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Abstract

We describe for the first time the theory of electromagnetic wave scattering in a turbulent plasma taking into account the nonlinear plasma responses due to the presence of turbulence. It is shown that the presence of turbulence in the form of Langmuir waves introduces an extra expression in the dielectric function. This extra term is the non-linear susceptibility of the plasma due to the turbulent wave fields. It is shown that this extra term in the dielectric function produces a different expression for the dynamic form factor S($\underline{k}$,ω) which is related to the cross section for the scattering of an electromagnetic wave by electrons.

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Introduction

Scattering of electromagnetic waves by plasma has become a very successful plasma diagnostic technique. The calculation of the differential cross section for the scattering of an electromagnetic wave by a totally ionized plasma has previously been carried out for a number of plasma conditions [1] including different electron and ion temperatures, uniform stationary magnetic field and a relative drift velocity between electrons and ions. Not much attention however, has been taken of plasma turbulence[2]. In the presence of turbulence the wave characteristics such as frequency and wavenumber can change dramatically due to additional terms in the plasma dispersion relation produced by the turbulence[3,4]. Therefore in the presence of strong turbulence the differential scattering cross section should be altered.

In the first section of the paper we calculate the change in the dispersion relation for ion-acoustic waves in the presence of high frequency Langmuir waves (see also [3,4]). In the second section the differential scattering cross section is then derived in the presence of strong turbulence.

Derivation of the Dispersion Relation for Ion-Acoustic waves in the presence of Langmuir Turbulence.

In deriving the dispersion relation for ion-acoustic waves in the presence of a turbulent Langmuir wave field we must include the nonlinear coupling or beating between the high frequency disturbance. The high frequency wave field can produce a low frequency force or ponderomotive force which modifies the dispersive characteristics of the low frequency wave. We can regard this effect as a "radiation" pressure term, more commonly known as the ponderomotive force, produced by the turbulent wave fields. This new pressure term must be added to the normal plasma pressure thus altering the equation of state for the plasma.

To describe this effect we start from the Vlasov and Poisson equations

$$\frac{\partial f_\sigma}{\partial t} + \underline{v}.\underline{\nabla} f_\sigma + \frac{q_\sigma}{m_\sigma}(\underline{E} + \underline{v} \times \underline{B}).\underline{\nabla}_v f_\sigma = 0 \tag{1}$$

$$\underline{\nabla}.\underline{E} = \frac{1}{\epsilon_o} \sum q_\sigma n_{o\sigma} \int f_\sigma(\underline{x}, \underline{v}, t) d^3\underline{v} \tag{2}$$

where σ (=i,e) represents ions or electrons and $f_\sigma(\underline{x},\underline{v},t)$ is the single particle distribution function for the species σ. It is normalized so that the local number density is given by

$$n_\sigma(\underline{x}, t) = n_{o\sigma} \int f_\sigma(\underline{x}, \underline{v}, t) d^3\underline{v} \tag{3}$$

and $n_{o\sigma}$ is the mean density of the plasma particles ($n_{oe} = n_{oi} = n_o$) To describe the nonlinear interaction between the high frequency wave fields and the low frequency waves we write

$$f_\sigma = f_{o\sigma} + f_{1\sigma} \tag{4}$$

where we have split the particle distribution function into an equilibrium part and an oscillating part $f_{1\sigma}$ due to the fields in the plasma, where the oscillating part has the same harmonic space and time dependence as the fields, e.g.

$$f_{1e} = \sum_l f_{1el} + f_{1es}$$

and

$$f_{1el} \propto expi(\underline{k}_l.\underline{x} - \omega_l t)$$

etc.

The Vlasov equation is now written as

$$\frac{\partial f_{1\sigma}}{\partial t} + \underline{v}.\frac{\partial f_{1\sigma}}{\partial \underline{x}} + \frac{q_\sigma}{m_\sigma}(\underline{E} + \underline{v} \times \underline{B}).\frac{\partial f_{o\sigma}}{\partial \underline{v}} = -\frac{q_\sigma}{m_\sigma}(\underline{E} + \underline{v} \times \underline{B}).\frac{\partial f_{1\sigma}}{\partial \underline{v}} \tag{5}$$

the left hand side (linear part) describes the linear modes and the right hand side, which is nonlinear in the wave quantities, produces a coupling between these linear modes.

We now derive the equation describing the low frequency ion acoustic wave in the presence of a turbulent high frequency Langmuir wave field.

Since we are considering a low frequency disturbance the ions as well as the electrons will respond to the low frequency fields set up. Starting from equation(5) for both electrons and ions we obtain the following expressions for f_{1e} and f_{1i}

$$f_{1e} = \frac{ie}{m_e}\frac{(\underline{E} + \underline{v} \times \underline{B})}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oe}}{\partial \underline{v}} + \frac{ie}{m_e}\frac{(\underline{E} + \underline{v} \times \underline{B})}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{1e}}{\partial \underline{v}} \tag{6}$$

$$f_{1i} = -\frac{ie}{m_i}\frac{(\underline{E} + \underline{v} \times \underline{B})}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oi}}{\partial \underline{v}} - \frac{ie}{m_i}\frac{(\underline{E} + \underline{v} \times \underline{B})}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{1i}}{\partial \underline{v}} \tag{7}$$

where (ω_s, k_s) represent the frequency and wavenumber of the low frequency mode in the presence of high frequency Langmuir waves $(\omega_s, \underline{k}_s)$ are no longer related by a simple dispersion relation. Substituting these expressions into the ion frequency component of Poisson's equation, namely

$$\underline{\nabla}.\underline{E} = \frac{n_o e}{\epsilon_o}\int (f_{1i} - f_{1e}) d^3\underline{v} \tag{8}$$

we obtain the following equation describing the low frequency field

$$\Big[1 + \chi_e(\omega_s, \underline{k}_s) + \chi_i(\omega_s, \underline{k}_s)\Big]E_s = -\frac{\omega_{pe}^2}{\mid \underline{k}_s \mid}\int \frac{(\underline{E} + \underline{v} \times \underline{B})}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{1e}}{\partial \underline{v}} d^3\underline{v}$$

$$-\frac{\omega_{pi}^2}{\mid \underline{k}_s \mid}\int \frac{(\underline{E} + \underline{v} \times \underline{B})}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{1e}}{\partial \underline{v}} d^3\underline{v} \tag{9}$$

Since we are dealing with an unmagnetized plasma and considering only longitudional waves we neglect the $\underline{v} \times \underline{B}$ terms. $\chi_{e,i}(\omega_s, \underline{k}_s)$ are the electron (ion) susceptibility defined by

$$\chi_{e,i}(\omega, \underline{k}) = \frac{\omega_{pe,i}^2}{k^2} \int \frac{\underline{k}}{(\omega - \underline{k}.\underline{v})} . \frac{\partial f_{oe,i}}{\partial \underline{v}} d^3 \underline{v} \tag{10}$$

For a Maxwellian plasma

$$\chi_e(\omega, \underline{k}) = \frac{1}{k^2 \lambda_{De}^2} \left[1 - \left(\frac{\omega}{\sqrt{2} k v_{Te}} \right)^2 \right] + \frac{i\sqrt{\pi}\omega}{k^3 \lambda_{De}^2 v_{Te}} \; for \; \frac{\omega}{k} \ll v_{Te}$$

$$\chi_i(\omega, \underline{k}) = -\frac{\omega_{pi}^2}{\omega^2} + \frac{i\sqrt{\pi}}{k^2 \lambda_{Di}^2} \frac{\omega}{k v_{Ti}} exp\left(-\frac{\omega}{\sqrt{2} k v_{Ti}} \right)^2 \; for \; \frac{\omega}{k} \ll v_i \tag{11}$$

In the absence of nonlinear coupling terms in equation (9) the dielectric function $\epsilon_s(\omega_s, \underline{k}_s)$ is given by

$$\epsilon_s(\omega_s, \underline{k}_s) = 1 + \chi_e(\omega_s, \underline{k}_s) + \chi_i(\omega_s, \underline{k}_s) \tag{12}$$

which results in the usual dispersion relation for ion-acoustic waves namely

$$\omega_s^2 = k_s^2 c_s^2 \; for \; k\lambda_{De} \ll 1$$

However in the presence of strong high frequency fields this result no longer holds. We now include terms on the right hand side of equation(9) which will produce a low frequency response resulting in

$$\left[1 + \chi_e(\omega_s, \underline{k}_s) + \chi_i(\omega_s, \underline{k}_s) \right] E_s = \sum_l \frac{\omega_{pe}^2}{| \underline{k}_s |} \int \frac{\underline{E}_l}{(\omega_s - \underline{k}_s.\underline{v})} . \frac{\partial f_{1el}^*}{\partial \underline{v}} d^3 \underline{v}$$

$$- \sum_l \frac{\omega_{pe}^2}{| \underline{k}_s |} \int \frac{\underline{E}_l^*}{(\omega_s - \underline{k}_s.\underline{v})} . \frac{\partial f_{1el}}{\partial \underline{v}} d^3 \underline{v} \tag{13}$$

substituting for f_{1el} given by

$$f_{1el} = \frac{ie}{m_e} \frac{\underline{E}_l}{(\omega_l - \underline{k}_l.\underline{v})} . \frac{\partial f_{oe}}{\partial \underline{v}}$$

into equation (13) yields

$$\epsilon_s(\omega_s,\underline{k}_s)\underline{E}_s = \sum_l i\frac{\omega_{pe}^2 e}{\mid \underline{k}_s \mid m_e}\int \frac{\underline{E}_l}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial}{\partial v}\left(\frac{\underline{E}_l^*}{(\omega_l - k_l.\underline{v})}.\frac{\partial f_{oe}}{\partial \underline{v}}\right)d^3\underline{v}$$

$$-\sum_l \frac{i\omega_{pe}^2 e}{\mid k_s \mid m_e}\int \frac{\underline{E}_l^*}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial}{\partial \underline{v}}\left(\frac{\underline{E}_l}{\omega_l - \underline{k}_l.\underline{v}}.\frac{\partial f_{oe}}{\partial \underline{v}}\right)d^3\underline{v} \tag{14}$$

Integrating 14 yields the equation for the potential $\phi_s = \frac{E_s}{ik_s}$

$$\epsilon_s(\omega_s,\underline{k}_s)\phi_s = -\frac{e}{\epsilon_o m_i}\frac{\omega_{pe}}{\underline{k}_s^2 c_s^2}\sum_l \frac{\epsilon_o \mid \underline{E}_l \mid^2}{\omega_l} \tag{15}$$

where $\frac{\epsilon_o|E_l|^2}{\omega_l} = \mathrm{N}_k$ is the wave action or plasma number density of the high frequency Langmuir wave field. The equation describing the high frequency turbulent wave field, which has a broad spectrum in k space, is the Liouville equation.

$$\frac{\partial N_k}{\partial t} + \frac{\partial \omega_l}{\partial \underline{k}}.\underline{\nabla} N_k - \frac{\partial \omega_l}{\partial \underline{x}}.\underline{\nabla}_k N_k = 0 \tag{16}$$

In a homogeneous plasma changes in the frequency ω_l arise only from the ion acoustic wave since

$$\frac{\partial \omega_l}{\partial x} = \frac{\omega_l}{2n_o}\frac{\partial}{\partial x}\int (f_{1e} - f_{1i})d^3\underline{v} \tag{17}$$

$$= \frac{\omega_l}{2n_o}\frac{\partial}{\partial x}\left[\int\left(\frac{ie}{m_e}\frac{\underline{E}_s}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oe}}{\partial \underline{v}} + \frac{ie}{m_i}\frac{\underline{E}_s}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oi}}{\partial \underline{v}}\right)d^3\underline{v}\right] \tag{18}$$

$$= \frac{\omega_l}{2n_o}\frac{\partial}{\partial x}\left\{\left[\int \frac{e}{m_e}\frac{\underline{k}_s}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oe}}{\partial \underline{v}}d^3\underline{v} + \frac{e}{m_i}\int \frac{\underline{k}_s}{\omega_s.\underline{k}_s.\underline{v}}.\frac{\partial f_{oi}}{\partial \underline{v}}d^3\underline{v}\right]\phi_s\right\} \tag{19}$$

$$= \frac{\omega_l}{2n_o}\frac{\partial}{\partial x}\phi_s\left\{\frac{e}{m_e}\int \frac{\underline{k}_s}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oe}}{\partial \underline{v}}d^3\underline{v} + \frac{e}{m_i}\int \frac{\underline{k}_s}{(\omega_s - \underline{k}_s.\underline{v})}.\frac{\partial f_{oi}}{\partial \underline{v}}d^3\underline{v}\right\} \tag{20}$$

$$\frac{\partial \omega_l}{\partial x} = \frac{\epsilon_o \omega_l \underline{k}_s^2}{2n_o^2 e}(\chi_e + \chi_i)\frac{\partial \phi_s}{\partial x} \tag{21}$$

where ϕ_s represents the ion acoustic potential perturbation. This potential perturbation causes a change in the action density N_k which is written as $N_{ko} + \delta N_k$ where N_{ko} is the equilibrium value and δN_k is the charge due to the presence of the ion wave. Using this expression for N_k the kinetic equation for δN_k is given by

$$\frac{\partial \delta N_k}{\partial t} + \frac{\partial \omega_k}{\partial \underline{k}}.\frac{\partial \delta N_k}{\partial \underline{x}} - \frac{\partial \omega_k}{\partial \underline{x}}.\frac{\partial N_{ko}}{\partial \underline{k}} = 0 \tag{22}$$

$$-i\left(\omega_s - \underline{k}_s.\frac{\partial \omega_l}{\partial \underline{k}}\right)\delta N_k = \frac{\partial \omega_l}{\partial \underline{k}}.\frac{\partial N_{ko}}{\partial \underline{k}} \tag{23}$$

using the expression

$$\frac{\partial \omega_l}{\partial \underline{x}} = \frac{\omega_{pe}}{2n_o} i\underline{k}_s \frac{\epsilon_o}{e}\frac{\omega_{pi}^2}{\omega^2} k_s^2 \phi_s \tag{24}$$

in equation (23) results in

$$\delta N_k = -\frac{\omega_k}{2m_i}\frac{ek_s^2\phi_s}{\omega_s^2}\frac{\underline{k}_s.\frac{\partial N_{ko}}{\partial \underline{k}}}{\omega_s - \underline{k}_s.\frac{\partial \omega_l}{\partial \underline{k}}} \tag{25}$$

Substituting equation (25) into equation (15) and changing the summation to an integral over k we obtain the following equation for ϕ_s.

$$\epsilon_s(\omega_s, \underline{k}_s)\phi_s = \frac{\omega_{pi}^2\omega_{pe}^2}{2n_o m_i c_s^2 \omega_s^2}\phi_s \int \frac{\underline{k}_s.\frac{\partial N_{ko}}{\partial \underline{k}}}{(\omega_s - \underline{k}_s.\frac{\partial \omega_l}{\partial \underline{k}})} d\underline{k} \tag{26}$$

which leads to

$$(1 + \chi_e + \chi_i + \chi_{Nl})\phi_s = 0 \tag{27}$$

where

$$\chi_{NL} = \frac{\omega_{pe}^2}{2\rho}\frac{\omega_{pi}^2}{\omega^4} k_s^2 \int \frac{\underline{k}_s.\frac{\partial N_{ko}}{\partial \underline{k}}}{(\omega_s - \underline{k}_s.\frac{\partial \omega}{\partial \underline{k}})} d\underline{k} \tag{28}$$

which is the wave susceptibility for $\omega_s \simeq k_s c_s$, or

$$\chi_{NL} = \frac{\omega_{pi}^2\omega_{pe}^2}{\rho c_s^2\omega_s^2}\int \frac{\underline{k}_s.\frac{\partial N_{k\alpha}}{\partial \underline{k}}}{(\omega_s - \underline{k}_s.\frac{\partial \omega_l}{\partial \underline{k}})}d\underline{k} \tag{29}$$

for $\omega \neq k_s c_s$, $\rho = n_o m_i$.

For small k values of turbulence the renormalisation of χ_{NL} is essential which alters the nonlinear response [3,4], but this effect will only be essential for very small angles of scattering and will be the subject of further investigation. We restrict ourselves here to the condition $sin^2\theta > m_e/m_i$ when the renormalization is not important.

For the case where ϕ_s varies as $e^{i(qx-\omega t)}$ the dispersion relation for the low frequency ion-modes can be obtained from equation (7) and is given by (4)

$$\omega^2 - c_s^2 q^2 - i\gamma_s\omega - \frac{\omega_{pe}^2}{\rho}q^2\int \frac{\underline{q}.\frac{\partial N_{k\alpha}}{\partial \underline{k}}}{\omega - q.\frac{\partial \omega_l}{\partial \underline{k}}}dk \tag{30}$$

The effect of the Langmuir turbulence on the propagation of the ion acoustic wave is to provide an extra pressure term due to the ponderomotive force of the plasmons in the plasma fluid. This extra pressure term can be described in terms of an effective temperature T_{eff} given by

$$kT_{eff} = \frac{\omega_{pe}^2}{n_o}\int_c \frac{\underline{q}.\frac{\partial N_{k\alpha}}{\partial \underline{k}}}{\omega - \underline{q}.\frac{\partial \omega_l}{\partial \underline{k}}}d\underline{k} \tag{31}$$

and an effective pressure $= n_o KT_{eff}$, K is the Boltzmann constant. This term is in general complex with the singularity being treated according to the Landau rule with

$$\frac{1}{\omega - \underline{q}.\frac{\partial \omega_l}{\partial \underline{k}}} = P\frac{1}{\omega - \underline{q}.\frac{\partial \omega_l}{\partial \underline{k}}} - i\pi\delta\Big(\omega - \underline{q}.\frac{\partial \omega_l}{\partial \underline{k}}\Big) \tag{32}$$

Using this expression the dispersion relation equation (27) describing the propagation of ion acoustic waves in the presence of Langmuir turbulence becomes

$$\omega^2 - q^2(KT_e - KT_{eff})/m_i - i(\gamma_s - \gamma_L^{\pm})\omega = 0 \tag{33}$$

$\gamma_L^{\pm}$ represents the effect of growth or damping and is given by the imaginary part of equation (29) namely

$$\gamma_L^{\pm} = \pi \frac{\omega_{pe}^2}{\rho c_s^2} q \int \left(\underline{q}.\frac{\partial N_{ko}}{\partial \underline{k}} \right) \delta \left(\pm q c_s - \underline{q}.\frac{\partial \omega_l}{\partial \underline{k}} \right) . d\underline{k} \tag{34}$$

This expression represents growth (+) or damping (-) of the low frequency ion mode in the presence of Langmuir turbulence. The analogy with the usual Landau damping (growth) of waves on particles is obvious.

As an example assume $N_{ko} = \frac{1}{2} k_o^3 N_o \delta(k - k_o)$, substituting this into (30) and integrating we obtain the dispersion relation.

$$\omega^2 - q^2 c_s^2 \left(1 + \frac{3}{2} \frac{q^2 \omega_{pi} N_o k_o^3}{(\omega - \underline{q}.\frac{\partial \omega_L}{\partial \underline{k}})^2} \right) = 0 \tag{35}$$

This expression represents the low frequency plasma response close to the ion-acoustic frequency. The second term in the brackets represents a change to the low frequency plasma wave due to the ponderomotive potential of the high frequency plasma turbulence. This equation also describes the modulational instability[2] of large amplitude Langmuir waves. We will consider here the case of a stationary turbulence when the growthrate of modulational instability is balances either by energy transfer or by linear damping. The result of this analysis shows that in the presence of high frequency Langmuir turbulence the low frequency dielectric response changes dramatically. Indeed in some cases the nonlinear frequency corrections can be as large as the linear terms giving rise to oscillating two stream and modulational type instabilities [3,5] of the Langmuir wave. In the modulational type instabilities the low frequency driven mode usually associated with the ion acoustic fluctuation spectrum can have zero frequency. The wave having only spatial structure in the plasma. Radiation scattered from such fluctuations is similar to the scattering from a stationary grating ie it will have zero frequency shift rather than being shifted by an amount equivalent to the ion-acoustic frequency ($\omega = qc_s$). The signature of the radiation scattering from ion-density fluctuations in the presence of turbulence will now contain information about the turbulence level since the frequency of these low frequency density fluctuations are now functions of the strength of the high frequency turbulence. For strong high frequency turbulence the "ponderomotive" frequency shift of the ion-acoustic spectrum can result in the scattered radiation signal to be peaked closer to the incident radiation frequency in some cases there could only be a single peak in the scattered signal indicating that the scattering element has zero frequency as in a stationary grating. If the structure is moving then this peak will be shifted by the amount corresponding to $\underline{q}.\underline{v}$ where $\underline{q}$ is the wavenumber and $\underline{v}$ the velocity of the zero frequency density fluctuation.

Calculation of the Dynamic Form Factor in a Turbulent Plasma.

We consider the re-radiation by an electron under the action of the electric field of a plane monochromatic plane wave of amplitude $\underline{E}o$ wavenumber $\underline{K}_o$ and frequency ω_o. Scattering by ions is less by the ratio m_i/m_e and will not be considered. The electron will have an initial velocity and be subject to the fluctuating microfields arising from the other charged particles of the plasma as well as the radiation field. We assume that the effect of the initial velocity and microfields is sufficiently weak so as not to effect the re-radiated wave except for a first order contribution to the phase of the scattered wave. Assume electron to be non-relativistic, $e\underline{E}_o/m_e\omega_o c \ll 1$ and also that $\hbar\omega_o \ll m_e c^2$ which means we neglect momentum transfer to the electron ie., the Compton regime is not considered.

For a plane electromagnetic wave of the form

$$\underline{E}_i(t) = \underline{E}_o expi(\underline{K}_o.\underline{x} - \omega_o t) + c.c. \tag{36}$$

where $\underline{K}_o = \omega_o/c\underline{\hat{n}}$, the acceleration of the electron under the action of this field is

$$\ddot{\underline{x}} = -\left(\frac{e}{m_e}\right)\underline{E}_o expi(\underline{K}_o.\underline{x}_j - \omega_o t) \tag{37}$$

where $\underline{x}_j = \underline{x}_j(t)$ is the position vector at time t. The acceleration depends upon time implicitly through $\underline{x}_j(t)$ as well as explicitly through the phase factor $\omega_o t$. Hence the fourier analysis of this acceleration will contain in addition to the frequency Ω_o frequencies characteristic of the electron motion in the absence of the radiation field. The scattered signal will thus contain additional frequencies characteristic of the electron motion. This is the essential property of scattering as a diagnostic. The scattered electric field is calculated from the equation of motion using the Lienard-Wiechert potentials. Which give the vector and scaler potentials of the scattered field at the position $\underline{R}$ of the detector. Using the results of Evans and Katzenstein[6] we can write down the field of the scattered radiation due to electrons which is

$$\underline{E}_{sc}(t) = \frac{r_o \underline{E}_o}{R} sin\theta/2 \int_{V_s} \delta n(\underline{x}, t) expi((\underline{K}_o - \underline{K}_s).\underline{x} - \omega_o t) d\underline{x} \tag{38}$$

Where $\underline{E}_{sc}$ is the scattered electric field amplitude, R is the distance from the scattering volume to the receiver, V_s is the scattering volume $(R \gg V_s)\delta n$ is the fluctuating electron density, $r_o = e^2/\epsilon_o m_e c^2$ is the classical electron radius and θ is the angle between the incident wavevector $\underline{k}_o$ and scattered wavevector $\underline{K}_s$. The scattered signal depends on the electron density fluctuations at a particular wavenumber $\underline{K} = \underline{K}_o - \underline{K}_s$ and involves essentially a spatial fourier analysis of the scattering medium.

We now construct the autocorrelation function (ACF) of the scattered radiation field which is

$$\varphi(\tau) =< \underline{E}_{sc}(\tau)\underline{E}^*_{sc}(t+\tau) > \tag{39}$$

$$= \frac{r_o^2 E_o^2}{R^2} sin^2\theta/2 \int_{V_s}\int < \delta n(\underline{x}^1, t+\tau)\delta n^*(\underline{x}, t) > expi(\underline{K}.(\underline{x} - x^1))d\underline{x}d\underline{x}^1 \tag{40}$$

where <> means the ensemble average. The Fourier transform of the ACF is just the power spectrum

$$< \underline{E}_{sc}(\omega_o, t)\underline{E}^*_{sc}(\omega_o t + \tau) >= \int < |E_{sc}(\omega_o + \omega)|^2 > exp(-i\omega t)d\omega \tag{41}$$

Therefore taking the Fourier transform gives

$$\int < |E_{sc}(\omega_o + \omega)|^2 > d\omega = \frac{r_o^2 E_o^2 V_s sin^2\theta/2}{R^2} \int < |\delta n(\underline{k}, \omega)|^2 > d\omega \tag{42}$$

and hence the differential scattering cross-section per unit volume, per unit solid angle, per unit incident power, per unit frequency interval is

$$\sigma(\omega_o + \omega)d\omega = r_o^2 sin^2\theta/2 < |\delta n(\underline{k}, \omega)|^2 > d\omega \tag{43}$$

where $< |\delta n(\underline{k}, \omega)|^2 >$ is the power spectrum of the electron density fluctuations. From equation (43) we can write the resulting cross section as

$$\sigma(\underline{k}, \omega) = \sigma_\tau S(\underline{k}, \omega) \tag{44}$$

where σ_τ is the Thompson cross section for the scattering of an electromagnetic wave by a free electron [6] and $S(\underline{k}, \omega)$ is known as the dynamic form factor which is the Fourier transform in time of the autocorrelation function of the spatial Fourier component of the electron density fluctuations.

$$S(\underline{k}, \omega) = \frac{1}{2\pi} \int < \delta n(\underline{k}, t)\delta n^*(\underline{k}, t+\tau) > expi(\omega - \omega_o)\tau d\tau \tag{45}$$

The dynamic form factor S $(\underline{k}, \omega)$ gives the frequency shifts resulting from the electron motion as well as the effect of correlations between the electrons due to the different plasma waves, $S(\underline{k}, \omega)$ can be $\gg 1$ resulting in scattering cross-sections much larger than that of free electrons. $S(\underline{k}, \omega)$ contains information about the eigen-frequencies of the different plasma modes, if one of these modes has their frequency significantly altered for example by the presence of turbulence this will be reflected in the value of $S(\underline{k}, \omega)$. The problem now remains to calculate $< |\delta n(\underline{k}, \omega)|^2 >$ for a plasma in the presence of high frequency turbulence. For this calculation we will follow the method of "dressed test particles" pioneered by Rosenbluth and Rostoker [1]. In this method the electrostatic potential ϕ_s of the plasma with a test charged particle of charge Q, mass m and velocity V_o is obtained using the Vlasov equations for both electrons and ions and Poisson's equation. We will however not neglect the second order terms f_{e1} and f_{i1} as has been done in all previous derivations of the dielectric coefficient $\epsilon(\omega, \underline{k})$ but use the results of (27) to write the equation for the potential ϕ_s of a test charge in the plasma which is

$$\epsilon(\omega, \underline{k})\phi(\underline{k}, t) = \frac{Q}{\epsilon_o k^2} expi(\underline{k}.(\underline{x}_o - \underline{v}_o t)) \tag{46}$$

where $\epsilon(\omega, \underline{k})$ is obtained from equation 27 and is given by

$$\epsilon(\omega, \underline{k}) = 1 + \chi_e + \chi_i + \chi_{NL} \tag{47}$$

The Fourier transform of the electron density or "shielding cloud" $\delta n(\underline{k}, t)$ is obtained by integrating the equation

$$f_{e1} = \frac{e}{m_e}\phi(\underline{k}, t)\frac{\underline{k}.\underline{\nabla}_v f_{oe}}{\underline{k}.(\underline{v} - \underline{v}_o)} \tag{48}$$

over velocity space and substituting the expression for $\phi(\underline{k}, t)$ from equation (46) resulting in

$$n_j(\underline{k}, t) = \frac{\chi_e}{1 + \chi_e + \chi_i + \chi_{NL}} \frac{Q_j}{e} expi\underline{k}.(\underline{X}_{oj} + \underline{v}_{oj} t) \tag{49}$$

which is the electron density fluctuation of the j test particle. The total electron density fluctuation is obtained by summing (49) over all particles of the plasma where we assume all particles are test particles[6] with the charge Q_j of the test particle, taking the value $-e$ for electrons and $+Ze$ for ions. In addition to the sum of expression (49) over all particles, the "self-fluctuation" of the test particle itself must be included if it is an electron [6]. We can now write the Fourier transform of the total electron density fluctuation of the plasma as

$$\delta n(\underline{k},t) = \sum_{j=1}^{n} \delta n_j(\underline{k},t) + \sum_{\ell=1}^{n/Z} \delta n_\ell(\underline{k},t) + \sum_{j=1}^{n} exp\{i\underline{k}.(\underline{x}_{oj} + \underline{v}_{oj})\}$$

$$= \sum_{j=1}^{n}\Big(1 - \frac{\chi_e}{1+\chi_e+\chi_i+\chi_{NL}}\Big) exp\{i\underline{k}.(\underline{x}_{oj} + \underline{v}_{oj}t)\}$$

$$-Z\sum_{\ell=1}^{n/Z}\Big(\frac{\chi_e}{1+\chi_e+\chi_i+\chi_{NL}}\Big) exp\{i\underline{k}.(\underline{x}_{o\ell} + \underline{v}_{o\ell}t)\} \tag{50}$$

Using equation (50) we obtain the autocorrelation function $< |\delta n(\underline{k},t)|^2 >$ which upon substitution into equation (45) and valuating the sums over configuration and velocity space weighted by the zero-order distribution function $f_{oe}(\underline{v})$ and $f_{oi}(\underline{v})$ for $\underline{v} = \frac{\omega}{|k|}\hat{\underline{k}}$ we obtain the dynamic form factor in the presence of high frequency turbulence given by

$$S(\underline{k},\omega) = \left|\frac{1+\chi_i+\chi_{NL}}{1+\chi_e+\chi_i+\chi_{NL}}\right|^2 f_e(\frac{\omega}{k}) +$$

$$Z\left|\frac{\chi_e}{1+\chi_e+\chi_i+\chi_{NL}}\right|^2 f_i(\frac{\omega}{k}) \tag{51}$$

This has to be compared to equation (43) of Evans and Katzenstein [6] which was calculated in the absence of turbulence. From equations (44) and (51) we can obtain an expression for the total scattering cross section of an electromagnetic wave by a turbulent plasma.

The effect of the turbulence can be considered as a radiation pressure term which contributes to the overall pressure in the plasma. The high frequency turbulence can change the dispersive properties of low frequency modes by introducing both a real frequency shift as well as an imaginary frequency which can either be growth or damping of the low frequency mode. It can also change the characteristic frequencies of the high frequency modes. The relative shift in this case is much smaller and would be more difficult to detect. The effects of the turbulence on damping the low frequency responce can also have significant effects in stabilizing instabilities. Although a number of papers have appeared on scattering from microscopic turbulence[7] this is the first time an attempt has been made to include the effects of turbulence on the scattering cross section. Most of the previous work has been concerned with the experimental observation of enhanced scattering from the large density fluctuations present in a turbulent plasma. Examples and more detailed investigations of specific cases will be presented in future work.

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