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J Bordes Chan Hong-Mo J Faridani J Pfaudler and Tsou Sheung Tsun

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CKM Matrix and Fermion Masses in the Dualized Standard Model

José BORDES

bordes@evalvx.ific.uv.es

*Dept. Fisica Teorica, Univ. de Valencia,
c. Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain*

CHAN Hong-Mo, Jacqueline FARIDANI

chanhm@v2.rl.ac.uk faridani@hephp1.rl.ac.uk

*Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon, OX11 0QX, United Kingdom*

Jakov PFAUDLER

jakov@thphys.ox.ac.uk

*Dept. of Physics, Theoretical Physics, University of Oxford,
1 Keble Road, Oxford, OX1 3NP, United Kingdom*

TSOU Sheung Tsun

tsou@maths.ox.ac.uk

*Mathematical Institute, University of Oxford,
24-29 St. Giles', Oxford, OX1 3LB, United Kingdom*

Abstract

A Dualized Standard Model recently proposed affords a natural explanation for the existence of Higgs fields and of exactly 3 generations of fermions, while giving at the same time the observed fermion mass hierarchy together with a tree-level CKM matrix equal to the identity matrix. It further suggests a method for generating from loop corrections the lower generation masses and nondiagonal CKM matrix elements. In this paper, the proposed calculation is carried out to 1-loop. It is found first that with the method suggested one can account readily for the masses of the second generation fermions as a 'leakage' from the highest generation. Then, with the Yukawa couplings fixed by fitting the masses of the 2 higher generations, one is left with only 2 free parameters to evaluate the CKM matrix and the masses in the

lowest generation. One obtains a very good fit to the CKM matrix and sensible values for the masses of d and e , though, for a valid reason, not of u . In addition, the fitted values of the Yukawa couplings and vacuum expectation values of the dual colour Higgs fields show remarkable features perhaps indicative of a deeper significance.

1 Introduction

Up to the present, the Standard Model has worked exceptionally well, there being no experimental fact we know which is demonstrably contradictory to its predictions. Nevertheless, the Standard Model contains in itself a number of unsatisfactory features, which are widely recognized as such. For example, at the more fundamental level, Higgs fields are introduced to break the electroweak symmetry and fermions are assumed to exist in three generations or families to fit observation without theoretical reasons being given for why it should be so. Compared with the intrinsic gauge structure and the existence of the gauge bosons in the theory, both of which have deep geometric significance, the assumptions about Higgs fields and fermion generations appear *ad hoc*. At the more practical level, this situation is reflected in the large number of independent parameters which have to be determined by experiment. Besides, these parameters exhibit some quite startling patterns which are still unexplained. In particular, there is first the so-called fermion hierarchy puzzle, namely that fermions of the same type but different generations have widely different masses. Take, for example, the three *U*-type quarks; the experimental values quoted in the latest data booklet [1] for the masses of *t*, *c*, and *u* respectively are 176 ± 5 GeV, 1.0 - 1.6 GeV, and 2 - 8 MeV, dropping by more than two orders of magnitude from generation to generation. Then secondly, there is the mixing problem, say, between the *U*-type and *D*-type quarks through the CKM matrix [2, 3], which though tantalisingly close to the identity matrix is yet not the identity, with its off-diagonal elements varying in magnitude from about 20 percent to about 3 permille [1]. These empirical facts, of course, are all of the greatest phenomenological significance and cry out for a theoretical explanation but are not given one in the Standard Model as usually formulated.

In the literature, answers to these questions are often sought for from beyond the Standard Model, but with, to our minds, no obvious great success. The difficulty is that, there being more freedom working outside the Standard Model framework, one often ends up by putting in more than one gets out. Recently, however, a suggestion was made for a solution of the above problems from within the framework of the Standard Model itself which, if at all possible, would at least have the advantage of economy and restraint. In this suggestion [4], one first made use of a newly discovered generalized electric-magnetic duality for Yang-Mills fields [5] together with a well-known

result of 't Hooft on confinement [6] to give a natural place to both Higgs fields and fermion generations, with Higgs fields appearing as frame vectors in internal symmetry space and fermion generations appearing as dual colour. As an immediate consequence, one then deduced that fermions will occur in exactly three generations and that the generation symmetry will be broken, as experimentally observed. Moreover, with one more simple assumption about the dual hypercharges of the dual Higgs fields, it was shown that there will be a fermion mass hierarchy and that the CKM matrix would be the identity at tree level, but that loop corrections would lift the above tree-level degeneracy to give small but nonzero values both to the lower generation fermion masses and to the off-diagonal CKM matrix elements.

The purpose of the present paper is to push further in this direction to make a first attempt at actually evaluating quark masses and CKM matrix elements for comparison with experiment. The calculation is here carried out to 1-loop level. Out of the many 1-loop diagrams we have examined, it turns out first that some, which affect only the normalization of the fermion mass matrix but not its orientation, are large due to the large dual gauge coupling and cannot be evaluated perturbatively. Since it is only the orientation in flavour space which is of the most interest to us as far as the CKM matrix is concerned, it is profitable at present to abandon calculating the normalization of the mass matrix and concentrate solely on its orientation. This has the benefit of allowing us to ignore those diagrams affecting only the normalization, reducing thus the number of free parameters in the problem. Secondly, it happens that of the remaining diagrams affecting the orientation of the fermion mass matrix, most are negligible if we put in the estimate for the dual gauge boson mass obtained from the absence of flavour-changing neutral decays. As pointed out already in [4], the exchange of the dual gauge bosons would give rise to FCNC effects, and experimental constraints put a lower bound on the lowest dual gauge boson mass of several 100 TeV.

What remains then is basically just the Higgs loop diagram which matters for our present investigation. This depends on a Yukawa coupling strength ρ , one for each fermion type, a mass scale m_T which may be identified as the highest generation mass, again one for each fermion type, and lastly the 3 vacuum expectation values (x, y, z) of the dual colour Higgs fields which are common to all fermion types. We ascertain first that the masses of the second generation fermions can indeed be obtained as a 'leakage' from the highest generation, as suggested in [4], with a Yukawa coupling strength ρ

of order unity for each fermion type. We then fixed their values by fitting these ρ 's to the empirical values of the masses of the second generation. Next, of the remaining parameters (x, y, z) , it was shown that the calculation is independent of their normalization to a high accuracy. With then only 2 free parameters, we had to calculate the CKM matrix and the fermion masses of the lowest generation. A very good fit to the absolute values of all CKM elements was obtained together with some ratios and products of these elements measured independently. In addition, in spite of the lack of knowledge on the scale-dependence of the normalization of the mass matrix, sensible estimates were obtained for all the lowest generation fermions except for the u -quark.

The result of the fit reveals also 2 intriguing features, namely (a) a close proximity of the normalized vector (x, y, z) to one of its fixed points $(1, 0, 0)$ to an accuracy of about one part in ten thousand, and (b) the near equality to a few percent accuracy of the fitted values of the ρ 's for all 3 fitted fermion types (i.e. U, D and the charge leptons L). We think these may be indicative of a hidden symmetry which we have not yet understood.

2 The Framework

We begin with a resumé of the dual framework on which the calculations are based, the details of which can be found in [4]. Generalized electric-magnetic duality as obtained in [5] implies that dual to colour in the Standard Model, there is also an $\widetilde{SU}(3)$ symmetry for dual colour. The charges of this dual symmetry are colour monopoles and its monopoles are colour charges. Using then the well-known result of 't Hooft [6], one concludes from the fact that colour is confined that dual colour $\widetilde{SU}(3)$ is spontaneously broken and Higgsed. The proposal was that this broken dual colour symmetry be identified with what is sometimes referred to in the literature as the "horizontal symmetry" relating the generations [7, 8].

Now it so happens that in the dual framework of [4] there are scalar fields occurring which have the right properties to play the role of Higgs fields, these being the frame vectors in the $\widetilde{SU}(3)$ internal space. They constitute altogether 3 dual colour triplets, which we denote by $\phi_{\tilde{a}}^{(a)}$, with $\tilde{a} = 1, 2, 3$ representing the dual colour which labels the 3 components of a triplet and $(a) = 1, 2, 3$ being just a label distinguishing the 3 triplets. We

want the vacuum expectation values of $\phi_a^{(a)}$ to form an orthogonal triad, as is appropriate for the 3 vectors which make up an $\widetilde{SU}(3)$ frame. We need therefore a Higgs potential which gives these vacuum expectation values as minima. The following was suggested [4]:

$$V[\phi] = -\mu \sum_{(a)} |\phi^{(a)}|^2 + \lambda \left\{ \sum_{(a)} |\phi^{(a)}|^2 \right\}^2 + \kappa \sum_{(a) \neq (b)} |\bar{\phi}^{(a)} \cdot \phi^{(b)}|^2, \quad (2.1)$$

with μ, λ and κ all positive. The minimum of V occurs when the $\phi^{(a)}$ are mutually orthogonal and $\sum_{(a)} |\phi^{(a)}|^2 = \mu/2\lambda$, independently of the individual lengths of the different $\phi^{(a)}$'s. Thus, a vacuum can be chosen as:

$$\phi_V^{(1)} = \zeta \begin{pmatrix} x \\ 0 \\ 0 \end{pmatrix}, \phi_V^{(2)} = \zeta \begin{pmatrix} 0 \\ y \\ 0 \end{pmatrix}, \phi_V^{(3)} = \zeta \begin{pmatrix} 0 \\ 0 \\ z \end{pmatrix}, \quad (2.2)$$

with

$$\zeta = \sqrt{\mu/2\lambda}, \quad (2.3)$$

and

$$x^2 + y^2 + z^2 = 1, \quad (2.4)$$

which will in general break both the $\widetilde{SU}(3)$ symmetry and the permutation symmetry between the different $\phi^{(a)}$'s. In our calculation here we shall use explicitly this potential although we recognize that it has no claim for uniqueness. We shall show, however, that the result will not depend much on its detailed properties.

As in [4], left-handed fermions are assigned to dual colour triplets and right-handed fermions to dual colour singlets. We have thus the Yukawa coupling term:

$$\sum_{[b]} Y_{[b]} \sum_{(a)} (\bar{\psi}_L)^{\bar{a}} \phi_a^{(a)} (\psi_R)^{[b]} + h.c., \quad (2.5)$$

where we have suppressed both colour and weak isospin indices which are irrelevant for our discussion here. Inserting then the vacuum expectation values given in (2.2) for the Higgs fields, we have at tree-level the following factorized fermion mass matrix:

$$m = \zeta \begin{pmatrix} x \\ y \\ z \end{pmatrix} (a, b, c), \quad (2.6)$$

where we have abbreviated the Yukawa couplings $Y_{[1]} = a, Y_{[2]} = b, Y_{[3]} = c$. The matrix being of rank 1, it follows that mm^\dagger has only one non-zero eigenvalue [9], namely $\rho^2\zeta^2$, with

$$\rho = \sqrt{|a|^2 + |b|^2 + |c|^2} \quad (2.7)$$

implying thus immediately a mass hierarchy with one fermion state much higher in mass than the other two. Furthermore, the first factor in m being given in terms just of the Higgs vacuum expectation values x, y, z , is independent of the fermion type, i.e. of whether it is U -type or D -type quarks or leptons that we are dealing with, although the second factor, given in terms of the Yukawa couplings a, b and c , does depend on the fermion type. As a consequence, one obtains that the CKM matrix, which depends only on the relative orientation of the first (left-handed) factors of respectively the U -type and D -type quarks, is at tree-level just the identity matrix. This was already discussed in detail in [4].

What we need to do now is to go beyond the tree level and look at loop corrections. As pointed out in [4], because of the special manner in which the fermions here are coupled to the dual gauge and Higgs bosons, loop corrections do not destroy the factorizability property of the tree-level mass matrix.¹ Nevertheless, they will modify the first (left-handed) factor in (2.6) and hence give rise to a nontrivial CKM matrix and nonzero masses to the lower generations fermions as explained in [4]. This is what we wish now to examine in detail.

3 One-Loop Diagrams

Our fermions carry in fact weak isospin and in the case of quarks also colour, so that in principle there will be loop corrections coming from colour gluon and electroweak boson loops. However, as far as the CKM matrix, or the lower generation fermion masses, are concerned, only those diagrams which rotate the mass matrix with respect to dual colour (i.e. the generation index) will matter. Since neither the colour gluons nor the gauge and Higgs bosons in the electroweak sector carry dual colour, they cannot rotate the generation

¹As a result, the mass matrix has two zero eigenvalues so that any θ -vacuum can be rotated away and the strong CP problem is avoided. [10]

index, and hence will leave both factors (2.6) of the mass matrix intact, affecting at most its normalization. As we shall see, there are other reasons why we cannot in any case concern ourselves with the normalization of the mass matrix. There is thus no point for us to consider gluon and electroweak boson loops any further. There remain then only those diagrams with dual gauge and Higgs boson loops, of which all those of 1-loop order are listed in Figure 1.

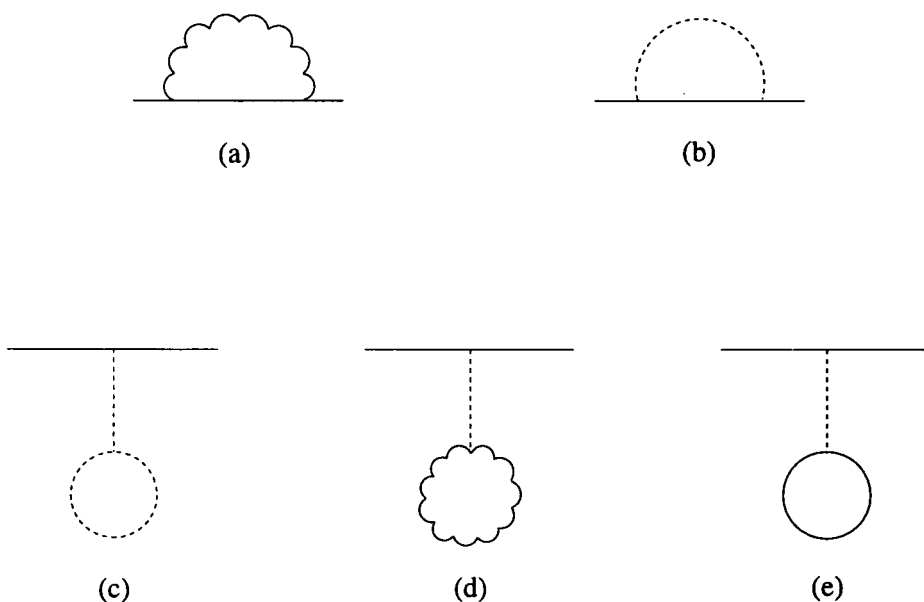


Figure 1: One loop corrections to the fermion mass matrix, where solid lines represent fermions, wavy lines dual gauge bosons and dotted lines dual colour Higgs fields.

Let us first write down the explicit expressions for the corrections to the fermion mass matrix arising from the diagrams in Figure 1. This has been done already in a general framework by Weinberg [12] who expressed the answer as a sum of five terms:

$$\delta m = -\Sigma^{(A1)} - \Sigma^{(A\phi)} - \Sigma^{(AT)} - \Sigma^{(\phi1)} - \Sigma^{(T1)}, \quad (3.1)$$

where:

$$\begin{aligned}
\Sigma^{(A1)} &= \frac{1}{16\pi^2} \sum_N \int_0^1 dx \{ -2m\bar{t}_N(1-x) + 4\gamma_4\bar{t}_N\gamma_4 m \} \ln \left(\mu_N^2 + \frac{m^2 x^2}{1-x} \right) \bar{t}_N, \\
\Sigma^{(A\phi)} &= \frac{1}{16\pi^2} \sum_N \frac{1}{\mu_N^2} \int_0^1 dx \{ (1-x)m[\gamma_4 m, \bar{t}_N]\gamma_4 + \gamma_4[\gamma_4 m, \bar{t}_N]m \} \\
&\quad \times \left\{ \ln \left(\frac{m^2 x^2}{1-x} \right) - \ln \left(\mu_N^2 + \frac{m^2 x^2}{1-x} \right) \right\} \gamma_4[\gamma_4 m, \bar{t}_N], \\
\Sigma^{(AT)} &= \frac{1}{32\pi^2} \sum_N \gamma_4[\bar{t}_N, [\bar{t}_N, \gamma_4 m]] \ln \mu_N^2, \\
\Sigma^{(\phi 1)} &= -\frac{1}{16\pi^2} \sum_K \int_0^1 dx \{ -(1-x)m\gamma_4 \bar{\Gamma}_K \gamma_4 + \bar{\Gamma}_K m \} \ln \{ m^2 x^2 + M_K^2(1-x) \} \bar{\Gamma}_K, \\
\Sigma^{(T1)} &= \frac{1}{32\pi^2} \Gamma_i M_{ij}^{-2} \left\{ f_{klj}(M^2 \ln M^2)_{kl} - 16 \text{Tr}(m^3 \ln m \Gamma_j) + 6 \sum_N (\bar{\theta}_N^2 \lambda)_j \mu_N^2 \ln \mu_N^2 \right\}.
\end{aligned} \tag{3.2}$$

As they are written, these formulae depend on the energy scale, the significance of which will be elucidated later.

The above formulae depend also on the following quantities, the explicit forms for which have yet to be specified: the fermion mass matrix m , the eigenvalues μ_N , $N = 0, \dots, 8$ of the dual gauge boson mass matrix, the Higgs boson mass matrix M_{ij} and its eigenvalues M_K , $K = 1, \dots, 9$, the fermion couplings to the dual gauge bosons \bar{t}_N and to the Higgs bosons Γ_i or $\bar{\Gamma}_K$, and then the Higgs bosons' couplings to themselves f_{klj} and to the dual gauge bosons $(\bar{\theta}_N^2 \lambda)_j$. We proceed to do so now.

The fermion mass matrix Weinberg defined somewhat differently from that given above in (2.6). Writing the Yukawa coupling in terms of the full fermion field ψ , thus:

$$\sum_{[b]} Y_{[b]} \sum_{(a)} \bar{\psi}^{\bar{a}} \phi_{\bar{a}}^{(a)} \psi^{[b]} + \text{h.c.}, \tag{3.3}$$

instead of the left- and right-handed components as in (2.5), one obtains a mass matrix of the following form:

$$m_{W'} = m \frac{1}{2} (1 + \gamma_5) + m^\dagger \frac{1}{2} (1 - \gamma_5), \tag{3.4}$$

containing factors in γ_5 . However, since the labels on the right-handed components have actually no physical significance, one can relabel them such as to make m hermitian and hence obtain the mass matrix in the form:

$$m_W = \rho \zeta \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z), \quad (3.5)$$

which has no γ_5 in it, and is essentially just the square root of mm^\dagger in terms of the m previously defined. We notice that m_W remains a factorized matrix, which property is crucial for our discussion later. In the calculations which follow, when no confusion is likely to occur, we shall drop the subscript W from the Weinberg mass matrix.

Next, the mass matrix for the dual gauge bosons has already been worked out in [4]. This 9×9 matrix is diagonal for $\alpha = N = 1, 2, 4, 5, 6, 7$, as labelled by the Gell-Mann matrices λ_α of $\widehat{SU}(3)$, with eigenvalues:

$$\begin{aligned} \mu_1, \mu_2 &= \frac{\tilde{g}_3 \zeta}{2} \sqrt{x^2 + y^2}, \\ \mu_4, \mu_5 &= \frac{\tilde{g}_3 \zeta}{2} \sqrt{z^2 + x^2}, \\ \mu_6, \mu_7 &= \frac{\tilde{g}_3 \zeta}{2} \sqrt{y^2 + z^2}. \end{aligned} \quad (3.6)$$

The remaining 3×3 nondiagonal block in μ^2 as labelled by λ_3, λ_8 and $\lambda_0 = \frac{2}{3}I$ reads as:

$$\begin{pmatrix} \frac{\tilde{g}_3^2}{4} \zeta^2 (x^2 + y^2) & \frac{\tilde{g}_3^2}{4\sqrt{3}} \zeta^2 (x^2 - y^2) & -\frac{\tilde{g}_1 \tilde{g}_3}{3} \zeta^2 (x^2 - y^2) \\ \frac{\tilde{g}_3^2}{4\sqrt{3}} \zeta^2 (x^2 - y^2) & \frac{\tilde{g}_3^2}{12} \zeta^2 (x^2 + y^2 + 4z^2) & -\frac{\tilde{g}_1 \tilde{g}_3}{3\sqrt{3}} \zeta^2 (x^2 + y^2 - 2z^2) \\ -\frac{\tilde{g}_1 \tilde{g}_3}{3} \zeta^2 (x^2 - y^2) & -\frac{\tilde{g}_1 \tilde{g}_3}{3\sqrt{3}} \zeta^2 (x^2 + y^2 - 2z^2) & \frac{4\tilde{g}_1^2}{9} \zeta^2 (x^2 + y^2 + z^2) \end{pmatrix}, \quad (3.7)$$

the eigenvalues of which we label by $\mu_N^2, N = 3, 8, 0$ with eigenvectors $C_{\alpha N}$ such that:

$$\sum_{\alpha, \beta=3,8,0} C_{\alpha N'} \mu_{\alpha\beta}^2 C_{\beta N} = \mu_N^2 \delta_{NN'}. \quad (3.8)$$

The diagonalization of this matrix we shall perform only with explicit values for the parameters.

The Higgs fields $\phi_a^{(a)}$ represent 9 complex degrees of freedom, which we write, following Weinberg's convention, as 18 real fields, thus:

$$\phi_a^{(a)} = \phi_{a1}^{(a)} + i\phi_{a2}^{(a)}. \quad (3.9)$$

From these and the potential $V[\phi]$ in (2.1), the Higgs boson mass matrix is given by:

$$M^2 = \left[\frac{\partial^2 V}{\partial \phi_{ar}^{(a)} \partial \phi_{bs}^{(b)}} \right]_{vacuum}, \quad (3.10)$$

which breaks up into 8 diagonal blocks as follows. First, there is a 3×3 block labelled by $\phi_{1,1}^{(1)}, \phi_{2,1}^{(2)}, \phi_{3,1}^{(3)}$:

$$8\lambda\zeta^2 \begin{pmatrix} x^2 & xy & xz \\ yx & y^2 & yz \\ zx & zy & z^2 \end{pmatrix}. \quad (3.11)$$

Second, there is a 2×2 block labelled by $\phi_{2,1}^{(3)}, \phi_{3,1}^{(2)}$:

$$4\kappa\zeta^2 \begin{pmatrix} y^2 & yz \\ zy & z^2 \end{pmatrix}. \quad (3.12)$$

Third, there is a 2×2 block labelled by $\phi_{2,2}^{(3)}, \phi_{3,2}^{(2)}$:

$$4\kappa\zeta^2 \begin{pmatrix} y^2 & -yz \\ -zy & z^2 \end{pmatrix}. \quad (3.13)$$

Fourth, there is a 2×2 block labelled by $\phi_{3,1}^{(1)}, \phi_{1,1}^{(3)}$:

$$4\kappa\zeta^2 \begin{pmatrix} z^2 & zx \\ xz & x^2 \end{pmatrix}. \quad (3.14)$$

Fifth, there is a 2×2 block labelled by $\phi_{3,2}^{(1)}, \phi_{1,2}^{(3)}$:

$$4\kappa\zeta^2 \begin{pmatrix} z^2 & -zx \\ -xz & x^2 \end{pmatrix}. \quad (3.15)$$

Sixth, there is a 2×2 block labelled by $\phi_{1,1}^{(2)}, \phi_{2,1}^{(1)}$:

$$4\kappa\zeta^2 \begin{pmatrix} x^2 & xy \\ yx & y^2 \end{pmatrix}. \quad (3.16)$$

Seventh, there is a 2×2 block labelled by $\phi_{1,2}^{(2)}, \phi_{2,2}^{(1)}$:

$$4\kappa\zeta^2 \begin{pmatrix} x^2 & -xy \\ -yx & y^2 \end{pmatrix}. \quad (3.17)$$

Finally, there is a 3×3 block labelled by $\phi_{1,2}^{(1)}, \phi_{2,2}^{(2)}, \phi_{3,2}^{(3)}$ the entries of which are all zero. All the first seven blocks are of rank 1 and have each only one nonzero eigenvalue, giving thus for M^2 altogether 11 zero modes, 9 of which, namely one each from each block except the first and last and all 3 from the last, are eaten up by the dual gauge bosons, leaving 2 from the first block. These 2 remaining zero modes come from an “accidental symmetry” of the vacuum, not from a symmetry of the potential (2.1), as explained in [4], and are therefore not eaten up by the gauge bosons. We are then left with 7 Higgs bosons coming one from each of the first seven blocks which we label in that order with their eigenvalues and eigenvectors each in its own block as follows:

$$\begin{aligned} K = 1 : & \quad 8\lambda\zeta^2(x^2 + y^2 + z^2) & (x, y, z), \\ K = 2 : & \quad 4\kappa\zeta^2(y^2 + z^2) & (y, z), \\ K = 3 : & \quad 4\kappa\zeta^2(y^2 + z^2) & (y, -z), \\ K = 4 : & \quad 4\kappa\zeta^2(z^2 + x^2) & (z, x), \\ K = 5 : & \quad 4\kappa\zeta^2(z^2 + x^2) & (z, -x), \\ K = 6 : & \quad 4\kappa\zeta^2(x^2 + y^2) & (x, y), \\ K = 7 : & \quad 4\kappa\zeta^2(x^2 + y^2) & (x, -y), \end{aligned} \quad (3.18)$$

while the two remaining zero (pseudo-Goldstone) modes coming from the first block will be assigned the following eigenvectors in the original basis of that block:

$$|v_8\rangle = -\beta \begin{pmatrix} y - z \\ z - x \\ x - y \end{pmatrix}; |v_9\rangle = \beta \begin{pmatrix} 1 - x(x + y + z) \\ 1 - y(x + y + z) \\ 1 - z(x + y + z) \end{pmatrix}, \quad (3.19)$$

with

$$\beta^{-2} = 3 - (x + y + z)^2. \quad (3.20)$$

Next, the couplings of the dual gauge bosons to the fermions are in Weinberg’s convention:

$$\bar{t}_N = t_N, \quad N = 1, 2, 4, 5, 6, 7,$$

$$\bar{t}_N = C_{3N}t_3 + C_{8N}t_8 + C_{0N}t_0, \quad N = 3, 8, 0, \quad (3.21)$$

where:

$$\begin{aligned} t_\alpha &= -\frac{\tilde{g}_3}{2}\lambda_\alpha\frac{1}{2}(1-\gamma_5), \quad \alpha = 1, \dots, 8, \\ t_0 &= \frac{2\tilde{g}_1}{3}\frac{1}{2}(1-\gamma_5). \end{aligned} \quad (3.22)$$

The coefficients C_{3N}, C_{8N}, C_{0N} are as defined in (3.8).

The couplings of the Higgs bosons to the fermions as extracted from the Yukawa couplings (3.3), expressed in terms of the real Higgs fields (3.9), and rotated to the basis where m is hermitian or where m_W has no γ_5 , are:

$$\begin{aligned} \Gamma_{11}^{(a)} &= \frac{1}{2}(1+\gamma_5)\rho \begin{pmatrix} x & y & z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2}(1-\gamma_5)\rho \begin{pmatrix} x & 0 & 0 \\ y & 0 & 0 \\ z & 0 & 0 \end{pmatrix}, \\ \Gamma_{12}^{(a)} &= \frac{i}{2}(1+\gamma_5)\rho \begin{pmatrix} x & y & z \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} - \frac{i}{2}(1-\gamma_5)\rho \begin{pmatrix} x & 0 & 0 \\ y & 0 & 0 \\ z & 0 & 0 \end{pmatrix}, \\ \Gamma_{21}^{(a)} &= \frac{1}{2}(1+\gamma_5)\rho \begin{pmatrix} 0 & 0 & 0 \\ x & y & z \\ 0 & 0 & 0 \end{pmatrix} + \frac{1}{2}(1-\gamma_5)\rho \begin{pmatrix} 0 & x & 0 \\ 0 & y & 0 \\ 0 & z & 0 \end{pmatrix}, \\ \Gamma_{22}^{(a)} &= \frac{i}{2}(1+\gamma_5)\rho \begin{pmatrix} 0 & 0 & 0 \\ x & y & z \\ 0 & 0 & 0 \end{pmatrix} - \frac{i}{2}(1-\gamma_5)\rho \begin{pmatrix} 0 & x & 0 \\ 0 & y & 0 \\ 0 & z & 0 \end{pmatrix}, \\ \Gamma_{31}^{(a)} &= \frac{1}{2}(1+\gamma_5)\rho \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & y & z \end{pmatrix} + \frac{1}{2}(1-\gamma_5)\rho \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & z \end{pmatrix}, \\ \Gamma_{32}^{(a)} &= \frac{i}{2}(1+\gamma_5)\rho \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ x & y & z \end{pmatrix} - \frac{i}{2}(1-\gamma_5)\rho \begin{pmatrix} 0 & 0 & x \\ 0 & 0 & y \\ 0 & 0 & z \end{pmatrix}, \end{aligned} \quad (3.23)$$

which are independent of the superscript (a) . Notice that the three indices $(a) = 1, 2, 3; \tilde{a} = 1, 2, 3; r = 1, 2$ here are to play together the role of the index $i = 1, \dots, 18$ in the formula for Σ^{T1} in (3.2). Alternatively, when expressed in terms of the basis formed by the eigenstates of the Higgs mass matrix M

as listed in (3.18) and (3.19), we have the same couplings in the form to be used in Σ^{ϕ^1} of (3.2):

$$\bar{\Gamma}_K = \bar{\gamma}_K \frac{1}{2}(1 + \gamma_5) + \bar{\gamma}_K^\dagger \frac{1}{2}(1 - \gamma_5), \quad (3.24)$$

where:

$$\bar{\gamma}_K = \rho |v_K\rangle \langle v_1|, \quad (3.25)$$

and:

$$\begin{aligned} |v_1\rangle &= \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \\ |v_2\rangle &= \frac{1}{\sqrt{y^2 + z^2}} \begin{pmatrix} 0 \\ y \\ z \end{pmatrix}, \\ |v_3\rangle &= \frac{i}{\sqrt{y^2 + z^2}} \begin{pmatrix} 0 \\ y \\ -z \end{pmatrix}, \\ |v_4\rangle &= \frac{1}{\sqrt{z^2 + x^2}} \begin{pmatrix} x \\ 0 \\ z \end{pmatrix}, \\ |v_5\rangle &= \frac{i}{\sqrt{z^2 + x^2}} \begin{pmatrix} -x \\ 0 \\ z \end{pmatrix}, \\ |v_6\rangle &= \frac{1}{\sqrt{x^2 + y^2}} \begin{pmatrix} x \\ y \\ 0 \end{pmatrix}, \\ |v_7\rangle &= \frac{i}{\sqrt{x^2 + y^2}} \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix}, \end{aligned} \quad (3.26)$$

while $|v_8\rangle$ and $|v_9\rangle$ are already given in (3.19).

There remains then for us to specify only the couplings $(\bar{\theta}_N^2 \lambda)_j$ of the gauge fields to the Higgs fields and the couplings f_{klj} of the Higgs fields to themselves, both occurring in the tadpole term Σ^{T^1} in (3.2). The calculations

for these are somewhat tedious, especially f_{klj} which has altogether $18 \times 18 \times 18$ entries, most of which are zero. Since the calculation is straightforward and their actual values will not be needed for our calculation later, here we shall give only those results which are relevant for our considerations. For instance, one does not need to include in the sum over K the zero modes (3.19) [12]. For the rest, it is easier to state the result in terms of the basis formed by the eigenstates (3.18) of the Higgs mass matrix than in terms of the original basis labelled by the three indices $(a) = 1, 2, 3, \bar{a} = 1, 2, 3, r = 1, 2$ corresponding together to the index i in Weinberg's formulae. In that case, the couplings $(\bar{\theta}_N^2 \lambda)_K$ for $K = 2, \dots, 7$ all vanish, leaving only:

$$(\bar{\theta}_N^2 \lambda)_{K=1} = -\zeta(D_{N1}^2 x^2 + D_{N2}^2 y^2 + D_{N3}^2 z^2), \quad (3.27)$$

with

$$\begin{aligned} D_{N1}^2 &= \frac{\tilde{g}_3^2}{4}, D_{N2}^2 = \frac{\tilde{g}_3^2}{4}, D_{N3}^2 = 0; & N &= 1, 2; \\ D_{N1}^2 &= \frac{\tilde{g}_3^2}{4}, D_{N2}^2 = 0, D_{N3}^2 = \frac{\tilde{g}_3^2}{4}; & N &= 4, 5; \\ D_{N1}^2 &= 0, D_{N2}^2 = \frac{\tilde{g}_3^2}{4}, D_{N3}^2 = \frac{\tilde{g}_3^2}{4}; & N &= 6, 7, \end{aligned} \quad (3.28)$$

and, for $N = 3, 8, 0$:

$$\begin{aligned} D_{N1}^2 &= \left(C_{3N} \frac{\tilde{g}_3}{2} + C_{8N} \frac{\tilde{g}_3}{2\sqrt{3}} - C_{0N} \frac{2\tilde{g}_1}{3} \right)^2 \\ D_{N2}^2 &= \left(-C_{3N} \frac{\tilde{g}_3}{2} + C_{8N} \frac{\tilde{g}_3}{2\sqrt{3}} - C_{0N} \frac{2\tilde{g}_1}{3} \right)^2 \\ D_{N3}^2 &= \left(-C_{8N} \frac{\tilde{g}_3}{\sqrt{3}} - C_{0N} \frac{2\tilde{g}_1}{3} \right)^2. \end{aligned} \quad (3.29)$$

Finally, in the same basis of eigenstates, of the Higgs fields self-couplings \bar{f}_{IJK} we need only those with $I = J$ and these are found to vanish except when $K = 1$, where:

$$\begin{aligned} \bar{f}_{111} &= 24\lambda\zeta, \\ \bar{f}_{221} &= 8\lambda\zeta + 8\kappa\zeta(y^2 + z^2), \end{aligned}$$

$$\begin{aligned}
\bar{f}_{331} &= 8\lambda\zeta + 16\kappa\zeta \left(\frac{y^2 z^2}{y^2 + z^2} \right) \\
\bar{f}_{441} &= 8\lambda\zeta + 8\kappa\zeta (z^2 + x^2), \\
\bar{f}_{551} &= 8\lambda\zeta + 16\kappa\zeta \left(\frac{z^2 x^2}{z^2 + x^2} \right) \\
\bar{f}_{661} &= 8\lambda\zeta + 8\kappa\zeta (x^2 + y^2), \\
\bar{f}_{771} &= 8\lambda\zeta + 16\kappa\zeta \left(\frac{x^2 y^2}{x^2 + y^2} \right).
\end{aligned} \tag{3.30}$$

With these, the specification of quantities entering in the expressions for the 1-loop diagrams in (3.2) is complete.

4 The Relevant Terms

Although the 1-loop diagrams detailed in the preceding section were all referred to formally as corrections, they need not all be small. In particular, the coupling of the dual gluon is given in terms of that of the usual colour gluon by the Dirac quantization condition [11]:

$$g\tilde{g} = 4\pi, \tag{4.1}$$

which means that for the usual colour coupling g having the observed value of around 1.18 at the Z mass, the dual colour coupling \tilde{g} is of order 10. Thus, loop diagrams such as Figure 1(a) and (d), in which the integrated momenta need not be low so that the propagator suppression by a high dual gluon mass is inoperative, can in fact take on very large values. They cannot then be treated perturbatively.

However, not all the diagrams in Figure 1 rotate the fermion mass matrix with respect to dual colour, which rotation is needed to give a nontrivial CKM matrix and nonzero masses to the two lower generations. Indeed, as we shall see, it turns out that the large contributions from Figure 1 will all affect only the normalisation of the mass matrix but not its orientation in dual colour, so that as far as the effects we seek are concerned, there are only small corrections to be considered. This is very fortunate, for otherwise one would not be able to calculate the lower generation quark masses and the CKM matrix perturbatively as we set out to do.

To see that this fortunate situation does indeed arise, consider first the last term in Σ^{T1} in (3.2) which represents the dual gauge boson tadpole diagram exhibited in Figure 1(d). This term is huge, being proportional to $\tilde{g}_3^2 \mu_N^2 \ln \mu_N^2$, where \tilde{g}_3 , as already mentioned above, is of order 10, and the dual gauge bosons, in order for their exchanges not to violate the very stringent experimental bounds on flavour-changing neutral current decays, have to have masses μ_N of the order of 100 TeV [13, 15]. However, in terms of the basis of Higgs mass eigenstates, this term appears as:

$$\frac{3}{16\pi^2} \sum_K \bar{\Gamma}_K M_K^{-2} \sum_N (\bar{\theta}_N^2 \lambda)_K \mu_N^2 \ln \mu_N^2. \quad (4.2)$$

Now, according to our previous result stated in (3.27), only the Higgs state $K = 1$ has nonvanishing coupling to the dual gauge bosons so that the sum over K in (4.2) has only the $K = 1$ term. By (3.24)-(3.26), however, $\bar{\Gamma}_1$ is itself proportional to the tree-level mass matrix m_W in (3.5), so that the whole diagram has the effect only of changing the normalization of the tree-level mass matrix as anticipated.

A similar conclusion is reached for the other terms Σ^{A1} , $\Sigma^{A\phi}$ and Σ^{AT} in (3.2), coming from the dual gauge boson loop. In contrast to the dual gauge boson tadpole studied in the above paragraph, these terms do rotate the fermion mass matrix but do so only through the mass matrix m itself on which these terms depend. Suppose then we expand these expressions in powers of m , then the leading term of order m will be just a scalar times the original tree-level mass matrix and can therefore only affect its normalization, not its orientation. The other terms in the expansion which rotate the mass matrix will be of order m^2/μ_N^2 times the mass matrix and hence much smaller, in fact even negligible, as we shall see later.

The fact that the normalization of the fermion mass matrix is affected by large loop corrections means of course that we cannot hope to calculate its value perturbatively but have to treat it as a parameter to be determined experimentally. It means in particular that the one nonzero eigenvalue of m corresponding to the mass of the highest generation which started at tree-level as $\rho\zeta$ can now no longer be given in terms of ρ , the Yukawa coupling, and ζ the vacuum expectation value of the Higgs fields, but has to be treated as a separate parameter, say m_T (where T labels the type of fermions under consideration, namely U or D for quarks and L for leptons), thus reducing the predictive power of the present calculation. On the other hand, since

the normalization cannot be predicted in any case, there is now no sense in calculating those diagrams which affect the normalization only. Hence, as mentioned already in the beginning of this section, one may safely ignore all those diagrams with loops of the ordinary colour gluons and the usual electroweak Higgs bosons.

A similar conclusion applies also to the dual Higgs tadpole diagram in Figure 1(c) representing the first term in Σ^{T1} of (3.2). In the diagonal basis for the Higgs mass matrix M , this term appears as:

$$\frac{1}{32\pi^2} \sum_K \sum_L \bar{\Gamma}_K M_K^2 \bar{f}_{LLK} (M^2 \ln M^2)_L, \quad (4.3)$$

where since, according to (3.30), \bar{f}_{LLK} vanish except for $K = 1$, there remains only one term in the sum proportional to the matrix $\bar{\Gamma}_1$, which is itself proportional to the tree-level mass matrix m . It will therefore affect only the normalization of m , not its orientation, and thus, by the logic above, can also be ignored. Given that in Figure 1, only the diagram (c) depends on the Higgs self-couplings f_{ijk} , this means that we can henceforth eliminate this coupling from our considerations.

The remaining terms in Figure 1, namely the terms of order m^2/μ_N^2 or higher in Σ^{A1} , $\Sigma^{A\Phi}$, and Σ^{AT} , the term $\Sigma^{\phi 1}$, and the second term in Σ^{T1} corresponding to the fermion-loop tadpole of Figure 1(e), the explicit expressions for which are given in (3.2), all rotate the mass matrix. However, as explained in ref. [4], they will leave the mass matrix in a factorized form with only the left-handed factor rotated. In other words, the correction Σ from these terms can each be written in the form:

$$\Sigma = m_T \begin{pmatrix} x_{1+} \\ y_{1+} \\ z_{1+} \end{pmatrix} (x, y, z) \frac{1}{2}(1 + \gamma_5) + m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x_{1-}, y_{1-}, z_{1-}) \frac{1}{2}(1 - \gamma_5). \quad (4.4)$$

Added to the tree-level mass matrix and symmetrized with respect to left and right this gives:

$$\Sigma = m_T \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} (x, y, z) \frac{1}{2}(1 + \gamma_5) + m_T \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x', y', z') \frac{1}{2}(1 - \gamma_5), \quad (4.5)$$

where $x' = x - \frac{1}{2}x_{1+} - \frac{1}{2}x_{1-}$ etc. Again, as in the tree-level mass matrix, the γ_5 terms in (4.5) can be rotated away by redefining the right-handed fermions

to give:

$$m'_W = \frac{m_T}{\zeta'^2} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} (x', y', z'). \quad (4.6)$$

Hence, in order to specify the effect of the loop corrections, we need only give for each the corresponding $x_1 = \frac{1}{2}x_{1+} + \frac{1}{2}x_{1-}$ etc.

The results of our calculation are as follows. From the sum of Σ^{A1} and Σ^{AT} in (3.2), we obtain:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = -\frac{1}{8\pi^2} \sum_N \bar{T}_N f_N(m^2) \bar{T}_N \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad (4.7)$$

where \bar{T}_N are just the dual gluon couplings \bar{t}_N listed in (3.21) without the $\frac{1}{2}(1 - \gamma_5)$ factor:

$$\bar{t}_N = \bar{T}_N \frac{1}{2}(1 - \gamma_5), \quad (4.8)$$

and $f_N(m^2)$ is the integral:

$$f_N(m^2) = \int_0^1 dx (1 - x) \ln \left[1 + \frac{(m^2/\mu_N^2)x^2}{1 - x} \right]. \quad (4.9)$$

From $\Sigma^{A\phi}$ (3.2), we have:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \frac{1}{16\pi^2} \sum_N \frac{1}{\mu_N^2} \left[-m^2 \bar{T}_N g_N^-(m^2) \bar{T}_N + \bar{T}_N m^2 g_N^+(m^2) \bar{T}_N \right] \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad (4.10)$$

with:

$$g_N^\pm(m^2) = \int_0^1 dx (1 \pm x) \left[\ln \left(\frac{m^2 x^2}{1 - x} \right) - \ln \left(\mu_N^2 + \frac{m^2 x^2}{1 - x} \right) \right]. \quad (4.11)$$

From $\Sigma^{\phi 1}$ in (3.2) we obtain:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = -\frac{1}{16\pi^2} \left[\sum_K A_K |v_1\rangle + \sum_K B_K |v_K\rangle \right], \quad (4.12)$$

where $|v_K\rangle$ are as listed in (3.26) and (3.19). The first term with:

$$A_K = -\rho^2 \langle v_K | F_K(m^2) | v_K \rangle \quad (4.13)$$

need not bother us, being proportional to $|v_1\rangle$ which is the same as for the tree-level mass matrix. For the second term, we have:

$$B_K = -\rho^2 \langle v_1 | F_K(m^2) | v_1 \rangle \langle v_K | v_1 \rangle + 2\rho^2 \langle v_1 | G_K(m^2) | v_1 \rangle \langle v_1 | v_K \rangle, \quad (4.14)$$

with

$$F_K(m^2) = \int_0^1 dx (1-x) \ln[m^2 x^2 + M_K^2 (1-x)], \quad (4.15)$$

and

$$G_K(m^2) = \int_0^1 dx \ln[m^2 x^2 + M_K^2 (1-x)]. \quad (4.16)$$

Lastly, from the fermion-loop tadpole term of Figure 1(e) as given in the second term of Σ^{T1} in (3.2), we obtain:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = -\frac{1}{\pi^2} \frac{\rho^2}{m_T} \sum_K M_K^{-2} \text{Tr}\{m^3 \ln m \bar{\gamma}_K\} |v_K\rangle, \quad (4.17)$$

with $\bar{\gamma}_K$ as given in (3.25). The sum here is to be taken only over the Higgs bosons with nonzero masses, namely over only $K = 1, \dots, 7$ with M_K given in (3.18).

Although the terms listed in the preceding paragraph all rotate the mass matrix and hence could contribute to the present calculation of the CKM matrix and lower generation masses, they are widely different in size. Thus, the terms (4.7) and (4.10) are both of order m^2/μ_N^2 where m is about 176 GeV for U -type quarks and 4.3 GeV for D -type quarks, while, as already mentioned before, the dual gauge bosons are bounded by experiment to have masses larger than 100 TeV, a bound that we shall be able to check later within the present framework for consistency. That being the case, the corrections due to these two terms are only of the order of 10^{-6} or less and are thus seen to be entirely negligible for calculating the lower generation quark masses or the CKM matrix to the present experimental accuracy. A similar conclusion is reached also for the term (4.17) which is of order $(m^2 \ln m)/M_K^2$, with Higgs boson masses M_K being estimated to be of order some tens of

TeV, again an estimate that we shall be able to check for consistency within the present framework. Hence, the end result of our analysis is that of all the 1-loop corrections we have evaluated, only the term (4.12) from the Higgs boson loop in Figure 1(b) is potentially large enough to give the right orders of magnitude for the lower generation quark masses or for the off-diagonal CKM matrix elements, and it is therefore to this term that we shall now direct our attention.

5 The Rotating Mass Matrix

The Higgs boson loop $\Sigma^{\phi 1}$ in (3.2) of Figure 1(b) not only rotates the fermion mass matrix but rotates it in a manner which depends on the renormalization scale. To see this, let us write the logarithm in (4.15) and (4.16) at any scale $\tilde{\mu}$ as a sum of its value at some given scale μ plus a scale-dependent term, thus:

$$\ln \left[\frac{m^2 x^2}{\tilde{\mu}^2} + \frac{M_K^2}{\tilde{\mu}^2} (1-x) \right] = \ln \left[\frac{m^2 x^2}{\mu^2} + \frac{M_K^2}{\mu^2} (1-x) \right] + \ln[\mu^2/\tilde{\mu}^2]. \quad (5.1)$$

If we change the scale from μ to $\tilde{\mu}$, then $\Sigma^{\phi 1}$ in (3.2) will change by the amount:

$$\left\{ -\frac{1}{32\pi^2} \sum_K m \gamma_4 \bar{\Gamma}_K \gamma_4 \bar{\Gamma}_K + \frac{1}{16\pi^2} \sum_K \bar{\Gamma}_K m \bar{\Gamma}_K \right\} \ln[\tilde{\mu}^2/\mu^2]. \quad (5.2)$$

Recalling (3.24), we can rewrite the first term within the curly brackets as:

$$-\frac{1}{32\pi^2} \sum_K m \{ \bar{\gamma}_K^\dagger \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) + \bar{\gamma}_K \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \}, \quad (5.3)$$

and the second term as:

$$\frac{1}{16\pi^2} \sum_K \{ \bar{\gamma}_K m \bar{\gamma}_K \frac{1}{2} (1 + \gamma_5) + \bar{\gamma}_K^\dagger m \bar{\gamma}_K^\dagger \frac{1}{2} (1 - \gamma_5) \}. \quad (5.4)$$

On substituting $\bar{\gamma}_K$ from (3.25) and summing over K we obtain for the first term:

$$-\frac{1}{32\pi^2} m_T \rho^2 \left\{ 3 \begin{pmatrix} x \\ y \\ z \end{pmatrix} (x, y, z) \frac{1}{2} (1 + \gamma_5) + \begin{pmatrix} x \\ y \\ z \end{pmatrix} (\tilde{x}_1, \tilde{y}_1, \tilde{z}_1) \frac{1}{2} (1 - \gamma_5) \right\}, \quad (5.5)$$

and for the second term:

$$\frac{1}{16\pi^2} m_T \rho^2 \left\{ - \begin{pmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{pmatrix} (x, y, z) \frac{1}{2} (1 + \gamma_5) - \begin{pmatrix} x \\ y \\ z \end{pmatrix} (\tilde{x}_1, \tilde{y}_1, \tilde{z}_1) \frac{1}{2} (1 - \gamma_5) \right\}, \quad (5.6)$$

with:

$$\tilde{x}_1 = \frac{x(x^2 - y^2)}{x^2 + y^2} + \frac{x(x^2 - z^2)}{x^2 + z^2}, \text{ cyclic}, \quad (5.7)$$

where we have kept only the contributions from $K = 3, 5, 7$ since those from $K = 1$ and the sum of $K = 2, 4, 6$ affect only the normalization of m , while those from $K = 8, 9$ both vanish. In principle, of course, a change in the normalization of m will get reflected also in its orientation, but this is of second order in smallness if the change in scale is small and can therefore be neglected.

The scale-dependent corrections (5.5) and (5.6) leave the mass matrix factorized, as expected from the arguments given in ref [4], but no longer hermitian. However, following the convention introduced above in (4.5) and (4.6), one can redefine the right-handed fermion fields again so as to make the corrected mass matrix m' hermitian. The net result then is that, apart from changes in the normalization which may be ignored, one obtains from these terms a rotation to the mass matrix which may be represented as a rotation to the vector (x, y, z) , thus:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \longrightarrow \begin{pmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \frac{5}{64\pi^2} m_T \rho^2 \begin{pmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{pmatrix} \ln[\tilde{\mu}^2/\mu^2], \quad (5.8)$$

which depends on the change in scale.

By iterating the formula (5.8) in small steps, one can evaluate the rotation in (x', y', z') over finite changes of scales. One arrives then at a picture similar to the familiar one of running coupling constants, except that here it is a normalized vector (x', y', z') that 'runs'. From (5.7), it is readily seen that for (x', y', z') equal to $(1, 0, 0)$ or $\frac{1}{\sqrt{3}}(1, 1, 1)$, the increment due to a change of scale vanishes. These 2 vectors are thus fixed points in the usual sense under changes of scales. It can also be seen from (5.7) that for other values of (x', y', z') (where we have adopted the convention $x' > y' > z'$), and for

decreasing energy scales, the increment satisfies the following inequalities:

$$\frac{\tilde{x}'}{\tilde{y}'} < \frac{x'}{y'}; \quad \frac{\tilde{y}'}{\tilde{z}'} < \frac{y'}{z'}, \quad (5.9)$$

which mean that for decreasing scales, the vector (x', y', z') will ‘run’ away from the fixed point $(1, 0, 0)$ towards, in general, the fixed point $\frac{1}{\sqrt{3}}(1, 1, 1)$, tracing out a trajectory as the scale decreases. These assertions are confirmed by the numerical calculation presented in Figure 2, where the spacing between points on a trajectory denotes the speed (in arbitrary units) at which the vector (x', y', z') runs as a function of $\ln \mu$. Since (x', y', z') is normalized by definition, only the values of x' and y' need be presented and the trajectories are bounded by the circle $x'^2 + y'^2 = 1$, while the convention adopted above, of $x' > y' > z'$, restricts the trajectories to within the line $x' = y'$ and the ellipse $y'^2 = (1 - x'^2 - y'^2)$. This figure gives us a very useful picture to which we shall often refer.

We recall that the vector (x', y', z') represents actually the two identical factors of the factorized mass matrix (4.6) at the 1-loop level, so that a rotating (x', y', z') implies a mass matrix with a scale-dependent orientation. Now, for such a case, it is not so obvious how the mass and state vector of each individual state ought to be defined. This ambiguity is not a peculiarity of the dualized standard model alone but arises already in the ordinary (i.e. nondualized) standard model where a mass matrix with scale-dependent orientation occurs by virtue of the non-diagonal CKM matrix in the renormalization group equation [4]; only there, the effect being small, its consequences can be neglected. The point is the following. At any scale, the mass matrix can of course be diagonalized and, being hermitian, its eigenvalues will be real and, if nondegenerate, their eigenvectors are orthogonal to one another so that the transformation matrix, say S , from the original basis to the new basis formed by the eigenvectors will be unitary. However, these eigenvalues are scale dependent, and cannot as yet be identified as the mass of the individual states. Usually, the actual mass of a state is defined as the value of the scale-dependent mass evaluated at the scale equal to its value at that scale: $m(\mu) = \mu$. Here, however, since the eigenvalues are different by assumption, there is no scale for which this criterion can be satisfied simultaneously for all of them. One can, of course, take each eigenvalue and evaluate it at the scale equal to its value, and hence define at this scale the corresponding eigenvector as the state vector of the particle with this value as its mass. But

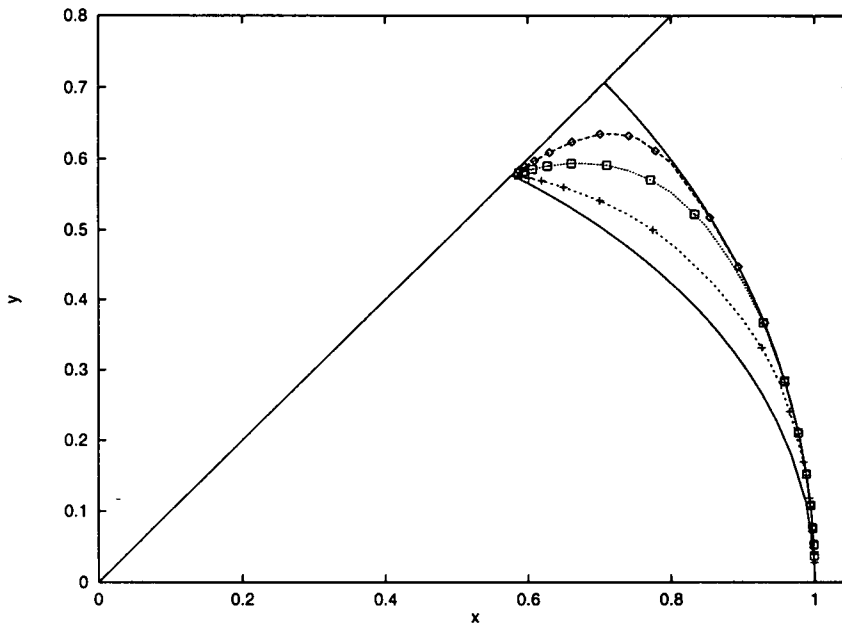


Figure 2: The ‘running’ of the vector (x', y', z')

since the orientation of the mass matrix is itself supposed to depend also on the scale, the state vectors so defined for the various particles at different scales will in general not be orthogonal to one another, so that the transformation S from the original basis to this new basis of state vectors will not in general be unitary. In fact, we do not know of a general prescription which can define, from a rotating mass matrix, the masses and state vectors of the individual particle states by the normal criterion which yet leaves the transformation matrix S unitary.

However, for a mass matrix which remains factorizable at all scales as the one considered here, it turns out that there is a way [4] in which masses and

state vectors can be defined in accordance to the normal criterion and which gives a unitary S matrix. To be specific, let us take first the U -type quarks as example. The mass matrix being factorizable, there is only one non-zero eigenvalue at any scale. Suppose we evaluate this nonzero eigenvalue at the scale equal to its value. Then, in accordance with the standard criterion above, we can define this value unambiguously as the mass of the top quark. The state vector of the top quark is thereby also defined uniquely as that eigenvector with the nonzero eigenvalue at the scale of the top mass, which in the present framework is just the vector (x', y', z') taken at the top mass. At this scale, the other two eigenvalues are zero, but they should not be regarded as the masses of the two lower generations for they are evaluated at the wrong scale. Furthermore, one does not know at this stage which two vectors should correspond to the 2 lower generations. However, being physically independent states, the 2 lower generations ought to have state vectors lying in the 2-dimensional subspace orthogonal to the top state vector, namely in the space spanned by two vectors with zero eigenvalues which we may choose as:

$$|v_2\rangle = -\beta \begin{pmatrix} y' - z' \\ z' - x' \\ x' - y' \end{pmatrix}, |v_3\rangle = \beta \begin{pmatrix} 1 - x'(x' + y' + z') \\ 1 - y'(x' + y' + z') \\ 1 - z'(x' + y' + z') \end{pmatrix}, \quad (5.10)$$

with

$$\beta = 1/\sqrt{3 - (x' + y' + z')^2}, \quad (5.11)$$

although at this stage one does not know which linear combinations of these two vectors should represent the c quark and which the u quark.

Suppose we now lower the scale μ by some small amount $\Delta\mu$. Then we know from (5.8) that the mass matrix will be rotated via a rotation of the vector (x', y', z') by a small amount proportional to $\ln(\mu^2/m_t^2)$. Hence, by repeated application of this procedure, one can evaluate the loop-corrected mass matrix \tilde{m}' at a scale some finite amount lower than the top mass. At the lower scale, because of the rotation, the vectors $|v_i\rangle$ are no longer eigenvectors of \tilde{m}' , so that in particular the mass submatrix:

$$\langle v_i | \tilde{m}' | v_j \rangle, \quad i, j = 2, 3, \quad (5.12)$$

which was zero at the top mass scale is here no longer the null matrix. However, being a submatrix of a rank 1 matrix, it is still of at most rank

1 and hence has at most one nonzero eigenvalue, the value of which will depend on the scale where the expression is evaluated. Applying then the same reasoning as for the top quark, we now vary the scale until the nonzero eigenvalue of (5.12) comes out equal to the scale itself, which value we shall call the charm mass for consistency. At the same time, the eigenvector $|\mathbf{v}'_2\rangle$ corresponding to this eigenvalue at this scale we define as the state vector of the c quark, which is, of course, by definition orthogonal to the state vector $|\mathbf{v}_1\rangle$ defined above for the t quark, as it should be. Furthermore, the state vector of the u quark is now also automatically given as the vector $|\mathbf{v}'_3\rangle$ which is orthogonal to both the top quark state vector $|\mathbf{v}_1\rangle$ and the charm quark state vector $|\mathbf{v}'_2\rangle$ already defined. At this stage, then, the state vectors of the 3 generations are all specified which we were unable to do before.

Finally, to find the mass of the u quark, we lower the scale again in small steps, applying repeatedly (5.8), to some scale. This scale we then vary until the value of

$$\langle \mathbf{v}'_3 | \tilde{m}' | \mathbf{v}'_3 \rangle \quad (5.13)$$

becomes equal to the scale itself, and this we define as the mass of the u quark, again in conformity to the procedure above. So now the masses of all three physical states are also defined, and they will all in general be nonzero.

We notice that the masses of all three generations t, c, u here are each defined using the normal criterion of evaluating the appropriate eigenvalue of the mass matrix at the scale equal to its value. Moreover, the three state vectors corresponding to the three generations so defined are also mutually orthogonal so that the matrix S transforming from the original “gauge basis” to the “physical basis” of state vectors is unitary as it ought to be. The actual values of the masses and state vectors so defined depend on the manner that the mass matrix rotates as a function of the energy scale, which in our present scheme depends in turn on the vacuum expectation values x, y, z of the (dual colour) Higgs fields and on the strength ρ_U of their Yukawa couplings to the U -type quarks, the values of which parameters have yet to be specified.

A similar procedure applied to the D -type quarks defines in turn the masses and state vectors of the b, s , and d quarks. The actual values of these quantities in the present scheme will depend on the same Higgs fields vacuum expectation values x, y, z as for the U -type quarks but in general a different Yukawa coupling strength ρ_D . Together with m_T , the normalization of the mass matrix for each quark type T , which may be identified with the

highest generation mass, there are altogether then 6 parameters, namely m_U , m_D , ρ_U , ρ_D , and the vector (x, y, z) which, being normalized to unit length, counts only as 2 parameters. With the remaining 4 parameters then, one is required to evaluate the 4 masses of the 2 lower generations m_c, m_u, m_s, m_d by the method described above, as well as the 4 relevant parameters of the CKM matrix in the manner outlined below.

By definition, the CKM matrix is the matrix giving the relative orientation of the physical state vectors of the 3 U -type quarks to those of the 3 D -type quarks. In terms of the notation introduced above, it is given as the matrix:

$$V_{ij} = \langle \mathbf{v}'_i | \mathbf{v}'_j \rangle, \quad (5.14)$$

where $i(j) = 1, 2, 3$ denote respectively $t(b), c(s), u(d)$, which in usual convention are arranged in the reversed order. Now, in the literature, the CKM matrix is often defined also as the overlap UD^\dagger between the matrix U which diagonalizes the mass matrix of the U -type quarks and the matrix D which diagonalizes the mass matrix of the D -type quarks. This definition is equivalent to that adopted above in terms of the physical states when the mass matrices do not rotate with the energy scale. When the mass matrices have scale-dependent orientations, however, the 2 definitions differ, since the vectors which diagonalize the mass matrices, as explained above, need no longer represent the physical states. Indeed, since the mass matrices are scale-dependent so will be their diagonalizing matrices U and D , and so also will be the CKM matrix which is defined as their overlap. On the other hand, the physical state vectors defined in the preceding section for the 3 generations of both the U -type and D -type quarks are all scale-independent, so that the CKM matrix defined as the transformation between the U physical basis to the D physical basis is also scale-independent. Here we shall evaluate the CKM matrix defined as the transformation matrix (5.14) between bases of physical states, which definition accords more with the philosophy adopted in this paper and seems to us also to correspond more to what is actually measured experimentally.

Before we proceed to numerical work, however, let us first note a qualitative feature of the present procedure which is of relevance both to our future calculation and to its comparison with experiment. The empirical CKM matrix, though near identity, has off-diagonal elements differing considerably in size, varying from around 20 percent for V_{cd} and V_{us} through a few percent

for V_{ts} and V_{cb} to just a few permille for V_{td} and V_{ub} . This variation may seem difficult to explain since if the matrix is due to some effect rotating the U -type mass matrix relative to the D -type mass matrix, one would expect the mixing elements to be of roughly the same order of magnitude. In the present scheme, however, there is a natural explanation for this variation. We recall that the state vectors of the two lower generations are to be defined through the running rotation of the mass matrices, so that these vectors get an extra kick in orientation in addition to that of the frames at the top and bottom mass. And it is this effect, having strictly to do with the special way that the physical states of the lower generations are here defined, which gives the Cabibbo angle a sort of special status among CKM matrix elements and hence, as we shall see, a particularly large value in comparison with the others as experimentally observed.

6 Numerical Results

To perform the calculation outlined in the preceding section, given any vector (x, y, z) for the Higgs fields vacuum expectation values which also doubles as the factor of the zeroth order fermion mass matrix, we face as our first task the evaluation of the 1-loop corrected vector:

$$|\mathbf{v}_1\rangle = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, \quad (6.1)$$

(properly normalized), for x_1, y_1, z_1 evaluated at the energy scale, respectively for the U - and D -type quark, of the top and bottom mass. This cannot be done by applying directly the formula (4.12) derived above for the following reason. The expression (4.12) depends on the masses of the Higgs bosons M_K , which in turn depend on the strength ζ of the Higgs vacuum expectation values and the Higgs self-couplings κ and λ . Of these parameters, λ is irrelevant since it occurs only in M_1 entering in A_1 and B_1 of (4.12) which are seen to affect only the normalization of (x_1, y_1, z_1) , not its orientation, while the other two occur only together in the combination $\kappa\zeta^2$, as seen in (3.18). From the lower bound on the dual gauge boson mass of around 400 TeV deduced from the absence of flavour-changing neutral currents effects in meson decays, one obtains from (3.7) an estimate of about 20 TeV for a

lower bound on ζ . Assuming the coupling κ to be of order unity, this gives an estimate for a lower bound on M_K also of around 20 TeV. Now the formula for (x_1, y_1, z_1) involves $\ln M_K^2$, which when evaluated directly at the top and bottom mass scales as we desired would be very large and violate the spirit of our present perturbative calculation. However, there is no real problem in this, for we can always evaluate first the correction (x_1, y_1, z_1) at the scale of the Higgs mass, say 20 TeV, and then use the formula (5.8) to ‘run’ the corrected vector by steps down to the scales of the top and bottom mass. At every step, then, the calculation would be perturbative for the correction is kept always small. This is in the spirit of the original Gell-Mann-Low idea [14] which led to the renormalization group equation.

A calculation done in this way, however, still leaves it dependent explicitly on the masses of the Higgs bosons. This would be a little awkward but for a happy and quite intriguing ‘accident’ to be elucidated later, for these masses are known only by the tree-level formulae (3.18) which are likely to be strongly renormalized, like the fermion masses, by e.g. the dual gluon loops. Because of this ‘accident’, however, it turns out that to a very good approximation we can put all the Higgs boson masses equal, say to a common scale M_H , even the value of which in the end does not really matter, but which we take at the moment to be 20 TeV. We need then to evaluate the formulae (4.12) for the common scale $M_H = M_K$ for all K . This expression is almost the same for U - and D -type quarks, and indeed even for leptons, differing only in the normalization m_T of the mass matrix. This difference is small, only of the order of m_T^2/M_H^2 which for M_H around 20 TeV, is less than 10^{-4} , as we have checked both by analytic and numerical calculations. It can thus be safely neglected. This means that whatever the correction due to (4.12) happens to be at the scale M_H (which is in fact quite small numerically), it will be the same for all the fermions. Therefore, in the present approximation when all M_K ’s are put equal, we can just start at the scale M_H with the same values of x', y', z' for all fermions, and simply ‘run’ them down to the mass of the highest generation for each fermion type to evaluate the vector $|v_1\rangle$ in (6.1) for each case.

The ‘running’ mechanism (4.12) and the starting values at M_H both being the same for all fermion types, the vector (x', y', z') will in this approximation ‘run’ along the same trajectory, only possibly at different speeds because the Yukawa couplings ρ_T may be different. In any case, since the m_T ’s are different, one would arrive at a different state vector for the highest

generation $|\mathbf{v}_1\rangle$ for the different fermion types. Starting with some input values for ρ_T and some initial values for (x', y', z') , say (x_I, y_I, z_I) , at the scale M_H , and applying (4.12) repeatedly in small steps, one would arrive at some value for $|\mathbf{v}_1\rangle$ for each fermion type. In principle, the steps should be infinitesimal, but in our numerical calculation we used typically about 500 steps for each decade of energy which we found were just about sufficient for the 1% accuracy that we wanted.

Having defined $|\mathbf{v}_1\rangle$ for each fermion type, we can now ‘run’ further down in energy scale to the second generation mass. As the mass matrix rotates in running, the mass will ‘leak’ into the second generation and give it a mass, as explained in the section above. The amount of leakage will depend on the value of the Yukawa coupling strength ρ_T and the range of energy run. Hence, the mass obtained from leakage at the mass scale of the second generation will in general be different for the different fermion types and different also from the actual input mass of the second generation. By adjusting the values of ρ_T , one can adjust the amount of leakage and hence ensure that the leaked mass obtained for the second generation is indeed the same as the input mass for each fermion type. Let us call these optimized values of ρ_T so obtained at this stage as the output ρ ’s.

These output ρ ’s, however, were determined starting from some vector $|\mathbf{v}_1\rangle$ for the highest generation, which in turn depended on the assumed input values of ρ_T used to run the initial vector (x_I, y_I, z_I) from the scale M_H down to the scale of the highest generation. Obviously, the input and output values of these ρ ’s need not be the same. We have thus to optimize again and adjust the input values of ρ_T until the output value is in each case the same as the output value of ρ_T obtained from it. This optimized value we now call the fitted ρ_T .

With these fitted values for ρ_T giving good second generation masses, we can now determine the state vectors $|\mathbf{v}'_i\rangle, i = 2, 3$ both for the second and the lowest generations, as we explained in the preceding section. Then, with the physical state vectors for all three generations and both U - and D -type quarks all determined, the CKM matrix easily follows from (5.14). Further, by running down to even lower energy scales, we can calculate the mass of the lowest generation by requiring that the ‘leaked’ mass in the lowest state in some scale be equal to the scale itself to which it is run. One sees thus that given any initial value (x_I, y_I, z_I) at the scale M_H , our program automatically determines for us the values of ρ_T which fit the masses of the

top 2 generations for each fermion type, and then gives us the CKM matrix and the lowest generation masses as the result. We have thus in effect just 2 real parameters left to adjust with which to calculate all these physical quantities.

We recall that the present scheme does not allow us to calculate the absolute normalization of the mass matrix, nor its variation with the energy scale, but only the orientation of the mass matrix. We are therefore more confident with our result on the CKM matrix which depends only on the orientation than on the fermion masses. The calculation of the fermion masses depends in principle on the scale-dependence not only of the normalization of the mass matrix but also of the ρ 's, which dependence, for lack of anything better, we have simply ignored. In our calculation therefore, we have concentrated on getting a good fit to the CKM matrix rather than the masses of the lowest generation.

With the whole calculation involving only real quantities, it is clear that we shall not be able to obtain any CP-violating phase in our CKM matrix. There are thus only 3 independent real parameters in the CKM matrix to calculate. We focus first our attention on the last row and column of V , namely that labelled by t and b . The state vectors of t and b , which in our notation were denoted by $|\mathbf{v}_1^U\rangle$ $|\mathbf{v}_1^D\rangle$ respectively, are not affected by the additional rotation of the physical states from the highest to the second generation, which, as explained in the last paragraph of the preceding section, is responsible for the particularly large value of the Cabibbo angle. Their relative orientation therefore give the measure of the relative rotation of the vectors (x', y', z') when run from the starting value at M_H to the respective highest generation mass. One sees that the difference in orientation between these two states are quite small, the off-diagonal elements being only of the order of a few percent in magnitude. However, the distances run from the starting point M_H to respectively the t and b mass are quite different, being only about 2 decades in energy for the t and nearly 4 decades for the b . Therefore, to end up with only a few percent difference in orientation, either the Yukawa couplings ρ_T must be so small as to give little running, which would contradict the sizeable amount of 'leakage' required to give the second generation mass, or the vectors (x', y', z') have to be near a fixed point so that the running is rather inefficient. We explored first the 'upper' fixed point $\frac{1}{\sqrt{3}}(1, 1, 1)$, but found no sensible solution. The 'lower' fixed point $(1, 0, 0)$, however, proved productive.

In the input initial values of (x_I, y_I, z_I) , for $x_I \sim 1$ and $y_I > z_I$ but both small, it is, crudely speaking, y_I which tells us how far down we are on the trajectory, and the relative size of z_I to y_I which tells us which trajectory we are on. By adjusting y_I , one can thus make the relative orientation between the t and b states, as exhibited in e.g. V_{ts} and V_{cb} , to be of the order of a few percent as required by experiment. Then, by adjusting z_I , to which V_{ts} and V_{cb} are quite insensitive, one can fit the Cabibbo angle V_{us} and V_{cd} to the empirical value of around 20 percent. The best result we have obtained so far in this exercise is shown below:

$$|V_{rs}| = \begin{pmatrix} 0.9755 & 0.2199 & 0.0044 \\ 0.2195 & 0.9746 & 0.0452 \\ 0.0143 & 0.0431 & 0.9990 \end{pmatrix}, \quad (6.2)$$

This is to be compared with the result below obtained from experiment [1]:

$$|V_{rs}| = \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix}. \quad (6.3)$$

The agreement is seen to be good. This we find encouraging, first that we can indeed adjust our parameters to obtain good values for the Cabibbo angle and the V_{ts} and V_{cb} elements, and second that once we have fitted these to approximately the right values, then V_{ub} and V_{td} automatically come out to be a few permille in magnitude as experimentally observed, which seem to indicate that the method we used for defining the lower generations states have somehow got the orientation right. We have calculated also with the same values of the parameters certain products and ratios of matrix elements which have been independently measured, obtaining:

$$\begin{aligned} |V_{ub}|/|V_{cb}| &= 0.0983, \\ |V_{td}|/|V_{ts}| &= 0.3310, \\ |V_{tb}^* V_{td}| &= 0.0142, \end{aligned} \quad (6.4)$$

to be compared with the values below quoted from the databook [1]:

$$\begin{aligned} |V_{ub}|/|V_{cb}| &= 0.08 \pm 0.02, \\ |V_{tb}|/|V_{ts}| &< 0.37, \\ |V_{tb}^* V_{td}| &= 0.009 \pm 0.003, \end{aligned} \quad (6.5)$$

The agreement is again reasonable.

These numbers were calculated with the following masses in GeV for the fermions in the 2 highest generations:

$$m_t = 176, m_b = 4.295, m_\tau = 1.777, m_c = 1.327, m_s = 0.173, m_\mu = 0.106, \quad (6.6)$$

where the central value was taken where such is given but otherwise the geometric mean of the upper and lower experimental bounds as listed in the databook [1]. We have included in the fit the charged leptons τ and μ which, though not entering into the CKM matrix, can be dealt with in the same manner as the quarks at the cost of only an extra ρ parameter. The initial values of (x_I, y_I, z_I) at the scale of $M_H = 20$ TeV chosen to fit the CKM elements were:

$$x_I = 0.999998, y_I = 0.002200, z_I = 0.000025. \quad (6.7)$$

The fitted ρ 's which emerged automatically from the requirement of consistency with the input masses (6.6) turned out then to be:

$$\rho_U = 3.4737, \rho_D = 3.4693, \rho_L = 3.4728, \quad (6.8)$$

which are encouragingly all of order unity.

One quite amazing feature of the parameters obtained from the fit is the close proximity to one another of the values of the Yukawa couplings ρ for all three fermion types, the spread of which in (6.8) is only around 1 part per mille. The actual values listed in (6.8) depend of course on the input values of the masses (6.6) of the fermions of the 2 higher generations. However, even if we vary these input masses to the utmost extremes allowed by the experimental bounds, the ρ 's are found by calculation to remain roughly equal, differing from one another always by less than 10 percent. At first sight, this may seem strange for the ratio of the second to highest generation mass differ considerably from fermion type to fermion type. For example, $(m_c/m_t) \sim 0.7$ percent, while $(m_s/m_b) \sim 4$ percent, a factor of 6 different, which would mean that the 'leakage' of mass by running from b to s must be several times stronger than that from t to c , suggesting that the coupling ρ which governs the speed of this running ought to be several times bigger for the D -type than for the U -type. The reason why this does not happen in the present calculation is that, the t quark being heavier than the b , lies

further down the trajectory depicted in Figure 2, i.e. nearer the fixed point $(1, 0, 0)$, so that the running rotation there is much less efficient than at the b mass which lies much higher on the trajectory. Hence, with about the same value for ρ one can still obtain widely different ‘leakages’ in the two cases. However, that the fitted values of ρ_T should come out so close to one another for all 3 fermion types is a bit of a surprise.

This approximate equality of the ρ ’s for all 3 fermion types is what we called our “happy accident” at the beginning of this section which gives us a number of practical advantages in our calculation. First, we recall that in the calculation reported above, we had made the simplifying assumption that all the Higgs bosons had the same mass M_H , which would be far from the truth if we believe the tree-level relations (3.18) given the very different values we need in (6.7) for x, y and z . To take Higgs mass splitting into account, one ought in principle to proceed as follows. One first goes up to the scale of the highest Higgs mass, which in the present case, according to (3.18) and (6.7), would be $M_4 \sim M_6 \sim$ several orders of magnitude higher than the lowest Higgs mass M_H of around 20 TeV. At this high scale, we have next to calculate the rotation to the original Higgs vacuum expectation values (x, y, z) due to the $K = 4, 5, 6, 7$ terms in the Higgs loop diagram (4.12), and then run the resulting (x', y', z') down to the scale of the lightest Higgs, namely $M_2 = M_H$. Then the result of this running has to be added to result of the rotation to the original (x, y, z) due to the $K = 2, 3$ terms in (4.12), and it is this sum that we have in principle to use as the initial vector (x_I, y_I, z_I) for our above calculation. If the Yukawa couplings ρ were different for the 3 fermion types, then (x_I, y_I, z_I) so obtained would be different also. Now, however, because of the “happy accident” of the ρ ’s being the same, (and the m_T -dependence of (4.12) being, as explained before, negligible), the resulting (x_I, y_I, z_I) of the above manoeuvre would be the same for the 3 fermion types. Hence, our ‘simplifying’ assumption made at the beginning of starting with the same (x_I, y_I, z_I) for all fermion types at scale M_H is now *a posteriori* entirely justified.

Further, this “happy accident” implies also that the calculation is actually independent of the scale M_H which we have so far chosen to be 20 TeV. To see this, recall that we were supposed to start with some (x_I, y_I, z_I) at M_H for all fermion types and run the vector, with the appropriate ρ ’s, down to respectively the t , b and τ mass values. If the ρ ’s were different, then starting with a different M_H , one would arrive at different values at the

highest generation mass for the 3 fermion types. Now that the ρ 's are the same, however, there is only one value of (x', y', z') at every point of the trajectory. One can therefore start at any point of the trajectory with some (x_I, y_I, z_I) for all 3 types of fermions and obtain the same answer. That this assertion holds even for approximately equal ρ 's has been checked numerically by repeating our calculation for various starting points M_H . It means that M_H can be removed altogether from our calculation as a parameter, leaving thus only the 2 ratios between (x_I, y_I, z_I) as the only parameters in the calculation, as we have claimed.

The other intriguing feature of the fit is the proximity of the fitted values in (6.7) of these (x_I, y_I, z_I) to the fixed point $(1, 0, 0)$. In contrast to the approximate equality of the ρ 's discussed above, this outcome is no accident but, as already explained before, is required by the smallness of all other off-diagonal CKM matrix elements besides the Cabibbo angle. Although the values of (x_I, y_I, z_I) at the arbitrary starting point M_H do not by themselves have much significance, we can deduce from them the vacuum expectation values (x, y, z) of the Higgs fields by running the scale backwards up to the highest Higgs mass and evaluating (4.12) there. Assuming the lowest Higgs mass 20 TeV, the tree-level formulae (3.18), and using (4.12), one obtains in this way for the vacuum expectation rough values of the Higgs fields:

$$x \sim 1, \quad y \sim 5 \times 10^{-5}, \quad z \sim 1 \times 10^{-8}, \quad (6.9)$$

which are very close indeed to the fixed point $(1, 0, 0)$

Though perhaps just fortuitous, the proximity of the fitted (x, y, z) to the fixed point $(1, 0, 0)$ and the near equality of the fitted ρ 's are so remarkable that it is tempting to consider the exciting possibility of the coincidence representing in fact a symmetry which is exact in some approximation and is only perturbed from it by an external agent. One possibility, for example, could be that if electroweak effects are neglected, then (x, y, z) is exactly $(1, 0, 0)$ and the ρ 's are exactly equal, and it is only the electroweak effects which give rise to the quantities's departure from the equilibrium values. At this stage, of course, the suggestion is a pure conjecture on our part, but it may be a worthwhile conjecture to entertain.

Having now determined the parameters of the problem, it is an easy matter to run the vector (x', y', z') further down in the energy scale and evaluate the masses of the lowest generation fermions following the method outlined in the

preceding section. Notice, however, that this calculation depends in principle on the scale-dependence both of the normalization of the mass matrix and also of the ρ 's, neither of which are calculable in the present framework. In fact, even the earlier calculation of the CKM matrix depends to some extent on these through fitting the ρ 's to the 2 higher generation fermion masses, but there, the change in scale not being too large, the change in normalization can be masked by adjusting the parameters in the fit, and hence not too noticeable. In calculating further the lowest generation masses, the effect of the neglect is compounded, and not too good results can be expected. Our calculation, using the values of the fitted parameters (6.7) and (6.8) and the same higher generation masses (6.6) and assuming constant ρ 's and mass matrix normalizations throughout the whole energy range of over 6 decades, gives:

$$m_u = 209 \text{ MeV}, m_d = 15 \text{ MeV}, m_e = 6 \text{ MeV}. \quad (6.10)$$

These mass values are fairly stable with respect to variations of the input masses for the 2 higher generations. For variations between the ranges given in the databook [1], the values obtained for the lowest generation lie in the range:

$$m_{\bar{u}} = 120 - 360 \text{ MeV}, m_d = 12 - 22 \text{ MeV}, m_e = 5 - 11 \text{ MeV}. \quad (6.11)$$

Apart from the mass of the u -quark, we regard these result as sensible given the crudeness of the assumption of no scale-dependence at all for either the Yukawa couplings ρ or the normalization of the mass matrix. It is perhaps interesting to understand technically why the mass for the u -quark turns out to be so much worse than in the other two cases. As explained above, the approximate equality of the ρ 's means that all 3 fermion types lie on the same trajectory, only differing in where the various physical states are placed. For the calculation here, these positions are shown in Figure 3. We notice there that the t -quark, being the heaviest fermion, lies of necessity the lowest on the trajectory, while the b and τ both lie higher up. For this reason, as already explained above, the running efficiency is much lower around the t mass than for the others so that even with the same value of ρ , the leakage from t to c is much smaller than that from b to s or from τ to μ . For the run from the second generation to the lowest, however, the U -type quark is now in the part of the trajectory where the running efficiency is high, while for the D -type quark and charged lepton, the lowest generation is already pressing

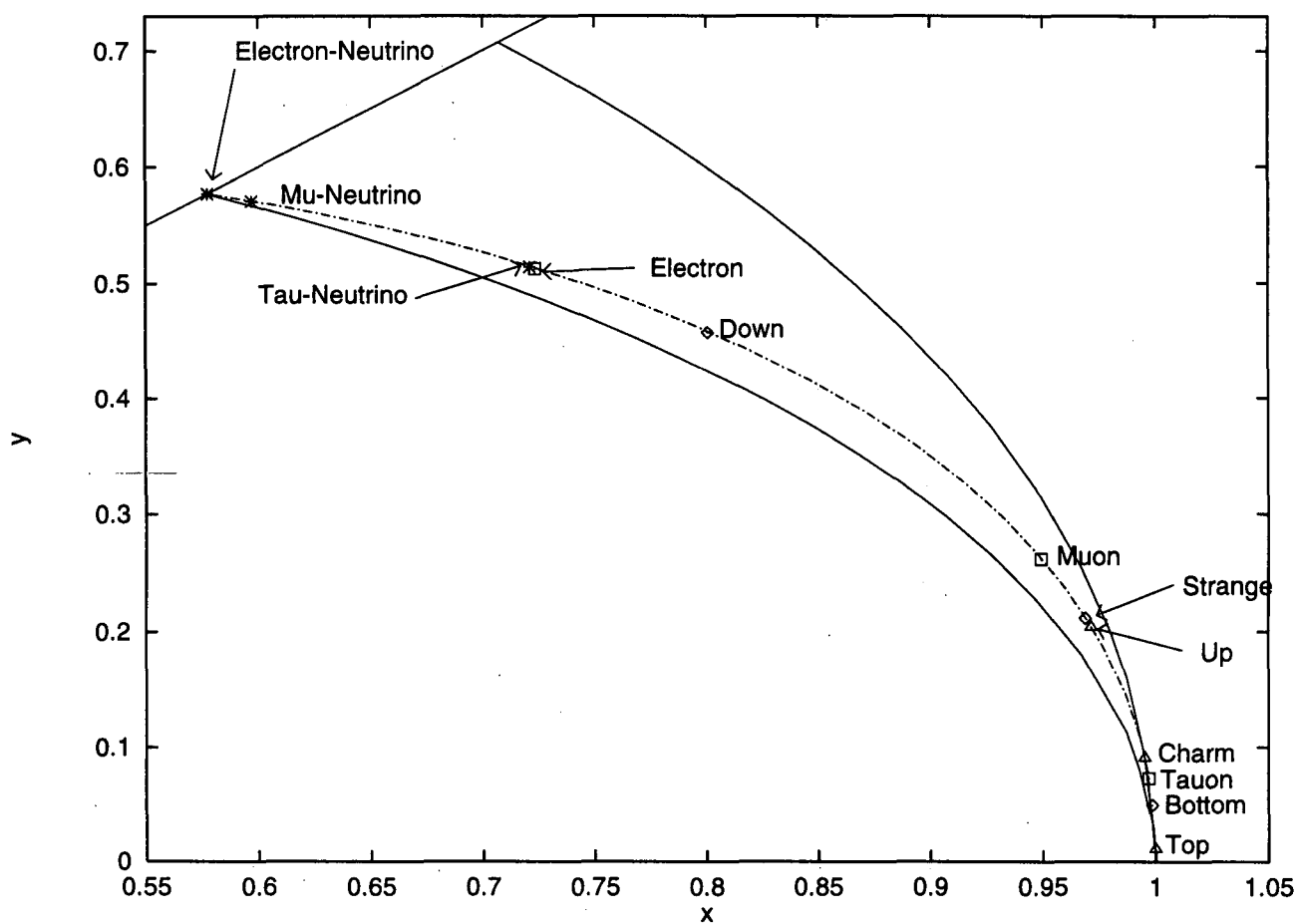


Figure 3: The locations of the various fermion states on the common trajectory

a little against the upper fixed point $\frac{1}{\sqrt{3}}(1, 1, 1)$ and losing running efficiency. Hence, we have the unwelcome large mass for the u -quark but not so large for d and e . When the scale-dependence of ρ and the normalization of the mass matrix are properly accounted for, a possible scenario may be that (x', y', z') will run faster along the trajectory so that all the lowest generation states will press against the upper fixed point $\frac{1}{\sqrt{3}}(1, 1, 1)$ and give lower masses for all of them, in particular for the u -quark. The investigation of this possibility, however, is beyond the scope of the present paper.

Finally, just as a matter of curiosity, let us apply the same sort of reasoning to the neutrino masses also. Assuming the same value of ρ for neutrinos as for the charged leptons, we can then in principle calculate the masses of all the neutrinos given any one of them. Or else, given the experimental upper bound of any neutrino, we can obtain upper bounds on the others. The strongest bounds obtained in this way, we found, comes from inputting the experimental bound < 0.17 MeV for the ν_μ mass quoted in [1]. Using the same fitted values of (x_I, y_I, z_I) in (6.7) we obtained:

$$m_{\nu_e} < 5 \text{ eV}, \quad m_{\nu_\tau} < 6 \text{ MeV}, \quad (6.12)$$

both of which, interestingly, are stronger than the experimental bounds: $m_{\nu_e} < 10 - 15 \text{ eV}$, $m_{\nu_\tau} < 24 \text{ MeV}$ given in [1]. We note that in Figure 3, the points representing the neutrinos all press quite tightly against the upper fixed point, especially for ν_e , which is why it gets such a stringent limit on its mass. These limits, however, should not be taken too seriously, since for the neutrinos, and indeed even for the charged leptons, there is much more than just the masses to be understood.

7 Conclusion and Remarks

In this paper, we set out to address the question whether the Dualized Standard Model scheme suggested in [4] is capable of giving reasonable CKM matrix elements and quark masses. The question has now, we think, been answered in the affirmative. Not only has one been able to fit the masses of the 2 higher generations sensibly with Yukawa coupling strengths all of order unity, but also with only 2 parameters then left to fit the absolute values of CKM matrix elements very well and give sensible estimates as well for the

fermion masses of the lowest generation except for the u -quark. This may not mean, of course, that the scheme is correct, but it is at least encouraging.

The calculation was done with dual Higgs and gauge boson masses consistent with existing bounds obtained from flavour-changing neutral currents decays [15, 13], namely $\sqrt{\kappa}\zeta = 20$ TeV, meaning Higgs masses of order several 10's of TeV and higher and gauge boson masses of order several 100's of TeV and higher. An estimate of these masses from the calculation, if available, would be of phenomenological significance, since it enters in FCNC decays [15, 16], and possibly also in understanding air showers from cosmic rays with energy greater than 10^{20} eV, namely those beyond the GZK spectral cutoff [17, 18, 15, 19]. However, unfortunately for this purpose though fortunately for the calculation, it turns out that the calculation is almost independent of the dual colour Higgs and gauge boson masses provided that they are large, so that no useful estimate for them can yet be made.

The calculation gave also a rather intriguing picture of how CKM mixing and lower generation fermion masses are generated, namely in terms of 'running' trajectories and fixed points. Two unexpected bonuses are the close proximity of the Higgs vacuum expectation values (x, y, z) to the fixed point $(1, 0, 0)$ and the near equality of the Yakawa coupling strengths ρ for the different fermion types. If one could find a theoretical reason why the ρ 's should be equal, or how (x, y, z) is given that miniscule departure from the fixed point $(1, 0, 0)$, one would be approaching a fit with a single parameter (the common ρ) to all CKM mixings and fermion mass splittings, which would be fantastic.

Of outstanding problems, we have identified two. One concerns the CP-violating phase in the CKM matrix which, as explained already, cannot be obtained in the present approach, at least not in first order. The other concerns the special properties of the mass matrix with scale-dependent orientation, also already discussed in section 5. The problem is that there does not seem to be a basis of state vectors with well-defined masses for which the mass matrix is exactly diagonal. In fact, this problem already figures in the ordinary (non-dualized) formulation of the Standard Model where a scale-dependent orientation is induced by the CKM matrix which cannot be made diagonal simultaneously with the mass matrix. The only difference is that the effect there is quite small, as shown in [4], and is for that reason often ignored. It seems to us that whichever description one chooses to adopt, whether in terms of the diagonal basis or the basis with definite masses, the

physics ought to be equivalent. However, the relation between the two descriptions and the physical consequences this implies have not been properly worked out.

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