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1 **Measurement and calculation of decay heat** 2 **in ISIS spallation neutron target**

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5 **Abstract**

6 Thermal powers from radioactive decays ('decay heat') within a proton-driven
7 neutron-producing tungsten target on the ISIS Spallation Neutron Source have been
8 measured. Very good agreement is found with calculations using the Monte Carlo
9 code MCNPX.

10 **1. Introduction**

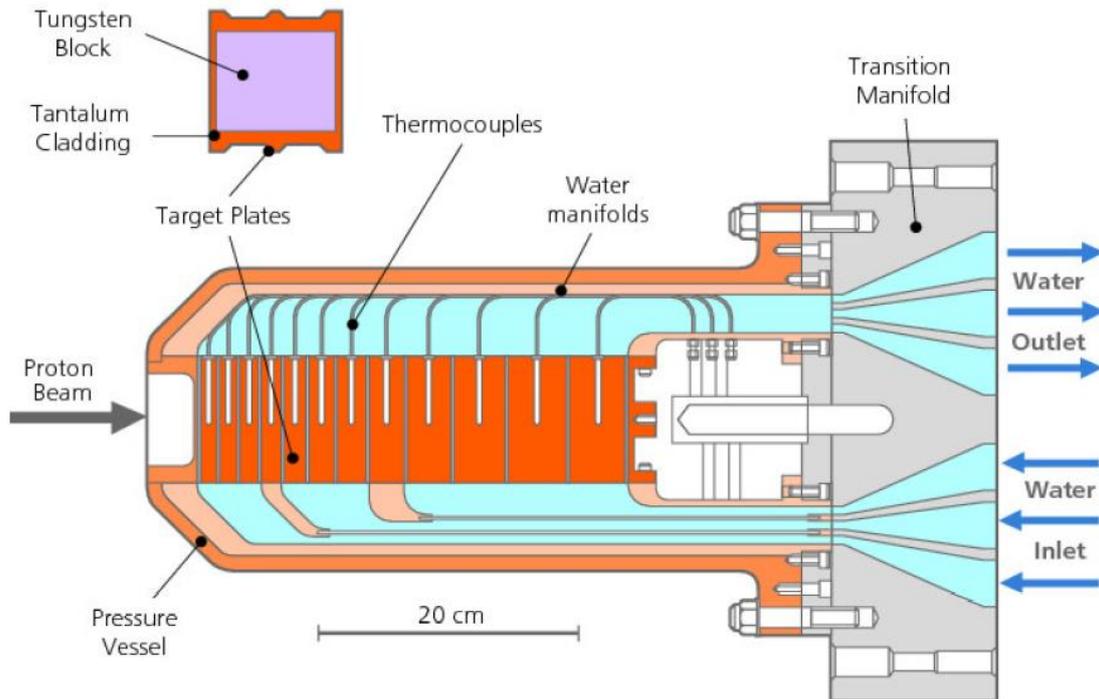
11 In highly irradiated targets on high-power particle accelerators it is often necessary to
12 know what the thermal power from radioactive decays ('decay heat') within the target
13 is once the particle beam from the accelerator has been switched off. Usually, decay
14 heats are calculated by Monte Carlo computer codes such as MCNPX [1], since decay
15 heats are not always easy to measure. In principle, decay heat may be measured
16 absolutely by calorimetry once a target has been removed from its operational
17 location, but on a working accelerator facility this may not be practical.

18 In the present publication, two measurements of the decay heat in the tungsten target
19 in the TS-1 target station on the ISIS Spallation Neutron Source are presented and
20 compared with Monte Carlo calculations using MCNPX.

21 **2. The ISIS TS-1 target and decay heat measurements**

22 The ISIS TS-1 target is a twelve-plate tantalum-clad tungsten target, irradiated by a
23 40-pulses-per-second (pps) 800-MeV $\sim 180\text{-}\mu\text{A}$ proton beam from the ISIS
24 synchrotron, and cooled by ~ 500 litres/minute of heavy water. During irradiation the
25 thermal power dissipated within the target is ~ 100 kW, and the target is instrumented
26 with thermocouples measuring the temperature of each plate. A schematic diagram of
27 the target is shown in Fig. 1.

28 The decay heat measurements were made essentially by switching off both the proton
29 beam and the flow of cooling water after a long irradiation, and recording the time
30 profiles of the thermocouple temperatures. Two such measurements were made, on
31 30 November 2016 and on 26 March 2018, in both cases at the ends of ~ 30 -day
32 irradiation campaigns. The temperature data are shown in Figs. 2a and 2b. The
33 vertical range spanned by the data in Fig. 2a is greater than the vertical range in
34 Fig. 2b because in Fig. 2a the water flow through the target was switched off
35 36 seconds after the beam was switched off, whereas in Fig. 2b the water flow and
36 beam were switched off at the same time; consequently in Fig. 2a there was more
37 time than in Fig. 2b for the bulk of the target to cool down before temperature rises
38 due to decay heat in the absence of water flow became evident.



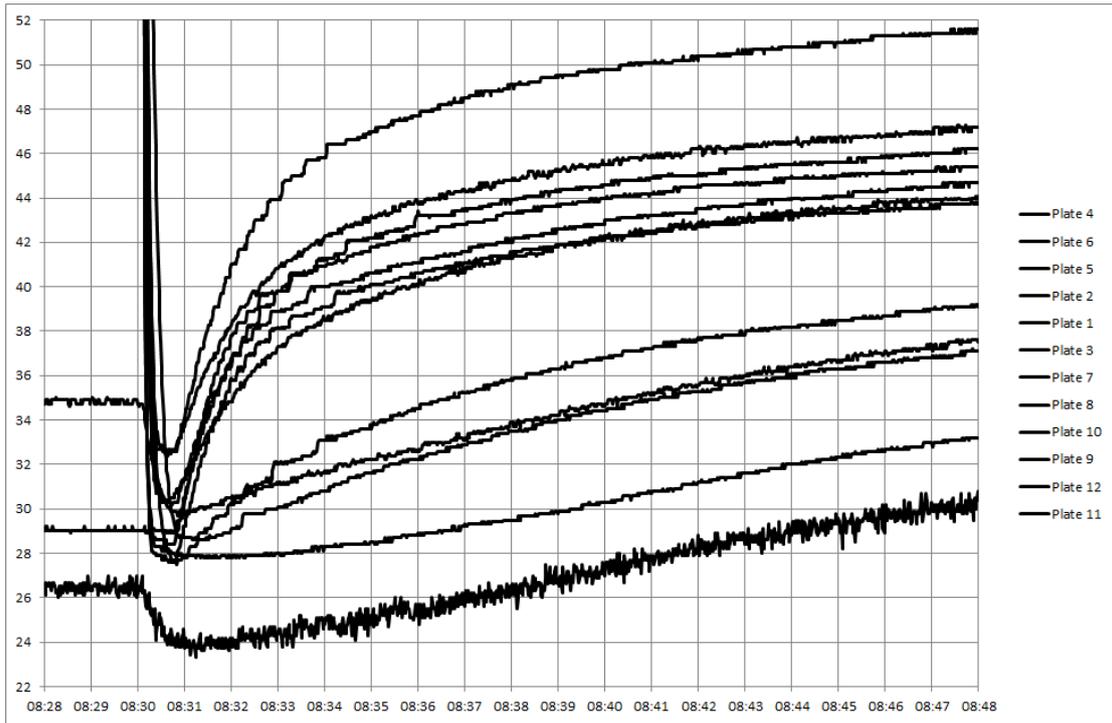
39

40 Fig. 1. Schematic diagram of ISIS TS-1 target. Along the direction of the incident beam
 41 dimensions are as follows: the thicknesses of tungsten in the target plates are 11.0, 11.0, 12.0,
 42 13.5, 15.0, 18.0, 21.0, 26.0, 34.0, 40.0, 46.0 and 46.0 mm; each plate has 2.0 mm of tantalum
 43 cladding on each side; and each plate is separated from its neighbours by 2.0 mm of water.
 44 The pressure vessel fits closely over the structures containing and separating the channels
 45 through which the cooling water flows.

46 3. Analysis

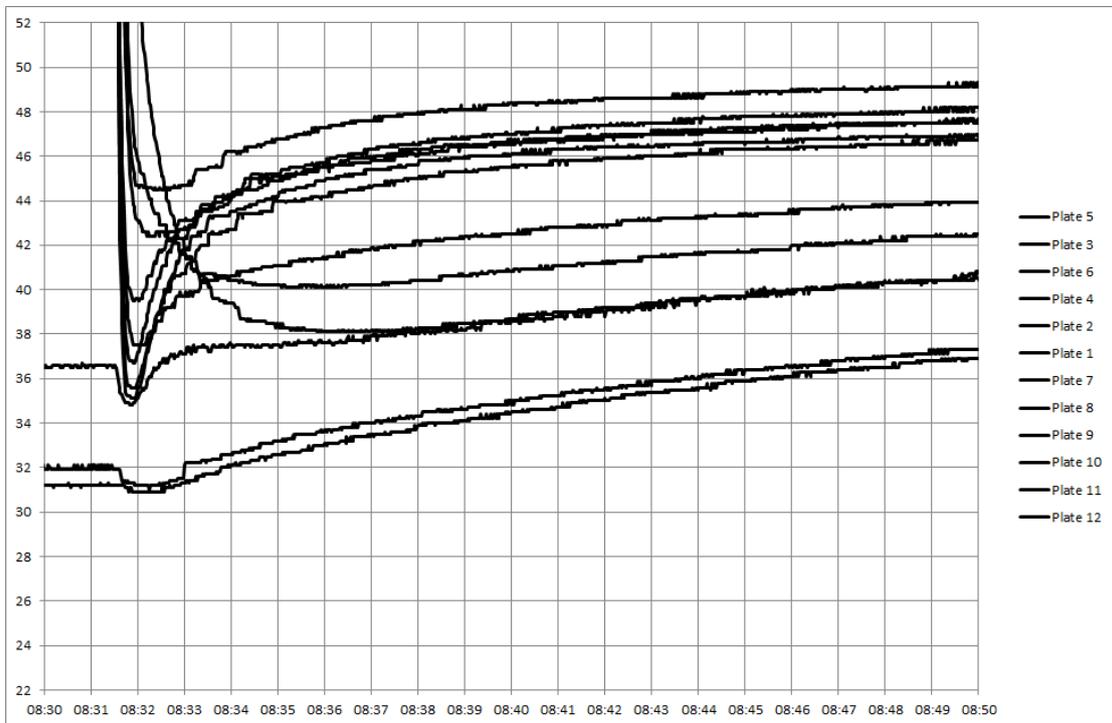
47 The simplest model for each of the target plates is that of an isolated mass m with
 48 specific heat c heated internally at a rate \dot{Q} and subject to Newton's Law of Cooling
 49 whereby heat flows out of the mass at a rate proportional to the difference between
 50 the temperature T of the mass and the (constant) temperature T_s of the surroundings.
 51 In such a case the temperature T is described by
 52 $T = T_s + (\dot{Q}/\alpha)(1 - \exp(-(\alpha/mc) t))$ where α is the constant of proportionality in
 53 the law of cooling, whereupon by fitting the expression $T = a + b(1 - \exp(-t/d))$
 54 to the data with a , b and d as three free parameters the internal rate of generation of
 55 heat \dot{Q} may be obtained as $\dot{Q} = mc b/d$. However, when such a function is fitted to
 56 the temperature data shown in Fig. 2 it soon becomes evident that whilst a reasonably
 57 good fit may be obtained for the first few tens of seconds after the water flow has
 58 been switched off, the function cannot fit the data at longer times; and the reason is,
 59 essentially, because there is more than one time constant present in the data.

60



61

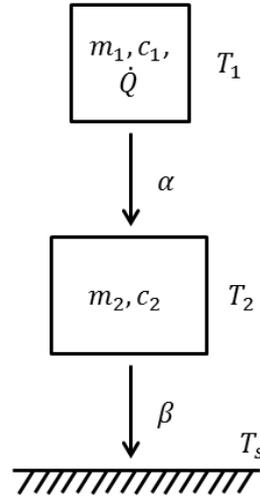
62 Fig. 2a. 30 November 2016 data. Target plate temperatures (°C) in the TS-1 target as a function of time when the proton beam and water flows were switched off; beam off,
 63 08:30:04; water off, 08:30:40. The order of the curves at 08:35 is the same as the list of plate
 64 numbers at the right-hand side.
 65



66

67 Fig. 2b. 26 March 2018 data. Target plate temperatures (°C) in the TS-1 target as a function
 68 of time when the proton beam and water flows were switched off simultaneously; beam and
 69 water off at 08:31:28. The order of the curves at 08:35 is the same as the list of plate
 70 numbers at the right-hand side. Same horizontal and vertical ranges as Fig. 2a.

71 A more realistic model is a ‘two-mass’ model, wherein a target plate at temperature T_1
 72 with mass m_1 , specific heat c_1 and internal heat source \dot{Q} is assumed to lose heat at a
 73 rate $\alpha(T_1 - T_2)$ to a surrounding assembly at temperature T_2 with mass m_2 and
 74 specific heat c_2 , and in turn this assembly is assumed to lose heat at a rate $\beta(T_2 - T_s)$
 75 to a thermal sink at temperature T_s , as shown diagrammatically in Fig. 3.



76

77 Fig. 3. Diagrammatic representation of two-mass model. Mass m_1 represents a target plate,
 78 mass m_2 represents the local surroundings (*e.g.* pressure vessel and associated flanges and
 79 manifolds), and the thermal sink represents the more distant surroundings.

80 For mass m_1 , $\dot{Q} = m_1 c_1 dT_1/dt + \alpha(T_1 - T_2)$,
 81 and for mass m_2 , $\alpha(T_1 - T_2) = m_2 c_2 dT_2/dt + \beta(T_2 - T_s)$.
 82 By re-arranging the first equation to give an equation for T_2 and then substituting into
 83 the second equation, the result is $d^2 T_1/dt^2 + a_1 dT_1/dt + a_2 T_1 = C$ where
 84 $a_1 = (\alpha m_2 c_2 + (\alpha + \beta) m_1 c_1) / (m_1 c_1 m_2 c_2)$, $a_2 = \alpha \beta / (m_1 c_1 m_2 c_2)$,
 85 $C = a_2 T_s + b \dot{Q}$, and $b = (\alpha + \beta) / (m_1 c_1 m_2 c_2)$. Using the machinery of the
 86 Laplace transform, the subsidiary equation corresponding to the second-order
 87 differential equation is, since C is constant,

88 $(p^2 + a_1 p + a_2) \bar{T}_1 = C/p + p T_1^{(0)} + \dot{T}_1^{(0)} + a_1 T_1^{(0)}$ whereupon by re-arranging as
 89 $\bar{T}_1 = (p^2 T_1^{(0)} + p(\dot{T}_1^{(0)} + a_1 T_1^{(0)}) + C) / (p(p^2 + a_1 p + a_2))$ to give the Laplace
 90 transform $\bar{T}_1 = \bar{T}_1(p) = \int_0^\infty \exp(-pt) T_1(t) dt$ of the temperature $T_1(t)$, re-writing
 91 in terms of partial fractions, completing the square in the quadratic term in the
 92 denominator, and then using the shifting theorem and the inverse transforms of the
 93 cosh and sinh functions, or by simply looking up a table of inverse transforms
 94 (*e.g.* [2]), the solution is

98 $T_1 = C/a_2$
 99 $+ (T_1^{(0)} - C/a_2 + (\dot{T}_1^{(0)} + (a_1/2)(T_1^{(0)} - C/a_2))/a_{12}) \exp((-a_1/2 + a_{12})t)/2$
 100 $+ (T_1^{(0)} - C/a_2 - (\dot{T}_1^{(0)} + (a_1/2)(T_1^{(0)} - C/a_2))/a_{12}) \exp((-a_1/2 - a_{12})t)/2$

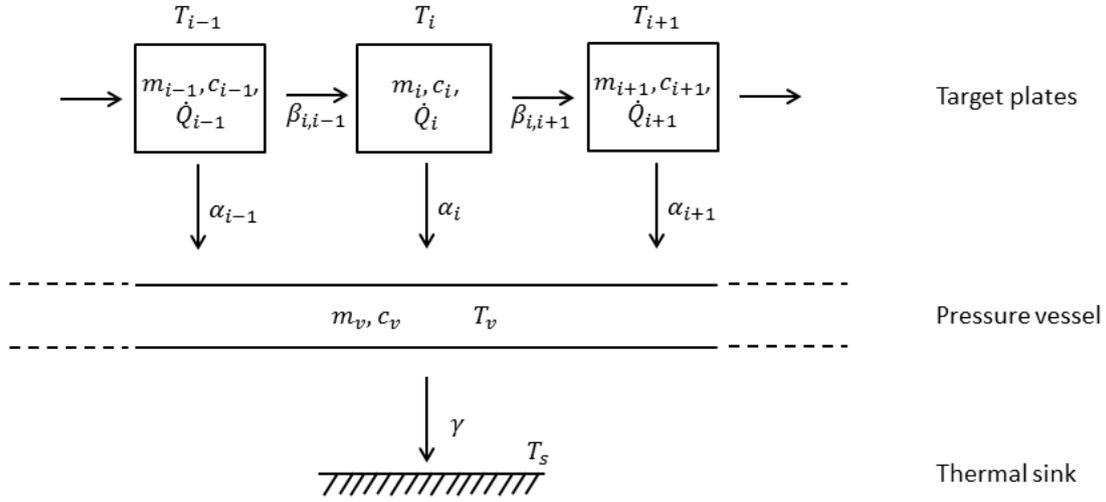
95 where $a_{12} = \sqrt{(a_1^2/4) - a_2}$, and $T_1^{(0)}$ and $\dot{T}_1^{(0)}$ are the initial value of T_1 and the

96 initial value of the rate of change of T_1 respectively. Two time constants

97 $1/((a_1/2) - a_{12})$ and $1/((a_1/2) + a_{12})$ are now evident.

101

102 However, it is obvious that in reality the target plates are not thermally isolated, and
 103 so the simple two-mass model described above was extended to take into account heat
 104 flow between plates as shown in Fig. 4.



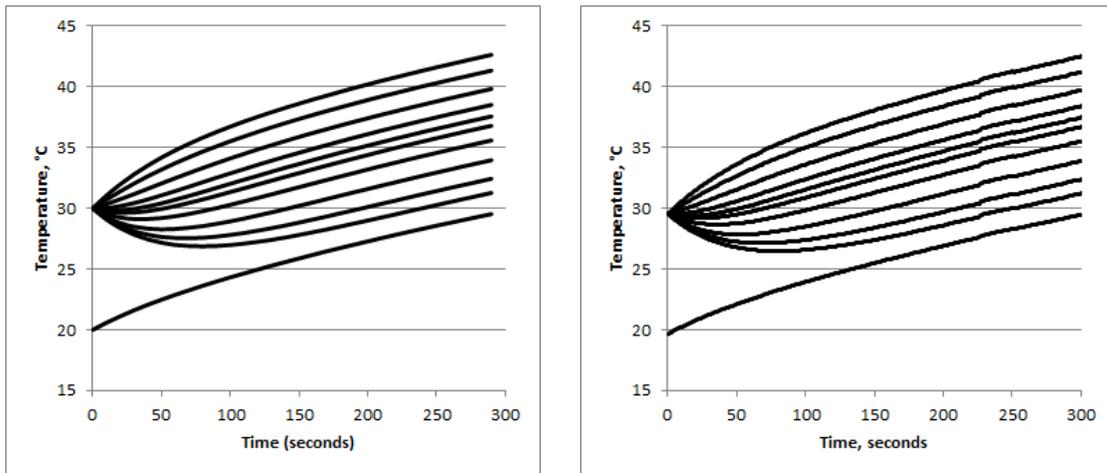
105
 106 Fig. 4. Extension of the two-mass model to take into account heat transfer between plates.

107 This extended two-mass model is described by the following set of $n + 1$ equations
 108 for n plates:

$$\begin{aligned}
 109 \quad & \dot{Q}_1 = m_1 c_1 \dot{T}_1 + \beta_{1,2}(T_1 - T_2) + \alpha_1(T_1 - T_v) \\
 110 \quad & \beta_{1,2}(T_1 - T_2) + \dot{Q}_2 = m_2 c_2 \dot{T}_2 + \beta_{2,3}(T_2 - T_3) + \alpha_2(T_2 - T_v) \\
 111 \quad & \vdots \\
 112 \quad & \beta_{i-1,i}(T_{i-1} - T_i) + \dot{Q}_i = m_i c_i \dot{T}_i + \beta_{i,i+1}(T_i - T_{i+1}) + \alpha_i(T_i - T_v) \\
 113 \quad & \vdots \\
 114 \quad & \beta_{n-2,n-1}(T_{n-2} - T_{n-1}) + \dot{Q}_{n-1} \\
 115 \quad & = m_{n-1} c_{n-1} \dot{T}_{n-1} + \beta_{n-1,n}(T_{n-1} - T_n) + \alpha_{n-1}(T_{n-1} - T_v) \\
 116 \quad & \beta_{n-1,n}(T_{n-1} - T_n) + \dot{Q}_n = m_n c_n \dot{T}_n + \alpha_n(T_n - T_v) \\
 117 \quad & \\
 118 \quad & \sum_{i=1}^n \alpha_i(T_i - T_v) = m_v c_v \dot{T}_v + \gamma(T_v - T_0)
 \end{aligned}$$

119 where the α 's represent thermal conductances between the plates and the pressure
 120 vessel, the β 's represent thermal conductances between pairs of plates, γ represents
 121 the thermal conductance between the pressure vessel and the surroundings or thermal
 122 sink s , and subscripts i and v refer to plate number and pressure vessel respectively.
 123 The solutions of this set of $n + 1$ coupled first-order linear differential equations can
 124 now be fitted to the plate temperature data as functions of time with the aim of
 125 extracting parameters of the fit, especially the decay heats \dot{Q}_i .
 126

127 The set of $n + 1$ differential equations was solved by DC04 from the Harwell
 128 Subroutine Library (HSL) [3]. But as a check, the equations for a 10-plate test case
 129 were also solved by the method of Laplace transforms — taking the transforms of the
 130 set of $n + 1$ equations above, solving for the resultant temperature transforms \bar{T}_i 's and
 131 \bar{T}_v using the HSL linear algebra routine ME05, and then numerically evaluating the
 132 inverse transform $T_i(t) = \int_{\gamma-i\infty}^{\gamma+i\infty} \exp(pt) \bar{T}_i(p) dp / 2\pi i$. A comparison of the two
 133 methods of solution for the 10-plate test case is shown in Fig. 5; the agreement is
 134 excellent, although a little numerical noise from the inverse Laplace transform is
 135 evident.



136

137 Fig. 5. Time development of plate temperatures by direct solution of coupled differential
 138 equations (left-hand side) and by Laplace transforms (right-hand side) (for which γ and ' ∞ '
 139 in the equation for the inverse transform were taken as 0.01 and 400 respectively). For both,
 140 the order of curves from top to bottom is plate 1, plate 2, plate 3, ... , plate 10, vessel. The
 141 parameters assumed were: m 's and c 's as in Table 1, $\beta = 5 \text{ W } ^\circ\text{C}^{-1}$ for all plates,
 142 $\gamma = 50 \text{ W } ^\circ\text{C}^{-1}$, $m_v = 50000 \text{ g}$, $T_v^{(0)} = 20^\circ\text{C}$, $T_s = 20^\circ\text{C}$, $T^{(0)} = 30^\circ\text{C}$ for all plates, $\alpha = 11,$
 143 $12, 13, \dots, 20 \text{ W } ^\circ\text{C}^{-1}$ for plates 1–10, and $\dot{Q} = 162, 154, 145, 137, 134, 135, 124, 102, 82,$
 144 and 62 W for plates 1–10 (these \dot{Q} 's were taken from preliminary Monte Carlo calculations).
 145 Some of the curves initially dip downwards because the internal heat is insufficient to prevent
 146 an initial cooling from the starting temperature.

147 Solutions to the set of $n + 1$ coupled first-order linear differential equations solved
 148 using DC04 were fitted to the target plate temperature data using the HSL
 149 minimisation routine VA04; plate numbers 11 and 12 were excluded because it was
 150 reasonably clear from Fig. 2 that decay heat within these two plates is insignificant. It
 151 was assumed that all the β 's had the same value. Parameters fitted were β , γ , m_v ,
 152 $T_v^{(0)}$, T_s , $T_1^{(0)}$, $T_2^{(0)}$, \dots , $T_{10}^{(0)}$, α_1 , α_2 , \dots , α_{10} , \dot{Q}_1 , \dot{Q}_2 , \dots , \dot{Q}_{10} , a total of
 153 35 parameters, and the function minimised was $\sum_{j,i} (T_{j,i}^{\text{data}} - T_{j,i}^{\text{fit}})^2 / \delta T_{j,i}^2$ where the
 154 sum is taken over temperature datum points j for plate numbers i and $\delta T_{j,i}$ was taken
 155 as 0.1°C for all j and i since the temperature data were recorded with a resolution of
 156 0.1°C . The temperature data for all ten plates were fitted simultaneously, although for
 157 a given i the only $T^{(0)}$, α and \dot{Q} parameters allowed to vary were $T_i^{(0)}$, α_i and \dot{Q}_i .
 158 Fixed parameters are given in Table 1. Since the Monte Carlo calculations described
 159 in Sect. 4 showed that during the 10 minutes immediately after irradiation ceased the

160 decay heat varied by less than 20%, it was assumed that over the 10-minute fitting
 161 interval the decay heat was constant. Results for $\dot{Q}_{\text{tot}} = \sum_i \dot{Q}_i$ after several iterations
 162 of VA04 are given in Table 2, and the uncertainties were obtained by repeatedly
 163 perturbing all the data points by normally distributed random numbers (from the HSL
 164 routine FA05) matched to the uncertainties in the data points and refitting, and then
 165 taking the standard deviations of the resultant sets of ‘perturbed’ values of \dot{Q}_{tot} . The
 166 greater uncertainty in \dot{Q}_{tot} for the 26 Mar. 2018 data is essentially because the
 167 (vertical) temperature range spanned by these data is less than it is for the
 168 30 Nov. 2016 data. A typical fit is shown in Fig. 6.

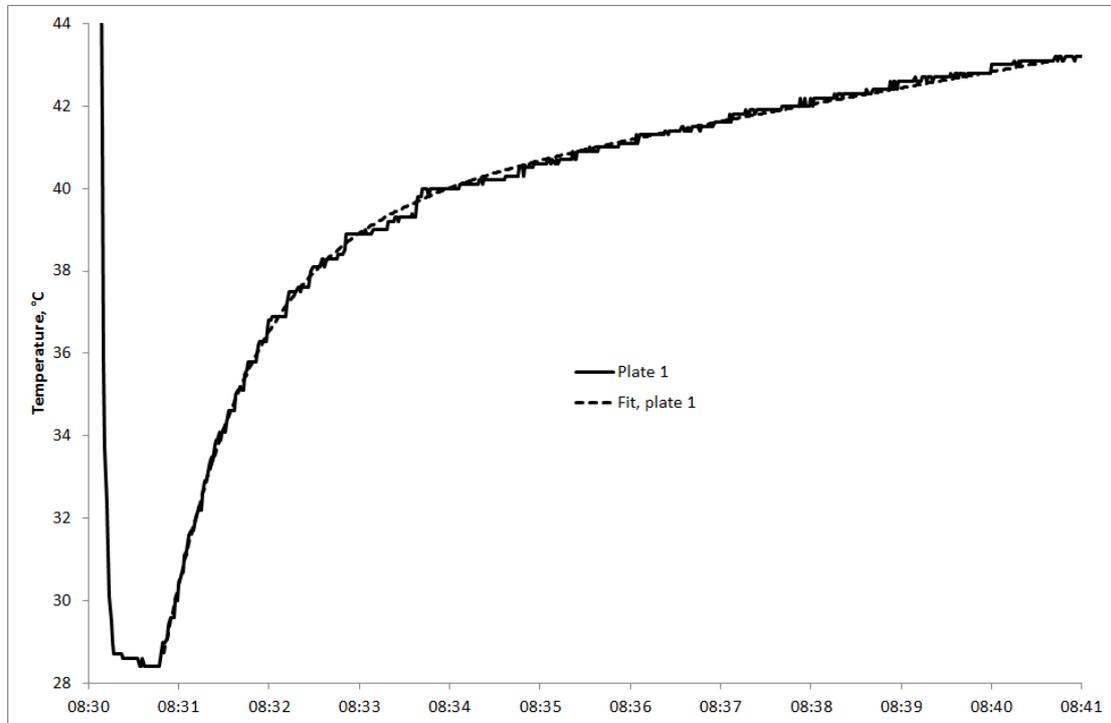
169 Whilst the number of parameters involved in each of the two fits, 35, is undoubtedly
 170 an unusually large number, it should be remembered that the temperature curve for
 171 plate i is fitted only by the three parameters $T_i^{(0)}$, α_i and \dot{Q}_i and by a tenth-share in the
 172 five parameters β , γ , m_v , $T_v^{(0)}$ and T_s , and so effectively each of the ten temperature
 173 curves is fitted by only ‘3½’ parameters, a much more modest number.

	m, g	$c, \text{J g}^{-1} \text{ } ^\circ\text{C}^{-1}$
Plate 1	3350	0.1366
Plate 2	3350	0.1366
Plate 3	3570	0.1365
Plate 4	3900	0.1364
Plate 5	4240	0.1363
Plate 6	4900	0.1362
Plate 7	5570	0.1360
Plate 8	6680	0.1359
Plate 9	8450	0.1358
Plate 10	9780	0.1357
Vessel		0.5

174 Table 1. Values of fixed parameters used in fitting the target plate temperature data. Specific
 175 heats c vary very slightly with plate numbers because c_W is slightly smaller than c_{T_a} and the
 176 tungsten plates become thicker towards the back of the target whereas the thickness of
 177 tantalum cladding remains the same. The pressure vessel is stainless steel, hence the different
 178 specific heat.
 179

Date	$\epsilon_{\text{rms}}, ^\circ\text{C}$	$\dot{Q}_{\text{tot}} = \sum_i \dot{Q}_i$, watts	
		Measured	Calculated
30 Nov. 2016	0.122	1220 ± 90	1130 ± 230
26 Mar. 2018	0.117	1350 ± 240	1320 ± 260

180 Table 2. Decay heats deduced from the two sets of target plate temperature measurements.
181 ϵ_{rms} is the root-mean-square value of the average deviations between fit and data. The
182 calculated values are from the Monte Carlo computer code MCNPX, and are discussed in
183 Sect. 4. Averaged-over-ten-plates values of the correlation coefficients amongst the fitted
184 $T^{(0)}$, α and \dot{Q} parameters were, for the 30 Nov. 2016 data, 0.48, -0.40 and -0.63 , and, for the
185 26 Mar. 2018 data, 0.94, -0.07 and -0.08 for \dot{Q} - α , \dot{Q} - $T^{(0)}$ and α - $T^{(0)}$ respectively; the signs
186 of these correlation coefficients are as expected, since, for example, for a given temperature, a
187 higher value of \dot{Q}_i can be partly compensated for by a higher value of α_i . Note that the
188 uncertainties on the calculated decay heats are likely to be almost entirely systematic, and are
189 therefore highly correlated.



190

191 Fig. 6. Fit to data, plate 1, 30 Nov. 2016. The root-mean-square deviation
192 between the fit and the data is 0.12°C .

193 4. Comparison with Monte Carlo calculations

194 Calculations were carried out using the MCNPX Monte Carlo code [1] in association
195 with the CINDER'90 transmutation code [4]. A very detailed model of the ISIS TS-1
196 target-reflector-and-moderators (TRAM) assembly was used (see Section 2 in
197 reference [5]) The model had been built using CombLayer [6], a set of C++ programs
198 which requires the user to effectively write the model geometry into the C++
199 construction system; this C++ code is then compiled into the final program which,
200 after running, produces the MCNPX input file. Detailed irradiation histories (proton

201 energies¹ and beam currents as functions of time since the target was first irradiated in
 202 March 2015) were used for the calculations; the total integrated proton beam currents
 203 were 1040 and 1779 milliamp-hours for the 30 Nov. 2016 and 26 Mar. 2018
 204 measurements respectively.

205 Table 2 also gives the calculated decay heats $\dot{Q} = \sum_k A_k q_k$ (where A_k is the activity
 206 of radionuclide k and q_k is its mean decay energy) calculated using
 207 MCNPX/CINDER'90 (no further MCNPX calculations were made after the
 208 CINDER'90 calculations), and it can be seen that the calculated values agree very
 209 well with the measured values. The 20% uncertainties given for the Monte Carlo
 210 results are plausible estimates of the uncertainties in the modelling and the nuclear
 211 data.

212 It is obvious that the decay heats do not scale with integrated beam current; this is
 213 simply because much of the decay heat is due to ¹⁸²Ta produced by neutron capture on
 214 the tantalum cladding around the tungsten in the target, and the half-life of ¹⁸²Ta,
 215 115 days, is much less than the overall irradiation times. Table 3 lists the
 216 radionuclides that contribute 1% or more to the calculated decay heat immediately
 217 after irradiation ceases, and it is clear that ¹⁸²Ta dominates the list.

Radionuclide	Half-life	% contribution to overall decay heat at $t_{\text{cool}} = 0$	Predominantly produced in
¹⁸² Ta	115 d	45.80	Ta
¹⁸⁷ W	23.7 h	4.81	W
¹⁶⁸ Lu	5.50 m	2.66	W
^{183m} W	5.20 s	2.10	W
¹⁶⁶ Lu	2.65 m	1.80	W
¹⁷⁶ Ta	8.08 h	1.77	W
¹⁶⁶ Tm	7.70 h	1.58	W
¹⁷⁰ Lu	48.1 h	1.52	W
¹⁶³ Lu	3.97 m	1.12	W
¹⁷¹ Hf	12.1 h	1.00	W

218 Table 3. Radionuclides in the irradiated target ordered by contribution ($\geq 1\%$) to overall decay
 219 heat immediately after irradiation ceases (from the MCNPX/CINDER'90 calculations for
 220 26 Mar. 2018). Also given are the parts of the target (*i.e.* the tungsten 'cores' of the plates, or
 221 the tantalum cladding) within which the radionuclides are predominantly produced.

222 5. Conclusion

223 Thermal powers from radioactive decays ('decay heat') in an ISIS tungsten target
 224 have been measured by fitting a coupled two-mass model to observed temperature
 225 dependences of the target plate temperatures when both the proton beam and the
 226 cooling water flow are switched off. Decay heats deduced from the measurements
 227 agree well with decay heat calculations using the Monte Carlo computer code
 228 MCNPX.

¹ During some of the time that the target was being irradiated the synchrotron was running at 700 MeV (rather than its usual 800 MeV) in order to reduce strain on elderly lattice dipoles.

229 **References**

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