Orbit distortion and correction in the EMMA non-scaling FFAG

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Abstract. The large variation in betatron tune over the energy range of the EMMA non-scaling FFAG, and the rapidity of the acceleration, result in novel features in the properties of orbit distortion. The crossing of many integer tune resonances is achieved through fast crossing. It is clear that standard harmonic correction is not applicable since the phase advance between lattice elements varies with momentum. Two correction methods that reduce orbit distortion due to transverse magnet misalignments are presented.

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INTRODUCTION

Bending and focusing in the EMMA FFAG is achieved using quadrupole doublets in which the beam pipe is offset from the magnet centres. There are 42 cells and in every second cell a 1.3 GHz rf cavity sits in the middle of the long drift (apart from at injection and extraction). For the purposes of this paper the 19 cavities are each set to 120kV allowing acceleration from 10-20 MeV to be completed in 6 turns. This is the maximum allowed voltage on each cavity in the current EMMA design. A novel 'serpentine' mode of acceleration is followed in which the rf frequency remains fixed [1], [2]. The details of the EMMA lattice are discussed in [3] and [4].

The quadrupoles are mounted on mechanical sliders which enable correction of misalignments in the horizontal direction. There is also the possibility to install vertical corrector magnets at various points in the lattice. The dipole field of these magnets will remain constant during the short acceleration time.

In this study the tracking code PTC (Polymorphic tracking code) [5], [6] and the beam optics code MAD-X [9] are employed to calculate the orbit distortion. PTC allows modelling of offset quadrupoles and includes an analytic model of the quadrupole fringe field [7].

ORBIT DISTORTION

A feature of a non-scaling FFAG with rapid acceleration is that the orbit distortion does not see the integer tune resonances that, in a slow cycling machine, are excited by magnet misalignments. As the tune in EMMA varies by approximately one integer for every turn, the concept of resonance is not applicable (Fig. 1). As is clear from the figure, the peaks in the closed orbit distortion that correspond to integer values of total tune do not appear when acceleration is included. The fact that there is no structure in the orbit distortion with acceleration that is related to integer tune values shows that a non-scaling FFAG with rapid acceleration does not experience resonances. Instead the orbit distortion is excited by random dipole kicks due to the magnet misalignments [8]. This orbit distortion we may refer to as the 'accelerated orbit distortion' to make a distinction with the closed orbit distortion that is calculated at fixed momentum.

In order to find the level of magnet misalignments that can be tolerated in EMMA, the amplification factor, defined as the ratio of the maximum orbit distortion to the standard deviation of the input misalignments, is calculated. The beam is tracked through 300 lattices, each with different random misalignments. The standard deviation of the magnet misalignments is increased from 1-150 μm and the maximum orbit distortion noted. A linear fit through this set of data yields the amplification factor (Fig. 2). This is found to be 89 in the horizontal plane and 72 in the vertical when operating at 120kV per cavity.

It should be noted that the injection phase space coordinates of the beam are found by calculating the closed orbit at injection energy. If the tune is too close to integer at the injection energy, a significant increase in the accelerated orbit distortion will result. Even if the injection tune is not close to integer, it should be noted that the closed orbit at injection energy does not give optimal initial conditions that minimise the accelerated orbit distortion. A method to optimise the injection parameters is presented in the next section of this paper, but at this point it is reasonable to assume that the closed orbit should be used.

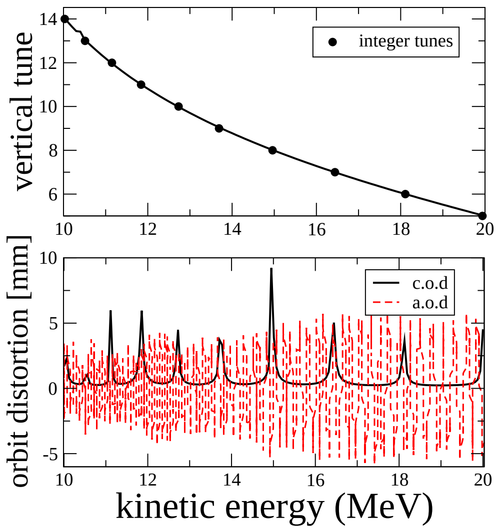

FIGURE 1. Vertical tune as a function of kinetic energy (top) and a comparison of the closed and accelerated orbit distortion in the vertical plane due to magnet misalignments (bottom). The peaks in the closed orbit distortion coincide with integer vertical tune (circles, top). It is clear that no such structure exists in the case with acceleration.

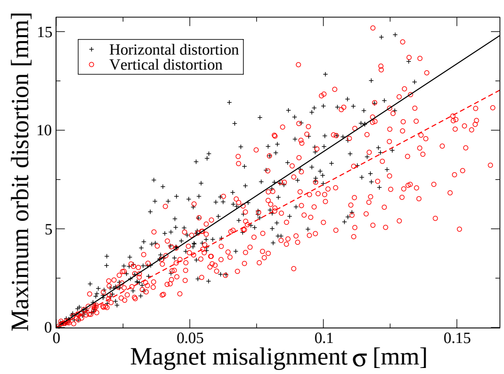

FIGURE 2. Dependence of maximum horizontal (pluses) and vertical (circles) accelerated orbit distortion on the standard deviation of random magnet misalignments. The lines show the linear fit to the data in the horizontal (solid) and vertical (dash) cases.

Since the orbit distortion is due to the cumulative effect of random dipole kicks at the misaligned quadrupoles, the amplification factor should scale with the square root of the number of quadrupoles, i.e. the number of turns taken to complete acceleration. Therefore a prediction of the amplification factor can be made at different acceleration rates, scaling from the already calculated value at 120kV per cavity. This scaling is applied to predict the amplification factor in the range 55kV – 180kV per cavity, corresponding to a range of 4 – 18 in the number of turns required to reach 20MeV. These predicted values are compared to the amplification factor calculated by tracking in 100 misaligned lattices (Fig.3). It is clear that the prediction is consistent with the tracking results (apart from a deviation in the horizontal case above 16 turns per cavity). The maximum voltage on each cavity in EMMA is about 120kV. As mentioned above, the horizontal amplification factor at this acceleration rate is about 90. This means that a magnet misalignment of 50 μm will lead to a maximum orbit distortion of 4.5mm. This level of orbit distortion is significantly above tolerance $\approx$ 1mm.

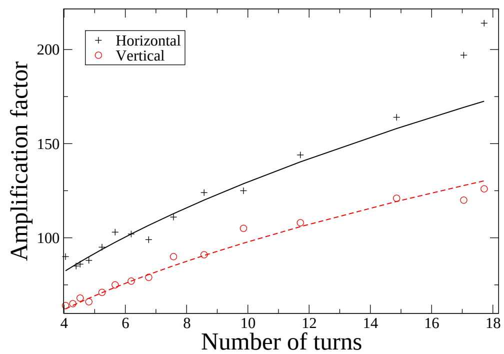

FIGURE 3. Dependence of the amplification factor on the number of turns required to complete acceleration corresponding to horizontal (pluses) and vertical (circles) misalignments. The amplification factor is calculated based on 100 random magnet misalignments with standard deviation in the range $1-60\mu$m. The lines show the predicted amplification factor based on the square root of the number of turns in the horizontal (solid) and vertical (dash) cases.

ORBIT CORRECTION

Conventional orbit correction relies on a constant phase advance between the corrector magnet and beam position monitor (BPM). This is shown in Eq. 1 where δx_i is the horizontal shift in the beam orbit at BPM i, β is the the betatron function, *ncorr* is the number of correctors, B_j and l_j is the dipole field and length of corrector j, $B\rho$ is the rigidity, v_x is the horizontal betatron tune and $\phi_i - \phi_j$ is the phase advance between corrector j and BPM i. In EMMA the phase advance $\phi_i - \phi_j$, and therefore δx_i, varies strongly with momentum (assuming that corrector magnet dipole field B_j remains constant during acceleration).

$$\delta x_i = \frac{\sqrt{\beta_i}}{2\sin(\pi v_x)} \sum_{j=1}^{ncorr} \frac{B_j l_j}{B\rho} \sqrt{\beta_j} \cos\left[v_x \pi + (\phi_i - \phi_j)\right] \quad (1)$$

Local Correction

The inclusion of horizontal sliders under each quadrupole enables correction of misalignments in that plane. Determining the magnet misalignments is the subject of this section. Since a quadrupole misalignment is equivalent to a dipole kick, the BPM measurements m_i will follow from Eq. 1.

$$m_i = m_{ideal} + \sum_{j=1}^{nquad} \Delta x_j R_{ij} + \sum_{j=1}^{ncorr} \theta_j T_{ij} \quad (2)$$

where Δx_j is the misalignment of quadrupole j, m_{ideal} is BPM measurement that would be made with perfectly aligned magnets and R_{ij} is the response coefficient that gives the closed orbit distortion at the measurement points due to the magnet misalignments, T_{ij} is the response matrix relating closed orbit distortion to corrector magnet kick angle $\theta_{ij} = \frac{B_j l_j}{B\rho}$ (as in Eq. 1). The beam optics code MAD-X [9] includes the MICADO correction algorithm [10] that will find the set of corrector kick angles θ_j that minimises the closed orbit distortion at the BPMs. The magnet misalignments can be calculated from the corrector kick angles by placing 'virtual' corrector magnets in the centre of each quadrupole and noting $\Delta x_j = \theta_j/(k_j l_j)$, where k_j is the normal quadrupole coefficient and l_j its length.

To begin, we assume that the BPMs have no errors associated with them. We also make the assumption that the BPM measurements m_{ideal} in the perfect lattice are known. The accuracy, i.e. the standard deviation of the absolute difference between the calculated and input misalignments, is shown in Fig. 4. A linear fit to the data implies that the standard deviation in the misalignment calculation error is about 2% of the standard deviation of the misalignments themselves.

BPM offset errors can be included in the MAD-X correction algorithm. The effect of their inclusion is shown in Fig. 5. As before, the standard deviation in the magnet misalignment calculation is shown, in this case against the

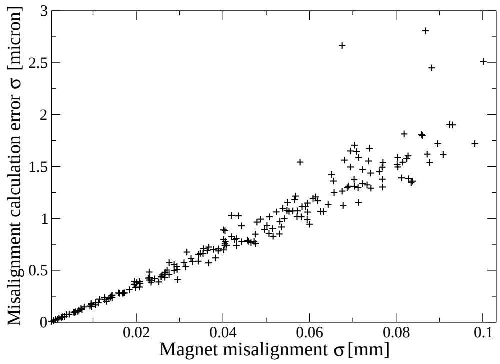

FIGURE 4. Dependence of standard deviation in magnet misalignment calculation on the standard deviation of the input magnet misalignments. Each data point uses a different misalignment error pattern. The BPM measurements are assumed to be perfect.

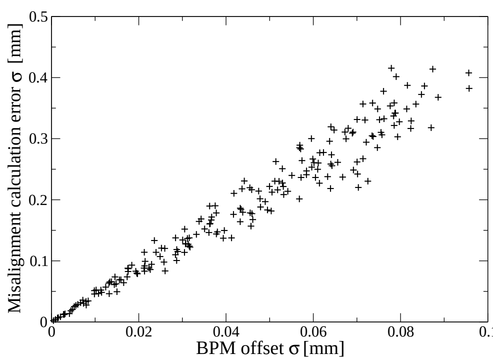

FIGURE 5. Dependence of standard deviation in magnet misalignment calculation on the standard deviation of the BPM offsets. Each data point uses a different misalignment error pattern. The magnets were misaligned with a standard deviation of 50μm.

standard deviation in the BPM offsets. In the figure it was assumed that the magnets are misaligned with a standard deviation of 50μm. In fact, the dependence is found to be so insensitive to the input magnet misalignments that any adjustments made would not be visible in the figure. It is clear that the calculation of magnet displacements is extremely sensitive to the level of BPM errors. The accuracy with which the magnetic misalignments should be determined is given by the amplification factor discussed above. Methods that reduce this sensitivity to BPM errors are currently under investigation.

Overall correction

As noted above, conventional harmonic correction with corrector magnets will not work in a non-scaling FFAG such as EMMA. Instead a method that, on average, reduces the accelerated orbit distortion calculated over the entire energy range is proposed. The parameters to be adjusted in this optimisation are the corrector magnet strengths and the injection phase space variables. In the PTC code differential algebra is used to construct Taylor maps of arbitrary order. A set of Taylor coefficients A_{ij} corresponding to the linear dependence of each BPM measurement y_i on each corrector magnet j that preceded it can then be created, i.e. $A_{ij} = \frac{\delta y_i}{\delta \theta_j}$. Note that the number of measurements is the product of the number of BPMs and the number of turns. We may write

$$A \cdot \theta = -y_{bpm} \tag{3}$$

where θ is the set of corrector strengths and y_{bpm} the measured distortion. The least squares problem can be solved for the corrector strengths θ via QR Decomposition.

The same method can be used to calculate the optimal injection phase space variables. Eq. 3 in this case becomes

$$A \cdot \zeta_{initial} = -\xi_{bpm} \tag{4}$$

where ζ is the set of transverse phase space coordinates. In this example we consider only the vertical component of BPM measurements and injection phase variables, i.e. $\zeta_{initial} = (y, y')$. Fig. 6 show how the vertical orbit distortion is, on average, reduced by optimising $\zeta_{initial}$. Alternative constraints may be preferred when determining $\zeta_{initial}$, for example it might be preferred to reduce the maximum orbit distortion rather than the mean value. Calculated as a mean over 100 misalignment error patterns, the orbit distortion standard deviation is improved by 36%.

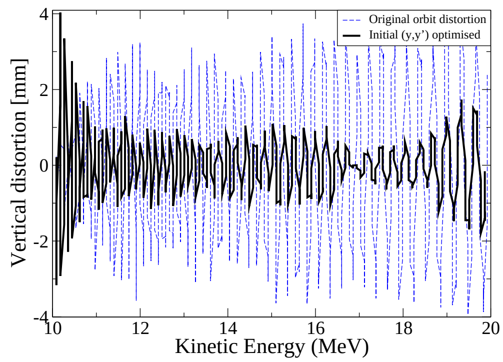

FIGURE 6. Vertical accelerated orbit distortion measured at BPMs against kinetic energy with initial (y, y') from the closed orbit at the injection energy (dash) or set to values optimised by overall correction (solid). The magnets were misaligned with a standard deviation of 50μm.

Having optimised the injection parameters, further improvements in the orbit distortion can be made by optimising the corrector magnet strengths (Eq. 3). It is of interest to determine the dependence of the orbit distortion reduction on the number of corrector magnets added. In the case of EMMA there are 16 points in the ring where a corrector magnet could be installed. The number of corrector magnets is increased from 1 to 16 and in each case the best corrector locations are found for each of 100 misalignment error patterns. The mean improvement in orbit distortion with number of correctors is shown in Fig. 7. It is apparent that each corrector magnet added produces a smaller improvement in orbit distortion than the last and that the bulk of the improvement is achieved by the injection optimisation. Two corrector magnets together with injection optimisation results in a mean 50% reduction in orbit distortion, while adding another 12 corrector magnets provide a mean 58% reduction.

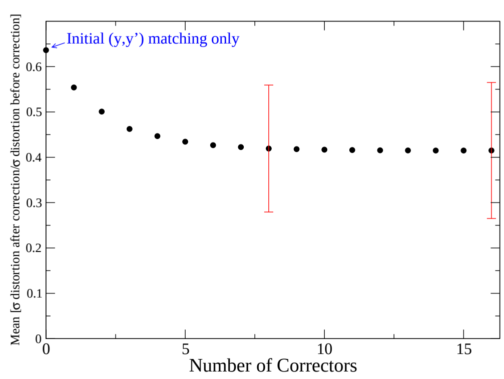

FIGURE 7. Mean ratio of orbit distortion σ with and without corrector magnet optimisation over 100 misalignment error patterns against number of correctors included. In each case the best set of corrector magnets from 16 locations was used. The first point shows the mean improvement with injection optimisation only. The error bars show 1σ over the 100 cases and are almost identical for each data point; just two are shown for clarity. The magnets were misaligned with a standard deviation of 50μm.

SUMMARY

It has been shown that in EMMA the amplification factor that relates the maximum orbit distortion to the level of transverse magnet misalignments is about 90 in the horizontal plane and 70 in the vertical plane at the maximum rate of acceleration (120kV per cavity). Given tolerances in orbit distortion of the order of 1mm, this figure places stringent requirements on orbit correction accuracy.

It was demonstrated that although traditional harmonic correction of orbit distortion with corrector magnets will not work in a non-scaling FFAG, other approaches can be adopted. In the horizontal plane, magnet sliders allow the misalignments to be corrected. The misalignments themselves are found by finding the closed orbit distortion at fixed energy and adapting the standard MAD-X correction algorithm. The accuracy of the misalignment calculation is very sensitive to the BPM errors. In the vertical plane the absence of sliders means that this method of local correction will not be convenient. However, vertical (and horizontal) orbit distortion can instead be reduced using the so called overall correction method. Optimisation of the injection phase space parameters results in a substantial reduction in orbit distortion ($\sim$ 40%). The orbit distortion can be further improved by including vertical corrector whose strengths are determined by the overall correction. However, the improvements that can can be gained in this way rapidly become insignificant with each corrector magnet added.

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