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Spin Motion Near Snake Resonances [1]

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Abstract. We give a brief account of on–going work on the loss of polarisation during acceleration close to so–called snake resonances in proton storage rings. We show that within the model studied here the polarisation can be preserved if the rate of acceleration is low enough.

INTRODUCTION

At SPIN2002 [1] we presented new results on features of the invariant spin field (ISF) at the so–called snake resonances in ring accelerators. This is the name given to the loss of polarisation in rings with Siberian Snakes, occurring during acceleration at certain vertical orbital tunes, namely tunes with fractional part $[Q_2] = 1/6,\ 5/6,\ 1/10,\ 3/10,\ 7/10,\ 9/10,\ 1/14,\ 3/14,\ 5/14,\ 9/14,\ 11/14,\ 13/14, \cdots$. In particular, we found that within our simple model, the ISF is unusual in that it is irreducibly discontinuous at snake–resonance tunes. Furthermore, its segments are close to horizontal at energies far from those of the parent resonance. In the meantime we have extended this work with the aim of understanding why polarisation should be lost during acceleration at such tunes. We concentrated on the representative tune $[Q_2] = 1/6$. The key outcome is that exactly at snake resonances the non–standard evolution, during acceleration, of initially vertical polarisation can be understood by invoking the concept of adiabatic invariance of the spin action and by considering the special geometry of the ISF together with the evolution of the eigentune of the appropriate multiturn spin map. The results of that study together with references to background material, an explanation of important concepts and our notation are available in [2]. In order to appreciate the contents of this paper, the reader should read [1, 2]. Thus we do not explain the concepts and notation again here.

Here, we give a brief account of further, and on–going, studies aimed at understanding spin motion near to, but not at, snake–resonance tunes. As in [1, 2] we work with a model ring containing two point–like Siberian Snakes at diametrically opposite sides of the ring with their snake axes at 0° and 90° to the longitudinal direction. Then the spin tune on the design orbit, ν_0, is 1/2 at all energies. The effect on the spin of vertical betatron motion is described by the so–called single resonance model. We illustrate our work by

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focussing on a $[Q_2]$ near $1/6$.

SPIN MOTION NEAR $[Q_2] = 1/6$

The reason for the non–standard behaviour of polarisation exactly at snake resonances was sketched above. But that case is special and, in fact, the depolarisation observed during acceleration near to, but not at, snake–resonance tunes has a different origin. In particular, in this case the ISF is close to the vertical at most large $|\delta|$ (i.e., small $\varepsilon/|\delta|$) so that an ensemble of vertical spins is then close to spin equilibrium in the sense explained in [2] and the spin actions J_s are close to 1. However, in contrast to the examples of non–snake–resonance tunes discussed in [2], for tunes near to, but not at, snake–resonance tunes, the ISF can be extremely complicated at small $|\delta|$ [1]. Moreover, as the tune approaches the snake–resonance tune, the level of complication, measured in terms of the Fourier spectrum with respect to the fractional normalised orbital phase $[\phi_2/2\pi]$, increases strongly at small $|\delta|$. For example, for $[Q_2] = 0.16667586...$, an irrational tune close to $[Q_2] = 1/6$, and for the resonance strength used in [2], namely $\varepsilon = 0.4$, the strongest harmonic is around 4300 at $\delta = 0$. Figure 1 shows the phase dependence of the vertical component of the ISF vector $\hat{n}$ at $\delta = 0$ and just before the 0° snake for this case. At large $|\delta|$ the strongest harmonics are around 1. Note that despite the complexity exhibited in figure 1, $\hat{n}_2(\phi_2)$ is, as required, a single valued function of ϕ_2 as can be seen by a detailed inspection. Our investigations show that the loss of polarisation usually seen in simulations of acceleration of an ensemble of initially vertical spins from large negative δ through zero to large positive δ is due to the loss of invariance of J_s in regions of δ where the ISF is varying very strongly with δ. Then, with $\varepsilon = 0.4$, an acceleration rate of less than 2 KeV per turn is needed to maintain the invariance of J_s. This is illustrated in figure 2 which shows the beam polarisation just before the 0° snake for acceleration rates of 2, 5 and 10 KeV per turn. Simulations are usually carried out at much higher rates of acceleration but far away from snake–resonance tunes the invariance of J_s can nevertheless be maintained. See, for example, Section 3.1 in [2]. At $[Q_2] = 0.16667586...$, $\varepsilon = 0.4$ and (say) $\delta \approx -3$, P_{lim} is small. So at $\delta \approx -3$ an ensemble of vertical spins would not be at equilibrium and, moreover, the initial vertical polarisation would not be preserved even at acceleration rates much lower than 2 KeV per turn.

Full details of these and further investigations will appear in the sequel to [2] announced therein.

SUMMARY

Our studies have led to a classification and a substantial clarification of spin motion during acceleration at and near snake–resonance vertical orbital tunes within our simple model. As in other parts of our work, the invariant spin field has been an essential tool.

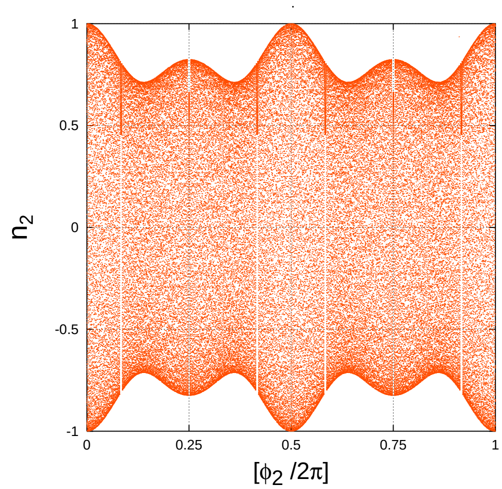

FIGURE 1. The vertical component of $\hat{n}(\phi_2)$ at $\delta = 0$ for $[Q_2] = 0.16667586...$ and $\varepsilon = 0.4$.

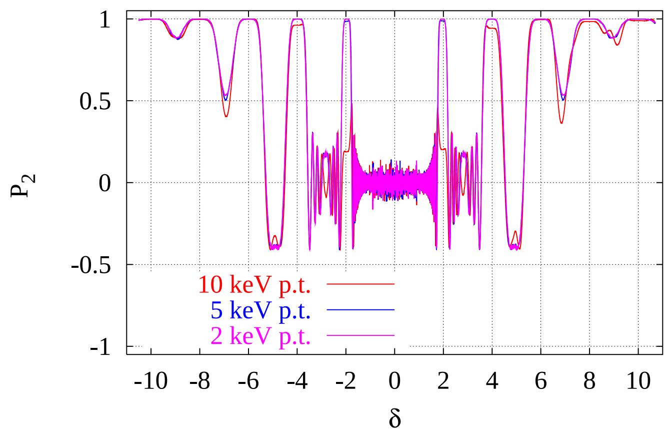

FIGURE 2. With each spin initially parallel to its ISF vector $\hat{n}$, the beam polarisation sampled turn–by–turn, for $[Q_2] = 0.16667586\ldots$ during acceleration from $\delta = -10.6$ to $\delta = +10.6$ at the rates of 2, 5 and 10 KeV per turn. $\varepsilon = 0.4$.

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