operators which can be constructed from the five fields $F\bar{F}hH\bar{H}$ by contracting the group indices in all possible ways, as discussed in Appendix 1. After the Higgs fields H and $\bar{H}$ develop VEVs at M_{GUT} of the form $\langle H^{ab}\rangle = \langle H^{41}\rangle = \nu_H$, $\langle \bar{H}_{ax}\rangle = \langle \bar{H}_{41}\rangle = \bar{\nu}_H$, the operators listed in the appendix yield effective low energy Yukawa couplings with small coefficients of order M_{GUT}^2/M^2. However, as in the simple example discussed previously, there will be precise Clebsch relations between the coefficients of the various quark and lepton component fields. These Clebsch relations are summarised in Table 7, where relative normalisation factor has been applied to each. The table identifies which SU(4) and SU(2) structures have been used to construct each individual operator by reference to Eqs. 86,87.

The $n = 1$ operators are formed from different group theoretical contractions of the indices in

$$O^{\alpha\rho y w}_{\beta\gamma x z} \equiv F^{\alpha a}\bar{F}_{\beta x}h^{y}_{a}\bar{H}_{\gamma z}H^{\rho w}. \tag{84}$$

It is useful to define some SU(4) invariant tensors C, and $\mathrm{SU}(2)_R$ invariant tensors R as follows:

$$\begin{aligned}
(C_1)^{\alpha}_{\beta} &= \delta^{\alpha}_{\beta} \\
(C_{15})^{\alpha\rho}_{\beta\gamma} &= \delta^{\gamma}_{\beta}\delta^{\rho}_{\alpha} - \frac{1}{4}\delta^{\alpha}_{\beta}\delta^{\rho}_{\gamma} \\
(C_6)^{\rho\gamma}_{\alpha\beta} &= \epsilon_{\alpha\beta\omega\chi}\epsilon^{\rho\gamma\omega\chi} \\
(C_{10})^{\alpha\beta}_{\rho\gamma} &= \delta^{\alpha}_{\rho}\delta^{\beta}_{\gamma} + \delta^{\alpha}_{\gamma}\delta^{\beta}_{\rho} \\
(R_1)^{x}_{y} &= \delta^{x}_{y} \\
(R_3)^{wx}_{yz} &= \delta^{x}_{y}\delta^{w}_{z} - \frac{1}{2}\delta^{x}_{z}\delta^{w}_{y},
\end{aligned} \tag{85}$$

where δ^{α}_{β}, $\epsilon_{\alpha\beta\omega\chi}$, δ^{x}_{y}, ϵ_{wz} are the usual invariant tensors of SU(4), $\mathrm{SU}(2)_R$. The SU(4) indices on $C_{1,6,10,15}$ are contracted with the SU(4) indices on two fields to combine them into $\underline{1}$, $\underline{6}$, $\underline{10}$, $\underline{15}$ representations of SU(4) respectively. Similarly, the $\mathrm{SU}(2)_R$ indices on $R_{1,3}$ are contracted with $\mathrm{SU}(2)_R$ indices on two of the fields to combine them into $\underline{1}$, $\underline{3}$ representation of $\mathrm{SU}(2)_R$.

The SU(4) structures in Table 7 are

I. $(C_1)^{\beta}_{\alpha}(C_1)^{\gamma}_{\rho}$

II. $(C_{15})^{\beta\chi}_{\alpha\sigma}(C_{15})^{\gamma\sigma}_{\rho\chi}$

	SU(2)	SU(4)	$Q\bar{U}h_2$	$Q\bar{D}h_1$	$L\bar{E}h_1$	LNh_2
O^A	I	I	1	1	1	1
O^B	II	I	1	-1	-1	1
O^C	I	II	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^D	II	II	$\frac{1}{\sqrt{5}}$	$\frac{-1}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^E	III	III	0	2	0	0
O^F	II	III	$\sqrt{2}$	$-\sqrt{2}$	0	0
O^G	III	IV	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0
O^H	IV	IV	$4/5$	$2/5$	$4/5$	$8/5$
O^I	V	V	0	0	0	2
O^J	VI	V	0	0	$\frac{4}{\sqrt{5}}$	$\frac{2}{\sqrt{5}}$
O^K	V	VI	$8/5$	0	0	$6/5$
O^L	IV	VI	$\frac{16}{5\sqrt{5}}$	$\frac{8}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$
O^M	III	I	0	$\sqrt{2}$	$\sqrt{2}$	0
O^N	V	III	2	0	0	0
O^O	V	IV	$\frac{2}{\sqrt{5}}$	0	0	$\frac{4}{\sqrt{5}}$
O^P	I	VI	$\frac{4\sqrt{2}}{5}$	$\frac{4\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$
O^Q	II	VI	$\frac{4\sqrt{2}}{5}$	$-\frac{4\sqrt{2}}{5}$	$-\frac{3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$
O^R	III	VI	0	$\frac{8}{5}$	$\frac{6}{5}$	0
O^S	VI	VI	$\frac{8}{5\sqrt{5}}$	$\frac{16}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$
O^T	IV	I	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$
O^U	VI	I	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$
O^V	V	I	$\sqrt{2}$	0	0	$\sqrt{2}$
O^W	III	II	0	$\sqrt{\frac{2}{5}}$	$-3\sqrt{\frac{2}{5}}$	0
O^X	IV	II	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$	$\frac{-6\sqrt{2}}{5}$
O^Y	VI	II	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{-6\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$
O^Z	V	II	$\sqrt{\frac{2}{5}}$	0	0	$-3\sqrt{\frac{2}{5}}$
O^a	I	III	$\sqrt{2}$	$\sqrt{2}$	0	0
O^b	IV	III	$\frac{4}{\sqrt{5}}$	$\frac{2}{\sqrt{5}}$	0	0
O^c	VI	III	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0	0
O^d	I	IV	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$
O^e	II	IV	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{5}}$	$-2\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$
O^f	VI	IV	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{8}{5}$	$\frac{4}{5}$
O^g	I	V	0	0	$\sqrt{2}$	$\sqrt{2}$
O^h	II	V	0	0	$-\sqrt{2}$	$\sqrt{2}$
O^i	III	V	0	0	2	0
O^j	IV	V	0	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$

Table 7: When the Higgs fields develop their VEVs, the $n = 1$ operators lead to the effective Yukawa couplings with Clebsch coefficients as shown.

$$\begin{array}{ll} \text{III.} & (C_6)^{\omega\chi}_{\alpha\rho}(C_6)^{\beta\gamma}_{\omega\chi} \\ \text{IV.} & (C_{10})^{\omega\chi}_{\alpha\rho}(C_{10})^{\beta\gamma}_{\omega\chi} \\ \text{V.} & (C_1)^{\beta}_{\rho}(C_1)^{\gamma}_{\alpha} \\ \text{VI.} & (C_{15})^{\gamma\chi}_{\alpha\sigma}(C_{15})^{\beta\sigma}_{\rho\chi}, \end{array} \tag{86}$$

and the SU(2) structures are

$$\begin{array}{ll} \text{I.} & (R_1)^{z}_{w}(R_1)^{x}_{y} \\ \text{II.} & (R_3)^{zq}_{wr}(R_3)^{xr}_{yq} \\ \text{III.} & \epsilon^{xz}\epsilon_{yw} \\ \text{IV.} & \epsilon_{ws}\epsilon^{xt}(R_3)^{sq}_{yr}(R_3)^{zr}_{tq} \\ \text{V.} & (R_1)^{z}_{y}(R_1)^{x}_{w} \\ \text{VI.} & (R_3)^{zq}_{yr}(R_3)^{xr}_{wq}. \end{array} \tag{87}$$

The operators are then given explicitly by contracting Eq.84 with the invariant tensors of Eq.85 given by Table 7 and Eqs.86,87.

Appendix 2. Review of Analysis of Ref.[17]

In ref.[17] we assumed that the Yukawa matrices at M_X are all of the form

$$\lambda^{U,D,E,N} = \begin{pmatrix} O(\epsilon^2) & O(\epsilon^2) & 0 \\ O(\epsilon^2) & O(\epsilon) & O(\epsilon) \\ 0 & O(\epsilon) & O(1) \end{pmatrix}, \tag{88}$$

where $\epsilon << 1$ and some of the elements may have approximate or exact texture zeroes in them. First, we examine closer the assumption that the operator in the (33) position of the Yukawa matrices is the renormalisable one. It has been suggested in the past that the large value of $\tan\beta$ required by the constraint

$$\lambda_t(M_{GUT}) = \lambda_b(M_{GUT}) = \lambda_\tau(M_{GUT}) \tag{89}$$

such as is predicted by the renormalisable operator, leads to some phenomenological problems. One such problem is that a moderate fine tuning mechanism is required to radiatively break the electro-weak symmetry in order to produce the necessary hierarchy of Higgs VEVs $v_1/v_2 \approx m_t/m_b$ [28],[29]. One could set about trying to extend the present model in a manner that would lead to an arbitrary choice of $\tan\beta$, for example by introducing extra Higgs bidoublets. This route has its disadvantages in that a low value of $\tan\beta$ has been shown [30] in most schemes to be inconsistent with $\lambda_b(M_{GUT}) = \lambda_\tau(M_{GUT})$ unification if the tau neutrino mass constitutes the hot dark matter requiring the Majorana mass of the right handed tau neutrino to be $M_R^{\nu_\tau} \sim O(10^{12})$ GeV. To a very good approximation, the largest diagonalised Yukawa coupling in λ^I is equal to its 33 entry λ^I_{33}. (One may obtain small $\tan\beta$ solutions consistent with m_b-m_τ unification and an intermediate neutrino scale, in specific models: Either large mixing in the $\mu - \tau$ charged leptonic sector has to occur [31] or the Dirac-type Yukawa coupling of the neutrino has to be very suppressed [32].)

To force things to work in a generic scheme, one solution could be to use a non-renormalisable operator in the 33 position which has some Clebsch factor $x > 1$ such that

$$\lambda_t(M_{GUT}) = x\lambda_b(M_{GUT}) = x\lambda_\tau(M_{GUT}). \tag{90}$$

Eq.90 would preserve the bottom-tau Yukawa unification, but lower the prediction of $\tan\beta$ due to the bigger contribution to the top Yukawa coupling. It may only be reasonable to examine $n = 1$ operators in this context since we know that the third family [17] Yukawa coupling is $\sim O(1)$ and higher dimension operators could be expected to provide a big suppression factor. Systematically examining the $n = 1$ operators we find that only the operator O^U_{33}, which leads to the prediction

$$\lambda_t(M_{GUT}) = 2\lambda_b(M_{GUT}) = 2\lambda_\tau(M_{GUT}) \tag{91}$$

can decrease $\tan\beta$. The change is minimal, from 56.35 to 55.19 for $\alpha_S(M_Z) = 0.117$ and $M_R^{\nu_\tau} = O(10^{12})$ GeV. The reason that the change is minimal is due to the fact that the Yukawa couplings are approximately at their quasi fixed points [33] and so even a large change to $\lambda_{t,b,\tau}(M_X)$ produces only a small change in $\lambda_{t,b,\tau}(m_t)$, which are the quantities that require a high $\tan\beta$ through the relations in Eq.60. Another possibility would be to include O^M_{33}, O^V_{33} which would allow arbitrary $\tan\beta$ (in particular intermediate $\tan\beta \sim 10-20$.) However, this would reduce the predictivity of the scheme as $\tan\beta$ would become an input. One might also be skeptical about whether a parameter ~ 1 could be generated by a non-renormalisable operator in a perturbative scheme. It would certainly require the heavy mass scales M to be very close to the VEVs $H, \bar{H}, \theta, \bar{\theta}$ and we might therefore naively expect large corrections to any calculation based on this model. We thus abandon these ideas and continue with the usual renormalisable operator in the 33 position of the Yukawa matrices that leads to Eq.89. We note in any case that a recent analysis [34] explains that in gauge mediated supersymmetry breaking models, the radiative mechanism of electroweak symmetry breaking can be such that no fine tuning occurs for large $\tan\beta$. In these models high $\tan\beta$ admits solutions of the hot dark matter problem in which the Yukawa couplings unify [30].

The hierarchy assumed in Eq.88 allows us to consider the lower 2 by 2 block of the Yukawa matrices first. In diagonalising the lower 2 by 2 block separately, we introduce corrections of order ϵ^2 and so the procedure is consistent to first order in ϵ. We found several maximally predictive ansatze that were constructed out of the operators whose Clebsch coefficients are listed in table 3 for the $n = 1$ operators. The explicit $n = 1$ operators in component form are listed in the Appendix 1. We label the successful lower 2 by 2 ansatze A_i:

$$A_1 = \begin{bmatrix} O^D_{22} - O^C_{22} & 0 \\ O^C_{32} & O_{33} \end{bmatrix} \tag{92}$$

$$A_2 = \begin{bmatrix} 0 & O^A_{23} - O^B_{23} \\ O^D_{32} & O_{33} \end{bmatrix} \tag{93}$$

$$A_3 = \begin{bmatrix} 0 & O^C_{23} - O^D_{23} \\ O^B_{32} & O_{33} \end{bmatrix} \tag{94}$$

$$A_4 = \begin{bmatrix} 0 & O^C_{23} \\ O^A_{32} - O^B_{32} & O_{33} \end{bmatrix} \tag{95}$$

$$A_5 = \begin{bmatrix} 0 & O_{23}^A \\ O_{32}^C - O_{32}^D & O_{33} \end{bmatrix} \tag{96}$$

$$A_6 = \begin{bmatrix} O_{22}^K & O_{23}^C \\ O_{32}^M & O_{33} \end{bmatrix} \tag{97}$$

$$A_7 = \begin{bmatrix} O_{22}^K & O_{23}^G \\ O_{32}^G & O_{33} \end{bmatrix} \tag{98}$$

$$A_8 = \begin{bmatrix} 0 & O_{23}^H \\ O_{32}^G - O_{32}^K & O_{33} \end{bmatrix}. \tag{99}$$

We now note that solutions A_{2-8} require a parameter $H_{23} \sim O(1)$ to attain the correct λ_μ and V_{cb}. Any calculation based on the hierarchy assumed in Eq.88 is therefore inconsistent and so we discard these solutions. We also note that O_{32} only has the effect of fixing V_{cb} to a good approximation and so can consist of any operator in Table 7 that has a different Clebsch coefficient for up quark and down quark Yukawa couplings. The precise operator responsible for V_{cb} has no bearing on the rest of the calculation and we therefore just make an arbitrary choice of O_{32}^C for the rest of this paper. We also note that for the phenomenologically desirable and predictive relation

$$\frac{\lambda_{22}^D(M_{GUT})}{\lambda_{22}^E(M_{GUT})} = 3, \tag{100}$$

to hold, we may replace $O_{22}^D - O_{22}^C$ in A_1 with $O_{22}^W + O_{22}^C$, $O_{22}^X + O_{22}^D$ or any other combination of two operators which preserves Eq.100 and allows λ_{22}^U to be smaller and independent of $\lambda_{22}^{D,E}$. In fact, the preferred solution is that the dominant operator in that position be O_{22}^W which does not give a contribution to the up quark mass. Then, a subdominant operator would be responsible for the entry λ_{22}^U and would therefore be suppressed naturally by one or more powers of ϵ.

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