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ScienceDirect
Journal of Magnetism and Magnetic Materials 310 (2007) 1549–1551
Journal of magnetism and magnetic materials
www.elsevier.com/locate/jmmm

Fractal dimension in percolating Heisenberg antiferromagnets

S. Itoh[a,*], R. Kajimoto[b], M.A. Adams[c], M.J. Bull[c], K. Iwasa[d], N. Aso[e], H. Yoshizawa[e], T. Takeuchi[f]

[a] Neutron Science Laboratory, High Energy Accelerator Research Organization, Tsukuba 305-0810, Japan
[b] Quantum Beam Science Directorate, Japan Atomic Energy Agency, Tokai 319-1195, Japan
[c] ISIS Facility, Rutherford Appleton Laboratory, Didcot, Oxon OX11 0QX, UK
[d] Department of Physics, Tohoku University, Sendai 980-8578, Japan
[e] Neutron Science Laboratory, Institute for Solid State Physics, University of Tokyo, Tokai 319-1106, Japan
[f] Low Temperature Center, Osaka University, Toyonaka 560-0043, Japan

Available online 9 November 2006

Abstract

We investigated static and dynamical properties in the three-dimensional percolating Heisenberg antiferromagnets, $RbMn_cMg_{1-c}F_3$, with the magnetic concentration close to the percolation threshold, $c_P = 0.312$, around the superlattice point well below T_N. In neutron diffraction experiment, the wave number dependence of the elastic scattering component was well fitted to q^{-x}. Magnetic fractons were also studied using inelastic neutron scattering, and the observed fractons showed the dispersion relation of q^z. The determined exponents, $x = 2.43 \pm 0.05$ and $z = 2.5 \pm 0.1$, were in good agreement with the fractal dimension ($D_f = 2.48$).

PACS: 61.43.Hv; 75.40.Cx; 75.40.G; 61.12.−q; 78.70.Nx

Keywords: Fractal; Fracton; Percolation; Antiferromagnet; Neutron scattering

It is generally accepted that the atomic connectivity of a percolating network takes the form of a fractal. A site-diluted antiferromagnet is recognized to be the simplest ideal percolating network for probing fractal nature. When the magnetic ions in a parent antiferromagnet are randomly replaced by nonmagnetic ions, the Néel temperature (T_N) decreases as the magnetic concentration (c) decreases, and T_N becomes zero at the critical concentration of the percolation threshold (c_P). At $c = c_P$, infinite spin clusters with a fractal geometry are present in the crystal.

The fractal structure is characterized by the self-similarity and the scattering law is proportional to q^{-D_f} (q: the wave number). In the two-dimensional (2D) percolating Ising antiferromagnet $Rb_2Co_cMg_{1-c}F_4$ [1] ($c = 0.6$ is very close to $c_P = 0.593$ [2]), the observed magnetic elastic scattering around the superlattice point was well fitted to q^{-x} with $x = 1.95 \pm 0.07$ in good agreement with $D_f = 1.896$ [2] for a square lattice.

Collective excitations on a fractal lattice are called fractons. In theories, the dispersion relation can be described by q^z with $z = D_f/d$, where d is the fracton dimension [3]. The fracton dimension is the exponent characterizing the density of states of fractons. A numerical simulation of the density of states showed that $d = 1$ for any Euclidean dimension [4]. Therefore, z is equal to D_f. However, the observed exponents for the three-dimensional (3D) system, $RbMn_cMg_{1-c}F_3$, as well as the 2D system, $Rb_2Mn_cMg_{1-c}F_4$, were much less than D_f [5,6].

We try to detect D_f in static and dynamical properties in the 3D Heisenberg antiferromagnets $RbMn_cMg_{1-c}F_3$ with c close to $c_P = 0.312$ [2] for a cubic lattice. The scattering law for 2D systems is close to a Lorentzian form, q^{-2}. The 3D system is a good example to see a clear difference between a fractal structure and thermal fluctuations. Also a high resolution inelastic neutron scattering experiment was performed to observe fractons clearly.

*Corresponding author. Tel./fax: +81 29 864 3202.
E-mail address: shinichi.itoh@kek.jp (S. Itoh).

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doi:10.1016/j.jmmm.2006.10.560

In order to observe a fractal structure, we performed neutron diffraction experiments at around the $(\frac{1}{2},\frac{1}{2},\frac{1}{2})$ magnetic superlattice point of a single crystal sample with $c = 0.31$ [7], on the PRISMA spectrometer installed at the pulsed neutron source of the ISIS Facility at the Rutherford Appleton Laboratory, as well as on the triple axis spectrometer, GPTAS, installed at the steady state neutron source, JRR-3M, at the Japan Atomic Energy Agency. The scans along $[hhh]$ were performed in the diffraction mode in the range $1.6\,\mathrm{K} \leqslant T \leqslant 100\,\mathrm{K}$ on PRISMA, and in the double axis mode in the range $25\,\mathrm{mK} \leqslant T \leqslant 3\,\mathrm{K}$ on GPTAS.

The critical scattering at $T > T_N$ was found to be well described by that in a homogeneous system, $RbMnF_3$ [8], and determined $T_N = 4.0 \pm 0.5\,\mathrm{K}$. From the empirical form for the c dependence of T_N, the magnetic concentration can be estimated as $c = 0.32$ [7]. In the vicinity of c_P, the crossover wave number is defined as $q_c = |c - c_P|^{\nu_G} a_0^{-1}$, where ν_G is a numerical constant ($\nu_G = 0.88$ for a cubic system [2]) and a_0 is the lattice spacing. The system is fractal at $q > q_c$ and homogeneous at $q < q_c$. For $c = 0.32$, q_c is estimated to be $0.0036\,\text{Å}^{-1}$.

At $T = 1.6\,\mathrm{K}$ ($< T_N$), the spectrum should be described by the magnetic fractal structure. The observed spectrum also includes the critical scattering, because the system is a Heisenberg spin system and the transverse susceptibility exists even well below T_N. We assumed that the spectrum at $T = 5\,\mathrm{K}$, just above T_N, is equal to the critical scattering component at $T = 1.6\,\mathrm{K}$, so that the magnetic elastic component at $T = 1.6\,\mathrm{K}$ is determined by subtracting the $T = 5\,\mathrm{K}$ spectrum from the $T = 1.6\,\mathrm{K}$ spectrum. The elastic component should be negligible at $T = 5\,\mathrm{K}$ just above T_N. The deduced elastic component was well fitted to $(q^2 + k^2)^{-x/2}$ convoluted with the instrumental resolution, as shown in Fig. 1(a). If x was fixed at D_f, $k = 0.004 \pm 0.001\,\text{Å}^{-1}$ was obtained in good agreement with q_c. If k was fixed at q_c, $x = 2.43 \pm 0.05$ was obtained in good agreement with D_f. Therefore, this confirmed that the spectrum at $T = 5\,\mathrm{K}$ is a good approximation for the critical scattering component of the spectrum at $T = 1.6\,\mathrm{K}$. As shown in Fig. 1(a), the q range where q^{-D_f} is shown is consistent with q_c.

In order to observe fractons, we performed an inelastic neutron experiment at around the magnetic superlattice point of a single crystal sample with $c = 0.4$ [9], on the IRIS spectrometer at ISIS with a high energy resolution of $\Delta E = 17.5\,\mu\mathrm{eV}$ (FWHM) in the range $1.5\,\mathrm{K} \leqslant T \leqslant 100\,\mathrm{K}$. The details of the scan are described in Ref. [9]. First, we determined $T_N = 20\,\mathrm{K}$ from the very sharp T dependence of the energy width of the magnetic critical scattering. From T_N, we estimated $c = 0.4$ by the empirical formula [7], which is identical to that for the crystal preparation, and $q_c = 0.028\,\text{Å}^{-1}$ was obtained.

The observed spectrum consisted of an elastic peak, nondispersive peaks and a dispersive peak. At $T = 100\,\mathrm{K}$, these signals disappear except for the incoherent elastic scattering, therefore, these signals are of magnetic origin. The origins of the nondispersive peaks are spin cluster excitations such as those from a dimer and the Ising cluster excitations (spin flip in a molecular field). By fitting the scattering function of the dispersive component with the Lorentzian scattering function, the peak positions are obtained as shown in Fig. 1(b). The peak positions were well fitted by q^z with $z = 2.5 \pm 0.1$. This value of z is in good agreement with $D_f = 2.48$, as predicted by theory. The q range for this observation as shown in Fig. 1(b) is consistent with $q_c = 0.028\,\text{Å}^{-1}$. In the early work [5], fractons were observed in the $c = 0.39$ system at $q \geqslant 0.2\,\text{Å}^{-1}$ with $\Delta E = 1\,\mathrm{meV}$ (its magnetic properties were almost identical to the present system). Since the dispersion relation should be flat at the

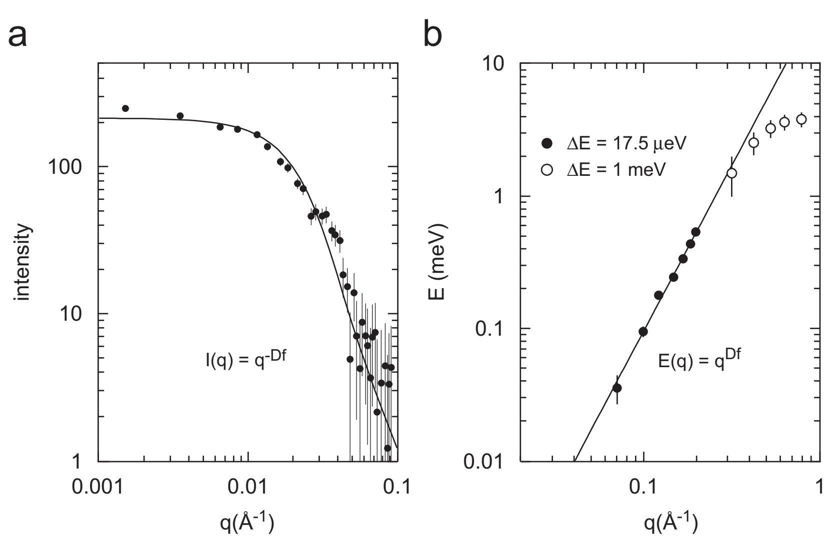

Fig. 1. Static and dynamical properties in $RbMn_{0.31}Mg_{0.69}F_3$. (a) The elastic component in the scattering function in the $c = 0.31$ sample at $T = 1.6\,\mathrm{K}$. The solid line is the fitted curve (see text). The flat intensity at low q comes from the instrumental resolution, not from q_c. (b) The dispersion relation of magnetic fractons in the $c = 0.4$ sample at $T = 1.5\,\mathrm{K}$ (closed circles). The solid line is the fitted line to q^z. The peak positions measured on the triple axis spectrometer are also plotted ($c = 0.39$ [5], open circles).

zone boundary due to continuity between zones, the predicted dispersion relation, q^z, can be clearly observed in the low-q and low-energy region.

In summary, we detected D_f in static and dynamical properties in the 3D percolating Heisenberg antiferromagnets $RbMn_cMg_{1-c}F_3$ with c close to $c_P = 0.312$. The elastic scattering and the fracton excitations were observed around the magnetic superlattice point well below T_N, we found that these properties are well described by D_f.

The pulsed neutron scattering experiments were performed under the Japan-UK Collaboration Programme on Neutron Scattering. This work was supported by a Grant-in-Aid for Creative Scientific Research (No. 16GS0417) from the Ministry of Education, Culture, Sports, Science and Technology of Japan.

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