MATCHING WITH SPACE CHARGE

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Abstract

This paper explores the possibility of performing matching in the presence of space charge to an acceptable and useful level. Space charge gives rise to a mismatch for beams at low energies. This mismatch can be very harmful for certain applications, for example the tomography diagnostic of the PITZ2 test line. In this case, the Twiss parameters at the start of the tomography section have to be as close as possible to the design ones. As can be shown by a thin lens approximation, all the Twiss parameters at the start of the tomography section are fully determined, as is the quadrupole strength, once the length of the FODO cells is chosen. With the presence of space charge it is necessary to introduce a modification to the original matching, itself performed with a standard optimizing routine. The idea is that this modification can only compensate for the linear part of space charge and it does so by changing the quadrupole strengths. The theory is verified by using an very simple test line consisting of just two quadrupoles and modeling it using GPT (General Particle Tracer). This results in modified values for the quadrupole strengths to accommodate the effect of space charge.

INTRODUCTION

The mismatch due to space charge can be very harmful for certain applications, for example the PITZ2 tomography diagnostics line [1, 2, 3]. By matching with space charge we mean introducing a modification to the original matching performed with for example, MAD8. The idea is that this modification can only compensate for the linear part of space charge and it does so by changing the quadrupole strengths. The article is divided into two, the first part explains the theoretical aspects of the problem and the second looks at practical examples of matching with space charge. In the second part, the matching is done with the GPT [4] tracking program as this is one of the few programs with which it is possible to do matching with space charge. Two examples are considered, one is an arbitrary model consisting of two quadrupoles separated by a drift and the second is the PITZ2 diagnostic line where as this technique has been applied as shown in [5].

SPACE CHARGE COMPENSATION

In thin lens approximation, if we consider two quadrupoles with focusing given by q_1 and q_2 ($q_i = \frac{1}{f_i}$, $i = $

1, 2) separated by a drift of length L_d, we have

$$\begin{pmatrix} 1 + q_2 L_d & L_d \\ 1 + q_1 + q_2 + q_1 q_2 L_d & 1 + q_1 L_d \end{pmatrix}.$$

The effect of the linear component of the space charge force may be written as

$$\begin{pmatrix} 1 & 0 & & \\ q_{sc} & 1 & & \\ & & 1 & 0 \\ & & q_{sc} & 1 \end{pmatrix}$$

which, as mentioned earlier, is defocusing in both planes. In order to try and prevent a mismatch, we wish to try to compensate for this effect by increasing the quadrupole strengths according to $q_i \to \tilde{q}_i = q_i + q_{isc}, i = 1, 2$. The focusing strength of this new system is (given by R_{21} and R_{43})

$$1 \pm q_1 \pm q_2 + L_d q_1 q_2 \pm q_{1sc} \pm q_{2sc}$$
$$+ L_d q_{1sc} q_2 + L_d q_{2sc} q_1 + L_d q_{1sc} q_{2sc}$$

where the sign depends on the plane and the extra terms should be used to compensate $q_{sc} = \frac{1}{f_{sc}}$ in both planes simultaneously. Hence we want to solve the following two equations simultaneously

$$\pm q_{1sc} \pm q_{2sc} + L_d q_{1sc} q_2 + L_d q_{2sc} q_1 + L_d q_{1sc} q_{2sc} + q_{sc} = 0$$

Subtracting the two equations gives $q_{1sc} = -q_{2sc}$ as expected and both equations reduce to

$$q_{1sc}^2 + (q_1 - q_2) q_{1sc} - \frac{q_{sc}}{L_d} = o$$

which has the solution

$$q_{1sc} = \frac{-(q_1 - q_2) \pm \sqrt{(q_1 - q_2)^2 + \frac{4q_{sc}}{L_d}}}{2} = -q_{2sc}.$$

Hence a compensation scheme is indeed possible theoretically (more details can be found in [5]) and we now look at how this is done in practice.

There are several programs which support matching optimisation routines in the presence of space charge. One of the most widely used ones is GPT. We can use this program in order to re-match several transfer lines in the presence of space charge. In all cases considered we shall see that the mismatch due to space charge may be removed completely provided the flow in the electron bunches is essentially laminar.

For a coasting beam with an elliptical beam cross section, the electric field inside the beam ($x^2/a^2 + y^2/b^2 < 1$) is given by

$$E_x(x,y) = \frac{4I}{\nu}\frac{x}{a(a+b)}, \quad E_y(x,y) = \frac{4I}{\nu}\frac{y}{b(a+b)}$$

where a and b are the transverse beams sizes, $I = I(s)$ is the current, and $\nu = \beta c$ the longitudinal velocity. If we assume that the average kinetic energy of the transverse motion or temperature of the beam is much less than the electrostatic potential energy, then we have a laminar beam and we can approximate the linear space charge effect to be given by a defocusing (in both x and y planes) quadrupole whose strength is given by

$$\frac{1}{F_x} = -\frac{4I(s)}{(\beta\gamma)^3 I_0}\frac{L}{a(a+b)}$$

with a similar expression for the vertical plane and where s is the longitudinal coordinate. The precise condition for laminarity may be derived from the Kapchinsky-Vladimirky (KV) envelope equations of motion which for a circular beam with $a = b$ imply

$$\frac{\epsilon}{\beta_{x,y}} \ll \frac{2I}{(\beta\gamma)^3 I_0}$$

where ϵ is the un-normalized transverse emittance, $\beta_{x,y}$ the beta function in either the x or y plane and γ is the relativistic factor. For details of the above derivations the reader is referred to [6] and [7]. From the above we can see that the beam parameters are crucial in determining the presence of a mismatch for a given charge. The simple model below illustrates that it is not always possible to eliminate a space charge-induced mismatch.

MODEL TRANSFER LINE

For illustration purposes, we have chosen a simple model consisting of a transfer line made up of two quadrupoles separated by a drift. The length of the transfer line is 1.2 m and the quadrupoles have length 10 cm and 20 cm and are located at 20 cm and 50 cm respectively. The requirements of the model, from the point of view of matching, is that there be a waist in both transverse planes at 1.0 m. This waist is shown in Fig. 1 in the absence of space charge. The initial distribution chosen for this model is a 1 nC bunch at 6 m from the gun of the PITZ2 diagnostic line [1] at 32 MeV. The reasons for this choice shall become apparent later. The bunch length is 6 mm (rms) and the transverse dimensions vary according to the desired quadrupole focusing in the line.

Now, the same transfer line was investigated in the presence of space charge and at various charges ranging from 1 to 20 nC. In order to achieve all these different charges, the charge of the initial distribution was scaled manually. The results without doing *any* kind of rematch are shown (i.e. with the quadrupole strengths kept the same irrespective of
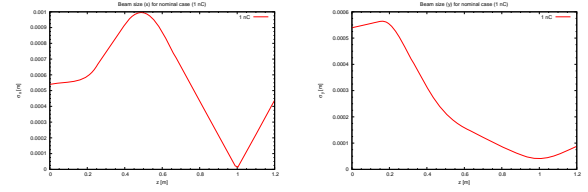

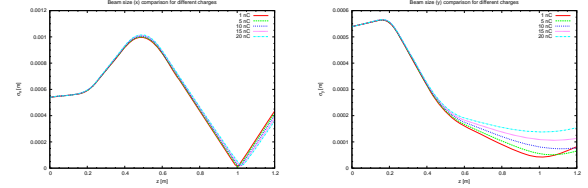Figure 1: Waist of simple model (horizontal and vertical) in the absence of space charge.

Figure 2: Waist of simple model in both planes for charges ranging from 1 to 20 nC with space charge and no compensation (quadrupole strengths identical to original match).

bunch charge) in Fig. 2. The waist is now no longer precisely at 1 m as can be seen from the horizontal plot of Fig. 2 and, even worse, it is almost no longer a waist in the vertical plane. Hence we clearly see that space charge induces a mismatch in the beam. The optimisation was therefore repeated, in the presence of space charge this time, and the results are shown in Fig. 3. Note how the waist is now
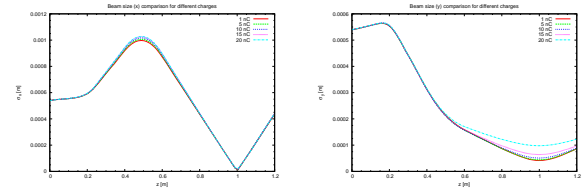

Figure 3: Waist of simple model in both planes for charges ranging from 1 to 20 nC with space charge and compensation.

back in the exact desired location in all cases apart from those with the highest charges (≥ 10 nC). This is because the space charge effect is so strong at those charges that it leads to a loss of laminarity in the bunch. This should be borne in mind in all cases, because it is only the dimensions of the bunch which ultimately dictate the effect of space charge. This may be seen from the rough formula due to Vinokurov used to calculate the defocusing strength due to space charge.

PITZ TOMOGRAPHY LINE

As a second example, we look at the PITZ2 tomography diagnostic line discussed in [8]. The original match for the

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PITZ2 tomography diagnostic line without space charge, as optimised using MAD8, is shown in Fig. 4 using the same initial distribution and a 1 nC charge.. However, when the
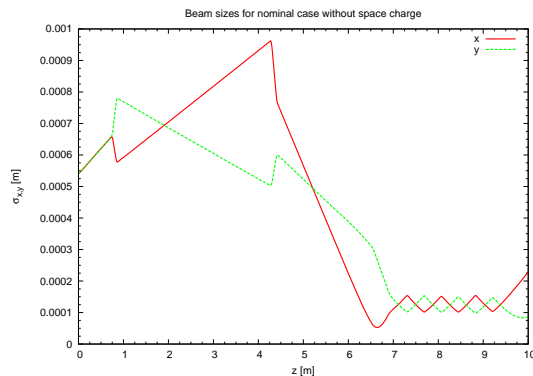

Figure 4: Beam sizes for the PITZ2 test line without space charge for a 1 nC bunch charge.

same line is considered in the presence of space charge, there is a considerable mismatch at the start of the tomography section. This mismatch, illustrated in Fig. 5, renders any tomography measurements very difficult if not useless. Fortunately, it is possible to rectify this mismatch by using
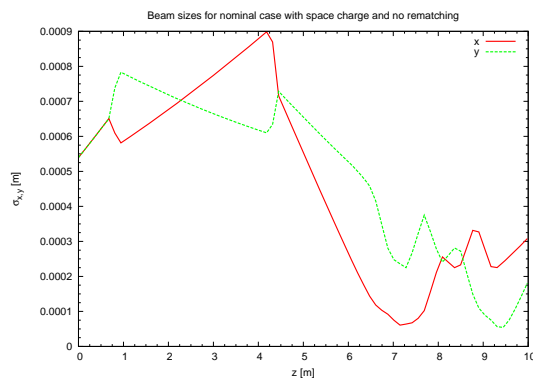

Figure 5: Beam sizes for the PITZ2 test line with space charge before entering the tomography diagnostic for a 1 nC bunch charge.

GPT to do some additional matching as shown in Fig. 6. Note that the results shown in Fig. 6 show the effect of space charge compensation in the matching section prior to the tomography module, not within the tomography module itself, as this was already dealt with in [2].

CONCLUSIONS

The effect of a space charge-induced mismatch in a diagnostic line was looked at and it was shown that, at least in some cases, this can be elliminated entirely. An initial theoretical approach shows that in order to compensate for the defocusing force due to space charge, one must increase the absolute value of the quadrupole strength.
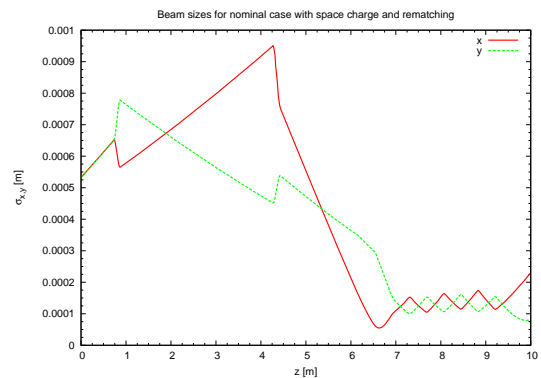

Figure 6: Beam sizes for the PITZ2 test line with the space charge-induced mismatch corrected for a 1 nC bunch charge.

This result was then compared to that produced by using GPT to correct for the influence of space charge in a simple model. In this case it was found that the mismatch may be removed by increasing the quadrupole strength. However, this is limited by the laminarity of the beam which has to be preserved. As shown, this condition is not straightforward to fulfil and, depending on beam parameters, it is quite possible that laminarity is lost. Nevertheless, the method was successfully applied to completely eliminate the space charge-induced mismatch in the PITZ2 diagnostic line and the resulting quadrupole strengths, while slightly different, remained within the specification of the magnets.

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