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Transverse Resistive Wall Wakefields of the ILC Positron Undulator Beam Tube and their Effects on an Orbit Dependant Emittance Growth[1]

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ABSTRACT

Previous work has calculated the longitudinal wakefields for the different bunch distributions, ILC parameters, vessel materials and DC, AC and anomalous skin effect impedances [1]. From these longitudinal wakes the transverse wakes can be simply be calculated via integration. The peak transverse kick is found to be 0.27 eV μm^{-1} m^{-1} for a 5.85 mm diameter copper vessel at 77 K. This has been used along with particle tracking in MAD and a simple Single Value Decomposition based correction algorithm to estimate an emittance growth of the electron beam caused by the undulator line. It was found that the emittance increase is due to the optics of the line, with the resistive wall wakefields being a negligible effect.

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1. Introduction

Previous work has calculated the monopole longitudinal resistive wall wakefields for the ILC undulator vacuum vessel [1]. These were calculated for a variety of different beam and vessel parameters. From the resistive wall monopole order longitudinal wake potential, $W_{\parallel}^{0}$, the dipole order transverse wake potential, $W_{\perp}^{1}$, can be derived using the following relation [2]:

$$W_{\perp}^{1}(s) = \frac{2}{b^2}\int_{0}^{s} W_{\parallel}^{0}(s')\,ds' .$$

And, for a given charge distribution, $\rho(s)$ and transverse wake potential, a transverse kick factor, $k_{\perp}$ can then be defined:

$$k_{\perp} = \int_{-\infty}^{\infty} \rho(s) W_{\perp}(s)\,ds ,$$

where, $\int_{-\infty}^{\infty} \rho(s)\,ds = 1$. This kick factor gives the strength of the transverse kick a particle receives as a function of its distance off axis per meter longitudinally. We can check this method for the DC conductivity wakes as an analytic expression for the dipole order wake function per unit length in the round resistive pipe has been derived [3]. For a distance r off-axis it is found to be:

$$\tilde{w}_{\perp}(s) = \frac{2 s_0 r}{3\pi\varepsilon_0 b^4}\left\{ e^{-\frac{s}{s_0}}\left[\sqrt{3}\sin\left(\frac{s\sqrt{3}}{s_0}\right) - \cos\left(\frac{s\sqrt{3}}{s_0}\right)\right] + \frac{12\sqrt{2}}{\pi}\int_{0}^{\infty} \frac{e^{-x^2\frac{s}{s_0}}}{x^6+8}dx \right\},$$

where, s_0 is a function of the pipe radius, b, σ_{DC} its DC conductivity, the speed of light, c, and the permittivity of free space, ε_0:

$$s_0 = \left(\frac{2cb^2\varepsilon_0}{\sigma_{DC}}\right)^{\frac{1}{3}} .$$

This expression needs to be convoluted with a charge distribution to obtain the wake potential:

$$W_{\perp}^{1}(s) = \int_{-\infty}^{\infty} \rho(s)\,\tilde{w}_{\perp}(s)\,ds .$$

Figure 1 shows a comparison of the two methods for the parameters given in Table 1.

Parameter	Unit	Value
rms Bunch Length	10^{-6} m	150
Vessel radius	mm	1, 2
Vessel Material		Copper
Bunch Distribution		Gaussian

Table 1: beam and vessel parameters for comparison of the two methods of calculating the transverse wake potential.

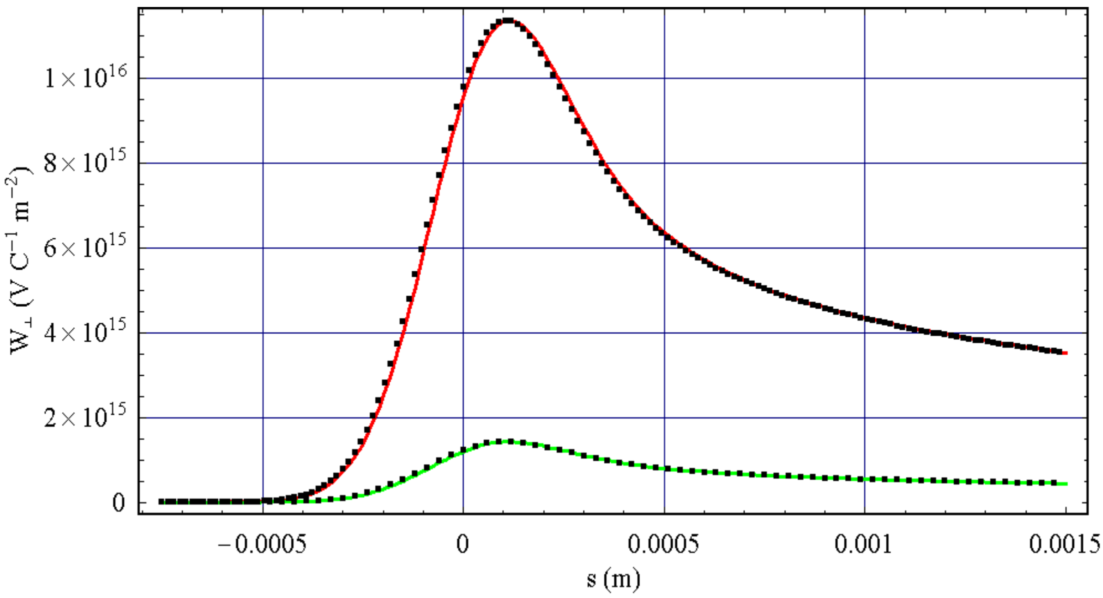

Figure 1: transverse wake potential for a Gaussian bunch calculated using analytic expression (points) and direct integration of the longitudinal wake for 1mm radius (red) and 2mm radius (green) vessels.

2. Results of Transverse Resistive Wake Calculations

Table 2 gives the range of parameters that the longitudinal wakes were previously calculated for. From these wakes all the transverse kick factors have been calculated as a function of the vessel radius. A full explanation of this calculation can be found in Ref. [1]. An example of the kick factor's dependence on the vessel radius for a copper vessel at 273 K with a 150 micron Gaussian or trapezium shaped bunch for the three different impedance models is shown Figure 2 and Figure 3. The proportionality to b^{-3} can clearly be seen.

Parameter	Unit	Value		
ILC Mode		Min	Nominal	Max
rms Bunch Length	10^{-6} m	150	300	500
Number of electrons	10^{10}	1	2	2
Beam Energy	GeV	150		
Vessel radius (min)	mm	2		
Vessel radius (nominal)	mm	2.925		
Vessel radius (max)	mm	5		
Vessel Material		Gold, Copper, Iron, Aluminium		
Impedance Models		DC, AC, ASE		
Temperature	K	77, 273		
Charge Distribution		Gaussian, Trapezium		

Table 2: range of ILC parameters for longitudinal wake calculation.

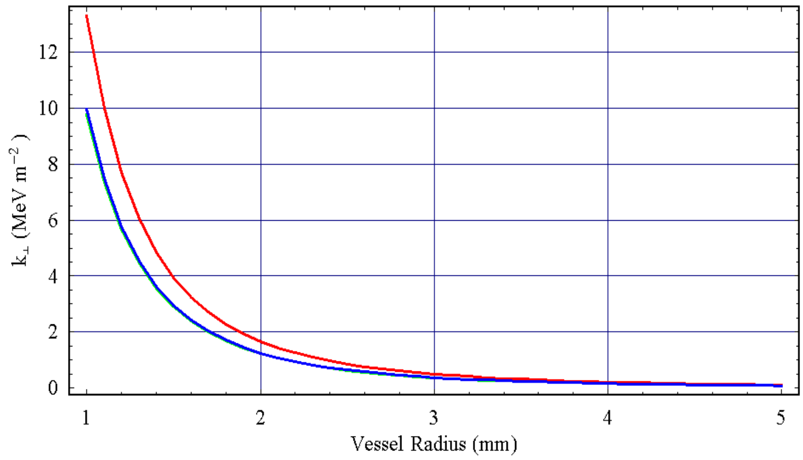

Figure 2: transverse kick factor for a Gaussian bunch per meter per meter offset for a copper vessel at 273 K.

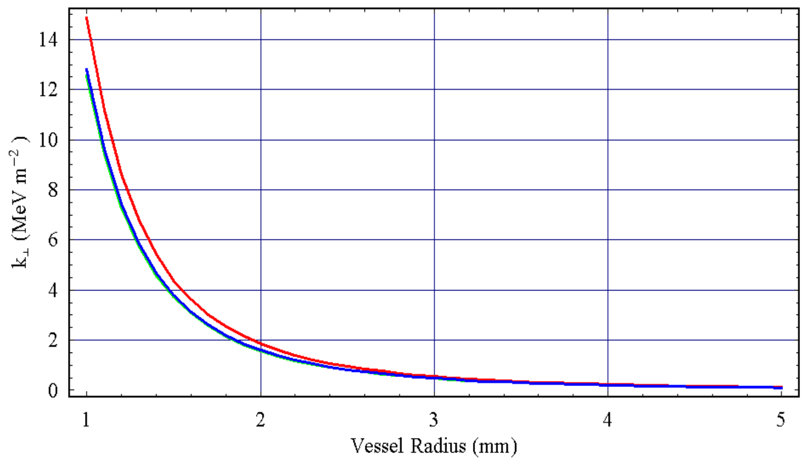

Figure 3: transverse kick factor for a Trapezium bunch per meter per meter offset for a copper vessel at 273 K.

From these graphs and similar ones for the other parameters the maximum possible kick can be found. This is shown for a Gaussian bunch and a 2.925 mm radius vessel[3] at 273 K and at 77 K in Figure 4 and Figure 5 respectively and for a trapezium shaped bunch in Figure 6 and Figure 7. The peak transverse kick is only 6 eV per micron offset for a trapezium bunch in an iron vessel at 273 K. For the ILC the undulator vessel will be made of copper and the charge distribution will be a Gaussian. The undulator will also be operating at 4.2 K and so we can assume that a transverse kick of 0.27 eV per meter per micron offset is a worst case example for a copper vessel at 77 K. This value should be compared with the ~150 GeV forward momentum of the particles.

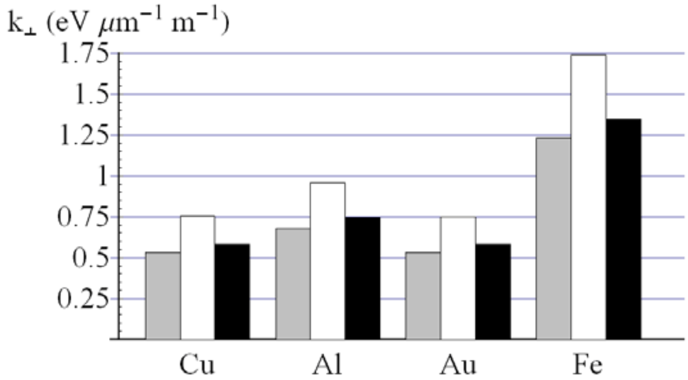

Figure 4: peak transverse kick for 2.95 mm radius vessels at 273 K and a 150 (grey), 300 (white) and 500 (blue) micron Gaussian bunch.

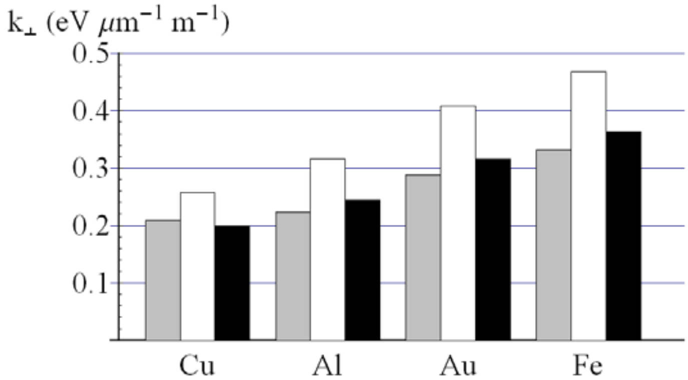

Figure 5: peak transverse kick for 2.95 mm radius vessels at 77 K and a 150 (grey), 300 (white) and 500 (black) micron trapezium bunch.

[3] The current "best-guess" for the radius of the positron undulator vessel.

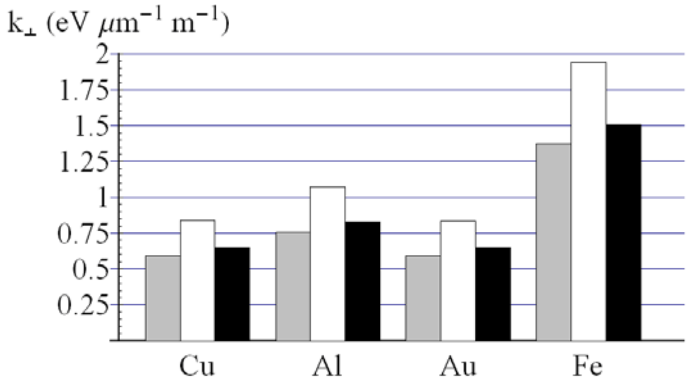

Figure 6: peak transverse kick for 2.95 mm radius vessels at 273 K and a 150 (grey), 300 (white) and 500 (black) micron trapezium bunch.

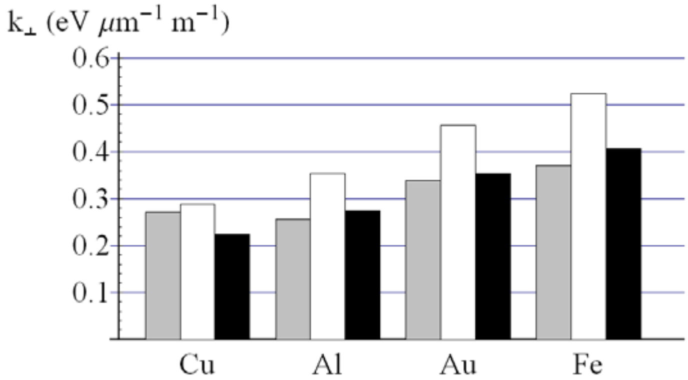

Figure 7: peak transverse kick for 2.95 mm radius vessels at 77 K and a 150 (grey), 300 (white) and 500 (black) micron trapezium bunch.

3. Emittance Growth due to Transverse Resistive Wakes

The transverse momentum kicks could cause an emittance growth of the electron beam passing through the undulator, leading to a loss in luminosity at the interaction point. To calculate the effect of this a model of the undulator lattice has been created in a particle tracking code and the effects of the transverse kick included systematically along the length of the lattice. A suitable undulator lattice is outlined in Figure 8 and Figure 9 using the current best guess at the known parameters [4].

The original NLC lattice proposal allowed only 143 m of undulator within a total line length of 246 m. The current undulator proposal calls for an undulator length in excess of 200 m. One limitation in the packing density of undulator in the NLC design is the number of cryo-modules between quadrupoles. To increase the usable length of undulator without compromising greatly on the total length of the line, this was increased to 3 cryo-modules between quadrupoles. The increased length of cryo-cell therefore required a re-matching of the linear lattice functions for the undulator section.

The new undulator and cryostat layout is illustrated in Figure 8 and Figure 9, illustrating the new lengths for the layout as well as showing one cell of the undulator lattice. In this example the quadrupole magnets are split in two, illustrating linear symmetry points in the lattice.

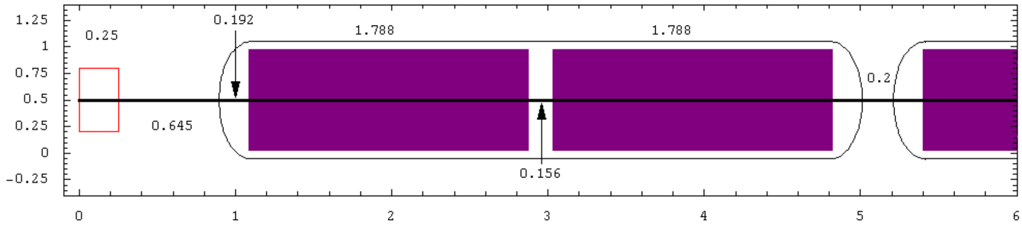

Figure 8: Layout of the components in the undulator cell, giving element separations in meters, quadrupole in red, undulator (in cryomodule) purple.

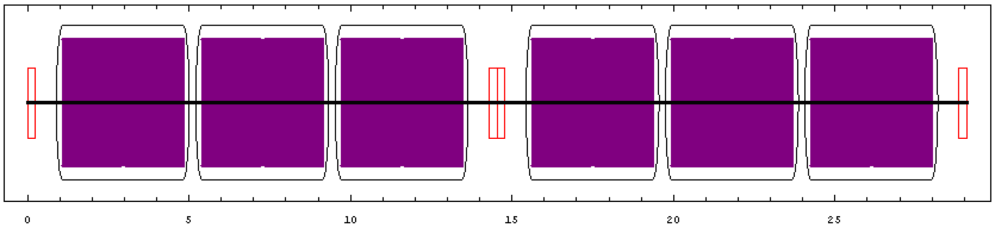

Figure 9: A representation of one undulator section period, showing 3 cryostats between quadrupoles, with 2 undulator sections in each cryostat, quadrupoles are red and the undulators purple.

The linear lattice functions for the undulator line are shown in Figure 10, the dispersion is zero. Compared to the original NLC design, the new design has higher beta functions in both planes due to the larger separation between quadrupoles. The magnitudes of the beta function in both transverse planes have been optimised to be as similar as possible within the given constraints. The new line has a usable undulator length of 214 m, with a total length of 290.76 m. Therefore, for a line increase of 45 m, there is a gain of 71 m of usable undulator.

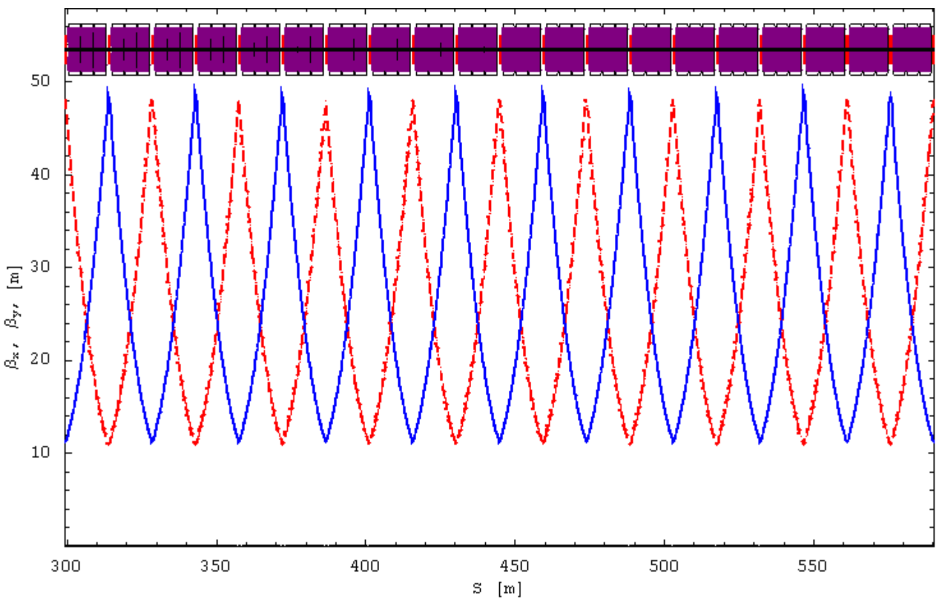

Figure 10: Linear lattice functions for the 10 period undulator line, showing the horizontal (red, dashed) and vertical (blue) beta functions.

The emittance growth due to vacuum chamber transverse wakefields was modelled in the MAD code by replacing each 1.788 m undulator section with 5 drift-kick-drift sections. Investigations showed that increasing the number of kicks per undulator did not affect the results. The magnitude of the each kick is given as 0.27 eV/μm/m, which for a 1.788 m/5 section of vacuum pipe and a 150 GeV nominal beam energy gives a 1.8 10^{-6} change in the normalised transverse momentum, in either plane.

The emittance growth was calculated by tracking beams of 500 particles, created with the nominal Twiss parameters and emittances, down the length of the undulator. The emittance at the end compared to that at the start of the undulator section was then calculated. The emittance was calculated using the standard method:

$$\varepsilon_x = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} ,$$

where the same applies for the vertical case, and < > is an average over all particles, and similarly for the ‘y’ direction. The values were averaged over 10 different beams.

No change in the emittance is seen for nominal kick strengths. By varying the magnitude of the kick it is possible to determine the margin of safety in the current design. It was found that there is no effect until the kick strength is increased to >5000 times the nominal value.

There is also an emittance increase due to trajectory errors at the start of the undulator section, this has been included with a simple trajectory correction system. As a first estimate, the trajectory system comprises 21 corrector magnets and BPMs situated at each quadrupole magnet. They are initially assumed to be zero length magnets, although this should not significantly affect the simulation. The correction system uses a singular value decomposition (SVD) based response matrix inversion technique. In the vertical plane the orbit is minimised along with the vertical dispersion. The relative weightings between trajectory and vertical dispersion correction is 0.5. This gives a good correction of both quantities. There is no correction of spurious horizontal dispersion. The emittance increases as functions of initial angle and displacement errors are shown in Figure 11. This is with no positional errors on the quadrupoles or BPMs.

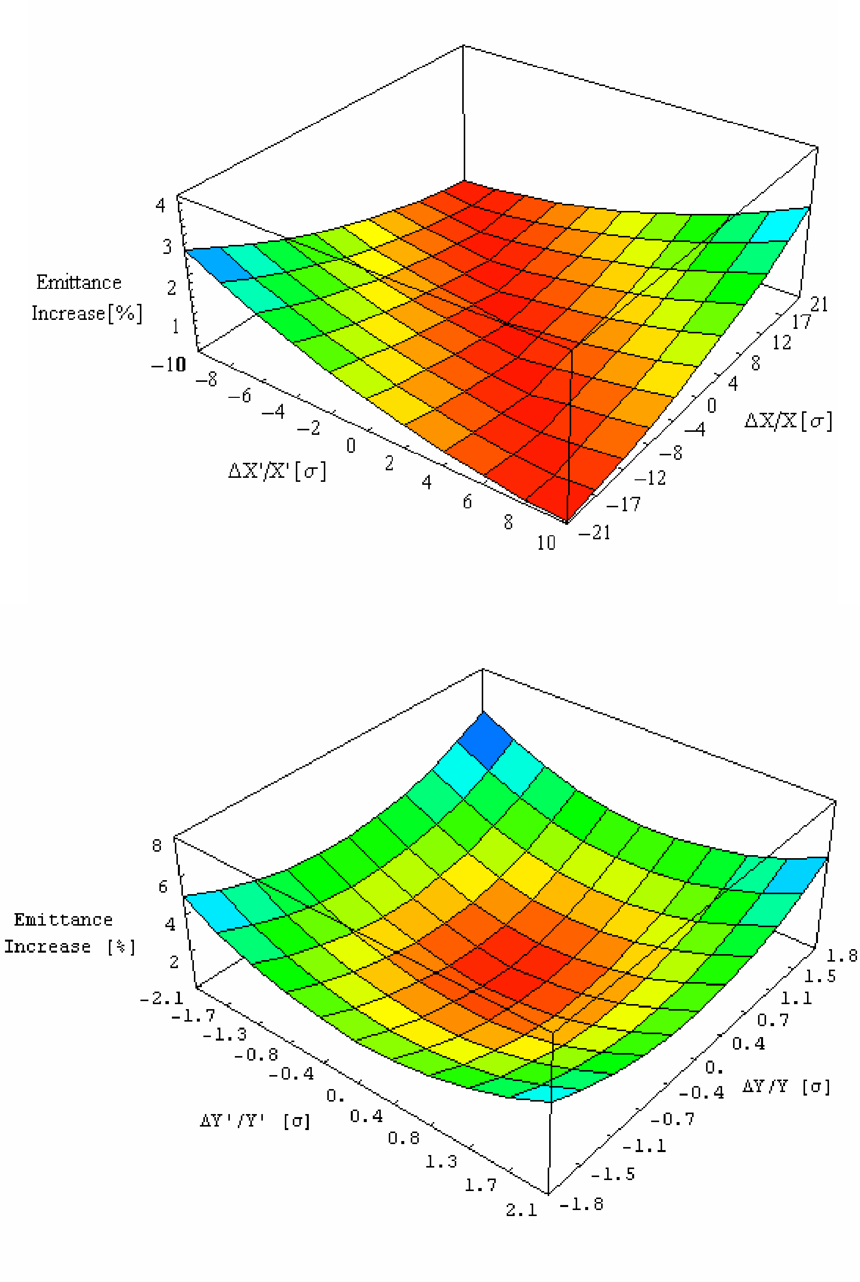

Figure 11 Percentage emittance increase due to trajectory errors (in units of sigma) and offsets in the horizontal (top) and vertical (bottom) planes.

To show the small effect of the wakefields compared to the errors due to the undulator optics a similar calculation has been performed in the vertical direction with wakefield kicks of 0 eV, shown in Figure 12. As can be seen the result is almost identical to Figure 11, indicating that the emittance increases are due to the optics of the line and not the wakefields.

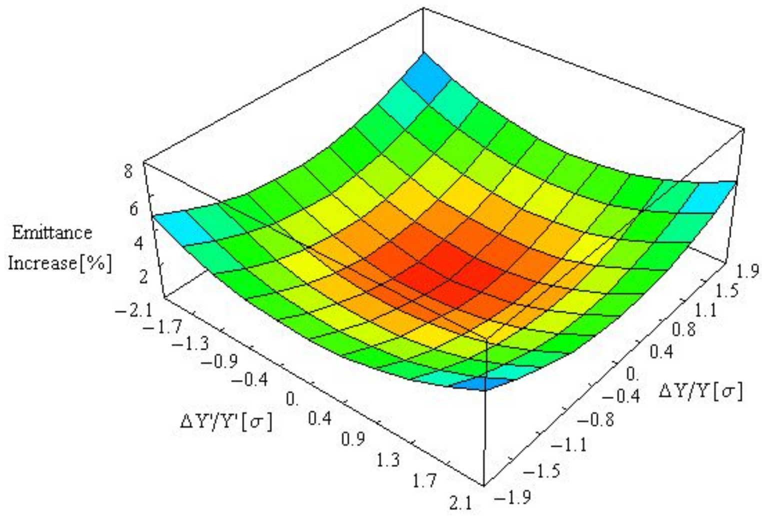

Figure 12: Percentage Emittance increase (vertical axis) due to trajectory errors (in units of sigma) with no wakefield kicks.

BPM-Quad Alignment Errors

The required precision of the BPM-to-Quad alignment will also affect the emittance increase seen in the undulator. Figure 13 shows the rms emittance growth over 50 seeds for varying BPM alignment errors (and no other errors). The final tolerance should include both the BPMs inherent resolution and the alignment tolerance of the BPM to the associated quadrupole, and these should be added in quadrature.

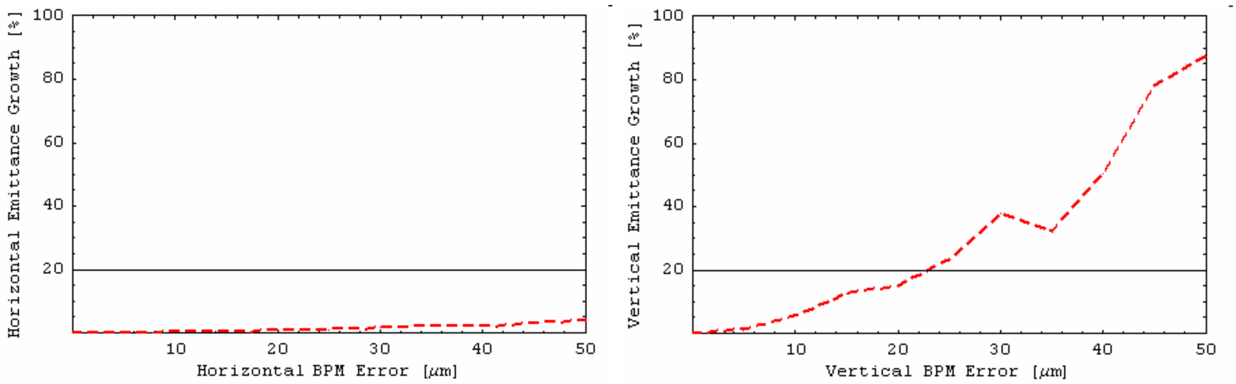

Figure 13: Emittance growth from BPM-to-Quad resolution without errors.

4. Conclusions

The transverse kicks are extremely small. In a pessimistic case of a 5.85 mm diameter copper vessel at 77 K with the worst ILC beam parameters the kick is still only ~0.27 eV per micron offset per meter. By combining this with the undulator lattice and the current dimensions of the undulator cryomodules an estimation of the emittance growth due to trajectory errors has been made.

σ_x	σ'_x	σ_y	σ'_y
27	13	0.99	1.2

Table 3: Individual trajectory errors (in units of sigma) required to induce a 1% emittance increase in the undulator line.

It has been shown that the emittance growth is due to the optics of the line and not the wakefield kicks. It is not certain that a line based on FODO cells is needed for the positron undulator insert and so it could be re-optimised if its effects on the beam emittance are too deleterious.

An estimation of the BPM-Quad resolution indicates that a 10 µm resolution results in a ~8% emittance growth in the vertical plane. It is possible to have BPM-Quad alignments of ~1 µm, although this is challenging.

The tolerances may be further improved by the use of more sophisticated correction algorithms more suitable to the ILC, such as dispersion free steering and kick minimisation, however, at the current level of detail the effects of the undulator line do not seem to have too significant an effect. None of the current modelling includes synchrotron radiation produced in the undulator section. Independent studies, without wakefields, have modelled this and shown that the effects of SR from the undulator and QUAD-BPM misalignments on the emittance growth can be tolerated [5]. Further studies should combine all these effects together with the most recent undulator lattice and undulator parameters.

5. References

1 D.J.Scott "*Longitudinal Resistive Wall Wakefields for the ILC Positron Undulator Vessel*" ASTEC-ID-040, Cockcroft-06-11, EUROTeV-Report-2006-084

2 K. Bane "*Wakefields of Sub-Picosecond Bunches*" SLAC-PUB-11829

3 K. Bane & M. Sands, '*The Short Range Resistive Wall Wakefield*' SLAC-PUB-95-7074

4 U.S. Linear Collider Steering Group: Accelerator Sub-committee, U.S. Linear Collider Technology Options Study, March 2004

5 See talks by D. Shulte and K.Kubo, "*ILC-LET Workshop*," Daresbury Laboratory, Jan 2007.