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Next-to-Leading Order Evolution of Polarized and Unpolarized Fragmentation Functions

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Abstract

We determine the two-loop 'time-like' Altarelli-Parisi splitting functions, appearing in the next-to-leading order Q^2-evolution equations for fragmentation functions, via analytic continuation of the corresponding 'space-like' splitting functions for the evolution of parton distributions. We do this for the case of unpolarized fragmentation functions and - for the first time - also for the functions describing the fragmentation of a longitudinally polarized parton into a longitudinally polarized spin-1/2 hadron such as a Λ baryon. Our calculation is based on the method proposed and employed by Curci, Furmanski and Petronzio in the unpolarized case in which we confirm their results.

1 Introduction

The spin structure of longitudinally polarized nucleons has been investigated in a number of experiments [1] in recent years by scattering polarized highly virtual space-like ($q^2 \equiv -Q^2 < 0$) photons off polarized targets, which provides access to the spin-dependent 'space-like' parton distributions of the nucleon. In contrast to this, nothing is known as yet about the corresponding polarized *'time-like'* parton densities, i.e., the functions describing the fragmentation of a longitudinally polarized quark or gluon into a longitudinally polarized (spin-1/2) hadron. In analogy with the space-like case these are defined by

$$\Delta D_f^h(z,Q^2) = D_{f(+)}^{h(+)}(z,Q^2) - D_{f(+)}^{h(-)}(z,Q^2)\,, \tag{1}$$

where $D_{f(+)}^{h(+)}(z,Q^2)$ ($D_{f(+)}^{h(-)}(z,Q^2)$) is the probability for finding a hadron h with positive (negative) helicity in a parton f with positive helicity at a mass scale Q, carrying a fraction z of the parent parton's momentum. We note that taking the sum instead of the difference in Eq. (1) one obtains the corresponding unpolarized fragmentation function.

Spin-dependent fragmentation functions appear equally interesting as the space-like distributions since they obviously contain information on how the spin of a fragmenting parton is transmitted to that of the produced hadron. The most likely candidate for a measurement of polarized fragmentation functions is the Λ baryon since its dominant decay $\Lambda \to p\pi^-$ is parity-violating and enables the determination of the Λ's polarization [2]. In [3] a strategy was proposed for extracting the ΔD_f^Λ ($f = q, \bar{q}$) in single-particle inclusive e^+e^- annihilation (SIA) $e^+e^- \to \Lambda X$. If the energy is far below the Z resonance, one longitudinally polarized beam is required in order to fix the polarization of the outgoing (anti)quark that fragments into the Λ and to obtain a non-vanishing twist-two spin asymmetry. At higher energies, no beam polarization is needed since the parity-violating coupling $q\bar{q}Z$ automatically generates a net polarization of the quarks. Apart from SIA, which plays the same fundamental role for the determination of fragmentation functions as deep-inelastic scattering (DIS) does for that of space-like parton distributions, the possibility of extracting the ΔD_f^Λ in semi-inclusive DIS (SIDIS) in the current fragmentation region, $ep \to e\Lambda X$, has also been studied theoretically recently [4, 5]. Here, either a longitudinally polarized lepton beam or a polarized nucleon target would be required. On the experimental side, ALEPH has reported a first measurement of the Λ polarization in Z decays recently [6], and a measurement of the ΔD_f^Λ in SIDIS appears possible for the HERMES experiment [7] and is planned by the COMPASS collaboration [8].

The polarized cross section for, say[1], the process $e^+\vec{e}^- \to \vec{\Lambda}X$ (the arrows denoting longitudinal polarization) can be written in the factorized form as a sum over convolutions of polarized hard subprocess cross sections with the *process-independent* fragmentation functions ΔD_f^Λ of (1),

$$\frac{d\Delta\sigma^{e^+e^-\to\Lambda X}(s,y,z)}{dydz} \equiv \frac{d\sigma(\lambda_e=+,\lambda_\Lambda=+,s,y,z)}{dydz} - \frac{d\sigma(\lambda_e=+,\lambda_\Lambda=-,s,y,z)}{dydz} \tag{2}$$

$$= \sum_{f=q,\bar{q},g} \int_z^1 \frac{d\xi}{\xi} \frac{d\Delta\hat{\sigma}^{e^+e^-\to fx}(s,y,\xi)}{dyd\xi} \Delta D_f^\Lambda(\frac{z}{\xi},Q^2)\,, \tag{3}$$

where λ_e, λ_Λ denote the helicities of the polarized electron and the Λ, and $s \equiv 2p_{e^+}\cdot p_{e^-}$, $z \equiv 2p_\Lambda \cdot q/Q^2$ with the momentum q of the time-like ($q^2 \equiv Q^2 > 0$) intermediate γ or Z. The variable y is defined by $y \equiv p_\Lambda\cdot p_{e^-}/p_\Lambda\cdot q$ and is related to the cms scattering angle θ of the produced Λ with respect to the incoming electron via $y = (1+\cos\theta)/2$. The polarized subprocess cross sections $d\Delta\hat{\sigma}^{e^+e^-\to fx}(s,y,\xi)/dyd\xi$ are defined in complete analogy with (2), and ξ is the partonic counterpart of z, $\xi \equiv 2p_f\cdot q/Q^2$. Again the corresponding expression for the unpolarized cross section is obtained by taking the sum instead of the difference in Eq. (2) and omitting all Δ's. Unlike the fragmentation functions, the hard subprocess cross sections are calculable in perturbative QCD. QCD can, however, predict the Q^2 dependence of the fragmentation functions via the Altarelli-Parisi equations [9], once a suitable non-perturbative hadronic input for the evolution has been found. In the leading order (LO), there is only one subprocess, namely $e^+e^- \to q\bar{q}$ via γ or Z exchange, and the fragmentation functions evolve according to the LO polarized (time-like) Altarelli-Parisi equations.

It is the main purpose of this paper to set up the complete next-to-leading order (NLO) framework of QCD for single-inclusive annihilation into a polarized hadron. This

[1]Since the Λ appears the most realistic candidate for measurements of spin-dependent fragmentation functions we write down all formulae below for this specific case. They apply, of course, equally well to any other final state (spin-1/2) hadron whose longitudinal polarization can be determined experimentally.

task first of all involves calculating the $\mathcal{O}(\alpha_s)$ corrections to the LO hard subprocess cross section, including the calculation of subprocesses that first appear at NLO. This was recently achieved in [10, 11]. However, knowledge of the underlying hard subprocesses to NLO accuracy cannot be the full story as becomes immediately obvious from the well-known fact that the corresponding corrections are factorization scheme dependent, i.e., depend on the convention adopted when subtracting the collinear singularities appearing in the calculation. For a fully consistent NLO calculation one also needs to perform the evolution of the polarized fragmentation functions in NLO, which requires knowledge of the polarized NLO evolution kernels in the (time-like) Altarelli-Parisi equations. Only when both types of NLO corrections, those to the subprocess cross sections *and* to the evolution kernels, are known does the NLO framework become complete and consistent, the factorization scheme dependencies cancelling out to the order considered whenever a physical cross section is calculated. This situation is of course completely the same as in the more familiar space-like case of, e.g., DIS structure functions.

As will be discussed below, it is possible to derive the polarized NLO time-like evolution kernels by analytic continuation of their space-like counterparts which have been calculated recently [12, 13, 14]. The procedure for doing this has first been worked out for the unpolarized non-singlet case in [15] and has also been used for the unpolarized singlet sector in [16]. We will pursue this method. The results we obtain refer to the $\overline{\mathrm{MS}}$ scheme and need to be combined with NLO corrections to the hard subprocess cross sections in the same scheme, as recently presented for SIA and SIDIS in [11]. Since the procedure of analytic continuation can also be applied to the hard subprocess cross sections we will provide a check on the results of [11] for SIA.

In view of the present lack of any experimental information on the ΔD_f^{Λ} one could argue that it is somewhat premature to set up the full NLO framework for their evolution and the processes in which they appear. On the other hand, it seems likely that data will become available in the future. Furthermore, the transition from the space-like to the time-like region in the polarized case appears interesting in itself: In LO the space-like and time-like Altarelli-Parisi evolution kernels are related to each other via an analytic continuation rule (ACR) [17] and also via the so-called Gribov-Lipatov relation (GLR) [18]. In the unpolarized case these relations were shown to be broken beyond leading order in the $\overline{\mathrm{MS}}$ scheme [15, 16, 19], and a similar feature is thus expected for the spin-dependent case. The NLO effects also appear interesting from a more physical point of view. For instance, one would expect [20] that to a first approximation polarized-Λ production in SIA proceeds just via strange quark fragmentation $\vec{s} \to \vec{\Lambda}$, i.e., is essentially sensitive to ΔD_s^{Λ}. NLO evolution on the other hand automatically generates non-vanishing non-strange fragmentation functions due to the existence of flavor non-diagonal quark-to-quark splitting functions. Also, the possibly important [10] role played by gluons is appreciated when going beyond the leading order.

The remainder of this paper is organized as follows. In section 2 we set the general framework for our calculations and briefly discuss the LO results. In sections 3 and 4 we present in some detail the determination of the NLO corrections for the unpolarized time-like situation by analytic continuation of their space-like counterparts. Even though neither the method of analytic continuation nor the final result of the calculation are new, the full calculation itself has never been documented before, and we also provide new insight in the breakdown of the ACR beyond LO. Furthermore, our findings in sections 3,4 are crucial for dealing with the polarized case, which is then done in the subsequent section. In section 6 we study an interesting supersymmetric relation obeyed by the NLO unpolarized and polarized time-like splitting functions. Section 7 briefly summarizes our work.

2 General framework and LO results

Let us first set the notation by collecting all ingredients for a NLO treatment of the cross section in Eq. (2). We begin by dealing with the hard subprocess cross sections. In analogy with the familiar space-like $g_1 \equiv g_1^{(S)}(x, Q^2)$ (where $x \equiv Q^2/2p \cdot q \leq 1$) we define a time-like structure function $g_1^{(T)}(z,Q^2)$ and write Eq. (3) as[2]

$$\frac{d\Delta\sigma^{e^+e^-\to\Lambda X}(s,y,z)}{dzdy} = \frac{6\pi\alpha^2}{Q^2}(2y-1)g_1^{(T)}(z,Q^2) \,. \tag{4}$$

To facilitate the further discussion, we adopt a combined treatment of the space-like and time-like situations and introduce the structure function $\mathcal{G}_1^{(U)}(\xi,Q^2)$, where the index U stands for either 'space-like' ($U=S$, $\mathcal{G}_1^{(S)} \equiv 2g_1^{(S)}$, $\xi = x$), or 'time-like' ($U=T$, $\mathcal{G}_1^{(T)} \equiv g_1^{(T)}$, $\xi = z$), and the parton distributions $\Delta f^{(U)}(\xi,Q^2)$ ($f = q,\bar{q},g$), where $\Delta f^{(S)} \equiv \Delta f$ (with the usual polarized hadronic parton densities Δf) and $\Delta f^{(T)} \equiv \Delta D_f^{\Lambda}$. In terms of the $\Delta f^{(U)}$ we can write $\mathcal{G}_1^{(U)}$ to NLO as

$$\mathcal{G}_1^{(U)}(\xi,Q^2) = \sum_q e_i^2 \left\{ \left[\Delta q^{(U)} + \Delta\bar{q}^{(U)}\right] \otimes \Delta C_q^{(U)} + \eta_U \Delta g^{(U)} \otimes \Delta C_g^{(U)} \right\}(\xi,Q^2) \,, \tag{5}$$

where the sum runs over the n_f active quark flavors, $\eta_S = 1/n_f$, $\eta_T = 2$ and $\otimes$ denotes the usual convolution. The hard subprocess cross sections $\Delta C_q^{(U)}$, $\Delta C_g^{(U)}$ are taken to have the perturbative expansion

$$\Delta C_i^{(U)}(\xi,\alpha_s) = \Delta C_i^{(U),(0)}(\xi) + \frac{\alpha_s}{2\pi}\Delta C_i^{(U),(1)}(\xi) \,, \tag{6}$$

where $\Delta C_q^{(U),(0)}(\xi) = \delta(1-\xi)$, $\Delta C_g^{(U),(0)}(\xi) = 0$.

To determine the Q^2 evolution of the space-like and time-like parton densities $\Delta f^{(U)}$ in Eq. (5) it is as usual convenient to decompose them into flavor singlet and non-singlet pieces by introducing the densities $\Delta q_\pm^{(U)}$ and the vector

$$\Delta\vec{v}^{(U)} \equiv \begin{pmatrix} \Delta\Sigma^{(U)} \\ \Delta g^{(U)} \end{pmatrix} \,, \tag{7}$$

where

$$\Delta q_\pm^{(U)} \equiv \Delta q^{(U)} \pm \Delta\bar{q}^{(U)} \,, \quad \Delta\Sigma^{(U)} \equiv \sum_q (\Delta q^{(U)} + \Delta\bar{q}^{(U)}) \,. \tag{8}$$

One then has the following non-singlet evolution equations ($q,\tilde{q}$ being two different flavors):

$$\frac{d}{d\ln Q^2}(\Delta q_+^{(U)} - \Delta\tilde{q}_+^{(U)})(\xi,Q^2) = \left[\Delta P_{qq,+}^{(U)} \otimes (\Delta q_+^{(U)} - \Delta\tilde{q}_+^{(U)})\right](\xi,Q^2) \,, \tag{9}$$

$$\frac{d}{d\ln Q^2}\Delta q_-^{(U)}(\xi,Q^2) = \left[\Delta P_{qq,-}^{(U)} \otimes \Delta q_-^{(U)}\right](\xi,Q^2) \,. \tag{10}$$

[2]For simplicity we restrict our considerations to pure photon exchange in the process $e^+e^- \to q\bar{q}$. Exchange of Z^0 and γZ^0 interference modify the angular dependence of the longitudinally polarized cross section and thus add new structure functions to its expression [10].

The two evolution kernels $\Delta P_{qq,\pm}^{(U)}(\xi, \alpha_s(Q^2))$ start to become different beyond LO as a result of the presence of transitions between quarks and antiquarks. The singlet evolution equation reads

$$\frac{d}{d\ln Q^2}\Delta\vec{v}^{(U)}(\xi, Q^2) = \left[\Delta\hat{P}^{(U)} \otimes \Delta\vec{v}^{(U)}\right](\xi, Q^2) \,. \tag{11}$$

We write the singlet evolution matrices for the space-like and time-like cases as

$$\Delta\hat{P}^{(S)} \equiv \begin{pmatrix} \Delta P_{qq}^{(S)} & \Delta P_{qg}^{(S)} \\ \Delta P_{gq}^{(S)} & \Delta P_{gg}^{(S)} \end{pmatrix} , \quad \Delta\hat{P}^{(T)} \equiv \begin{pmatrix} \Delta P_{qq}^{(T)} & 2n_f \Delta P_{gq}^{(T)} \\ \frac{1}{2n_f}\Delta P_{qg}^{(T)} & \Delta P_{gg}^{(T)} \end{pmatrix} . \tag{12}$$

The qq-entries in (12) are expressed as

$$\Delta P_{qq}^{(U)} = \Delta P_{qq,+}^{(U)} + \Delta P_{qq,PS}^{(U)} \,. \tag{13}$$

$\Delta P_{qq,PS}^{(U)}$ which vanishes in LO is called the 'pure singlet' splitting function since it only appears in the singlet case. To NLO, all splitting functions in (9)-(13) have the perturbative expansion

$$\Delta P_{ij}^{(U)}(\xi, \alpha_s) = \left(\frac{\alpha_s}{2\pi}\right) \Delta P_{ij}^{(U),(0)}(\xi) + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta P_{ij}^{(U),(1)}(\xi) \,. \tag{14}$$

Just like their unpolarized counterparts, the polarized space-like and time-like splitting functions are equal in LO:

$$\Delta P_{ij}^{(T),(0)}(\xi) = \Delta P_{ij}^{(S),(0)}(\xi) \,. \tag{15}$$

Eqs. (15) are manifestations of the so-called Gribov-Lipatov relation (GLR) [18] which connects space-like and time-like structure functions within their respective physical regions ($\xi < 1$) and is known to be broken beyond LO in the unpolarized case [15, 19]. Recalling that for $x < 1$ [9, 21]

$$\begin{aligned}
\Delta P_{qq}^{(S),(0)}(x) &= C_F \frac{1+x^2}{1-x} \,, \\
\Delta P_{qg}^{(S),(0)}(x) &= 2T_f \left[2x - 1\right] \,, \\
\Delta P_{gq}^{(S),(0)}(x) &= C_F \left[2 - x\right] \,, \\
\Delta P_{gg}^{(S),(0)}(x) &= 2C_A \Big[\frac{1}{1-x} - 2x + 1\Big] \,,
\end{aligned} \tag{16}$$

where

$$C_F = \frac{4}{3},\ C_A = 3,\ T_f = T_R n_f = \frac{1}{2}n_f,\ \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_f \,, \tag{17}$$

it becomes obvious that the space-like and time-like LO quantities are also directly related by analytic continuation through $x = 1$:

$$\begin{aligned}
&\Delta P_{qq,\pm}^{(T),(0)}(z) = -z\Delta P_{qq,\pm}^{(S),(0)}(\frac{1}{z}) \, , \\
&\Delta P_{qq}^{(T),(0)}(z) = -z\Delta P_{qq}^{(S),(0)}(\frac{1}{z}) \, , \;\; \Delta P_{gq}^{(T),(0)}(z) = \frac{C_F}{2T_f} z\Delta P_{qg}^{(S),(0)}(\frac{1}{z}) \, , \\
&\Delta P_{qg}^{(T),(0)}(z) = \frac{2T_f}{C_F} z\Delta P_{gq}^{(S),(0)}(\frac{1}{z}) \, , \;\; \Delta P_{gg}^{(T),(0)}(z) = -z\Delta P_{gg}^{(S),(0)}(\frac{1}{z}) \, ,
\end{aligned} \tag{18}$$

where $z < 1$. For future convenience we have explicitly written out the singlet as well as the non-singlet sector even though all LO quark-to-quark splitting functions coincide, $\Delta P_{qq,+}^{(U),(0)} = \Delta P_{qq,-}^{(U),(0)} = \Delta P_{qq}^{(U),(0)}$. Eqs. (18) represent the analytic continuation or Drell-Levy-Yan relation (ACR) [17] to LO which we cast into the generic form

$$\Delta P_{ij}^{(T),(0)}(z) = z\mathcal{AC}\left[\Delta P_{ji}^{(S),(0)}(x = \frac{1}{z})\right] \, , \tag{19}$$

where the operation $\mathcal{AC}$ analytically continues any function to $x \to 1/z > 1$ and correctly adjusts the color factor and the sign depending on the splitting function under consideration, cf. Eqs. (18). The LO relations (18) are based on symmetries of tree diagrams under crossing, and one therefore has to expect that they are in general no longer valid when going to NLO, depending on the regularization and the factorization/renormalization prescriptions used in the NLO calculation. This is exactly what happens in dimensional regularization in the $\overline{\rm MS}$ scheme as was shown in [15] for the unpolarized non-singlet case. Fortunately, as was also demonstrated in [15], the breaking of the ACR arising beyond LO is essentially due to kinematics and can therefore be rather straightforwardly detected within the method used in [15, 16, 22] to calculate splitting functions. We will now first collect the findings of [15] concerning the connection between the space-like and time-like flavor non-singlet configurations in the unpolarized case and analyze in detail their extension to the singlet sector made in [16]. Afterwards we will apply the results to the polarized case.

3 NLO results for the unpolarized case

Eqs. (4)-(19) above have been written down for the polarized case, but they all apply equally well to the unpolarized one when all Δ's are removed and, obviously, the unpolarized LO splitting functions $P_{ij}^{(S),(0)}$ as calculated in [9],

$$\begin{aligned}
P_{qq}^{(S),(0)}(x) &= C_F \frac{1+x^2}{1-x} , \\
P_{qg}^{(S),(0)}(x) &= 2T_f \left[x^2 + (1-x)^2\right] , \\
P_{gq}^{(S),(0)}(x) &= C_F \left[\frac{1+(1-x)^2}{x}\right] , \\
P_{gg}^{(S),(0)}(x) &= 2C_A \Big[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\Big] ,
\end{aligned} \tag{20}$$

(for $x < 1$), are used in Eq. (16). Furthermore, in the unpolarized case with pure photon exchange one needs to introduce two independent structure functions $\mathcal{F}_1^{(U)}$, $\mathcal{F}_2^{(U)}$ (see, e.g., [23, 24, 25]) with short-distance cross sections $C_{i,1}^{(U)}$, $C_{i,2}^{(U)}$ $(i = q, g)$, respectively.

In [15, 16, 22] the unpolarized NLO evolution kernels for the space-like situation were calculated using a method [26] that is as close as possible to parton model intuition since it is based explicitly on the factorization properties of mass singularities in the light-like axial gauge. The general strategy here consists of a rearrangement of the perturbative expansion which makes explicit the factorization into a part which does not contain any mass singularity and another one which contains all (and only) mass singularities. More explicitly, $M_{j,k}$ $(j = q, g,\ k = 1, 2)$, the contribution of virtual (space-like) photon–quark or photon–gluon scattering to the structure functions $\mathcal{F}_k^{(S)}$ on parton-level, is expanded into two-particle irreducible (2PI) kernels. In the light-cone gauge these 2PI kernels have been proven to be finite as long as the external legs are kept unintegrated, such that all collinear singularities originate from the integrations over the momenta flowing in the lines connecting the various kernels [26]. This allows for projecting out these singularities [15], and $M_{j,k}$ can thus be written in the factorized form

$$M_{j,k} = \sum_{i=q,g} C_{i,k}^{(S)} \otimes \Gamma_{ij}^{(S)} , \tag{21}$$

where the $C_{i,k}^{(S)}$ are finite (and obviously depend on the hard process considered), whereas the $\Gamma_{ij}^{(S)} \equiv \Gamma_{ij}^{(S)}(x, \alpha_s, 1/\epsilon)$ contain just the mass singularities (which appear as poles in

ϵ when using dimensional regularization, $d = 4 - 2\epsilon$) and are process-*in*dependent. The $\Gamma_{ij}^{(S)}$ are to be convoluted with bare ('unrenormalized') parton densities which must cancel their poles. As was shown in [15], the $\overline{\mathrm{MS}}$ scheme Altarelli-Parisi [9] kernels one is looking for, appear order by order as the residues of the $1/\epsilon$ poles in $\Gamma_{ij}^{(S)}$,

$$\Gamma_{ij}^{(S)}(x, \alpha_s, \frac{1}{\epsilon}) = \delta(1-x)\delta_{ij} - \frac{1}{\epsilon}\left[\left(\frac{\alpha_s}{2\pi}\right) P_{ij}^{(S),(0)}(x) + \frac{1}{2}\left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(S),(1)}(x) + \ldots\right] + \mathcal{O}(\frac{1}{\epsilon^2}) . \tag{22}$$

The NLO contribution to the hard short-distance cross sections in the $\overline{\mathrm{MS}}$ scheme is obtained by calculating the full ('bare') subprocess cross sections $\hat{C}_{i,k}^{(S),(1)}(x, \frac{1}{\epsilon})$ ($i = q, g$, $k = 1, 2$) and subtracting off the poles:

$$C_{i,k}^{(S),(1)}(x) = \hat{C}_{i,k}^{(S),(1)}(x, \frac{1}{\epsilon}) + \frac{1}{\epsilon}\left(\frac{Q^2}{4\pi\mu^2}\right)^{-\epsilon} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} P_{qi}^{(S),(0)}(x) , \tag{23}$$

where μ is the arbitrary mass scale to be introduced in dimensional regularization.

In the time-like region one can repeat the above procedure and introduce analogous quantities $\Gamma_{ji}^{(T)}(z, \alpha_s, 1/\epsilon)$ that contain all final-state mass singularities arising in a fragmentation process. It turns out [15] that the task of establishing the connection between $\Gamma_{ij}^{(S)}$ and $\Gamma_{ji}^{(T)}$ via analytic continuation can be reduced to understanding the differences between the 2PI kernels in the space-like and time-like situations. These essentially amount [15] to relative extra phase space factors of $(k_2 \cdot n/k_1 \cdot n)^{-2\epsilon}$ in the time-like case, where k_1 and k_2 are the momenta of the particles entering or leaving a 2PI kernel, respectively. Here n is the vector specifying the light-cone gauge and the longitudinal direction, i.e., $(k_2 \cdot n/k_1 \cdot n)^{-2\epsilon} \equiv \zeta^{-2\epsilon}$ with ζ to be interpreted as the fraction of the momentum k_1 transferred to the particle with k_2. In the unpolarized case a further difference arises from the spin-average factor for initial-state gluons which is $(d-2)^{-1} = 1/2(1-\epsilon)$ in d dimensions. As apparent from (18), the off-diagonal splitting functions interchange their roles during the transition from the space-like to the time-like situation. In particular, the space-like $P_{qg}^{(S)}$ which includes the spin-averaging factor $(d-2)^{-1}$ gives rise to the time-like $P_{gq}^{(T)}$ which should just have the spin-average $1/2$, and vice versa for $P_{gq}^{(S)}$, $P_{qg}^{(T)}$. These effects have to be taken into account along with those coming from the $(k_2 \cdot n/k_1 \cdot n)^{-2\epsilon}$ terms mentioned above. Consequently, all this gives on aggregate for $z < 1$

$$\Gamma_{qq,\pm}^{(T)}(z, \alpha_s, \frac{1}{\epsilon}) = -z^{1-2\epsilon}\Gamma_{qq,\pm}^{(S)}(\frac{1}{z}, \alpha_s, \frac{1}{\epsilon}) ,$$

$$\Gamma^{(T)}_{qq}(z,\alpha_s,\frac{1}{\epsilon}) = -z^{1-2\epsilon}\Gamma^{(S)}_{qq}(\frac{1}{z},\alpha_s,\frac{1}{\epsilon})\,,\quad \Gamma^{(T)}_{gq}(z,\alpha_s,\frac{1}{\epsilon}) = \frac{C_F}{2T_f}z^{1-2\epsilon}(1-\epsilon)\Gamma^{(S)}_{qg}(\frac{1}{z},\alpha_s,\frac{1}{\epsilon})\,,$$
$$\Gamma^{(T)}_{qg}(z,\alpha_s,\frac{1}{\epsilon}) = \frac{2T_f}{C_F}\frac{z^{1-2\epsilon}}{1-\epsilon}\Gamma^{(S)}_{gq}(\frac{1}{z},\alpha_s,\frac{1}{\epsilon})\,,\quad \Gamma^{(T)}_{gg}(z,\alpha_s,\frac{1}{\epsilon}) = -z^{1-2\epsilon}\Gamma^{(S)}_{gg}(\frac{1}{z},\alpha_s,\frac{1}{\epsilon})\,. \quad (24)$$

We also include now the corresponding relations for the hard subprocess cross sections $\hat{C}^{(U),(1)}_{i,k}$ *before* subtraction of their pole terms (see Eq. (23)):

$$\begin{aligned}
\hat{C}^{(T),(1)}_{q,k}(z,\frac{1}{\epsilon}) &= -z^{1-2\epsilon}\hat{C}^{(S),(1)}_{q,k}(\frac{1}{z},\frac{1}{\epsilon})\,,\\
\hat{C}^{(T),(1)}_{g,k}(z,\frac{1}{\epsilon}) &= \frac{C_F}{2T_f}z^{1-2\epsilon}(1-\epsilon)\hat{C}^{(S),(1)}_{g,k}(\frac{1}{z},\frac{1}{\epsilon})\,.
\end{aligned} \quad (25)$$

It becomes obvious that higher pole terms in the expression for $\Gamma^{(S)}_{ij}$ in (22) will generate additional contributions to the single pole of $\Gamma^{(T)}_{ji}$ when they are combined with the factors $z^{-2\epsilon}$ or $(1-\epsilon)^{\pm 1}$ in (24), e.g.,

$$\frac{1}{\epsilon^2}z^{-2\epsilon} = \frac{1}{\epsilon^2} - \frac{2}{\epsilon}\ln z + \mathcal{O}(1)\,. \quad (26)$$

In the same way the pole terms in $\hat{C}^{(S),(1)}_{i,k}(x,\frac{1}{\epsilon})$ will give rise to extra finite contributions to the $\hat{C}^{(T),(1)}_{i,k}$ that remain after the pole is subtracted. Following [15] we separate all such ACR-violating contributions by writing

$$\begin{aligned}
\Gamma^{(T)}_{ij}(z,\alpha_s,\frac{1}{\epsilon}) &= z\mathcal{AC}\left[\Gamma^{(S)}_{ji}(x=\frac{1}{z},\alpha_s,\frac{1}{\epsilon})\right] + \Gamma^{\epsilon}_{ij}(z,\alpha_s,\frac{1}{\epsilon})\,,\\
\hat{C}^{(T),(1)}_{i,k}(z,\frac{1}{\epsilon}) &= z\mathcal{AC}\left[\hat{C}^{(S),(1)}_{i,k}(x=\frac{1}{z},\frac{1}{\epsilon})\right] + C^{\epsilon,(1)}_{i,k}(z)\,,
\end{aligned} \quad (27)$$

where, as before, $k=1,2$, $i,j=q,g$ (or '$ij=qq,\pm$' for the non-singlet case). We have extended the notation $\mathcal{AC}[...]$ for the analytic continuation (see Eq. (19)) to the short-distance cross sections, its action here being obvious from Eq. (25). One can now go through the NLO calculation [22] of the $\Gamma^{(S),(1)}_{ij}$ graph by graph to pick up the $1/\epsilon^2$ pole terms and thus to extract the contributions to the $\Gamma^{\epsilon}_{ij}(z,\alpha_s,1/\epsilon)$ that break the ACR. For this purpose we present in Fig. 1 the basic topologies for all NLO diagrams involved here, where the notation is as introduced in [15, 22][3]. For topologies (cd),(e),(fg) the higher pole terms are necessarily proportional to the pole terms in the renormalization constants as listed in [15, 22]. The corrections to the ACR coming from these graphs are therefore

[3]We note that the remaining topologies ((b),(jk)) introduced in [15, 22] do not possess higher pole terms and thus do not contribute to the Γ^{ϵ}_{ij}.

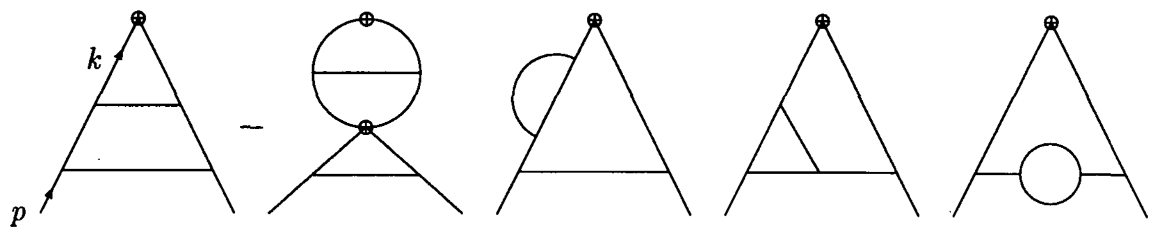

Figure 1: Basic topologies of the diagrams that contribute to Γ^{ϵ}_{ij} (as defined in Eq. (27)) in NLO.

easily detected upon insertion into Eq. (24). The higher pole terms of the genuine ladder graph (h), which is quadratic in the LO 2PI kernels, are given by the convolution of the LO splitting function for the upper rung with that corresponding to the lower one, and again the result is straightforwardly obtained. The only subtlety arises for the subtraction graphs (i) whose contributions to $\Gamma^{(S),(1)}_{ij}$ are proportional to [13]

$$\text{graph(i)} \sim \frac{1}{\epsilon^2}\left((1-z)^{-\epsilon} P^{(S),(0)}_{ik,4-2\epsilon}\right) \otimes P^{(S),(0)}_{kj} \,, \tag{28}$$

where $P^{(S),(0)}_{ik,4-2\epsilon}$ denotes the $(d = 4-2\epsilon)$-dimensional LO splitting function standing for the upper part of the diagram, the factor $(1-z)^{-\epsilon}$ arising from phase space. The lower part of graph (i) is represented by $P^{(S),(0)}_{kj}$ which is the usual *four*-dimensional LO splitting function. Application of the rules (24) to topology (i) is then to be understood as to include the kinematical $z^{-2\epsilon}$ corrections only in the kernel representing the *upper* part of the diagram [15], which gives

$$\text{graph(i)} \sim z\mathcal{AC}\left[\frac{1}{\epsilon}\left(-2\ln z P^{(S),(0)}_{ik}\right) \otimes P^{(S),(0)}_{kj}\right] \tag{29}$$

as the contributions to the Γ^{ϵ}_{ij}. If required, the spin-averaging factors $(1-\epsilon)^{\pm 1}$ also have to be taken into account. Contrary to all other topologies, adjusting the spin-averaging in graphs (i) generates corrections also to the ACR for the *diagonal* NLO singlet splitting functions $P^{(U),(1)}_{qq,PS}$ and $P^{(U),(1)}_{gg}$. As an example, Fig. 2 shows the graph of topology (i) for the case of the 'pure singlet' function $P^{(U),(1)}_{qq,PS}$ for both the space-like and the time-like situations. According to Eq. (28), the contribution of this graph in the space-like case

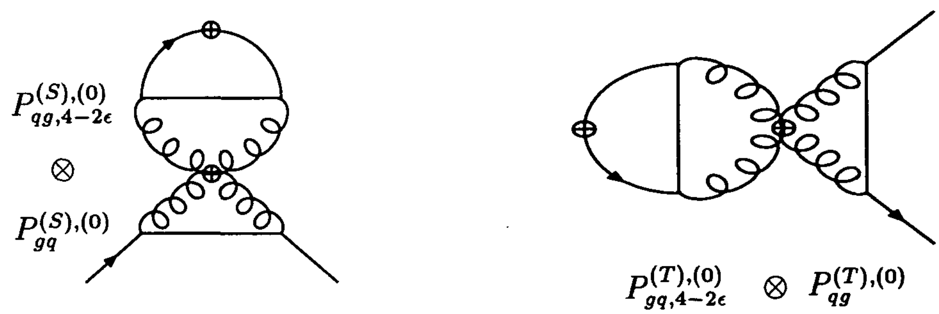

Figure 2: Graphs of topology (i) contributing to $P^{(U),(1)}_{qq,PS}$ in the space-like (left) and time-like (right) situations.

involves the convolution of the d-dimensional splitting function $P^{(S),(0)}_{qg,4-2\epsilon}$ – which contains the spin-averaging factor $(1-\epsilon)^{-1}$ – with the *four*-dimensional function $P^{(S),(0)}_{gq}$. In the time-like case this obviously turns into the convolution of the d-dimensional $P^{(T),(0)}_{gq,4-2\epsilon}$ with the *four*-dimensional $P^{(T),(0)}_{qg}$, and *no* factor $(1-\epsilon)^{-1}$ is involved. The reversed situation appears in the $C_F T_f$ part of $P^{(U),(1)}_{gg}$. It is straightforward to account for these effects.

We have now collected all ingredients for calculating the corrections to the ACR arising from the $z^{-2\epsilon}$ terms (and, if applicable, the spin-averaging factors) in Eqs. (24), i.e., for the functions Γ^{ϵ}_{ij} and $C^{\epsilon,(1)}_{i,k}$ in (27). Following Eqs. (22),(23) we keep just the residues of the $1/\epsilon$ poles in Γ^{ϵ}_{ij} and subtract the pole terms from the 'bare' subprocess cross sections. We then rewrite Eq. (27) as

$$P^{(T),(1)}_{ij}(z) = zAC\left[P^{(S),(1)}_{ji}(x=\frac{1}{z})\right] + P^{\epsilon,(1)}_{ij}(z)\,, \tag{30}$$

$$C^{(T),(1)}_{i,k}(z) = zAC\left[C^{(S),(1)}_{i,k}(x=\frac{1}{z})\right] + C^{\epsilon,(1)}_{i,k}(z)\,, \tag{31}$$

where, using the unpolarized LO splitting functions [9] of Eq. (20), we have

$$\begin{aligned}
P^{\epsilon,(1)}_{qq,\pm}(z) &= \beta_0 P^{(S),(0)}_{qq}(z)\ln z\ , \\
P^{\epsilon,(1)}_{qq,PS}(z) &= -C_F T_f \frac{4}{9z}\left[(1-z)(38+47z+38z^2)+3(1+z)(4+11z+4z^2)\ln z\right]\ , \\
P^{\epsilon,(1)}_{gq}(z) &= 2(C_F-C_A)P^{(S),(0)}_{gq}(z)\Big[-4S_1(z)+2\ln(1-z)\ln z+\ln(1-z)\Big] \\
&+ \frac{C_F^2}{2z}\Big[-4z(3-z)\ln z-20+28z-5z^2\Big] \\
&+ \frac{C_A C_F}{9z}\Big[6\left(4z^3+15z^2+18z+19\right)\ln z+(1-z)(235+55z+64z^2)\Big]\ ,
\end{aligned}$$

$$
\begin{aligned}
P_{qg}^{\epsilon,(1)}(z) &= \frac{4}{3}T_f P_{qg}^{(S),(0)}(z)(1-2\ln z) \\
&+ 2(C_F - C_A)P_{qg}^{(S),(0)}(z)\Big[4S_1(z) - 2\ln(1-z)\ln z + \ln(1-z)\Big] \\
&+ C_F T_f\left[-4(2-z)(1-2z)\ln z - 5 + 28z - 20z^2\right] \\
&+ \frac{2C_A T_f}{9z}\Big[6(-4-z-46z^2+9z^3)\ln z - 64 - 24z - 114z^2 + 169z^3\Big] \ , \\
P_{gg}^{\epsilon,(1)}(z) &= -P_{qq,PS}^{\epsilon,(1)}(z) + \beta_0 P_{gg}^{(S),(0)}(z)\ln z \ , \\
C_{q,k}^{\epsilon,(1)}(z) &= 2\ln z P_{qq}^{(S),(0)}(z) \ , \\
C_{g,k}^{\epsilon,(1)}(z) &= (2\ln z + 1)P_{gq}^{(S),(0)}(z) \ .
\end{aligned} \tag{32}
$$

For the off-diagonal $P_{ij}^{\epsilon,(1)}$ in (32) we have introduced the function [16]

$$
S_1(z) \equiv \int_0^{1-z} dy \frac{\ln(1-y)}{y} = -\mathrm{Li}_2(1-z) \tag{33}
$$

with the Dilogarithm function Li_2.

The final step is to determine the analytic continuation of the space-like NLO splitting functions $P_{ji}^{(S),(1)}$ as published in [15, 16, 22] and of the short-distance cross sections $C_{i,k}^{(S),(1)}$ (see, e.g., [25]) by using the operation $\mathcal{AC}[...]$ defined above. This is a straightforward task apart from two subtleties [15]. Firstly, one has to recall that – as a result of the finiteness of the 2PI kernels in the light-cone gauge – the $1/\epsilon$ poles in the expression for $\Gamma_{ji}^{(S)}$ originate from the final integration over the momentum k of the off-shell particle emerging from the uppermost kernel (see Fig. 1 for notation). This momentum obviously satisfies $k^2 < 0$ in the space-like case, but becomes *time-like* ($k^2 > 0$) when dealing with the $\Gamma_{ij}^{(T)}$. In other words, the transition from the space-like to the time-like situation does not only involve analytically continuing to $x > 1$ but also to $k^2 > 0$, crossing the threshold at $k^2 = 0$. Since the $1/\epsilon$ poles arise from terms like $(-k^2)^{-1-\epsilon}$ (for the space-like case) which are integrated over $-k^2$ down to $-k^2 = 0$, factors of $(-1)^{-\epsilon}$ will appear in the virtual integrals when going to $k^2 > 0$. Using $\mathrm{Re}[(-1)^{-\epsilon}] \approx 1 - \epsilon^2\pi^2/2$ one realizes that this will result in extra π^2-contributions when multiplied by double pole terms present in intermediate stages of the calculations [15]. The latter arise from the emission of soft gluons in the qqg vertex. A similar feature is present for the $qq\gamma$ vertex [23] and thus appears in the quark short-distance cross section $C_{q,k}^{(U),(1)}$. Here it is the transition

$q^2 < 0 \to q^2 > 0$ that is responsible for the effect [15] which only affects the endpoint contributions at $z = 1$ to be discussed below.

The other subtlety concerns the analytic continuation to $x > 1$ of terms $\sim \ln^i(1-x)$ ($i = 1, 2$) appearing in the $P_{ji}^{(S),(1)}(x)$ and in the $C_{i,k}^{(S),(1)}$. To find the correct answer for this, one has to go through the relevant real (phase-space), virtual, and convolution integrals in the limit $x > 1$. It turns out (see also [15]) that all integrals can be smoothly continued through $x = 1$ via $\ln^i(1-x) \to \ln^i(x-1)$, the only relevant exception being one of the scalar three-point functions with a light-cone gauge propagator which, for $x < 1$, was given in Eq. (A.15) of ref. [22]. This particular three-point function only contributes to the NLO splitting functions, but not to the hard subprocess cross sections $C_{q,k}^{(S),(1)}$. Upon recalculation of the function for $x > 1$ one finds that the correct continuation yields $\ln^2(1-x) \to \ln^2(x-1) + \pi^2$ in this case. When combining this result with that for the crossing of the $k^2 = 0$ threshold discussed above, one arrives at the interesting finding that the correct analytic continuation to $x > 1$ of *all* terms $\sim \ln^i(1-x)$ ($i = 1, 2$) in the NLO *splitting functions* $P_{ji}^{(S),(1)}(x)$ is effectively obtained by simply substituting

$$\ln(1-x) \longrightarrow \ln(x-1)\ , \tag{34}$$

$$\ln^2(1-x) \longrightarrow \ln^2(x-1) - \pi^2\ . \tag{35}$$

For the non-singlet case, in which no $\ln^2(1-x)$ terms appear in the space-like NLO splitting function, this result is in agreement with the conclusion drawn in ref. [15] that the extra π^2 terms stemming from the threshold at $k^2 = 0$ and from the three-point function cancel each other. In case of the NLO short-distance cross sections $C_{i,k}^{(S),(1)}$ there are again only single powers of $\ln(1-x)$, and Eq. (34) provides their correct analytic continuation.

Combining everything, we arrive at the final result for $z < 1$ for the NLO ($\overline{\rm MS}$ scheme) non-singlet and singlet time-like splitting functions $P_{ij}^{(T),(1)}(z)$ and the NLO time-like short-distance cross sections $C_{i,k}^{(T),(1)}(z)$. The result for the NLO splitting functions is in complete agreement with that of [15, 16], apart from a known misprint[4] in [16]. The

[4]The term $(10 - 18x - 16x^2/3)\ln x$ in the $C_F T_f$ part of $P_{qq}^{(T),(1)}(x)$ in [16] must correctly read [27] $(-10 - 18x - 16x^2/3)\ln x$.

endpoint contributions, i.e., the terms $\sim \delta(1-z)$, to the diagonal splitting functions can be obtained from the fermion number and energy-momentum-conservation conditions [15, 16] and are exactly the same as in the space-like situation. In case of the NLO time-like quark short-distance cross section the endpoint contributions differ from those in the space-like situation by $C_F \pi^2 \delta(1-z)$ which is just the effect of the above mentioned π^2-correction when crossing the threshold at $Q^2 = 0$ [15]. Taking this into account, the results for the $C_{i,k}^{(T),(1)}(z)$ agree with those in, e.g., [25][5]. Since all the unpolarized expressions have appeared in the literature we do not repeat them here. We only note that, in contrast to the leading order (cf. Eq. (15)), the NLO differences $P_{ij}^{(T),(1)}(\xi) - P_{ij}^{(S),(1)}(\xi)$ and $C_{q,k}^{(T),(1)}(\xi) - C_{q,k}^{(S),(1)}(\xi)$ are non-zero (note that here $\xi \leq 1$ in both the space-like and the time-like functions), i.e., in addition to the ACR the GLR is also broken beyond LO [15], as we anticipated in the introduction.

4 The breaking of the ACR revisited

Before addressing the polarized case which we are mainly interested in, let us return for a moment to Eqs. (30)-(32). The rather simple structure of the $P_{ij}^{\epsilon,(1)}(z)$ and its transparent origin suggest that there could be a more straightforward way of linking the time-like and the analytically continued space-like NLO splitting functions, than going through Fig. 1 graph by graph and picking up the higher pole terms. The starting point for such considerations is to notice that Eq. (19) (when adapted to the *un*polarized case, i.e., with the Δ's omitted and the $P_{ji}^{(S),(0)}$ as given in (20)) only states that the *four*-dimensional LO splitting functions obey the ACR. The rule must break down for the $(d = 4 - 2\epsilon)$-dimensional counterparts, $P_{ij,4-2\epsilon}^{(U),(0)}$, of the $P_{ij}^{(U),(0)}$ as an immediate consequence of Eq. (24). We write down a LO analogue of Eq. (30) in d dimensions,

$$P_{ij,4-2\epsilon}^{(T),(0)}(z) = z\mathcal{AC}\left[P_{ji,4-2\epsilon}^{(S),(0)}(x = \frac{1}{z})\right] + P_{ij}^{\epsilon,(0)}(z) \qquad (36)$$

(for $z < 1$), the main difference being that the LO $P_{ij}^{\epsilon,(0)}(z)$ are only $\mathcal{O}(\epsilon)$ and not $\mathcal{O}(1)$:

$$P_{qq}^{\epsilon,(0)}(z) \quad = \quad (-2\ln z)\epsilon P_{qq}^{(S),(0)}(z) \; ,$$

[5] There is a typographical error in the first equation of appendix II in [25]: the prefactor of the $\ln x$ term should correctly read $3(1+x^2)/(1-x)$.

$$
\begin{aligned}
P_{gq}^{\epsilon,(0)}(z) &= (-2\ln z - 1)\epsilon P_{gq}^{(S),(0)}(z) \ , \\
P_{qg}^{\epsilon,(0)}(z) &= (-2\ln z + 1)\epsilon P_{qg}^{(S),(0)}(z) \ , \\
P_{gg}^{\epsilon,(0)}(z) &= (-2\ln z)\epsilon P_{gg}^{(S),(0)}(z) \ . \qquad (37)
\end{aligned}
$$

As already seen from the example of Eq. (28), the pieces $\sim \epsilon$ in the d-dimensional LO splitting functions result in finite contributions in the calculation of the *NLO* splitting functions. One therefore suspects that the breakdown of the ACR beyond leading order in the $\overline{\mathrm{MS}}$ scheme, as expressed by Eqs. (30)-(32), is entirely driven by the corresponding breaking in the part $\sim \epsilon$ of the d-dimensional *LO* splitting functions, cf. Eqs. (36),(37). If this is indeed the case, then the functions $P_{ij}^{(T),(1)}$, $C_{i,k}^{(T),(1)}$ and

$$
\begin{aligned}
\tilde{P}_{ij}^{(T),(1)} &\equiv zAC\left[P_{ji}^{(S),(1)}(x=\frac{1}{z})\right] \ , \\
\tilde{C}_{i,k}^{(T),(1)} &\equiv zAC\left[C_{i,k}^{(S),(1)}(x=\frac{1}{z})\right] \ , \qquad (38)
\end{aligned}
$$

respectively, should be simply related by a factorization scheme transformation[6]. The general form of such a transformation can be determined from the condition that is must leave any physical quantity such as, e.g., $\mathcal{F}_1^{(T)}$ or $\mathcal{F}_2^{(T)}$ invariant, and reads

$$
\begin{aligned}
P_{qq,\pm}^{(T),(1)} &\longrightarrow P_{qq,\pm}^{(T),(1)} - \frac{\beta_0}{2} z_{qq}^{(T)} \ , \\
\hat{P}^{(T),(1)} &\longrightarrow \hat{P}^{(T),(1)} - \frac{\beta_0}{2}\hat{Z}^{(T)} + \left[\hat{Z}^{(T)}, \hat{P}^{(T),(0)}\right]_{\otimes} \ , \\
C_{i,k}^{(T),(1)} &\longrightarrow C_{i,k}^{(T),(1)} - z_{iq}^{(T)} \ , \qquad (39)
\end{aligned}
$$

where the subscript '$\otimes$' denotes convolution when evaluating the commutator. Again, $\hat{P}^{(T),(0)}$ and $\hat{P}^{(T),(1)}$ are the (unpolarized) LO and NLO evolution matrices, respectively (cf. Eq. (12)), and $z_{qq}^{(T)}$ and the 2×2 matrix $\hat{Z}^{(T)}$ generate the transformation. In analogy with Eq. (12) we set[7]

$$
\hat{Z}^{(T)} \equiv \begin{pmatrix} z_{qq}^{(T)} & 2n_f z_{gq}^{(T)} \\ \frac{1}{2n_f} z_{qg}^{(T)} & z_{gg}^{(T)} \end{pmatrix} \ . \qquad (40)
$$

[6]For the non-singlet case this possibility was already hinted at in [15].

[7]For our purposes, we do not need to distinguish between the non-singlet $z_{qq}^{(T)}$ and the qq entry in the singlet matrix $\hat{Z}^{(T)}$ even though these could be chosen differently in principle.

According to Eq. (37) one now expects that the choice

$$z_{ij}^{(T)}(z) = (2\ln z + a_{ij}) P_{ij}^{(S),(0)}(z) \tag{41}$$

with the logarithms being of kinematical origin and the a_{ij} resulting from the adjustment of the spin-averaging factors,

$$a_{qq} = a_{gg} = 0 \ , \quad a_{gq} = 1 \ , \quad a_{qg} = -1 \ , \tag{42}$$

transforms all time-like NLO ($\overline{\mathrm{MS}}$) quantities to a scheme in which they satisfy the ACR, i.e., in which

$$\begin{aligned} P_{ij}^{(T),(1)} &= \tilde{P}_{ij}^{(T),(1)} = z\mathcal{AC}\left[P_{ji}^{(S),(1)}(x=\frac{1}{z})\right] , \\ C_{i,k}^{(T),(1)} &= \tilde{C}_{i,k}^{(T),(1)} = z\mathcal{AC}\left[C_{i,k}^{(S),(1)}(x=\frac{1}{z})\right] . \end{aligned} \tag{43}$$

This indeed turns out to be the case as one finds upon insertion of the $z_{ij}^{(T)}$ in (41) into Eq. (39) and comparison with (32). We emphasize that the space-like NLO quantities on the right-hand-sides of Eq. (43) have not been transformed and are still in the $\overline{\mathrm{MS}}$ scheme. Eq. (43) therefore links quantities referring to different factorization schemes. This is perfectly legitimate since one is free to choose the factorization schemes independently for the space-like and time-like cases[8]. On the other hand, it does not really appear sensible from a physical point of view, and it actually turns out [27] that the transformed time-like NLO splitting functions of (43) do no longer obey the energy-momentum-conservation condition. Anyway the above scheme transformation is not meant to be used in any practical calculation, it just serves to identify the breakdown of the ACR beyond LO as a mere matter of convention and provides a very transparent and remarkably simple way of obtaining the correct $\overline{\mathrm{MS}}$ time-like splitting functions from the analytically continued space-like ones. We note that in [19] the unpolarized NLO time-like splitting functions and short-distance cross sections were calculated using the cut vertex method. In this formalism the validity of the ACR occurs quite naturally if certain renormalization conditions are imposed [28], and the results of [19] therefore correspond to the $\tilde{P}_{ij}^{(T),(1)}$, $\tilde{C}_{i,k}^{(T),(1)}$ in (43) rather than to the $\overline{\mathrm{MS}}$ scheme results.

[8]For instance, one can choose to factorize initial- and final-state collinear singularities differently in any higher order calculation.

5 NLO results for the polarized case

The extension of our results in sections 3 and 4 to the spin-dependent case is rather straightforward now. We first write down Eqs. (30),(31) for the polarized case,

$$\Delta P_{ij}^{(T),(1)}(z) = z\mathcal{AC}\left[\Delta P_{ji}^{(S),(1)}(x=\frac{1}{z})\right] + \Delta P_{ij}^{\epsilon,(1)}(z)\,, \tag{44}$$

$$\Delta C_{i}^{(T),(1)}(z) = z\mathcal{AC}\left[\Delta C_{i}^{(S),(1)}(x=\frac{1}{z})\right] + \Delta C_{i}^{\epsilon,(1)}(z)\,, \tag{45}$$

where $z < 1$ and where we have recalled from section 2 that contrary to the unpolarized case there is only one longitudinally polarized structure function for pure photon exchange, $\mathcal{G}_1^{(U)}$. For the space-like situation, NLO ($\overline{\mathrm{MS}}$) results for the short-distance cross sections $\Delta C_{i=q,g}^{(S),(1)}$ (i.e., the coefficient functions for $g_1^{(S)}$) have first been published quite some time ago [29, 30, 31], whereas the corresponding $\overline{\mathrm{MS}}$ splitting functions have been calculated only fairly recently via the OPE [12] (where they appear as the anomalous dimensions) and in [13, 14], where the method of [26, 15] was used. To be more precise, use of dimensional regularization in such NLO calculations for the polarized case implies to choose a prescription for dealing with the Dirac matrix γ_5 and the Levi-Civita tensor $\epsilon_{\mu\nu\rho\sigma}$ in $d \neq 4$ dimensions, which poses a non-trivial problem. In [12] the 'reading point' prescription of [32] with a fully anticommuting γ_5 was chosen, whereas [14] adopted the original definition for γ_5 of [33] (HVBM scheme) which is widely considered to be the most consistent method. A crucial feature in both [12] and [13, 14] was that the genuine ('naive') $\overline{\mathrm{MS}}$ scheme result for the non-singlet NLO splitting function $\Delta P_{qq,+}^{(S),(1)}(x)$ (cf. Eq. (10)) possessed the disagreeable property of having a non-zero first moment (x-integral), in obvious conflict with the conservation of the non-singlet axial current [34, 35] which demands that the first moment of the non-singlet quark combination $\Delta q_+^{(S)}$ be independent of Q^2. This effect was clearly due to the γ_5 prescriptions chosen and could be removed by a finite renormalization in [12] or, equivalently, by a factorization scheme transformation in [13, 14] generated by the difference of the d-dimensional LO quark-to-quark splitting functions for the unpolarized and polarized (HVBM scheme) cases[9],

$$\Delta P_{qq,4-2\epsilon}^{(S),(0)}(x) - P_{qq,4-2\epsilon}^{(S),(0)}(x) = 4C_F\epsilon(1-x) \; . \tag{46}$$

It turned out that both [12] and [13, 14] then arrived at the same final result for the space-like polarized NLO splitting functions and also for the coefficient functions $\Delta C_q^{(S),(1)}$ and $\Delta C_g^{(S),(1)}$ for which the previous results of [29, 30] and [31], respectively, were confirmed. In a strictly technical sense of the word, the results of [12, 13, 14] thus do not correspond to the $\overline{\rm MS}$ scheme. On the other hand, the 'γ_5-effect' described above has been known to occur in the HVBM scheme for some time [37, 38, 35, 39] and is purely artificial in the sense that it is related to helicity non-conservation at the quark-gluon tree-level vertex in $d \neq 4$ dimensions as expressed by the non-vanishing of the rhs of Eq. (46). Since physical requirements like the conservation of the non-singlet axial current serve to remove the effect in a straightforward and obvious way, the final results of [12, 13, 14] are nevertheless usually regarded as the 'real' conventional $\overline{\rm MS}$ scheme results.

The reason for going into this discussion is the following. If we use the final results of [12, 13, 14] for the $\Delta P_{ji}^{(S),(1)}$ and the $\Delta C_i^{(S),(1)}$ to obtain their time-like counterparts via Eqs. (44),(45), the factorization scheme transformation generated by (46) and performed in the space-like situation will obviously also affect the time-like result. On the other hand, in the case of the time-like NLO quantities, there appears to be no obvious requirement that enforces a certain value for, say, the first moment $\int_0^1 \Delta P_{qq,+}^{(T),(1)}(z)dz$. Thus in principle one would be allowed equally well to use the space-like 'naive' $\overline{\rm MS}$ scheme results in (44),(45), i.e., those that possess the wrong (non-vanishing) value for the integral of $\Delta P_{qq,+}^{(S),(1)}(x)$. However, taking into consideration the origin of the above 'γ_5-effect' as a pure artefact of the dimensional calculation in a certain γ_5 prescription, we decide against this latter option and will use the final results of [12, 13, 14], i.e., the 'real' $\overline{\rm MS}$ scheme results for the space-like case in what follows. This choice of factorization scheme appears most sensible since it actually turns out that Eq. (46) remains completely unchanged

[9] As was also shown in [13, 14, 35], the scheme transformation corresponding to (46) is needed at the same time to bring the first moment of the quark non-singlet coefficient function into agreement with the value given by the Bjørken sum-rule [36].

when transformed to the time-like situation,

$$\Delta P_{qq,4-2\epsilon}^{(T),(0)}(z) - P_{qq,4-2\epsilon}^{(T),(0)}(z) = 4C_F\epsilon(1-z) \;, \tag{47}$$

implying that the unphysical helicity non-conservation at the quark-gluon vertex in $d \neq 4$ dimensions also takes place in the time-like case if one uses the HVBM prescription for γ_5 [11]. Our choice obviously has implications for factorizing collinear singularities in NLO calculations of other cross sections with polarized final state particles: The 'real' $\overline{\mathrm{MS}}$ scheme factorization counterterm for all collinear poles coming from polarized (time-like) quark-to-quark transitions should be taken as [11]

$$-\frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\left(\frac{M_F^2}{4\pi\mu^2}\right)^{-\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[\Delta P_{qq}^{(T),(0)}(z) + 4C_F\epsilon(1-z)\right] \otimes \Delta\sigma_{4-2\epsilon}^{LO} \;,$$

where M_F is the factorization scale and $\Delta\sigma_{4-2\epsilon}^{LO}$ is some appropriate polarized Born-level cross section in d dimensions.

After these precautions we can turn to the analytic continuation to $x > 1$ of the space-like NLO quantities which is required for Eqs. (44),(45) and works in exactly the same way as for the unpolarized case studied in section 3. The other ingredients to Eqs. (44),(45), $\Delta P_{ij}^{\epsilon,(1)}(z)$ and $\Delta C_i^{\epsilon,(1)}(z)$, are also straightforwardly calculated following the lines of section 3. The situation is facilitated by the fact that in the polarized case there are obviously no complications due to the gluon spin-averaging factors. Thus we only have to keep track of the $z^{-2\epsilon}$ terms. We find:

$$\begin{aligned}
\Delta P_{qq,\pm}^{\epsilon,(1)}(z) &= \beta_0 \Delta P_{qq}^{(S),(0)}(z)\ln z = P_{qq,\pm}^{\epsilon,(1)}(z) \;, \\
\Delta P_{qq,PS}^{\epsilon,(1)}(z) &= -12C_F T_f\left[(1+z)\ln z + 2(1-z)\right] \;, \\
\Delta P_{gq}^{\epsilon,(1)}(z) &= 4(C_F - C_A)\Delta P_{gq}^{(S),(0)}(z)\left[-2S_1(z) + \ln(1-z)\ln z\right] \\
&+ C_F^2\left[(4-5z)\ln z - 4(1-z)\right] + 8C_A C_F\left[(1+z)\ln z + 2(1-z)\right] \;, \\
\Delta P_{qg}^{\epsilon,(1)}(z) &= -\frac{8}{3}T_f \Delta P_{qg}^{(S),(0)}(z)\ln z + 4(C_F - C_A)\Delta P_{qg}^{(S),(0)}(z)\left[2S_1(z) - \ln(1-z)\ln z\right] \\
&+ 2C_F T_f\left[(5-4z)\ln z + 4(1-z)\right] + \frac{4}{3}C_A T_f\left[(10z-23)\ln z - 24(1-z)\right] \;, \\
\Delta P_{gg}^{\epsilon,(1)}(z) &= -\Delta P_{qq,PS}^{\epsilon,(1)}(z) + \beta_0 \Delta P_{gg}^{(S),(0)}(z)\ln z \;, \\
\Delta C_q^{\epsilon,(1)}(z) &= 2\ln z\,\Delta P_{qq}^{(S),(0)}(z) \;, \\
\Delta C_g^{\epsilon,(1)}(z) &= 2\ln z\,\Delta P_{gq}^{(S),(0)}(z) \;.
\end{aligned} \tag{48}$$

As for the unpolarized case in section 4 it turns out that all these terms can also be fully accounted for by a factorization scheme transformation, i.e., the $\overline{\mathrm{MS}}$ scheme $\Delta P_{ij}^{(T),(1)}$ and $\Delta C_i^{(T),(1)}$ and the corresponding analytically continued space-like functions,

$$\begin{aligned} \Delta\tilde{P}_{ij}^{(T),(1)} &\equiv z\mathcal{AC}\left[\Delta P_{ji}^{(S),(1)}(x=\frac{1}{z})\right] , \\ \Delta\tilde{C}_i^{(T),(1)} &\equiv z\mathcal{AC}\left[\Delta C_i^{(S),(1)}(x=\frac{1}{z})\right] , \end{aligned} \quad (49)$$

respectively, are related via Eqs. (39),(40) (with Δ's everywhere in (39),(40)) if one chooses

$$\Delta z_{ij}^{(T)}(z) = 2\ln z \Delta P_{ij}^{(S),(0)}(z) . \quad (50)$$

Now we finally insert everything into Eqs. (44),(45) and arrive at the final results for the time-like NLO quantities which, in case of the splitting functions, are conveniently expressed as differences with respect to the space-like situation, at the same time indicating the breakdown of the GLR:

$$\Delta P_{ij}^{(T),(1)}(z) = \Delta P_{ij}^{(S),(1)}(z) + \Delta_{ij}(z) , \quad (51)$$

where the $\Delta P_{ij}^{(S),(1)}(z)$ are found in [12, 13, 14] and

$$\begin{aligned} \Delta_{qq,\pm}(z) &= C_F^2 \frac{\ln z}{1-z}\left[4\left(1+z^2\right)\ln(1-z) - \left(1+3z^2\right)\ln z + z^2 + 4z + 1\right] , && (52) \\ \Delta_{qq,PS}(z) &= 4\, C_F T_f\,(\ln z - 3)\left[(1+z)\ln z + 2(1-z)\right] , && (53) \\ \Delta_{gq}(z) &= \frac{1}{3} C_F^2 \left[(2-z)\left[-24S_1(z) - 4\pi^2 + 6\ln^2(1-z) + 12\ln(1-z)\ln z \right.\right. \\ & \qquad \left.\left. -3\ln^2 z + 15\ln z\right] + (15z-6)\ln(1-z) - 33z + 54\right] \\ &+ \frac{1}{9} C_A C_F \left[6(2-z)\left[12S_1(z) + 2\pi^2 - 3\ln^2(1-z)\right]\right. \\ & \qquad \left. +(6-39z)\ln(1-z) - 18(z+4)\ln^2 z + 72(3z-1)\ln z - 71z + 4\right] \\ &+ \frac{4}{9} C_F T_f \left[3(2-z)\ln(1-z) + z + 4\right] , && (54) \\ \Delta_{qg}(z) &= \frac{8}{9} T_f^2 \left[-3(2z-1)(\ln(1-z) + \ln z) - 4z - 1\right] \\ &+ \frac{2}{3} C_F T_f \left[(2z-1)\left[24S_1(z) + 4\pi^2 - 6\ln^2(1-z) - 3\ln^2 z\right]\right. \\ & \qquad \left. +(6z-15)\ln(1-z) + 30\ln z - 78z + 57\right] \\ &+ \frac{2}{9} C_A T_f \left[6(2z-1)\left[-12S_1(z) - 2\pi^2 + 6\ln(1-z)\ln z \right.\right. \end{aligned}$$

$$
\begin{aligned}
& +3\ln^2(1-z)\Big] - (6z-39)\ln(1-z) + 36(1+z)\ln^2 z \\
& -3(26z+11)\ln z + 284z - 217\Big] \ , \qquad (55) \\
\Delta_{gg}(z) &= 4\,C_F T_f \left[6(1-z) + 6\ln z + (1+z)\ln^2 z\right] \\
&- \frac{8}{3} C_A T_f \frac{2z^2-3z+2}{1-z}\ln z \\
&+ \frac{2}{3} C_A^2 \frac{\ln z}{1-z}\Big[12(2z^2-3z+2)\ln(1-z) + 6(3z-4)\ln z \\
& -26z^2 + 63z - 26\Big] \ . \qquad (56)
\end{aligned}
$$

For the short-distance cross sections we obtain

$$
\begin{aligned}
\Delta C_q^{(T),(1)}(z) &= C_F \left[(1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ + 2\frac{1+z^2}{1-z}\ln z - \frac{3}{2}\frac{1}{(1-z)_+} + \frac{1}{2}(1-z) \right. \\
& \left. + \left(-\frac{9}{2} + \frac{2}{3}\pi^2\right)\delta(1-z)\right] \ , \qquad (57) \\
\Delta C_g^{(T),(1)}(z) &= C_F \left[(2-z)\ln(z^2(1-z)) - 4 + 3z\right] \ , \qquad (58)
\end{aligned}
$$

in agreement with the results of [11] for the corresponding choices $\Delta \tilde{f}_q^D(z) = -4(1-z)$ and $\Delta \tilde{f}_g^D(z) = 0$ in Eq. (14) of that paper[10]. The '+'-prescription in (57) is defined as usual via

$$
\int_0^1 dz f(z)\,(g(z))_+ \equiv \int_0^1 dz\,(f(z) - f(1))\,g(z) \ . \qquad (59)
$$

For numerical evaluations of our results it is convenient to have the Mellin-moments of the expressions above which are defined by

$$
f[n] \equiv \int_0^1 z^{n-1} f(z) dz \qquad (60)
$$

and are presented in the appendix. Fig. 3 provides a comparison of our results for the NLO ($\overline{\mathrm{MS}}$ scheme) time-like polarized and unpolarized splitting functions in Mellin-n space. It is interesting to observe that all LO and NLO time-like splitting functions obey $\Delta P_{ij}^{(T),(k)}[n] \to P_{ij}^{(T),(k)}[n]$ ($k = 0, 1$; $i, j = q, g$) as $n \to \infty$, i.e., as $z \to 1$, except for $\Delta P_{gq}^{(T),(1)}[n]$. A similar observation was made for the space-like quantities where, again, only $\Delta P_{gq}^{(S),(1)}[n]$ does not fulfil $\Delta P_{ij}^{(S),(k)}[n] \to P_{ij}^{(S),(k)}[n]$ as $n \to \infty$ [40]. Finally, the

[10] Note that our definition for the gluonic short-distance cross section differs by a factor of 2 from the one used in [11].

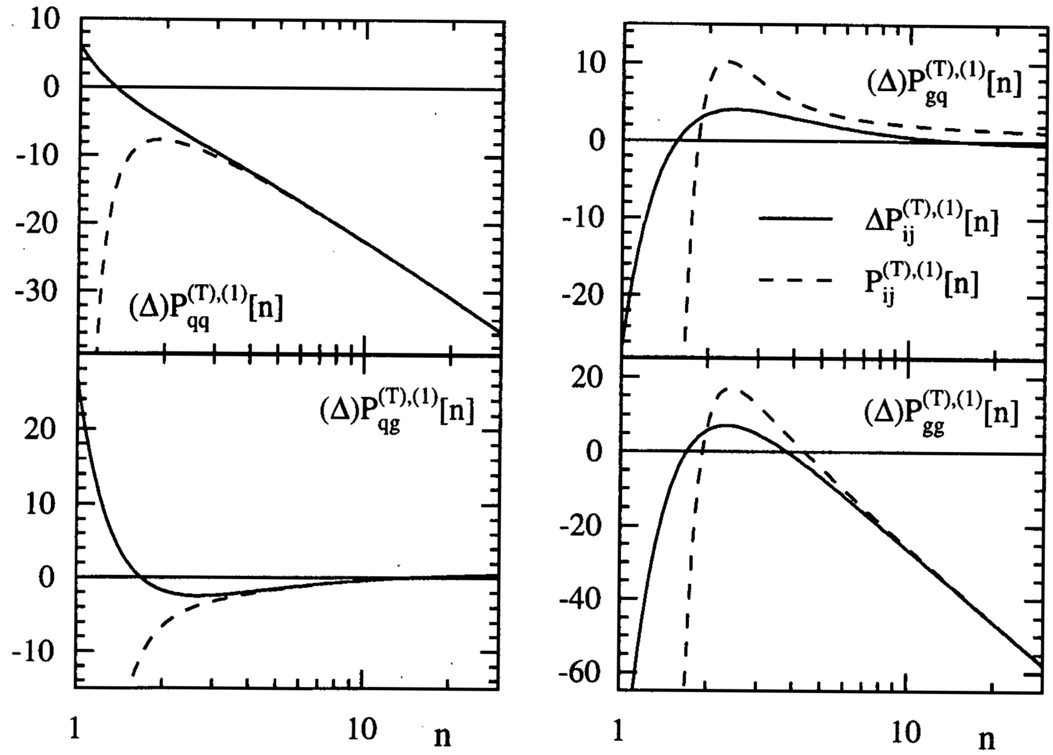

Figure 3: Comparison of the spin-dependent NLO ($\overline{\text{MS}}$) time-like singlet splitting functions $\Delta P_{ij}^{(T),(1)}[n]$ as functions of Mellin-n according to Eqs. (A.1)-(A.6) with the corresponding unpolarized ones, as taken from [27], for $f = 3$ flavors.

values for the first ($n = 1$) moments of the polarized NLO singlet quantities are given by

$$\Delta P_{qq}^{(T),(1)}[1] = 3C_F T_f \ , \quad \Delta P_{gq}^{(T),(1)}[1] = -3C_A C_F + C_F^2 \left(\frac{3}{2} - \pi^2\right) \ ,$$
$$\Delta P_{qg}^{(T),(1)}[1] = 2T_f \beta_0 \ , \quad \Delta P_{gg}^{(T),(1)}[1] = -\frac{5}{6}C_A^2 - 4C_F T_f - \frac{1}{3}C_A T_f - \frac{1}{3}\pi^2 C_A \beta_0 \ ,$$
$$\Delta C_q^{(T),(1)}[1] = 0 \ , \quad \Delta C_g^{(T),(1)}[1] = -\frac{29}{4}C_F \ . \tag{61}$$

6 A supersymmetric property of the NLO time-like splitting functions

In this section we finally very briefly address a relation that is conjectured for an $\mathcal{N} = 1$ supersymmetric Yang-Mills theory and connects all singlet splitting functions in a remarkably simple way in the limit $C_F = N_C = 2T_f \equiv N$ (cf. [41]). In, e.g., the unpolarized case it reads

$$\delta[\hat{P}^{(U),(i)}(\xi)] \equiv P_{qq}^{(U),(i)}(\xi) + P_{gq}^{(U),(i)}(\xi) - P_{qg}^{(U),(i)}(\xi) - P_{gg}^{(U),(i)}(\xi) \equiv 0 \; . \qquad (62)$$

In LO ($i = 0$), the relation is satisfied by the both the unpolarized and the polarized splitting functions and, trivially, in both the space-like and the time-like situations. Beyond LO, one can expect it to continue to hold *only* if the regularization method adopted respects supersymmetry. One therefore anticipates that the NLO ($\overline{\mathrm{MS}}$) space-like and time-like splitting functions of dimensional regularization will not satisfy (62), in agreement with the findings in [16, 12]. However, one regularization method that is applicable to supersymmetry is dimensional *reduction* [42], a variant of dimensional regularization. The scheme essentially consists of performing the Dirac-algebra in *four* dimensions and of continuing only momenta to d ($d < 4$) dimensions. In order to match the ultraviolet (UV) sectors of dimensional regularization and dimensional reduction, specific counterterms need to be introduced [43, 44] in the latter which include a finite renormalization of the strong charge. Once this is done, all remaining differences between the results for a NLO quantity in dimensional regularization and in dimensional reduction can only be due to the effects of mass singularities. They are fully accounted for [44, 45, 39] by the differences between the d-dimensional LO splitting functions (as to be obtained in dimensional *regularization*) and the *four*-dimensional ones (corresponding to dimensional *reduction*). In other words, the breakdown of the supersymmetric relation for dimensional regularization is entirely blamed on the breakdown of this relation in the $\sim \epsilon$-parts of the d-dimensional LO splitting functions of dimensional regularization. This feature was exploited in [45, 14] to transform the space-like unpolarized and polarized NLO splitting functions of $\overline{\mathrm{MS}}$ dimensional regularization to dimensional reduction via a simple factorization scheme transformation and to establish the validity of Eq. (62) for the obtained quantities[11]. We will now extend the considerations of [45, 14] to the time-like situation. In the unpolarized case, one finds for the NLO $\overline{\mathrm{MS}}$ splitting functions of dimensional regularization in the limit $C_F = N_C = 2T_f \equiv N$ [16]:

$$\delta[\hat{P}^{(T),(1)}(z)] = N^2 \left[-\frac{2}{3z} + \frac{13}{6} + \frac{5}{3}z - z^2 + (-1 + 2z + 4z^2)\ln z - \frac{1}{2}\delta(1-z) \right] \qquad (63)$$

[11] The supersymmetry relation for the space-like NLO kernels was proved prior to [45, 14] in the OPE calculations of [46, 12] in which the transition from dimensional regularization to dimensional reduction occurs as a finite renormalization.

where we have included the endpoint contribution. The corresponding expression for the polarized NLO splitting functions can be obtained from Eqs. (51)-(56) and the space-like results of [12, 13, 14],

$$\delta[\Delta\hat{P}^{(T),(1)}(z)] = N^2\left[\frac{11}{6}-\frac{7}{6}z+(1-z)\ln z-\frac{1}{2}\delta(1-z)\right] \ . \qquad (64)$$

What we need to do is to perform factorization scheme transformations (39) of these results, with the $(\Delta)z_{ij}^{(T)}$ to be determined from the parts $\sim\epsilon$ of the time-like d-dimensional LO splitting functions as obtained in dimensional regularization. For our purposes we only need to consider (39) in the combination appearing on the lhs of Eq. (62) and in the supersymmetric limit:

$$\delta[(\Delta)\hat{P}^{(T),(1)}] \longrightarrow \delta[(\Delta)\hat{P}^{(T),(1)}]-\frac{\beta_0}{2}\delta[(\Delta)\hat{Z}^{(T)}]+\left((\Delta)P_{qg}^{(T),(0)}+(\Delta)P_{gq}^{(T),(0)}\right)\otimes\delta[(\Delta)\hat{Z}^{(T)}] \ , \qquad (65)$$

where now $\beta_0 = 3N$. The calculation of the parts $\sim\epsilon$ in the d-dimensional LO time-like splitting functions yields[12]

$$\delta[\hat{Z}^{(T)}(z)] = N\left(1-2z+2z^2-\frac{1}{3}\delta(1-z)\right) \ , \qquad (66)$$

$$\delta[\Delta\hat{Z}^{(T)}(z)] = N\left(1-z-\frac{1}{3}\delta(1-z)\right) \ . \qquad (67)$$

Upon insertion of Eqs. (63),(66) (or (64),(67)) into (65) one finds that the resulting transformed $\delta[\hat{P}^{(T),(1)}]$ (or $\delta[\Delta\hat{P}^{(T),(1)}]$, respectively) vanishes identically. We thus have demonstrated the validity of the $\mathcal{N}=1$ supersymmetric relation also for the time-like NLO polarized and unpolarized evolution kernels in dimensional reduction. Apart from being interesting of its own, this finding also provides evidence for the correctness of our results in sections 3-5.

7 Summary

We have presented a calculation of the unpolarized and polarized time-like NLO splitting functions, needed for the NLO Q^2-evolution of (spin-dependent) fragmentation functions.

[12] As discussed in the previous section the difference (47) between the LO polarized and unpolarized time-like quark-to-quark splitting functions arising in the HVBM scheme is already accounted for in Eqs. (52)-(58). To obtain the result in (67) one therefore has to use $\Delta P_{qq,4-2\epsilon}^{(T),(0)}(z) = P_{qq,4-2\epsilon}^{(T),(0)}(z)$ (see also [14]).

The starting point for our considerations were [15, 16, 22, 13, 14] in which the corresponding space-like quantities were calculated within a method based on the factorization properties of mass singularities in the light-cone gauge. As was shown in [15] for the non-singlet case one can then determine the time-like counterparts via analytic continuation to $x > 1$, which is also the way we have pursued. It turned out that beyond LO there are certain terms arising from phase space (and, for the unpolarized case, from the gluon spin-averaging in $d \neq 4$ dimensions) which prevent the analytic continuation relation (ACR) of [17] between the space-like and time-like splitting functions from remaining intact. The same statement applies to the connection between the space-like and time-like short distance cross sections of electroproduction and e^+e^- annihilation, respectively. Nevertheless, the corrections to the ACR are rather straightforwardly calculable within the method of [15]. Even more, we were able to show that in both the unpolarized and the polarized cases one can transform the results to a factorization scheme, different from the $\overline{\text{MS}}$ scheme, in which the breakdown of the ACR does not occur. In the unpolarized case our final $\overline{\text{MS}}$ results confirm those of [15, 16] obtained within the same method, whereas in the polarized case our results are entirely new. Finally we have shown that, when transformed to dimensional reduction, both our unpolarized and polarized results for the time-like NLO splitting functions satisfy a simple relation motivated from supersymmetry.

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Appendix

In this appendix we present the Mellin-n moments (as defined in Eq. (60)) of our NLO results for the polarized case in Eqs. (52)-(58). The corresponding n moments for the unpolarized time-like splitting functions and short-distance cross sections can be found in

[27]. As in (51) we write

$$\Delta P_{ij}^{(T),(1)}[n] = \Delta P_{ij}^{(S),(1)}[n] + \Delta_{ij}[n] \,, \tag{A.1}$$

where the $\Delta P_{ij}^{(S),(1)}[n]$ can be found in a compact analytic form[13] in [40] and the $\Delta_{ij}[n]$ are given by:

$$\begin{aligned}
\Delta_{qq,\pm}[n] &= C_F^2 \left[4\mathcal{S}_1(n) - \frac{3n^2+3n+2}{n(n+1)}\right] \left[-2\mathcal{S}_2(n) + \frac{\pi^2}{3} + \frac{2n+1}{n^2(n+1)^2}\right] , & \text{(A.2)} \\
\Delta_{qq,PS}[n] &= 4\,C_F T_f \frac{(n+2)(3n+1)}{n^3(n+1)^3} , & \text{(A.3)} \\
\Delta_{gq}[n] &= C_F^2 \left[-2\frac{n+2}{n(n+1)}\left(-\mathcal{S}_1^2(n) + \mathcal{S}_2(n) + \frac{\pi^2}{3}\right) - \frac{3n^3+n^2-18n-8}{n^2(n+1)^2}\mathcal{S}_1(n)\right. \\
&\quad \left. + \frac{7n^5+22n^4+7n^3-24n^2-22n-4}{n^3(n+1)^3}\right] \\
&+ C_A C_F \left[-2\frac{n+2}{n(n+1)}(\mathcal{S}_1^2(n) - 3\mathcal{S}_2(n)) + \frac{(n+2)(11n-1)}{3n(n+1)^2}\mathcal{S}_1(n)\right. \\
&\quad \left. - \frac{(67n^4+101n^3+34n^2+144n+72)\,(n+2)}{9n^3(n+1)^3}\right] \\
&+ \frac{4}{9}\,C_F T_f \left[-3\frac{n+2}{n(n+1)}\mathcal{S}_1(n) + \frac{(n+2)(5n+2)}{n(n+1)^2}\right] , & \text{(A.4)} \\
\Delta_{qg}[n] &= \frac{8}{9}\,T_f^2 \left[3\frac{n-1}{n(n+1)}\mathcal{S}_1(n) - \frac{5n^3-3n^2+7n+3}{n^2(n+1)^2}\right] \\
&+ 2\,C_F T_f \left[2\frac{n-1}{n(n+1)}(-\mathcal{S}_1^2(n) + 3\mathcal{S}_2(n)) + \frac{3n^2+5}{n(n+1)^2}\mathcal{S}_1(n)\right. \\
&\quad \left. - \frac{7n^5+7n^4-5n^3+5n^2+4n-2}{n^3(n+1)^3}\right] \\
&+ \frac{2}{9}\,C_A T_f \left[6\frac{n-1}{n(n+1)}(3\mathcal{S}_1^2(n) - 3\mathcal{S}_2(n) - \pi^2)\right. \\
&\quad -3\frac{11n^3-12n^2+37n+12}{n^2(n+1)^2}\mathcal{S}_1(n) \\
&\quad \left. + \frac{67n^5+34n^4+32n^3+98n^2+249n+72}{n^3(n+1)^3}\right] , & \text{(A.5)} \\
\Delta_{gg}[n] &= -8\,C_F T_f \frac{n^3+3n^2-1}{n^3(n+1)^3} + \frac{4}{3}\,C_A T_f \left[-2\mathcal{S}_2(n) + \frac{\pi^2}{3} + 4\frac{2n+1}{n^2(n+1)^2}\right] \\
&+ C_A^2 \left[4\mathcal{S}_1(n) - \frac{11n^2+11n+24}{3n(n+1)}\right] \left[-2\mathcal{S}_2(n) + \frac{\pi^2}{3} + 4\frac{2n+1}{n^2(n+1)^2}\right] & \text{(A.6)}
\end{aligned}$$

[13]Note that the results presented in [40] need to be divided by -8 in order to bring them to our normalization for the NLO splitting functions. Also note that the definition of the $\pm$-components in the non-singlet sector (see Eqs. (8)-(10)) occurs in a reversed notation in [40], i.e., as $\mp$.

with

$$\mathcal{S}_k(n) \equiv \sum_{j=1}^{n} \frac{1}{j^k} \,. \tag{A.7}$$

The analytic continuation of the $\mathcal{S}_k$, required for a numerical Mellin inversion, is well-known [47]. For the moments of the polarized NLO time-like short-distance cross sections we find

$$\begin{aligned}
\Delta C_q^{(T),(1)}[n] &= C_F \Bigg[\mathcal{S}_1^2(n) + 5\mathcal{S}_2(n) + \left(\frac{3}{2} - \frac{1}{n(n+1)}\right) \mathcal{S}_1(n) + \frac{3}{(n+1)^2} - \frac{1}{2(n+1)} \\
&\quad - \frac{1}{n} - \frac{2}{n^2} - \frac{9}{2} \Bigg] , \qquad \text{(A.8)} \\
\Delta C_g^{(T),(1)}[n] &= C_F \Bigg[- \frac{2+n}{n(n+1)} \mathcal{S}_1(n) - \frac{4}{n} - \frac{4}{n^2} + \frac{3}{n+1} + \frac{3}{(n+1)^2} \Bigg] . \qquad \text{(A.9)}
\end{aligned}$$

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