RAL-TR-1998-067
IFUM-633-FT
hep-ph/9809451

Matrix Element Corrections to Parton Shower Simulations of Heavy Quark Decay

G. Corcella

Dipartimento di Fisica, Università di Milano and INFN, Sezione di Milano
Via Celoria 16, 20133 Milano, Italy

and

M.H. Seymour

Rutherford Appleton Laboratory, Chilton,
Didcot, Oxfordshire. OX11 0QX. U.K.

Abstract

Parton showers are accurate for soft and/or collinear emission, but for a good description of the whole of phase space they need to be supplemented by matrix element corrections. In this paper, we discuss matrix element corrections to the decay $t \to Wb$ and apply our results to the HERWIG Monte Carlo event generator. The phenomenological results show marked improvement relative to previous versions and agree well with the exact first-order matrix-element calculation.

RAL-TR-1998-067
IFUM-633-FT
September 1998

1 Introduction

Heavy quark physics is at present one of the main subjects of theoretical and experimental particle physics[1] and one of the objectives of future experiments at the LHC and NLC. Although total cross sections and other inclusive quantities can be reliably calculated using fixed-order perturbative QCD, for analyses that require a detailed description of final state properties one needs to sum large logarithmic contributions to all orders and include non-perturbative hadronization effects. Monte Carlo event generators use parton shower models to perform this resummation and phenomenologically-inspired models of the hadronization process.

The parton shower approach relies on the universality of the QCD matrix elements in the dominant regions of phase space: soft and/or collinear emission[2]. Multi-parton final states are built up by starting from a few well-separated hard partons and sequentially adding one extra parton at a time, distributed according to the soft and collinear factorization formulae. In the collinear limit it is straightforward to formulate this as a probabilistic Markov chain, as the factorization formulae are directly written in terms of probability distributions. In the soft limit, the factorization theorem applies at the amplitude level, and many interfering amplitudes contribute equally, so it would appear that no probabilistic approach could be formulated. However, the remarkable result[3] is that the interference is entirely destructive outside angular-ordered regions of phase space. Therefore parton shower algorithms can be extended to correctly treat both regions, simply by using opening angle as the ordering variable within the collinear approach. This is the basis of coherence-improved parton shower algorithms such as HERWIG[4].

Coherence is also important in setting the initial conditions of the parton shower: each hard parton may only radiate into an angular region extending as far as its 'colour partner'[5]. The assignment of colour partners is controlled by the hard matrix element, and thus the properties of the parton shower and the resulting jets are influenced by the kinematics of the hard scattering, giving a perturbative origin to coherent phenomena like the string effect[6].

It is straightforward to incorporate quark masses into a coherence-improved parton shower algorithm[7]. In heavy quark production, the mass terms result in an angle-dependent suppression that can be approximated by an angular cutoff: the massive quark radiates like a massless one at large angles, and not at all at small angles, $\theta \lesssim m_q/E_q$. Heavy quark decay acts like a new hard process in which gluons are emitted coherently by the heavy quark and by its lighter quark decay product. In principle, this separation into production and decay phases is only valid for gluon energies above the decay width of the quark, which for top quarks is of order 1 GeV, and below that the coherence of the whole production and decay chain should be taken into account[8]. In practice however, 1 GeV is close enough to the cutoff that terminates the parton shower that these effects may be neglected in the parton shower and only need to be incorporated in the non-perturbative hadronization phase.

Although the soft and collinear regions dominate the emission probabilities, many experimental observables are most sensitive to hard non-collinear emission. A poignant example is the kinematic reconstruction of the mass of decaying top quarks, in which the treatment of additional jets can make a sizeable difference, leading to an uncertainty in the final result that is almost as large as the experimental uncertainty[9]. Unfortunately, there is no guarantee that a parton shower algorithm will describe such emission well. It should, however, be well-described by the next-order (but still tree-level) matrix element. Thus, if one wants to have a good description of the bulk properties of events and the rare hard wide-angle emissions simultaneously, one must combine the parton shower and matrix element approaches to provide a process-specific 'matrix element correction' to the parton shower.

In general there are two shortcomings to be corrected: it is possible that owing to the angular-ordering condition, there are regions of phase space that are not covered at all by the parton shower (which we call 'dead zones'); and it is possible that even in the region it does cover, the parton shower is not a good approximation to the exact matrix element. General solutions to both these problems (which we call the 'hard' and 'soft' corrections respectively) were described in Ref. [10], and have been incorporated into HERWIG for e^+e^- annihilation[11] and DIS[12]. Similar issues were discussed for the JETSET/PYTHIA parton shower algorithm in Ref. [13].

In this paper we discuss the implementation of matrix element corrections to heavy quark decay in HERWIG and show first results. In section 2 we recall the relevant features of HERWIG's parton shower algorithm. In sections 3 and 4 we discuss the hard and soft matrix element correction respectively. In section 5 we present some phenomenological distributions for top quark decay and compare them with the exact QCD results and those obtained from the previous version of HERWIG. Finally in section 6 we make some concluding remarks.

2 The parton shower algorithm

For the decay[1] $t \to bW$, the elementary probability to emit an additional gluon is given by

$$dP = \frac{d\Gamma(t \to bWg)}{\Gamma_0}, \tag{1}$$

where Γ_0 is the width for the Born process. In the soft and collinear limits, this can be approximated by the universal elementary probability for the emission of an additional parton in any hard process:

$$dP = \frac{dq^2}{q^2}\,\frac{\alpha_S}{2\pi}\,P(z)dz\,\frac{\Delta_S(q^2_{\rm max},q^2_c)}{\Delta_S(q^2,q^2_c)}\ . \tag{2}$$

The HERWIG parton shower is ordered according to the variable $q^2 = E^2\xi$, where E is the energy of the emitter, $\xi = \frac{q_1 \cdot q_2}{E_1 E_2}$ and $q_{1,2}$ being the four-momenta of the produced partons; z is the energy fraction of the emitted parton in the showering frame. For

[1]In fact we use exactly the same method for all heavy (bottom and above) quarks, but for clarity we always use the language of top quark decay. Note also that although we describe the decay as $t \to bW(g)$, we actually use the full three-body matrix elements for $t \to bf\bar{f}'$ so that the $f\bar{f}'$ mass distribution and angular correlations are correct.

massless partons we have $\xi = 1 - \cos\theta$, where θ is the opening angle of emission, so that ordering in q^2 corresponds to ordering in angles.

The crucial quantity $\Delta_S(q^2, q_c^2)$ is the Sudakov form factor, representing the probability that no *resolvable* radiation is emitted from a parton whose upper limit on emission is q^2. The resolution requirement, which in the case of HERWIG is a cutoff on transverse momentum, is q_c^2. Thus the ratio of form factors appearing in Eq. (2) represents the probability that the emission considered is the first i.e. the highest in q^2. In diagrammatic language, this actually sums up the contributions of virtual and unresolvable real emission to all orders.

Using HERWIG's definitions for q^2 and z, the results of the parton shower are not Lorentz covariant, although they become so in the exactly soft or collinear limits in which the algorithm is formally valid. Nevertheless, the extrapolations away from those limits depend on the frame in which the shower is developed. HERWIG uses a different frame for each jet i, in which $\xi_{i\max} = 1$ and hence, since $q^2 = E^2\xi$, $E_i = q_{i\max}$. After parton showering, the momenta of the jets cannot be identical to the momenta of the partons that initiated them, since the latter are on mass-shell, so some momentum must be transferred between jets to conserved energy-momentum overall. The precise details become irrelevant in the soft and collinear limits in which the algorithm is valid, but are again important for the extrapolation away from that limit that we are concerned with here. The procedure is again frame-dependent and is performed by Lorentz boosting each jet along its direction in the rest-frame of the hard process that produced it.

Colour coherence in the hard process dictates the value of $q^2_{\max}$, which for colour-connected partons i and j are related by $q_{i\max}q_{j\max} = p_i \cdot p_j$. In principle, this is the only requirement, and one has a free choice for the individual values $q_{i\max}$ and $q_{j\max}$, corresponding to different choices of frame, but in most cases, symmetric limits are chosen: $q^2_{i\max} = q^2_{j\max} = p_i \cdot p_j$. However, for the special case of top quark decay, we choose $E_t = q_{t\max} = m_t$ and therefore $E_b = q_{b\max} = \frac{p_t \cdot p_b}{m_t}$, corresponding to the top rest-frame, in which the top quark itself does not radiate.

Because of this choice, and the fact that HERWIG only radiates into the angular-ordered region $\xi < 1$, there is no radiation in the W hemisphere, while in the full matrix element such radiation is suppressed but not absent. Therefore the dead zone actually includes part of the soft singularity, in contrast to the e^+e^- and DIS cases in which soft emission from one or other jets covers the whole of the angular range. This poses some additional problems, which we describe heuristically here, and in more detail in the following sections.

Since HERWIG's dead zone includes part of the soft singularity, the total amount of emission into it, if calculated naïvely, is infinite. Therefore the hard correction method of Ref. [10] cannot be used. The correct solution to this problem is to modify HERWIG's parton shower algorithm so that it populates the backward direction. Unfortunately, this is far from straightforward to do. Instead, we look for simpler, more approximate, solutions.

If we apply a cutoff on the gluon energy then, provided the total probability of emitting into the dead zone above the cutoff is small, the hard correction can be used. Although this is only an approximation we can easily check, by varying the cutoff, how good it is. In section 5 we show that varying the cutoff in a reasonable range gives a negligible effect on physical distributions.

However, the fact that we apply an energy cut in the hard correction has implications for the soft correction. The general idea of HERWIG's approach to modelling emission from heavy quarks[7] is that in the soft limit, the emission pattern is similar to that from a light quark at large angles, but is smoothly suppressed at small angles, according to the replacement

$$\frac{1}{\theta^2} \longrightarrow \frac{\theta^2}{(\theta^2 + 1/\gamma^2)^2}, \tag{3}$$

where γ is the Lorentz factor of the emitting quark. This is approximated by a step function at $\theta = 1/\gamma$. While this does not give a very good description of the angular distribution of emitted gluons, it does give a good approximation to the total amount of gluon emission, which is important for the total energy loss of the heavy quark for example. In other words, we get the amount of gluon emission about right, but tend to put it in the wrong place. For top decay, this is at its most extreme, since we work in its rest-frame in which the neglected angular region is the whole of the W hemisphere.

If we were to apply a soft matrix element correction in the usual way, we would decrease the amount of emission in the b hemisphere but, because we use a cutoff on the gluon energy in the hard correction, for gluons below this cutoff there would be no corresponding increase in the W hemisphere. We would therefore get the total amount of emission wrong, and hence quantities like the b quark's energy spectrum would be poorly described. To avoid this, we use the same cutoff in the soft correction, and only correct the distribution of gluons with energy above it. Below the cutoff, the standard parton shower description is used alone.

In summary, we can construct the matrix element corrections as the combination of hard and soft corrections in exactly the usual way, but only in the region of gluon energies above a given cutoff, $E_{\min}$. Below the cutoff, we use the parton shower uncorrected. We use 2 GeV as the default value of the cutoff.

Finally, as discussed in Ref. [10], one should include a form factor in the region covered by the hard correction, to ensure smooth matching at the boundary. Since we approach the soft singularity in the top decay case, this could in principle be more important than in other cases. However, we find that even with a cutoff as small as 1 GeV, the form factor is never more than a few percent below unity, so does not significantly affect any physical distributions. We therefore neglect it for simplicity, as is already done in the e^+e^- and DIS cases.

3 Hard Corrections

The main step in constructing both the hard and soft matrix element corrections is to relate the variables generated by HERWIG, z and ξ, to the kinematic variables we use in the matrix element calculation. We parametrize the phase space of the process

$t(q) \to W(p_1)b(p_2)g(p_3)$ in terms of the variables[2]

$$x_1 = 1 - \frac{2p_2 \cdot p_3}{m_t^2} = \frac{2p_1 \cdot q}{m_t^2} - a \ , \tag{4}$$

where we have defined[3] $a = m_W^2/m_t^2$, and

$$x_3 = \frac{2p_3 \cdot q}{m_t^2} \ . \tag{5}$$

The phase space limits for the decay $t \to bWg$ are as follows:

$$\frac{a\, x_3}{1-x_3} + (1 - x_3) \ < \ x_1 \ < \ 1, \tag{6}$$

$$0 \ < \ x_3 \ < \ 1 - a. \tag{7}$$

In order to express x_1 and x_3 in terms of ξ and z, we observe that the Lorentz boost of the b quark's parton shower takes place in the top rest frame, so that the mass, m, of the b–g jet and its transverse momentum, q_t, relative to the b-W axis are conserved. We also define the energy of the b quark in the showering frame to be $\sqrt{c}m_t$ with, because HERWIG uses the top rest frame, $c = (1-a)^2/4$. In terms of the showering variables, we obtain:

$$m^2 = 2z(1-z)\xi \, c\, m_t^2, \tag{8}$$

$$q_t^2 = \frac{z^2(1-z)^2}{1-2z(1-z)\xi}\xi(2-\xi)\, c\, m_t^2, \tag{9}$$

and in terms of the matrix element variables:

$$m^2 = m_t^2(1-x_1), \tag{10}$$

$$q_t^2 = m_t^2 \frac{(1-x_1)(x_1 + x_3(2 - x_1 - a) - x_3^2 - 1)}{(x_1+a)^2 - 4a}. \tag{11}$$

They can be combined to give:

$$z = \frac{1}{2}\left\{1 + \frac{\sqrt{(x_1 - (1-c))/c}}{\sqrt{(x_1+a)^2 - 4a}}(2x_3 + x_1 - 2 + a)\right\}, \tag{12}$$

$$\xi = \frac{2(1-x_1)((x_1+a)^2 - 4a)}{(1-x_1)(2x_3 + x_1 - 2 + a)^2 + 4c(1-x_3)(x_1 + x_3 - 1) - 4a\, c\, x_3}. \tag{13}$$

The region not populated by HERWIG can be calculated from the condition $\xi > 1$:

$$0 \ < \ x_3 \ < \ 1 - a, \tag{14}$$

$$(1 - x_3) + \frac{a\, x_3}{1 - x_3} \ < \ x_1 \ < \ x_{1\max}, \tag{15}$$

[2]To simplify the formulae in this paper, we neglect the b mass, although we do include it in HERWIG, and hence in all figures that we show.

[3]Recalling that we use the three-body matrix elements for $t \to bf\bar{f}'$, a is actually defined to be $(p_f + p_{\bar{f}'})^2/m_t^2$, which varies from event to event, but this difference is unimportant here.

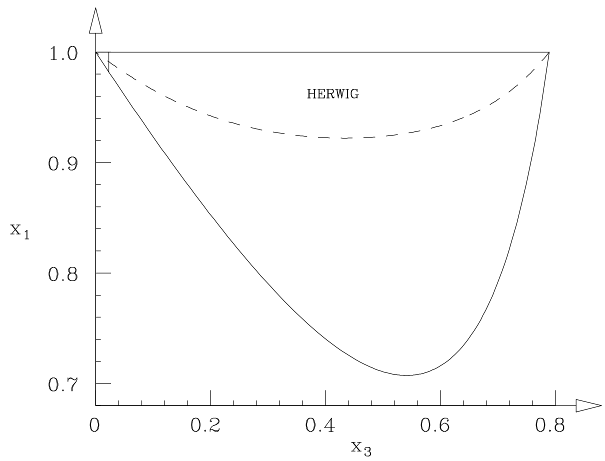

Figure 1: Phase space limits on $x_1 = \frac{2p_1 \cdot q}{m_t^2} - a$ and $x_3 = \frac{2p_3 \cdot q}{m_t^2}$ in the decay $t \to bWg$ (solid) and the edge of the region covered by HERWIG (dashed). Also shown, barely visible in the top-left corner, is a 2 GeV cutoff on the gluon energy.

with $x_{1\max}$ being the solution of the cubic equation

$$\begin{aligned} x_1^3 + (3+2a-4x_3)x_1^2 + (a^2 - 4(1-x_3)(2-c-x_3) - 2a(3+2x_3))x_1 \\ +a(4-a)(2-c) + (1-c)(2(1-x_3)-a)^2 &= 0. \end{aligned} \tag{16}$$

Such a solution can be expressed in the following form:

$$x_{1\max} = 2\Re\left\{\left(r + i\sqrt{q^3 - r^2}\right)^{1/3}\right\} - \frac{1}{3}(3+2a-4x_3), \tag{17}$$

where we have defined

$$q = \frac{1}{9}(3+2a-4x_3)^2 - \frac{1}{3}\left[a^2 - 4(1-x_3)(2-c-x_3) - 2a(3+2x_3)\right], \tag{18}$$

and

$$\begin{aligned} r &= \frac{1}{6}(3+2a-4x_3)\left[a^2 - 4(1-x_3)(2-c-x_3) - 2a(3+2x_3)\right] \\ &\quad -\frac{1}{2}\left(a(4-a)(2-c) + (1-c)\left[2(1-x_3)-a\right]^2\right) - \frac{1}{27}(3+2a-4x_3)^3. \end{aligned} \tag{19}$$

In Fig. 1 we plot the full phase space limits and HERWIG's limits. Note that the collinear limit is $x_1 = 1$ and the soft limit is $x_3 = 0$ and that, as anticipated, HERWIG's missing region includes part of the soft limit, which we avoid by using the cutoff $x_3 > 2E_{\min}/m_t$.

In the dead zone we use the exact differential width, which, continuing to neglect the b mass, is:

$$\begin{aligned} \frac{1}{\Gamma_0}\frac{d^2\Gamma}{dx_1\,dx_3} = {} & \frac{1}{(1-a)\left(1+\frac{1}{a}-2a\right)} 2\,\frac{\alpha_s}{2\pi}\,C_F \frac{1}{x_3^2(1-x_1)}\Big\{ (1+\tfrac{1}{a}-2a)\left[(1-a)x_3 - (1-x_1)(1-x_3) - x_3^2\right] \\ & +(1+\tfrac{1}{2a})x_3(x_1+x_3-1)^2 + 2x_3^2(1-x_1)\Big\}, \end{aligned} \tag{20}$$

It is straightforward to use standard techniques to generate this distribution within the allowed phase space region.

A final point involves the scale at which we evaluate α_s. As is well known, using the transverse momentum of the emitted gluon sums up an important class of next-to-leading logarithmic corrections[14]. However, we should be careful in using the transverse momentum in the backward hemisphere, because in the W direction it is zero, even though the gluon is not collinear to any coloured parton. We instead use the Durham-like generalization of transverse momentum,

$$\mu^2 = 2E_g^2(1-\cos\theta_{bg}) = \frac{x_3(1-x_1)}{2-x_1-x_3-a}m_t^2. \tag{21}$$

4 Soft Corrections

According to Ref. [10], the soft correction is implemented by multiplying the emission probability for every emission *that is the hardest so far* by a correction factor that is simply the ratio of HERWIG's differential distribution to the matrix element one. The only non-trivial part of this is in calculating the Jacobian factor $J(x_1, x_3)$ of the transformation $(z, \xi) \to (x_1, x_3)$. HERWIG's distribution is then given by

$$\frac{d^2\Gamma}{dx_1 dx_3} = \frac{d^2\Gamma}{dz d\xi}\, J(x_1, x_3), \tag{22}$$

where $d^2\Gamma/dzd\xi$ is given by the elementary emission probability given in Eq. (2). J can be simply calculated from the relations given earlier, but we have not found a particularly compact form so we do not reproduce it here.

We recall that we only apply the soft correction for $x_3 > 2E_{\min}/m_t$.

5 Results

The exact matrix element for $e^+e^- \to (bW^+)\,(\bar{b}W^-)\,g$ via top quarks was calculated in Refs. [15,16] and compared to HERWIG versions 5.8 and 5.9 respectively. In fact, serious bugs have been found in the treatment of top decays in both versions, so no firm conclusion could be reached about the need for matrix element corrections.

In this section, we repeat the analysis of Ref. [16] in which they consider a centre-of-mass energy only slightly above threshold, $\sqrt{s} = 360$ GeV, so that essentially all gluon emission is associated with the top quark decays. We work at the parton level (the final state of the parton shower, without hadronization or b decay) and cluster all partons into three jets using the $k_\perp$ algorithm[17]. Then the jets are required to be hard, $E_{Tj} > 10$ GeV, and well-separated[4], $\Delta R > 0.7$. We force the W decays to be leptonic and do not include their decay products in the jet clustering.

In Figs. 2 and 3, we show the differential distribution with respect to ΔR, the separation of the closest pair of jets in the event, and $\log y_3$, where y_3 is the value of the

[4] Note that the text of Ref. [16] says that the angular cut is $\Delta R > 0.4$, but it is clear from their Fig. 7(a) that it is actually $\Delta R > 0.7$.

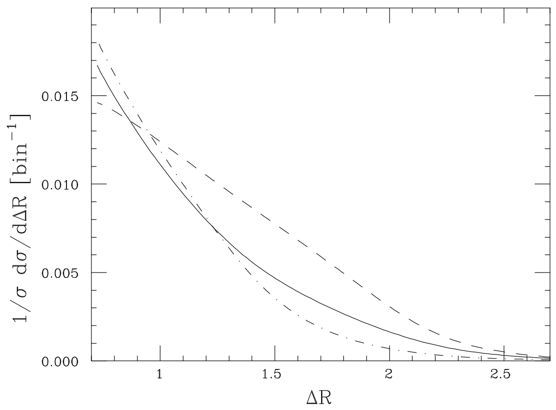

Figure 2: Differential distribution of invariant opening angle, $\Delta R^2 = \Delta\eta^2 + \Delta\phi^2$ for three-jet $e^+e^- \to t\bar{t}$ events at $\sqrt{s} =$ 360 GeV, according to HERWIG version 5.9 (dashed), 6.0 (dot-dashed) and the version with matrix element corrections, 6.1 (solid).

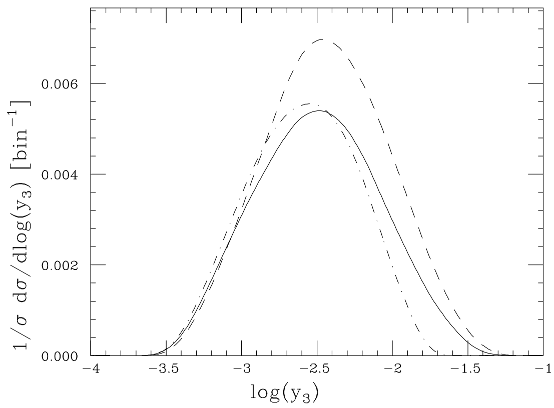

Figure 3: As Fig. 2 but for the distribution of Durham jet algorithm measure, $y_3 = \min_{ij}[\frac{2}{s}\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})]$.

cutoff in the $k_\perp$ algorithm at which the three jets would be merged to two, according to the last public version, 5.9, version 6.0, which has the bug that was found in 5.9 fixed, and a preliminary version that includes our matrix element corrections, 6.1. The version 5.9, which was used in Ref. [16], clearly gives too much gluon radiation. This is easily explained by the bug which was corrected in version 6.0[5], which is seen to have much less radiation at large angles and y_3 values.

The only difference for top decays between versions 6.0 and 6.1 is in the matrix element correction discussed here. Therefore Figs. 2 and 3 directly show its effect. At large angle and y_3 the rate of gluon emission is greatly enhanced due to the hard correction, which fills the missing region of phase space. At smaller angles and y_3 values, the soft correction gives a small reduction, showing that in the uncorrected version the deficit at large angles was at least partially compensated by a surfeit at small angles.

We turn now to a comparison with the tree-level matrix element results of Ref. [16]. In order to remove dependence on the electroweak production process (for example the fact that HERWIG uses the cross section for on-shell top quarks while the matrix element calculation implicitly includes the top width) we compare the differential distribution normalized to the total $e^+e^- \to t\bar{t}$ cross section. Even after doing this, the normalization of a tree-level calculation is still rather arbitrary, since the scale of α_s is not fixed. We choose to set α_s equal to its value at a typical transverse momentum of jets passing the cuts, $\alpha_s(\approx 30 \text{ GeV}) \approx 0.145$. Results are shown in Figs. 4 and 5. After applying matrix corrections the agreement is very good at large y_3 and ΔR. As y_3 is reduced, we probe lower momentum scales, where α_s is getting larger in HERWIG, while it is fixed in the matrix element calculation, so HERWIG lies above the calculation. Finally, at very small y_3, Sudakov suppression from multiple emission would be expected, which is

[5]The wrong frame had been used for the b parton shower: heavy quark decay was treated like all other processes and showered in the frame in which $E_b = \sqrt{p_t \cdot p_b} \approx 110$ GeV instead of the top rest frame in which $E_b = \frac{p_t \cdot p_b}{m_t} \approx 70$ GeV.

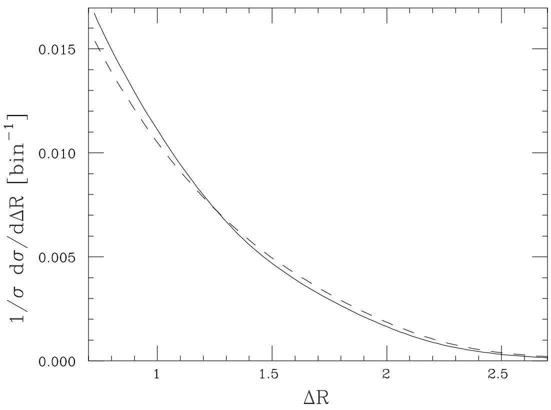

Figure 4: As Fig. 2 but from HERWIG 6.1 (solid) and the tree-level calculation with $\alpha_s = 0.145$ (dashed).

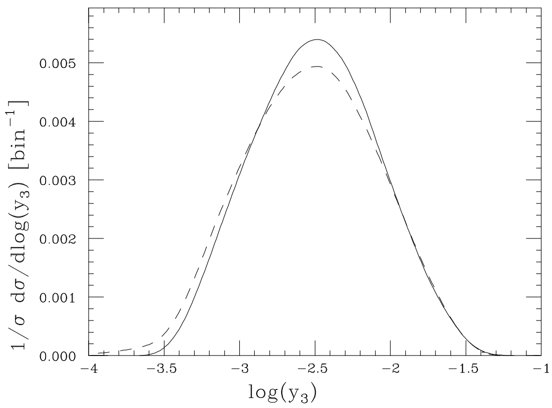

Figure 5: As Fig. 3 but from HERWIG 6.1 (solid) and the tree-level calculation with $\alpha_s = 0.145$ (dashed).

included in HERWIG but not in the matrix element calculation, so HERWIG lies below the calculation.

Finally, in Figs. 6 and 7 we show the dependence on the arbitrary cutoff on soft gluon energies we introduced in the hard matrix element correction. It is clearly insignificant. It is worth noting that the actual fraction of events that have an emission in the dead zone generated by the hard matrix element correction varies considerably with the cutoff, from 2.4% at 5 GeV to 3.8% at 1 GeV, so the fact that the physical distribution after cuts is unaffected is a non-trivial test of the self-consistency of the procedure. The default cutoff value, 2 GeV, was used for the previous plots.

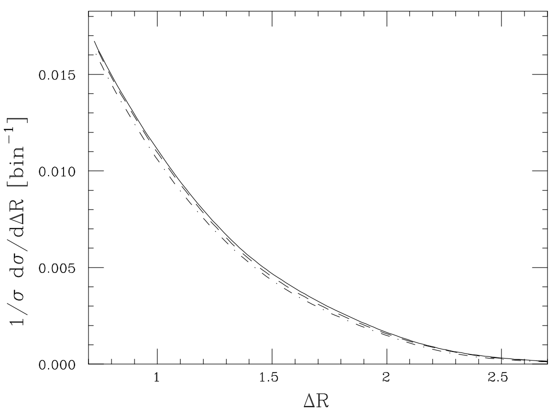

Figure 6: As Fig. 2 but from HERWIG 6.1 with gluon energy cutoff set to 1 GeV (dashed), 2 GeV (solid) and 5 GeV (dot-dashed).

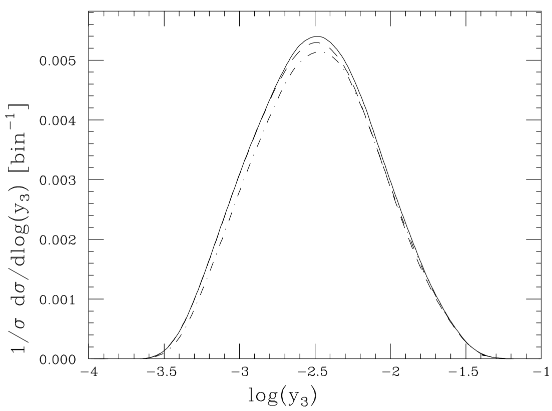

Figure 7: As Fig. 3 but from HERWIG 6.1 with gluon energy cutoff set to 1 GeV (dashed), 2 GeV (solid) and 5 GeV (dot-dashed).

6 Conclusions

We have discussed HERWIG's parton shower treatment of top quark decay. We applied the exact matrix-element result for emission in the region of phase space that it does not populate and for the hardest emission in the usually-populated region. Since the missing region includes part of the soft singularity, we have had to apply an arbitrary cutoff on the energy of gluons emitted in this region. The dependence on this cutoff is however negligible after applying jet cuts.

HERWIG's predictions are considerably improved by this correction, and agree well with the exact leading-order perturbative results. Where there are differences, they can be understood as advantages of the parton shower approach. We therefore feel confident that HERWIG now gives a reliable treatment of top quark decays.

It is clearly interesting to ask how much these corrections affect the top mass reconstruction from final state properties at the Tevatron. However, before doing so it is essential to also implement matrix element corrections to the top production processes. This work is in progress.

Acknowledgements

We are grateful to Yuri Dokshitzer, Michelangelo Mangano, Pino Marchesini, Stefano Moretti and Bryan Webber for fruitful discussions of these and related topics and to Tim Stelzer for providing the computer code to calculate the matrix element results in figures 4 and 5.

References

1. S. Frixione, M.L. Mangano, P. Nason and G. Ridolfi, *Heavy Quark Production*, to be published in *Heavy Flavours II*, ed. A.J. Buras and M. Lindner, World Scientific [hep-ph/9702287]

2. A. Bassetto, M. Ciafaloni and G. Marchesini, Phys. Rep. 100 (1983) 201
G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298

3. G. Marchesini and B.R. Webber, Nucl. Phys. B238 (1984) 1

4. G. Marchesini et al. Comput. Phys. Commun. 67 (1992) 465

5. G. Marchesini and B.R. Webber, Nucl. Phys. B310 (1988) 461
R.K. Ellis, G. Marchesini and B.R. Webber, Nucl. Phys. B286 (1987) 643; Erratum ibid. B294 (1987) 1180

6. Ya.I. Azimov, Yu.L. Dokshitzer, V.A. Khoze and S.I. Troyan, Phys. Lett. B165 (1985) 147

7. G. Marchesini and B.R. Webber, Nucl. Phys. B330 (1990) 261

8. V.A. Khoze and T. Sjöstrand Phys. Lett. B328 (1994) 466

9. The CDF Collaboration (F. Abe et al.) Phys. Rev. Lett. 80 (1998) 2779
The D0 Collaboration (B. Abbott et al.) Phys. Rev. D58 (1998) 052001

10. M.H. Seymour, Comput. Phys. Commun. 90 (1995) 95

11. M.H. Seymour, Z. Phys. C56 (1992) 161

12. M.H. Seymour, *Matrix Element Corrections to Parton Shower Simulation of Deep Inelastic Scattering*, contributed to 27th International Conference on High Energy Physics (ICHEP), Glasgow, 1994, Lund preprint LU-TP-94-12, unpublished

13. J. André and T. Sjöstrand, Phys. Rev. D57 (1998) 5767

14. S. Catani, G. Marchesini and B.R. Webber, Nucl. Phys. B349 (1991) 635

15. L.H. Orr, T. Stelzer and W.J. Stirling, Phys. Lett. B354 (1995) 442

16. L.H. Orr, T. Stelzer and W.J. Stirling, Phys. Rev. D56 (1997) 446

17. S. Catani, Yu.L. Dokshitzer, M. Olsson, G. Turnock and B.R. Webber, Phys. Lett. B269 (1991) 432