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ABSTRACT

The possibility that the electrons producing discrete aurora are stochastically accelerated by waves is investigated. It is demonstrated that the lower-hybrid waves seen on auroral field lines have the right properties to account for the electron acceleration. It is further shown that the lower-hybrid wave power measured on auroral field lines can be generated by the streaming ions observed at the boundary of the plasma sheet, and that this wave power is sufficient to account for the electron distributions observed close to the atmosphere.

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INTRODUCTION

In this paper we describe in detail the model outlined by Bingham et al [1984] in which it was proposed that the streams of electrons that are associated with discrete aurorae are accelerated by electrostatic waves.

The first direct measurements of the electrons that produced a discrete aurora were made by McIlwain [1960] who, after careful examination of the data, deduced that the electron spectrum responsible for producing a bright active auroral arc was more sharply peaked than a Maxwellian. The spectrum was described as consisting of nearly mono-energetic electrons with about 6 keV energy. The sharpness of the peak argued against it having been generated by a statistical process, and acceleration by electric fields parallel to the earth's magnetic field was suggested as a possible cause. This was followed by the work of Albert [1967], Evans [1968] and Hoffmann and Evans [1968], who, in order to account for their observations of "near mono-energetic electrons" and field-aligned electrons, suggested that the electrons traversed a magnetic-field-aligned potential difference which accelerated them to energies of a few keV. This interpretation has since become almost a tenet of auroral physics.

The auroral particle observations that have been made since 1968 have, however, led to the realization that the electron distributions cannot be accounted for by invoking a process as straightforward as one in which all electrons traverse the same potential difference [O'Brien, 1970; Whalen and McDiarmid, 1972]. It was pointed out by Hall and Bryant [1974] that the monotonic, rather than stepped, nature of the electron angular distributions, and the shape of the energy spectrum peak, which is too broad for the electrons to be described as "mono-energetic", indicated a stochastic acceleration process. To account for the wide range of energies that exhibit field alignment, and the wide range of angles over which field alignment extends, Hall and Bryant [1974] introduced the possibility that the accelerating electric field was essentially a time-varying phenomenon, with major fluctuations on timescales of milliseconds or less. A wave-particle interaction process was suggested by Bryant et al, [1978] as the mechanism. In order to accommodate another common feature of the spectrum - the rise in "temperature" of the high-energy tail with increasing peak energy [Burch et al, 1976; Bryant, 1981] - the degree of acceleration is required to be energy dependent as well as time-varying. Bryant [1983] has summarized the discrepancies between the observed electron distributions and those expected for acceleration through a potential difference. Wave-particle interactions have been invoked to account for some of these discrepancies, such as the flatness of the peak on its low energy side [Bryant et al, 1978; Lotko and Maggs, 1979; Johnstone, 1980; Maggs and Lotko, 1981; Kaufmann and Ludlow, 1981], and multiple peaks [Hoffmann and Lin, 1981; Arnoldy, 1981].

The conclusions drawn from particle observations that the acceleration mechanism is stochastic and energy-dependent has led authors such as Bryant [1978], Whalen and Daly [1979] and Hall [1980] to consider the possibility that the electrons are stochastically accelerated by plasma waves. Kaplan and Tsytovich [1973] have shown that interactions between electrostatic plasma waves and electrons can be very efficient in accelerating electrons to produce non-thermal tails. The effectiveness of electrostatic waves for accelerating electrons has been recognized during the search for current drivers in Tokamak fusion-devices [Boyd et al, 1976].

In this paper we will consider the possibility of electron acceleration by lower-hybrid waves, which are observed to be very intense on auroral field lines, Scarf et al [1973]. Lin and Hoffmann [1979] have found that inverted-V electron streams are associated more closely with regions of low-frequency turbulence having the same spatial extent [Temerin, 1981], than with electrostatic shocks. Gurnett and Frank [1977] show that the maximum wave energy resides at frequencies close to the lower-hybrid frequency. Scarf et al [1973], using results obtained from OGO 5, found that the spectrum peaked near the lower-hybrid frequency with a normalized energy density (normalization is with respect to plasma thermal energy density) of 10^{-4} to 10^{-3}, corresponding to electric field strengths between 0.2 - 0.5 Vm^{-1}. Mozer et al [1980] have reported that the lower-hybrid waves are one of the most important auroral zone waves observed by the S3-3 satellite; the waves having field strengths of the order of 100 mV/m, the most intense being seen in the region of steep density gradients which correspond to the boundary between the plasma sheet and tail lobes. Bryant et al [1972] first demonstrated that the electron streams accosiated with discrete aurora were formed at this boundary. This was confirmed by the measurements reported in Bryant et al [1977]. Further confirmation has been provided by Winningham et al [1975], who have shown that diffuse aurora is the footprint on the atmosphere of particles precipitating from the plasma sheet, and that discrete aurora is normally located at the poleward boundary of diffuse aurora. Measurements at the plasma sheet boundary at altitudes of 10-20 R_E have shown that the boundary is not a sudden transition from plasma-sheet to magnetotail plasma, but that there is a boundary layer within which ions stream towards the earth [Decoster and Frank 1979], and where there are highly-structured electron streams [Parks et al, 1979; Williams, 1981]. Lyons and Evans [1984] have demonstrated that the ion streams are associated with the production of discrete aurora. The mechanism described in the present paper concentrates on acceleration of electrons by parallel electric fields of waves rather than by d.c. electric fields which could also play a role in shaping the electron distribution functions.

Lower-hybrid waves can be driven unstable by a number of sources of free energy: e.g. differential electron-ion drift, cross-field currents due to density, temperature and magnetic field gradients, $\underline{E} \times \underline{B}$ drift, where $\underline{E}$ and $\underline{B}$ are the ambient electric and magnetic fields, ion loss-cone distributions with a positive value of $\partial f_\perp / \partial v_\perp$, where $f_\perp$ is the perpendicular ion distribution function, and momentum coupling between inter-penetrating plasmas. The most promising free-energy source is the earthward streaming ion flow in the boundary plasma sheet [Decoster and Frank, 1979; Williams, 1981]. The power carried by the ion stream at 20 R_E is 2 orders of magnitude higher than that carried by the electrons just above the atmosphere associated with discrete aurora [Hall et al, 1984,]. The process described here can be considered as a continuous transfer of ion power via the waves to the electrons. The wave power observed at any point is then an equilibrium level between production by the ions and loss to the electrons.

In this paper we consider various wave generation processes and calculate their growth rates and saturation levels. A quasi-linear theory of wave-particle interactions is then used to describe the acceleration, and to calculate the effective temperature and density of the accelerated particles, together with the length of the acceleration region.

LOWER-HYBRID WAVES

It is well known [Kaplan and Tsytovich 1973] that the development of non-thermal tails by stochastic acceleration requires high phase velocities in at least one direction. Lower-hybrid waves have phase velocities along the field lines ranging from just above the electron thermal velocity, (v_{Te}), to greater than the speed of light. In laboratory plasmas, lower-hybrid waves have been shown to be extremely effective in accelerating electrons parallel to $\underline{B}$ and in producing high energy tails in the electron distribution function [Boyd et al, 1976]. Numerical simulations by McBride et al [1972] and Tanaka and

Papadopolous [1983] show how large amplitude lower-hybrid waves are generated, and how effective these waves are in forming high-energy tails. A number of laboratory experiments using ion beams injected along [Ioffe et al, 1961], and perpendicular to [Barrett et al, 1972], the magnetic field, demonstrate the role that the ion beams play in generating the waves and, consequently, in accelerating electrons.

The dispersion relation for electrostatic waves in a magnetized plasma is [Stix, 1962]:

$$1 + \frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pi}^2}{\omega^2} = 0 \qquad (1)$$

where $k_\perp$ and $k_\parallel$ are the perpendicular and parallel components of the wave number with respect to the ambient magnetic field $\underline{B}$, $k^2 = k_\perp^2 + k_\parallel^2$, $\omega_{ce,i}$ are the electron, ion angular gyrofrequencies, and ω_{pe} is the plasma angular frequency. For $k_\parallel^2 \ll k_\perp^2$ solutions to (1) represent lower-hybrid waves.

Their frequency is given by:

$$\omega = \omega_{LH}\left[1 + (k_\parallel/k)^2(m_i/m_e)\right]^{\frac{1}{2}} \qquad (2)$$

where $\omega_{LH} = \omega_{pi}/(1 + \omega_{pe}^2/\omega_{ce}^2)^{\frac{1}{2}}$ is the lower-hybrid resonance frequency and ω_{pi} is the ion plasma frequency.

These waves have k nearly perpendicular to $\underline{B}$. There is, however, an electric-field component, and wave number $k_\parallel < (m_e/m_i)^{\frac{1}{2}}k$, parallel to $\underline{B}$, m_e/m_i being the electron ion mass ratio. The group velocities parallel and perpendicular to the magnetic field can be obtained from (2) and are given by:

$$V_{g\parallel} = \frac{\omega_{LH}}{k}\left[1 - k_\parallel^2/k^2\right]\beta \tag{3}$$

$$V_{g\perp} = -\frac{\omega_{LH}}{k}\frac{k_\parallel k_\perp}{k^2}\beta \tag{4}$$

$$\text{where} \quad \beta = \frac{k_\parallel}{k}\frac{m_i}{m_e}\left[1 + \frac{k_\parallel^2}{k^2}\frac{m_i}{m_e}\right]^{-\frac{1}{2}}$$

From equations (3) and (4), and using the condition $k_\parallel^2 \ll k_\perp^2$, we obtain the ratio $v_{g\parallel}/v_{g\perp} \simeq -k^2/k_\parallel k_\perp$ from which it follows that $v_{g\parallel} \gg v_{g\perp}$. Therefore, most of the energy flows parallel to the magnetic field. The phase velocity along the field is given by $v_{Ph\parallel} = \omega/k_\parallel$. Using the relation $k_\parallel \lessdot (m_e/m_i)^{\frac{1}{2}} k$, where $k \simeq \omega/c_s$ and c_s is the ion acoustic speed, we find that $v_{Ph\parallel} \gtrdot c_s(m_i/m_e)^{\frac{1}{2}}$; i.e. $v_{Ph\parallel} \gtrdot v_{Te}$. Thus the phase velocity parallel to $\underline{B}$ is greater than the electron thermal velocity. Characteristics of lower-hybrid waves are shown in Fig. 1.

A wave-spectrum measured on auroral field lines is plotted in Fig. 2 (Gurnett and Frank 1977). In the acceleration region the waves have a broad range of frequencies. In the boundary of the plasma sheet there is a broad range of frequencies, indicating a broad range of parallel phase velocities which can resonate with electrons of matching velocities and consequently accelerate them via Cherenkov resonance.

EXCITATION AND GROWTH RATES

As already noted, there are a number of free-energy sources on auroral field lines which can generate non-thermal levels of lower-hybrid waves. All ultimately derive their power from the solar wind. One model which transfers energy from the solar wind to the magnetosphere uses magnetic-field reconnection (see review by Sagdeev, 1979). The interaction between the solar wind and the magnetosphere compresses the plasma sheet transversely, the tail then narrows, leading to the tearing instability and the formation of a reconnection point. This in turn leads to enhanced particle flows [Decoster and Frank, 1979; Frank et al, 1976; Hones et al, 1972; Williams, 1981], with the ions convecting earthwards and carrying the bulk of the energy. As the plasma sheet is compressed, the magnetic field increases, leading to the enhancement of temperature and density gradients between the plasma sheet and tail lobes. These conditions excite several mechanisms which can drive lower-hybrid waves unstable; namely differential electron-ion drift, $\underline{E} \times \underline{B}$ drift, cross-field currents due to density, temperature and magnetic field gradients, and also loss cone-type distributions on auroral field lines.

Various names are given to the processes which result in the growth of the waves. If gradients drive the instability, the process is known as the lower-hybrid drift instability; if ion convection is the driver, it is called the modified two-stream instability. In both cases there is a threshold requirement of $v_d > v_{Ti}$ where v_d is the electron/ion relative drift speed, and v_{Ti} $(= (KT_i/m_i)^{\frac{1}{2}})$ is the ion thermal speed.

In constructing a model for wave growth and electron acceleration we will concentrate on the streaming ions in the geomagnetic tail [Decoster and Frank, 1979]. The acceleration mechanism then consists of excitation, by the streaming ions, of lower-hybrid waves which transfer their energy to the electrons.

The streaming ions can be regarded as the free-energy source from which the lower-hybrid waves are excited by an instability which is driven by either a cross-field current, or by an ion-loss cone distribution [Mikhalovskii, 1974]. The first of the two mechanisms is called the modified two-stream instability.

MODIFIED TWO-STREAM INSTABILITY

The modified two-stream instability is described by the dispersion relation [McBride et al, 1972]:

$$\epsilon(\omega, \underline{k}) = 1 + (2\omega_{pe}^2/k^2 v_{Te}^2)\left[1 + \zeta_e Z(\zeta_e) I_o(\lambda) e^{-\lambda}\right]$$

$$+ 2\omega_{pi}^2/k^2 v_{Ti}^2 \left[1 + \zeta_i Z(\zeta_i)\right] = 0 \qquad (5)$$

where $\zeta_e = \omega/k_\parallel v_{Te}$, $\zeta_i = (\omega - \underline{k}.\underline{v}_d)/kv_{Ti}$, $k_\parallel = k\cos\theta$, $Z(\zeta)$ is the plasma dispersion function (Fried and Conte, 1962), and $I_o(\lambda)$ is the zeroth order bessel function with $\lambda = k^2 v_{Te}^2 \sin^2\theta/2\omega_{ce}^2$. In the fluid approximation, where resonant particles are unimportant, equation (5) reduces to the following simplified dispersion relation:

$$1 + k_\perp^2 \omega_{pe}^2/k^2\omega_{ce}^2 - \omega_{pi}^2/(\omega - \underline{k}.v_d)^2 - k_\parallel^2 \omega_{pe}^2/k^2\omega^2 = 0 \qquad (6)$$

which describes modes for which $kv_{Te}/\omega_{ce} \ll 1$, $kv_{Ti} < |\omega - \underline{k}.v_d|$ and $k_\parallel v_{Te} < \omega$. Equation (6) is similar to the two-stream or Buneman-instability dispersion relation (Krall and Trivelpiece, 1973), but is, in addition, valid for particle drift velocities much less than the electron thermal velocity.

The term $k_\perp^2\omega_{pe}^2/k^2\omega_{ce}^2$ is due to the adiabatic polarization drift of electrons across the magnetic field. The term $k_\parallel^2\omega_{pe}^2/k^2\omega^2$ arises because the electrons are free to accelerate under an applied force only along the magnetic field, with the electrons behaving as if they had an effective mass $m_{eff} = k^2 m_e/k_\parallel^2$, which is very much greater than m_e for $k/k_\parallel \gg 1$. By writing equation (6) in the following form:

$$\left[(\omega^2 - \omega_{LH}^2)\right]\left[(\omega - \underline{k}.\underline{v}_d)^2 - (k_\parallel^2 m_i/k^2 m_e)\omega_{LH}^2\right] = \frac{m_i}{m_e}\frac{k_\parallel^2}{k^2}\omega_{LH}^2 \qquad (7)$$

we can identify the two waves which couple to produce the modified two-stream instability. They are $\omega = \omega_{LH}$, the lower-hybrid mode; and $\omega = k_\perp v_d - (k_\parallel/k)\,(m_i/m_e)^{\frac{1}{2}}\omega_{LH}$, the Doppler-shifted electron mode. For $\omega_{pe} < \omega_{ce}$, which is the situation in the auroral zone, the electron mode is just the Doppler-shifted electron plasma oscillation, propagating almost perpendicular to B with frequency $\omega \simeq k_\perp v_d - (k_\parallel/k)\omega_{pe}$. In this system the lower-hybrid mode is the positive-energy wave, while the Doppler-shifted electron mode is a negative-energy wave. Equation (7) can be solved analytically by writing $\omega = \omega' + \frac{1}{2}k_\perp v_d$ to give growth rates which agree very well with the numerical solution of equation (5) shown in Fig. 3. The solution of (7) for a wave number $k = \sqrt{3}\omega_{LH}v_d$ and real frequency $Re\omega = \sqrt{3}\omega_{LH}/2$ is:

$$\gamma_{gmax} \simeq \frac{1}{2}\omega_{pi}\Big/\left(1+\omega_{pe}^2/\omega_{ce}^2\right)^{\frac{1}{2}} = \frac{1}{2}\omega_{LH} \qquad (8)$$

The modified two-stream instability occurs for $k_\parallel/k \simeq (m_e/m_i)^{\frac{1}{2}}$ such that the wave propagates at an angle of about 1.4^0 with respect to the magnetic field. As $k_\parallel/k$ increases, the lower-hybrid mode no longer couples to the Doppler-shifted electron mode and the instability develops into the ion-acoustic instability. Numerical solutions of equation (5) for conditions found in the auroral zones, where $\omega_{pe} < \omega_{ce}$ and $T_i > T_e$, are shown in Fig. 3. An important consequence of this instability, as already pointed out by Barrett et al (1972), is that unlike other electrostatic instabilities, such as the ion-acoustic instability, the instability is insensitive to the electron-ion temperature ratio T_e/T_i, and can take place even if $T_i > T_e$. The reason the modified two-stream instability can exist for $T_i \simeq T_e$ is the finite or cut-off value of $Re\omega$ as $k_\parallel/k \to 0$. This ensures that the phase velocity of the wave is always greater than the bulk thermal velocity of the ions, thus preventing strong Landau damping. This is even the case when $T_i \gg T_e$. In the auroral zone, where the plasma density $n_o \simeq 10cm^{-3}$, the maximum growth rate is $\gamma_{gmax} \simeq 2 \times 10^3\ s^{-1}$, and the time required for 10 e-foldings is of the order of 5×10^{-3} sec, this corresponds to a distance, L, of about 300-1000km along the field lines for waves with phase velocity three times the thermal velocity. The effect of the instability is to heat the ions primarily in the perpendicular direction, and to accelerate the electrons parallel to B. Recent particle-in-cell simulations of the modified two-stream instability by Tanaka and Papadopolous [1983] confirm the threshold and growth rates given by the linear theory. They also show that 60% of the initial ion-stream energy is transferred via lower-hybrid waves to electrons, producing a high-energy tail extending to about $7v_{Te}$. Their results demonstrate the efficiency of transfer of energy from ions to electrons.

ION LOSS-CONE INSTABILITY

Another important mechanism which can transfer the kinetic energy of the ion-stream, flowing earthward from the tail, to lower-hybrid waves is the electrostatic ion-loss-cone instability (Rosenbluth and Post, 1965). A dominant characteristic of the ion distributions in the auroral zone is a loss-cone type distribution, with a positive value $\partial f_i(v_\perp)/\partial v_\perp$, where $f_i(v_\perp)$ is the ion distribution function integrated over the parallel velocity.

The gyrating ions transfers perpendicular energy from their gyro motion to lower-hybrid waves through Landau resonance on the positive slope of the distribution. This instability is similar to the modified two-stream instability with a beam-type distribution. Both excite the same mode, the lower-hybrid mode, whose frequency and wavelength satisfy the following inequalities:

$$\omega_{ci} \ll \omega \ll \omega_{ce} \quad , \quad \mathrm{Im}\,\omega \gg \omega_{ci}$$

$$k_\perp \rho_e \ll 1 \ll k_\perp \rho_i \quad \text{and} \quad k_\parallel \ll k_\perp$$

where $\rho_{e,i}$ is the electron, ion gyroradius. These assumptions are appropriate for plasmas of moderate densities with $\omega_{pi} \gg \omega_{ci}$.

When these inequalities are satisfied we can neglect the influence of the magnetic field on the ion motion. The instability is described by the kinetic equations for ions and electrons, together with Poisson's equation:

$$\left(\frac{\partial}{\partial t} + \underline{v}\cdot\underline{\nabla} + \frac{e}{m_i}\nabla\phi\cdot\frac{\partial}{\partial \underline{v}}\right) f_i(\underline{x},\underline{v},t) = 0 \qquad (9)$$

$$\left(\frac{\partial}{\partial t} + \underline{v}.\underline{\nabla} - \frac{e}{m_e}\left(\underline{\nabla}\phi + \underline{v} \times \underline{B}\right).\frac{\partial}{\partial \underline{v}} \right) f_e(\underline{x},\underline{v},t) = 0 \tag{10}$$

$$\underline{\nabla}.\underline{E} = -\frac{1}{\epsilon_0}\sum_{j=i,e} e_j \int f_j(\underline{x},\underline{v},t)\, d^3\underline{v} \tag{11}$$

where $\underline{E} = -\underline{\nabla}\phi$, and ϕ is assumed to have the space-time dependence $[\phi \sim \exp i(\omega t + k_{\parallel} z + k_{\perp} x)]$. In the linear approximation, equations (9) to to 11) reduce to the dispersion relation for the electrostatic lower-hybrid mode given by:

$$\epsilon(\omega,\underline{k}) = 1 + \omega_{pe}^2/\omega_{ce}^2 - \omega_{pe}^2 k_{\parallel}^2/\omega^2 k^2 + \frac{\omega_{pi}^2}{k^2}\int \frac{\underline{k}.\partial f_i/\partial \underline{v}}{\omega - \underline{k}.\underline{v} + i0}\, d^3\underline{v} \tag{12}$$

In deriving equation (12) we have assumed that the perpendicular wavelength is larger than the electron gyroradius, $k_{\perp}\rho_e << 1$, and that the parallel phase velocity is larger than the electron thermal velocity, an assumption commonly referred to as the cold plasma approximation. The growth rate and frequency of the unstable modes can be obtained from the equation:

$$\gamma_g = \operatorname{Im}\epsilon(\omega,\underline{k}) \Big/ \frac{\partial \operatorname{Re}\epsilon(\omega,\underline{k})}{\partial \omega_k} \tag{13}$$

To solve (13) we must carry out the ion integrals in equation (12). Defining $\int dv_{i\parallel} f_i(v_{i\perp}^2, v_{i\parallel}^2) = f_{\perp}(v_{\perp}^2)/\pi$, and integrating over the parallel velocities, the dispersion relation now becomes:

$$\omega_{pe}^2/\omega_{ce}^2 + 1 = \frac{\omega_{pe}^2}{\omega^2}\frac{k_{\parallel}^2}{k^2} + \frac{\omega_{pi}^2}{k^2}\int dv_{\perp} \frac{\partial f_{\perp}(v_{\perp}^2)}{\partial v_{\perp}} \frac{\omega}{(\omega^2 - k_{\perp}^2 v_{\perp}^2)^{\frac{1}{2}}} \tag{14}$$

Rosenbluth and Post (1965) have shown that this dispersion relation is unstable for loss-cone distribution functions for $k_\parallel \neq 0$. The fastest growing waves have frequencies:

$$\mathrm{Re}\,\omega \simeq \omega_{LH} \tag{15}$$

for $k_\parallel / k_\perp \approx (m_e/m_i)^{\frac{1}{2}}$.

The physical nature of this instability is very similar to that of the two-stream fluid instability discussed previously. The free energy comes from the gyromotion of the ions. The general effect of the instability would be to convey energy from perpendicular to parallel motion, forcing them into the loss cone. This instability is convective in nature, and therefore it is important to determine the exponentation length along the field lines. Solving equation (14) for the growth length at a given real ω, yields:

$$k_\parallel^2 = k^2\left[\omega^2(\omega_{pe}^2 + \omega_{ce}^2)/\omega_{pe}^2\,\omega_{ce}^2 - (m_e/m_i)\, y^2 F(y)\right] \tag{16}$$

$$\text{where} \quad F(y) = -2\int_0^\infty dx\,\frac{\partial\psi}{\partial x}\,\frac{1}{(1-x/y^2)^{\frac{1}{2}}}\,, \quad y = \omega^2/2k_\perp^2 v_{Ti}^2,$$

$$x = v_\perp^2/2v_{Ti}^2 \quad \text{and} \quad \psi = 2v_{Ti}^2 f_{i\perp}(v_{i\perp})\,.$$

Taking the square root, and expanding equation (16) using the condition $k^2 v_{Ti}^2 > \omega_{pi}^2$, yields the following:

$$k_\parallel = k\left[\omega(\omega_{pe}^2 + \omega_{ce}^2)^{\frac{1}{2}}/\omega_{pe}\,\omega_{ce}\right] - \frac{1}{2\sqrt{2}}(m_e/m_i)^{\frac{1}{2}}\, y F(y)\, \omega_{LH}/v_{Ti} \tag{17}$$

The spatial growth rate is obtained from the negative imaginary part of equation (17), i.e. the negative imaginary part of yF(y). Rosenbluth and Post (1965) analysed the function yF(y) for a distribution function expected in mirror magnetic fields, namely:

$$f_{i\perp} = (v_\perp^2 - v_\parallel^2)\exp(-m_i v_\perp^2/2KT_i) \qquad \text{for} \quad v_\perp > |v_\parallel| \qquad (18)$$

$$f_{i\perp} = 0 \qquad \text{for} \quad v_\perp < |v_\parallel| \qquad (19)$$

Using this function in equation (17), the fastest spatial growth along the field is found to be:

$$k_{\parallel\,growth} \simeq 0.1 \left(\frac{m_e}{m_i}\right)^{\frac{1}{2}} \frac{\omega_{LH}}{v_{Ti}} \qquad (20)$$

for a real frequency equal to the lower-hybrid frequency. This corresponds to an exponentiation length of

$$L \approx 500\ (1 + \omega_{pe}^2/\omega_{ce}^2)^{\frac{1}{2}}\, V_{Ti}/\omega_{pi} \qquad (21)$$

In the auroral zone this length is about 1000 km. For more extreme distributions, it may be much shorter.

SATURATION LEVEL

So far we have discussed the instabilities which could be responsible for the generation of lower-hybrid waves. To estimate the wave amplitude expected from such instabilities we have to consider the saturation mechanism. An important process common to both instabilities is the transfer of a substantial part of the ion kinetic energy to lower-hybrid waves which accelerate electrons in the tail of the distribution function. The equilibrium level expected when the system is marginally stable, i.e. when there is a balance between production and loss, can be obtained by considering specific saturation mechanisms. Simulations by McBride et al (1972) and Tanaka et al (1983) show that the waves saturate as a result of trapped ions. This increases the parallel phase velocity of the wave which stays longer in phase with the electron, thus increasing the efficiency of the acceleration process. Analytically, the waves are found to saturate at an energy density given by:

$$W_{LH}/n_o KT_e \simeq 0.05\left(1+\omega_{pe}^2/\omega_{ce}^2\right) \qquad (22)$$

where W_{LH} $(= \frac{1}{2}\,\epsilon_o |E|^2)$ is the energy density of the lower-hybrid wave with an electric field E. On auroral field lines where $\omega_{pe} < \omega_{ce}$, the saturation level for the waves given by equation (22) is 5×10^{-2}. This value is to be compared to experimental values of the normalized energy density of 10^{-3} - 10^{-4} obtained by Scarf et al (1973). These experimental values correspond to electric field strengths of between 0.2 and 0.5 Vm^{-1}, and power fluxes of 0.1 to 0.6 mW/m^2 at $3R_e$.

STATISTICAL ACCELERATION OF ELECTRONS BY LOWER-HYBRID TURBULENCE

When a microinstability develops, fluctuating fields generated by the instability scatter plasma particles and cause diffusion of the velocity distribution function. If the spectrum of the fluctuating fields is characterized by sufficiently small values of the parallel wavenumber $k_{\parallel}$, such that the phase velocity of the wave parallel to $\underline{B}$ is greater than the electron thermal speed, then the waves resonate with and accelerate only electrons moving parallel to $\underline{B}$ with velocities greater than v_{Te}.

The statistical acceleration of electrons in the tail of the distribution function due to the resonant interaction can be demonstrated by solving the one dimensional quasi-linear diffusion equation (Davidson, 1972):

$$\frac{\partial f_e}{\partial t} = \frac{\partial}{\partial v_{\parallel}} D_{\parallel} \frac{\partial f_e}{\partial v_{\parallel}} \qquad (23)$$

where f_e is the electron distribution function, and $D_{\parallel}$ is the quasi-linear diffusion operator resulting from the Landau wave-particle interaction, $D_{\parallel} = \Delta v^2/\tau$. Δv is the change in velocity in time τ, and where $D_{\parallel}$ is given by

$$D_{\parallel} = 16\pi^2 \frac{e^2}{m_e^2} \frac{k_{\parallel}}{\omega} \epsilon_{k_{\parallel}} \qquad (24)$$

and $\epsilon_{k\parallel}$ is the wave energy density per unit wavenumber

$$\frac{1}{2}\epsilon_o E_{\parallel rms}^2 = \int_0^\infty \epsilon_{k_\parallel} dk_\parallel \qquad (25)$$

and $E_{\parallel\, rms}$ is the rms value of the lower-hybrid field. For an acceleration region of finite length the effective range of $k_\parallel$ is

$$k_m \leq k_\parallel \leq k_o$$

where k_m is determined by the scale size of the plasma, and k_o is determined by the strong Landau damping that takes place for $k_\parallel = \omega_{LH}/2v_{Te}$. The total wave energy density is thus:

$$W_{LH} = (k_o - k_m)\epsilon_k \simeq k_o\epsilon_k \qquad (26)$$

By substituting equation (24) into equation (23) and integrating, it can be shown that the asymptotic solution is given by:

$$f_e = \frac{n_o}{\sqrt{4\pi D_\parallel t_o}} \exp\left(-v_\parallel^2/4D_\parallel t_o\right) \qquad (27)$$

where $t_o \approx L/\langle v\rangle$ is the maximum time the wave and electron interact, and $\langle v\rangle$ is the average electron velocity. Equation (27) shows that a tail is established with a velocity equivalent to $(4D_\parallel t_o)^{\frac{1}{2}}$. The temperature of this tail is found to be:

$$KT_{TAIL} \simeq 2m_e D_\parallel t_o \qquad (28)$$

The ratio of the electron tail temperature to the thermal temperature is now given by:

$$\frac{T_{TAIL}}{T_e} \simeq 4\pi \frac{W_{LH}}{n_o KT_e} \frac{k_\parallel}{k_o} \omega_{pe} t_o \qquad (29)$$

An important parameter is the length needed to accelerate a particle from v_1 to v_2. This was shown to be (Bingham et al, 1984):

$$L_{acc} = \frac{\omega_{LH}}{4\pi\omega_{pi}^2} \frac{n_o KT_e}{W_{LH}} \frac{(\Delta v)^2}{V_{Te}^2} \frac{k_o}{k_\parallel} \langle v \rangle \qquad (30)$$

where $\Delta v = v_2 - v_1$. Using results obtained by Scarf et al (1973), ie a normalized energy density of 10^{-4} - 10^{-3}, and field strengths between 0.2 and 0.5 Vm^{-1}, we find that the distance required to accelerate electrons from v to 2v, where v = 2 x 10^{-7} ms^{-1}, is between $0.3R_e$ to $2R_e$, where R_e is the earths radius.

One of the drawbacks of the analytic quasi-linear diffusion theory is that it allows for resonant acceleration only through Landau resonance. However, as shown by numerical simulations (Tanaka and Papadopolous, 1981), non-linear effects play an important role in tail formation. One non-linear effect is produced by the trapped ions which have been shown to lead to a positive frequency shift, which increases the parallel phase velocity of the waves, which in turn make the waves much more efficient in accelerating electrons than Landau damping alone. The waves due to the non-linear effects have characteristics, such as the phase-velocity increase, which are found in an auto-resonant accelerator.

Particle-in-cell simulations carried out by Dawson (1985), on the process of electron acceleration by waves around the lower-hybrid frequency, are shown in Fig. 4 plotted together with experimental results obtained from rocket measurements (Bryant et al, 1981). There is excellent agreement between experiment and simulations.

Sometimes the experimentally observed electron distributions have regions where $\partial f_e/\partial v_\parallel > 0$, i.e. regions with a positive slope as depicted in Figs 5 and 6. This feature again cannot be explained by the quasi-linear theory presented above. However, quasi-linear diffusion theory does predict a large velocity anisotropy, i.e. $T_{tail} > T_e$. A numerical study by Papadopoulos et al (1977) has demonstrated how an initially flat-tail distribution will evolve towards a bump-in-the-tail distribution through the anomalous Doppler-resonance instability (Kadomtsev and Pogutse, 1968). We can understand this instability physically as follows. Let us assume that the flat-tail distribution has already been established with $T_{tail} > T_e$. The streaming electrons in the tail then generate waves under anomalous Doppler-effect conditions such that:

$$\omega = k_{\parallel} v_{\parallel} - \omega_{ce} \qquad (31)$$

with negligible Landau-damping, i.e.

$$\omega / k_{\parallel} \gg v_{Te} \qquad (32)$$

It follows from (31) and (32) that only particles with velocities $v_{e\parallel} > v_m = (\omega_{ce}/\omega_{pe}) v_{Te}/k_{\parallel} \lambda_{De}$ can be resonant with the waves, Kadomtsev and Pogutse (1968) have shown that, for $\omega_{ce}/\omega_{pe} > 1$, this interaction leads to pitch-angle scattering for particles with $v_{e\parallel} > v_m$, and to particles with velocities $v_{e\parallel} < v_m$ being accelerated faster than particles with $v_m > v_{e\parallel}$, and a pile-up of particles will occur near v_m. As a consequence the high-energy electron tail will develop a positive slope near $v_{e\parallel} = v_m$. Fig. 6 shows an experimentally observed distribution function showing the features expected from anomalous Doppler resonance such as the bump-in-the-tail and the pitch-angle-scattered higher energy particles. The particles in the beam are expected to be more closely field-aligned than the higher energy ones, as is indeed found. Using the experimentally-obtained parameters, it is found that $v_m = 6 \times 10^7$m/s, in good agreement with the measured position of the beam.

CONCLUSION

We have examined the possibility of electron acceleration by lower-hybrid waves on auroral field lines and have shown it is feasible that the streams of accelerated electrons (inverted V's) associated with discrete aurora could be produced as a result of a continuous, evolutionary, process along an auroral flux tube. The generation of the lower-hybrid waves has been considered to be due to instabilities driven by the streaming ions produced in the tail of the magnetosphere by magnetic reconnection processes. The power in the ion-streams and in the waves associated with the boundary plasma sheet is found to be sufficient to account for the power and the degree of acceleration.

Two linear instability mechanisms have been studied, namely the modified two-stream instability and the ion loss-cone instability. Energy transfer takes place in a suitably short distance with an e-folding length along the field lines of about 400 km. An analytic theory for energy transfer from waves, based on the quasi-linear wave-particle diffusion model, has demonstrated the effectiveness of the waves in producing a high energy tail. The observed wave energy density appears to be sufficient to accelerate electrons over a distance of the order of $0.3R_e$ - $2R_e$. Simulations have shown that non-linear processes such as particle trapping can enhance the efficiency of tail formation still further.

FIGURE CAPTIONS

Figure 1. Lower-hybrid wave characteristics.

Figure 2. Spectrum of waves in the magnetosphere at $4R_e$, from Gurnett and Frank (1977).

Figure 3. Growth rate of lower-hybrid waves, on auroral field lines, due to the modified two-stream instability.

Figure 4. Auroral electron distributions. Dots represent experimental values, and the solid line represents results from a simulation code (J.M. Dawson, private communication).

Figure 5. Auroral electron distributions measured at different locations within an arc. Bryant (1983).

Figure 6. Pitch-angle dependence of an auroral electron distribution function showing a positive slope for electrons with small pitch angles. (X represents electrons with 75^0 pitch angles; ● represents electrons with 15^0 pitch angles (Bryant (1983)).

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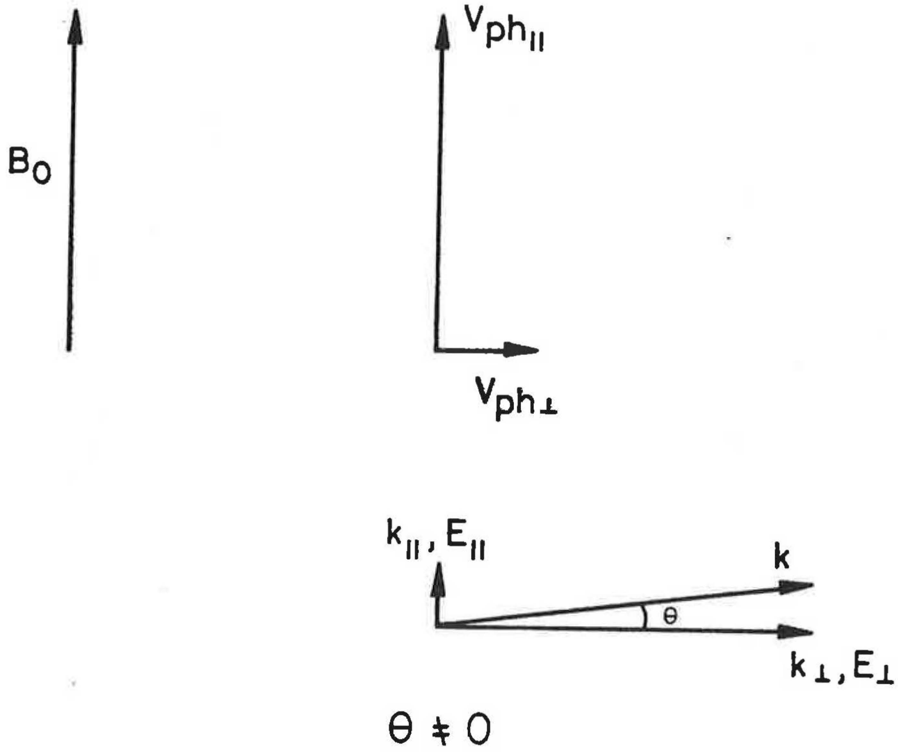
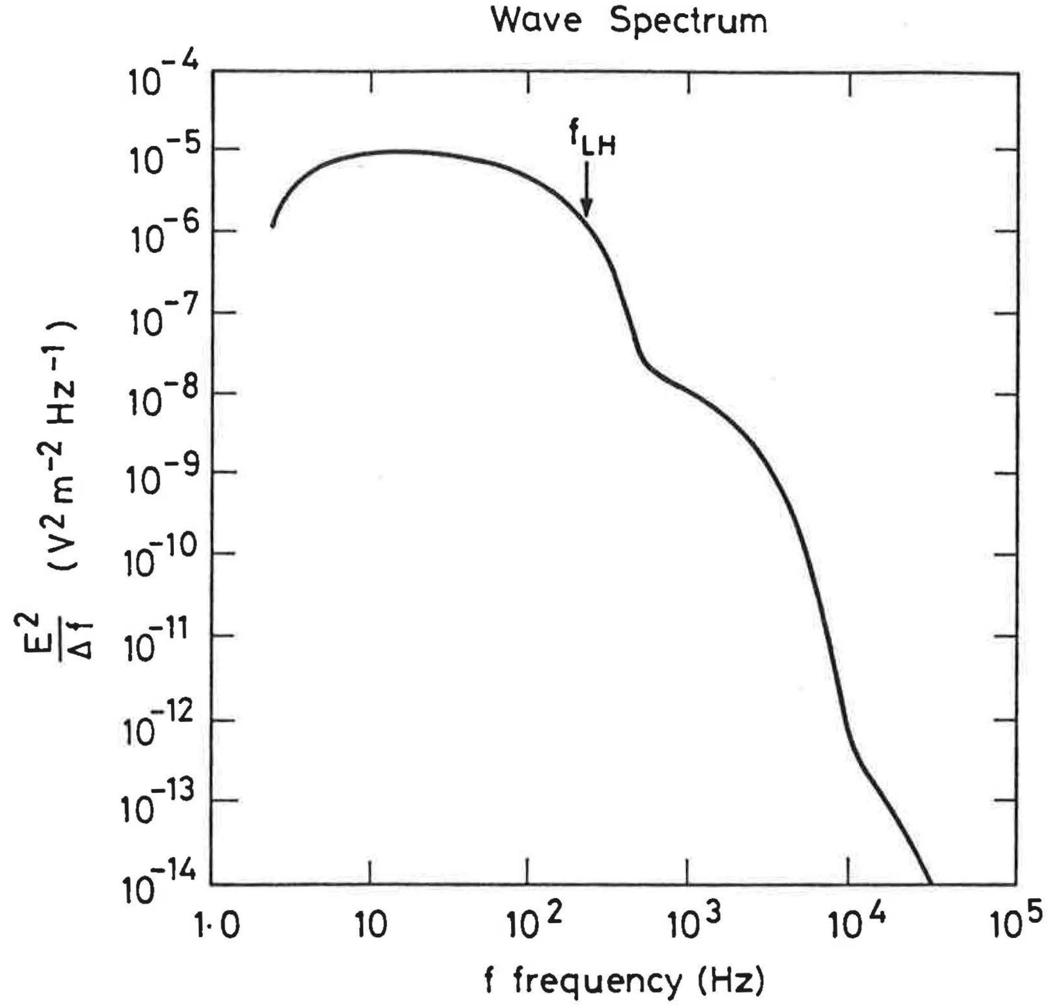
Lower hybrid waves

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10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
$\frac{E^2}{\Delta f}$ ($V^2 m^{-2} Hz^{-1}$)
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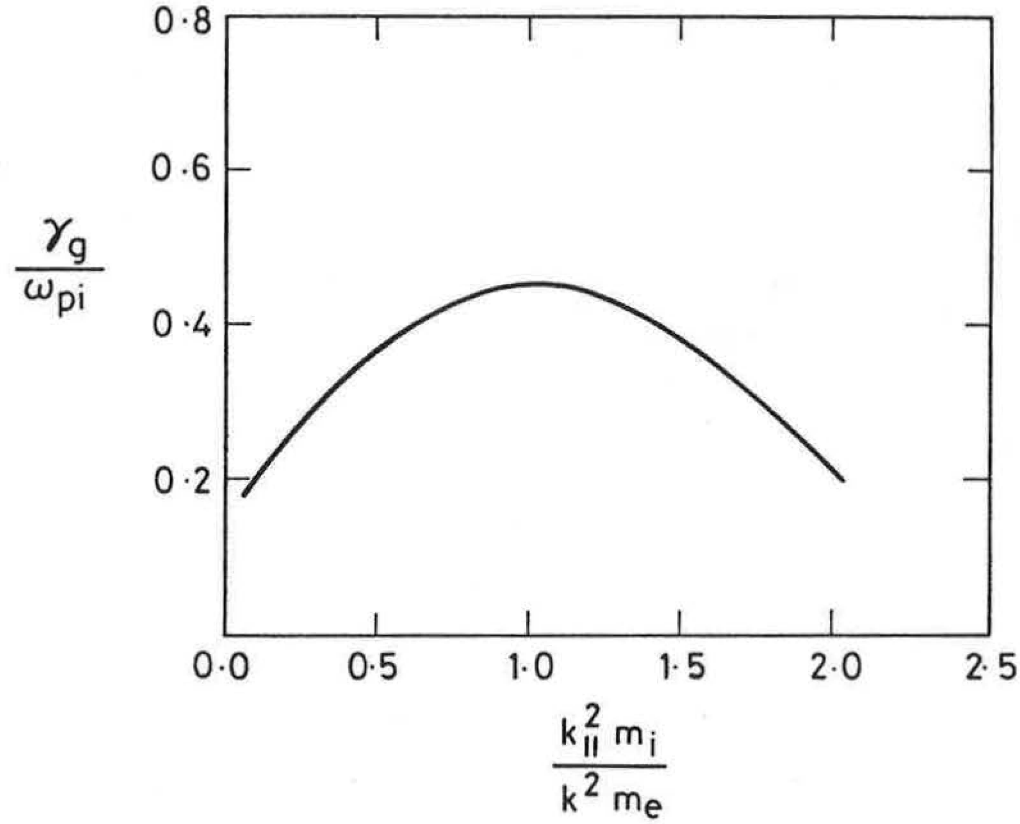
10
10^2
10^3
10^4
10^5
f frequency (Hz)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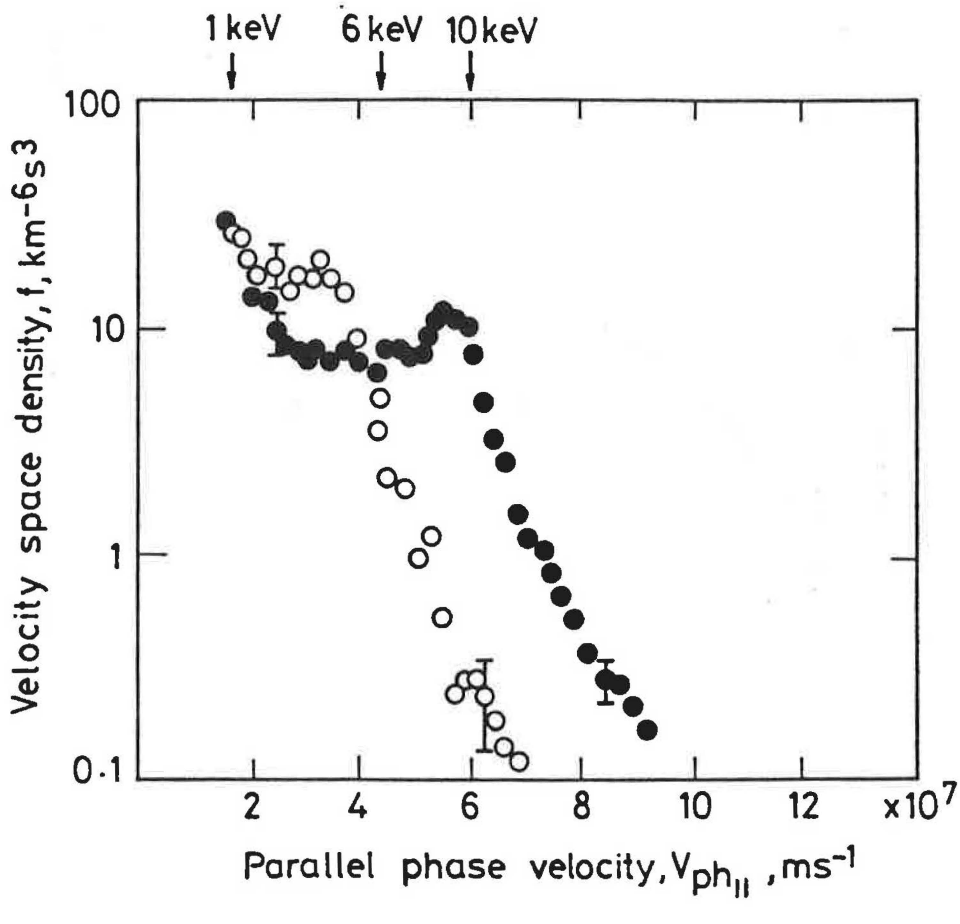

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10 keV
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10
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12
x10^7
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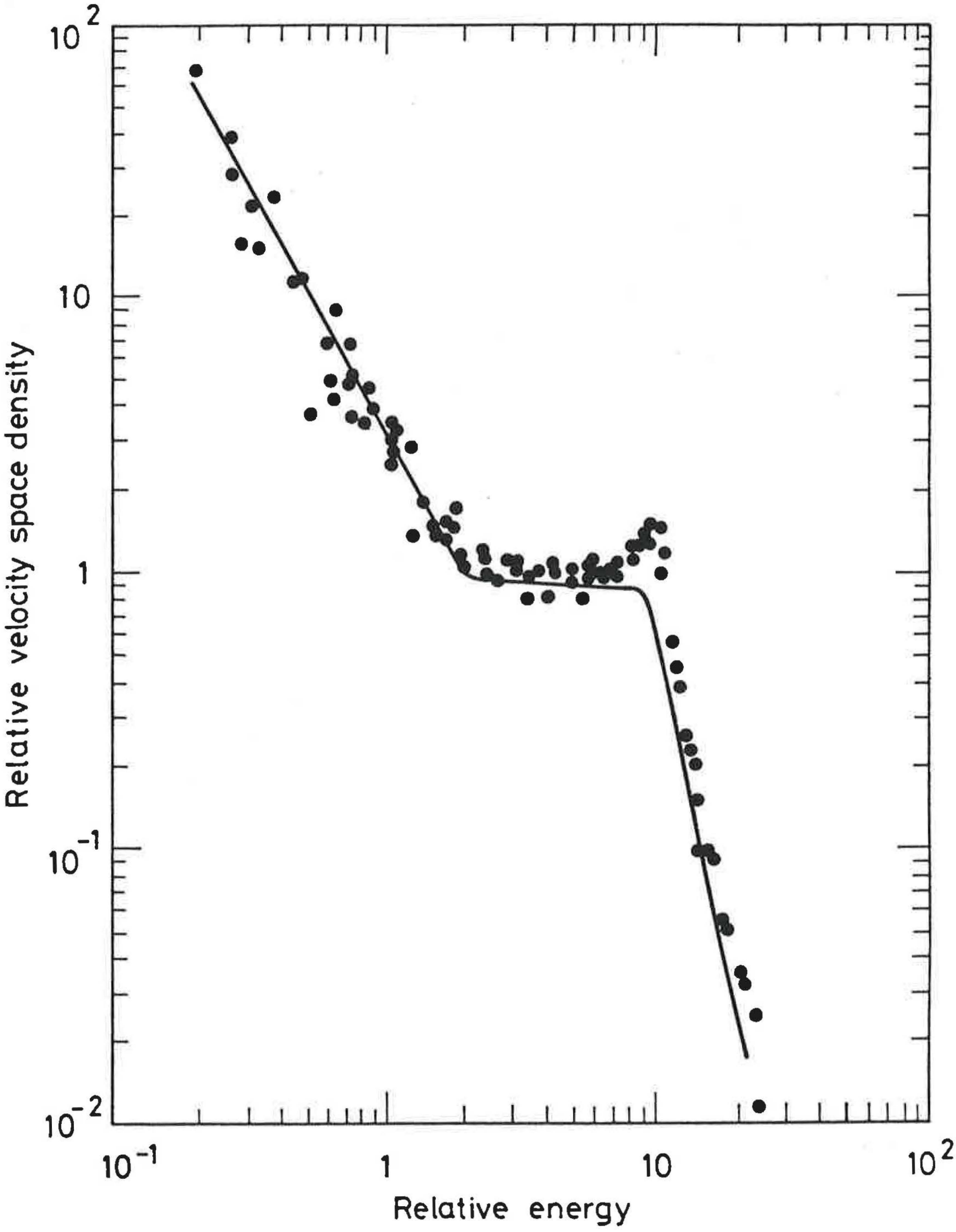

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10^{-1}
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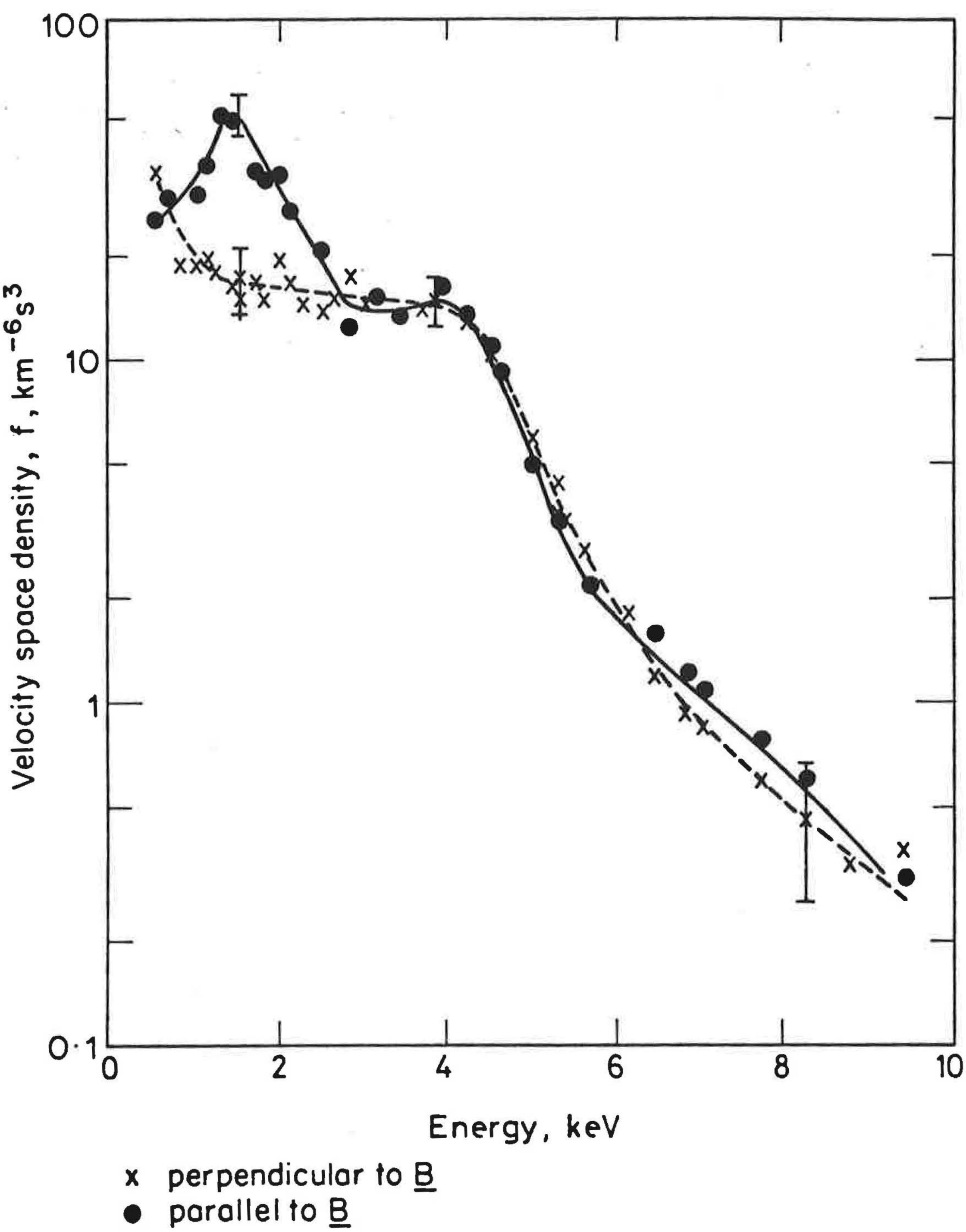
Relative velocity space density
10^{-1}
1
10
10^2
Relative energy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Energy, keV
x perpendicular to B
● parallel to B