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ABSTRACT

The minimal supersymmetric Standard Model with explicit R-parity nonconservation contains bilinear terms involving the left-handed lepton superfields and the Higgs chiral multiplet with hypercharge $Y = +1$, which cannot in general be rotated away in the presence of soft-supersymmetry-breaking interactions. These bilinear lepton-number-violating terms are found to give rise to non-zero vacuum expectation values of the scalar neutrinos. This leads to nonuniversal and flavour-violating tree-level couplings of the W and Z bosons to charged leptons and neutrinos. The parameter space of this novel scenario is systematically analyzed and further restricted by a number of laboratory, astrophysical, and cosmological constraints. The possibility that our minimal model can account for the KARMEN anomaly is examined.

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1 Introduction

The minimal supersymmetric Standard Model (MSSM) with the particle content of the Standard Model (SM), including their supersymmetric (SUSY) partners, conserves the discrete quantum number R, known as R parity. Under this symmetry, the SM particles are even whereas their superpartners are odd. The quantum number of R parity may conveniently be expressed as [1–4].

$$R = (-1)^{3B+L+2S},\qquad(1.1)$$

where B, L, and S are the baryon number, the lepton number, and the spin of a particle, respectively. Evidently, nonconservation of R parity results, in general, in baryon ($\not{B}$) and lepton ($\not{L}$) number violating terms. However, the usually considered models retain only the lepton-number-violating terms in order to preserve the stability of the proton. In the presence of both, $\not{L}$ and $\not{B}$ terms, the proton decay might render the model phenomenologically not viable.

The explicit breaking of R parity in a SUSY model means also that we introduce additional coupling constants into the Lagrangian, but not additional fields, *i.e.* the field content of the model is the same as in the MSSM. This property of the model has some interesting consequences. One important implication is that the explicit lepton-number violation allows, in principle, for a mixing of the left-handed neutrinos with the gauginos and higgsinos. As a consequence, such a mixing, if present, will generate neutrino masses. This mechanism to render neutrinos massive is quite distinctive from the standard options, such as the introduction of right-handed neutrinos or the extension of the Higgs sector by adding an exotic Higgs triplet. However, to make the afore-mentioned mixing possible, one should ensure that the sneutrinos acquire vacuum expectation values (vev's). By a careful examination of the scalar potential of the R-parity broken SUSY model, one can show that the appearance of non-vanishing vev's for the scalar neutrinos is not always a direct consequence of the theory. In fact, if a term of the form $\varepsilon_i \varepsilon_{ab} \hat{L}_i^a \hat{H}_1^b$ is neglected in the superpotential of the model, where $\hat{L}_i$ are the chiral lepton fields and $\hat{H}_1$ is the super-Higgs fields with hypercharge $Y = 1$, then insistence of non-zero vev's of the sneutrino fields can lead, in some cases, to a fine-tuning relation among the original parameters of the scalar potential. For instance, this would happen if we assumed the scalar mass terms $m_{ij} L_i^\dagger L_j +$ H.c. to be diagonal in i, j. On the other hand, the retention of the ε_i terms leads unavoidably to non-zero vev's of the sneutrinos (w_i). It is also easy to see that in the presence of soft-SUSY breaking parameters, the ε_i terms cannot be rotated away by a unitary transformation. This is consistent with the observation that such a term will be

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generated radiatively, even if one chooses $\varepsilon_i = 0$ at the Planck mass scale [3]. Therefore, one could argue that models which break the R parity explicitly, in which the ε_i terms are neglected, may be considered to be not general and hence incomplete. This fact is expected to influence the analytic expressions of the mass and mixing matrices, especially when new parameters enter the theory. Indeed, we will show how the ε_i terms change the phenomenological predictions of the model under consideration.

Models with an explicit mixing term between $\hat{L}_i$ and $\hat{H}_1$ have been considered in the past [3]. More recently, such models have been re-examined by paying special attention to CP violation in the scalar potential as well as to neutrino masses [5,6]. In Refs. [3,5,6], it was assumed that all individual lepton numbers, L_i, are broken. It has been known for some time that lepton-number-violating interaction can erase the existing baryon asymmetry in the universe (BAU) if $\not{L}$ interactions are in thermal equilibrium with the $B + L$-violating sphalerons. The most stringent constraints on w_i and ε_i arise from such considerations. However, it has been realized [7] that one can evade the erasure of the BAU if we demand that one individual lepton number is conserved. No constraints on lepton-number-violating couplings will then follow. The most conservative constraints on lepton-number-violating couplings can then be inferred from an analysis of the absence of possible new-physics phenomena in various experiments.

In R-parity broken models, there are in general two sources that can produce lepton-number- and lepton-flavour-violating interactions. Those that are induced by the ε_i and w_i parameters and give rise, e.g., to tree level off-diagonal Z-boson decays, and those that are proportional to the so-called λ and λ' couplings in the superpotential. The literature pertinent to constraints on the λ and λ' couplings is immanent [8–12]. Here, we will study effects of lepton-number and/or lepton-flavour violation on the ordinary known matter, which can be induced by W- and/or Z-boson interactions in an R-parity broken model. Therefore, one may consider our study complementary to investigations of limits on the λ and λ' couplings mentioned above. In an $\not{R}$ SUSY model, direct limits on w_i are mostly obtained from upper bounds on light neutrino masses, e.g. using the laboratory bound $m_{\nu_\tau} < 31$ MeV [13]. However, we find that better limits may be deduced by the nonobservation of non-SM processes, such as the decay of a muon into three electrons. Moreover, cosmological and astrophysical implications of a massive light neutrino for our model will be discussed in Section 5.7.

The paper is organized as follows. In Section 2, we will give a detailed discussion of the R-parity broken SUSY model where the emphasis will be put on the scalar potential of the model. Further analytic results on this topic are relegated in Appendix A. Section 3 treats the mixing between the neutralinos and neutrinos as well as that between the charginos

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and charged leptons. In Section 4, we derive the W- and Z-boson interaction Lagrangians in the seesaw approximation. In Section 5, we analyze a number of low-energy processes that can be induced by the non-SM couplings present in $\not{R}$ SUSY models. We then discuss new constraints that may be derived by laboratory experiments together with constraints coming from cosmology and astrophysics. In addition, we investigate the possibility that our R-parity broken SUSY model can explain the KARMEN anomaly. Our numerical results are presented in Section 6. We draw our conclusions in Section 7.

2 The R-parity violating SUSY model

In this section, we set up our definition and notation and outline in some more detail the scalar potential of the MSSM with explicit R-parity breaking terms. Our main concern will be to argue that the retention of a special term in the superpotential, which is usually not taken into account, leads *naturally*, *i.e.* without any fine tuning problems, to non-zero vev's of the sneutrinos. As a consequence, neutrinos acquire masses through mixing with gauginos and higgsinos and the standard W/Z-boson interactions with fermions get modified, *e.g.* one gets tree-level off-diagonal Z decays. The novelty in our approach is the afore-mentioned term in the superpotential whose coupling strength ε_i enters the W/Z interaction Lagrangians.

2.1 Superpotential

We write the full superpotential W as consisting of an R-parity conserving part (W_0) and R-parity violating term $(W_{\not{R}})$, *i.e.*

$$W \;=\; W_0 + W_{\not{R}} \,. \tag{2.1}$$

Let then $\hat{L}_i$ $(\hat{E}_i^C)$ and $\hat{Q}_i$ $(\hat{U}_i^C, \hat{D}_i^C)$ denote the lepton and quark doublets superfields (lepton and quarks $SU(2)$ singlets) with generation index i, respectively and let $\hat{H}_{1,2}$ be the super-Higgs fields. With the usual $U(1)_Y$ quantum number assignment, $Y(\hat{L}_i) = -1$, $Y(\hat{E}_i^C) = 2$, $Y(\hat{Q}_i) = 1/3$, $Y(\hat{D}_i^C) = 2/3$, $Y(\hat{U}_i^C) = -4/3$, $Y(\hat{H}_1) = -1$, $Y(\hat{H}_2) = 1$, the standard form for W_0 is

$$W_0 \;=\; \varepsilon_{ab}\left[h_{ij}\hat{L}_i^a\hat{H}_1^b\hat{E}_j^C + h'_{ij}\hat{Q}_i^a\hat{H}_1^b\hat{D}_j^C + h''_{ij}\hat{Q}_i^a\hat{H}_2^b\hat{U}_j^C + \mu\hat{H}_1^a\hat{H}_2^b\right] , \tag{2.2}$$

where a, b are $SU(2)$ group indices.

The explicit breaking of R parity can be introduced through $W_{\not{R}}$, which in its most general form is given by [2,3]

$$W_{\not{R}} = \varepsilon_{ab}\left(\lambda_{ijk}\hat{L}_i^a\hat{L}_j^b\hat{E}_k^C + \lambda'_{ijk}\hat{L}_i^a\hat{Q}_j^b\hat{D}_k^C + \varepsilon_i\hat{L}_i^a\hat{H}_2^b\right) + \lambda''_{ijk}\hat{U}_i^C\hat{D}_j^C\hat{D}_k^C. \quad (2.3)$$

Unlike the MSSM, the R-parity broken SUSY model allows for explicitly broken lepton ($\not{L}$) and baryon ($\not{B}$) number interactions. To be more precise, the terms in Eq. (2.2) proportional to $\lambda_{ijk} = -\lambda_{jik}$, λ'_{ijk} and ε_i violate lepton number, whereas the baryon number is explicitly broken by the λ''_{ijk}-term ($\lambda''_{ijk} = -\lambda''_{ikj}$). The presence of both $\not{B}$- and $\not{L}$-type of terms in the Lagrangian leads to unsuppressed proton decay. Therefore, at the most, we can retain either the L-violating or the B-violating terms in (2.2). To account for this fact, hereafter we will set $\lambda''_{ijk} = 0$.

Let us now comment on the term $\varepsilon_{ab}\varepsilon_i\hat{L}_i^a\hat{H}_2^b$ which is usually not taken into account in $W_{\not{R}}$ by using the argument of field redefinitions to rotate away such bilinears. It has been discussed in some detail in [5,6] that this argument may not be valid once we add to the Lagrangian soft-SUSY breaking terms. Indeed, we are unable to absorb the ε_i terms by using an orthogonal transformation of the $\hat{H}_1$ and $\hat{L}_i$ fields. The omission of such terms is therefore not justified. We note here that exactly these terms are in principle responsible for a non-zero vev's of the sneutrinos. The ε_i terms force the vev's of the sneutrino fields to assume non-zero values. We will return to this point while discussing the Higgs potential.

The λ-terms in $W_{\not{R}}$ lead to the interaction Lagrangian

$$\mathcal{L}_\lambda = \lambda_{ijk}\left[\tilde{\nu}_{iL}\bar{e}_{jR}e_{jL} + \tilde{e}_{jL}\bar{e}_{kR}\nu_{iL} + \tilde{e}_{kR}^*\bar{\nu}_{iL}^C e_{jL} - (i \leftrightarrow j)\right] + \text{H.c.}, \quad (2.4)$$

where e_i denote charged leptons enumerated by generation index i and tilded symbols denote as usual the superpartners. Correspondingly, the λ' interaction Lagrangian reads

$$\begin{aligned}\mathcal{L}_{\lambda'} = \lambda'_{ijk}\big(&\tilde{\nu}_{iL}\bar{d}_{kR}d_{iL} + \tilde{d}_{jL}\bar{d}_{kR}\nu_{iL} + \tilde{d}_{kR}^*\bar{\nu}_{iL}^C d_{jL} - \tilde{e}_{iL}\bar{d}_{kR}u_{jL} - \tilde{u}_{jL}\bar{d}_{kR}e_{iL}\\ &-\tilde{d}_{kR}^*\bar{e}_{iL}^C u_{jL}\big) + \text{H.c.}, \quad (2.5)\end{aligned}$$

where d_i (u_i) are down-type (up-type) quarks.

Since the main concern of the present paper will be to constrain the lepton-number-violating couplings, it is worth discussing briefly constraints on the L-violating couplings which come from baryogenesis. It has been known for some time that non-perturbative anomalous B-violating interactions of the electroweak theory can wash out the baryon asymmetry generated initially at the GUT scale. This gives rise to severe constraints on lepton-number-violating couplings, such as the trilinear couplings λ_{ijk} and λ'_{ijk}. In this context, it has been noticed that to evade such limits is sufficient to have one individual

lepton number, L_i, conserved [7]. The latter will be assumed throughout this work. In particular, we have

$$\lambda_{ijk} = 0, \quad \text{for} \quad i \neq j \neq k,$$

$$\lambda_{iki} \neq 0, \quad \text{for} \quad \not{L}_k \ L_i,$$

$$\lambda'_{ijk} \neq 0, \quad \text{for} \quad \not{L}_i, \tag{2.6}$$

where $\not{L}_i$ (L_i) indicates which lepton number is violated (conserved). Choosing L_i as the conserved lepton number, we then have four independent λ couplings. For instance, conserving L_e, we are left with non-zero λ_{121}, λ_{131}, λ_{232}, and λ_{323}. This conservation of a separate lepton number, i.e. $\Delta L_e = 0$, has profound consequences for exotic lepton-number-violating processes, which can, in principle, occur at the tree level in an R-parity broken SUSY model. If one L_i is strictly conserved, then no tree-level λ-dependent interaction can contribute, e.g., to the process $\mu \rightarrow eee$. The latter would proceed via a sneutrino mediated diagram only if all individual lepton numbers were broken. If $\Delta L_i = 0$ for only one lepton number L_i, the bound on the λ couplings derived via $\mu \rightarrow eee$ in [8] does not further apply. The reason is that the process $\mu \rightarrow eee$ is still possible at the tree level, however through a diagram containing off-diagonal Z couplings (see discussion in Section 5.1). In the subsequent section, we will also address the question whether having $\Delta L_i = 0$ for one lepton number L_i at the level of Lagrangian, this particular symmetry gets broken spontaneously through a vev of the sneutrino field, w_i, with the very same flavour index i. Indeed, we will argue that this is not the case, when the couplings ε_i are taken into account.

Laboratory constraints on λ and λ' couplings have been put in [9] and [10,11]. The detectability of possible direct R-parity-violating signals through the λ and λ' couplings at the CERN Large Electron Positron (LEP) collider, planned to operate at 200-GeV centre of mass energies (LEP-2), has been discussed in [12].

We note here that there are low-energy processes to which both type of $\not{R}$ interactions —those induced by the trilinear $\not{R}$ λ- and λ'-dependent couplings and those emanating from non-zero ε_i and w_i parameters— will contribute. We will see that a combined analysis is not necessary, i.e. deriving limits first on ε_i and w_i from, say, $\mu \rightarrow eee$, and then applying the so obtained results to put constraints on the λ and λ' couplings.

2.2 The scalar potential

With the superpotential given, it is straightforward to construct the scalar potential of an R-parity broken SUSY model [1]. Using the convention that symbols without a hat,

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say A, are the spin-zero content of a chiral superfield $\hat{A}$, we first write down the relevant soft-SUSY breaking terms

$$
\begin{aligned}
V_{soft} \;=\; & m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2 + (m_{L_{ij}}^2 L_i^\dagger L_j \;+\; \text{H.c.}) \\
& -(m_{12}^2 \varepsilon_{ab} H_1^a H_2^b \;+\; \text{H.c.}) + (\kappa_i' \varepsilon_{ab} H_2^a L_i^b \;+\; \text{H.c.}) \\
& +(\mu_{+ij}^2 E_i^* E_j \;+\; \text{H.c.}) + [\mu_{ij}'(H_1^\dagger L_i) E_j \;+\; \text{H.c.}] \\
& +(\kappa_{ijk}' \varepsilon_{ab} L_j^a L_j^b E_k \;+\; \text{H.c.}) \,.
\end{aligned}
\tag{2.7}
$$

In Eq. (2.7), E_i are the positively charged scalar singlet fields.

The full scalar potential (without squarks) can be written as the sum of five terms,

$$
V_{Scalar} \;=\; V^{2H} + V^L + V^{\not{L}} + V_+^L + V_+^{\not{L}} \,,
\tag{2.8}
$$

where V^{2H} is the usual Higgs potential of the MSSM, V^L and $V^{\not{L}}$ contain the slepton doublets (both L-conserving and L-violating parts, the latter connected with R-parity breaking couplings), and finally V_+^L and $V_+^{\not{L}}$ refers to the part of the potential which contains the charged scalars E_i. The standard MSSM potential, V^{2H}, reads

$$
\begin{aligned}
V^{2H} \;=\; & \mu_1^2 \phi_1^\dagger \phi_1 + \mu_2^2 \phi_2^\dagger \phi_2 + \frac{1}{2}\lambda_1(\phi_1^\dagger \phi_1)^2 + \frac{1}{2}\lambda_1(\phi_1^\dagger \phi_2)^2 \\
& +\lambda_3(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) - (\lambda_3 + \lambda_1)(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\
& +\lambda_6(\phi_1^\dagger \phi_2) + \lambda_6^*(\phi_2^\dagger \phi_1) \,,
\end{aligned}
\tag{2.9}
$$

where we have used the notation $\varphi_i \equiv L_i$, $\phi_2 \equiv H_2$, and $\phi_1 \equiv -i\tau_2 H_1^*$ [τ_2 being the Pauli matrix, $(i\tau_2)_{ab} = \varepsilon_{ab}$]. The new parameters in Eq. (2.9), which depend on the couplings entering the superpotential, are the soft-SUSY breaking parameters given in (2.7), as well as the $SU(2)_L$ coupling constant g and the corresponding $U(1)_Y$ coupling constant g'. These parameters are related as follows:

$$
\begin{aligned}
\mu_1^2 &= m_1^2 + |\mu|^2, \quad \mu_2^2 = m_2^2 + |\mu|^2 + \varepsilon_i \varepsilon_i^* \,, \\
\lambda_1 &= \frac{1}{4}(g^2 + g'^2), \quad \lambda_3 = \frac{1}{2}g^2 - \lambda_1, \quad \lambda_6 = -m_{12}^2 \,.
\end{aligned}
\tag{2.10}
$$

The lepton-number-conserving scalar potential containing the slepton doublet fields φ_i reads

$$
\begin{aligned}
V^L \;=\; & (\mu_{L_{ij}}^2 \varphi_i^\dagger \varphi_j \;+\; \text{H.c.}) + \frac{1}{2}\lambda_1(\sum_i \varphi_i^\dagger \varphi_i)^2 + \lambda_1(\phi_1^\dagger \phi_1)(\varphi_i^\dagger \varphi_i) \\
& -\lambda_1(\phi_2^\dagger \phi_2)(\varphi_i^\dagger \varphi_i) + (\lambda_3 + \lambda_1)(\phi_2^\dagger \varphi_i)(\varphi_i^\dagger \phi_2) \\
& +\big[\kappa_{jk} - (\lambda_3 + \lambda_1)\delta_{jk}\big](\phi_1^\dagger \varphi_k)(\varphi_j^\dagger \phi_1) + \kappa_{nmij}(\varphi_i^T \tau_2 \varphi_j)(\varphi_n^T \tau_2 \varphi_m)^\dagger \,.
\end{aligned}
\tag{2.11}
$$

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Here, we have used the definitions

$$\mu^2_{L_{ij}} = m^2_{L_{ij}} + \varepsilon^*_i \varepsilon_j, \quad \kappa_{jk} = \kappa^*_{kj} \equiv h^*_{ji} h_{ki},$$

$$\kappa_{nmij} = -\kappa_{mnij} = -\kappa_{nmji} = \kappa^*_{ijnm} \equiv \lambda^*_{nmk} \lambda_{ijk}. \tag{2.12}$$

V^L given in Eq. (2.11) may be contrasted with the corresponding potential of the MSSM. In the MSSM, the term proportional κ_{nmij} is absent and $\mu^2_{L_{ij}}$ should be replaced by the diagonal mass parameters $\mu^2_{L_i} = m^2_{L_i} + |\varepsilon_i|^2$.

Finally, the lepton-number-violating part of the potential is given by

$$\begin{aligned}
V^{\cancel{L}} &= [i\kappa_i(\phi^T_1 \tau_2 \varphi_i) + \text{H.c.}] + [i\kappa'_i(\phi^T_2 \tau_2 \varphi_i) + \text{H.c.}] \\
&\quad - i\kappa_{nmj}(\phi^\dagger_1 \varphi_j)(\varphi^T_n \tau_2 \varphi_m)^\dagger,
\end{aligned} \tag{2.13}$$

with

$$\kappa_i \equiv \mu^* \varepsilon_i, \quad \kappa_{nmj} \equiv \lambda^*_{nmk} h_{jk}, \tag{2.14}$$

and κ'_j is a soft-SUSY breaking parameter from Eq. (2.7). The parts V^{2H}, V^L and $V^{\cancel{L}}$ are sufficient to determine the minimization conditions of the potential. The explicit form of the remaining contributions to V_{Scalar}, V^L_+ and $V^{\cancel{L}}_+$, is given in Appendix A. At the minimum, the fields take the values

$$\langle \phi_1 \rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle \varphi_i \rangle = \begin{pmatrix} w_i \\ 0 \end{pmatrix}, \tag{2.15}$$

and the minimization conditions are found to be

$$\begin{aligned}
\mu^2_1 v^*_1 + \lambda_1 v^*_1 \left(|v_1|^2 - |v_2|^2 + \sum_k |w_k|^2\right) + \lambda^*_6 v^*_2 - \sum_k \kappa_k w_k &= 0, \\
\mu^2_2 v^*_2 - \lambda_1 v^*_2 \left(|v_1|^2 - |v_2|^2 + \sum_k |w_k|^2\right) + \lambda_6 v^*_1 - \sum_k \kappa'_k w_k &= 0, \\
\sum_j \mu^2_{L_{ij}} w^*_j + \lambda_1 w^*_i \left(|v_1|^2 - |v_2|^2 + \sum_k |w_k|^2\right) - \kappa_i v_1 - \kappa'_i v_2 &= 0, \tag{2.16}
\end{aligned}$$

where the last equation is valid for every generation index i. It is clear from Eq. (2.16) that $w_i \neq 0$ unless $\kappa_i = \kappa'_i = 0$.

To further elucidate this point, let us first consider the case of κ_i and κ'_i not identical to zero (i.e. the case of the R-parity broken SUSY model with $\varepsilon_i \neq 0$), and assume $w_i = 0$ for some generation index i. From Eq. (2.16), we then get a 'fine-tuning' relation among the original parameters of the potential (no summation convention)

$$(\mu^2_1 + \mu^2_2)\kappa^*_i \kappa'^*_i = \lambda^*_6 \kappa^{*2}_i + \lambda_6 \kappa'^{*2}_i. \tag{2.17}$$

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Analogous to our $\not{R}$ model is the limiting case of the MSSM, without explicit R-parity breaking, if one sets $\kappa_i = \kappa'_i = 0$ and replaces $\mu^2_{L_{ij}}$ by the diagonal coupling $\mu^2_{L_i}$. Again, the attempt to maintain $w_i \neq 0$ yields the 'fine-tuning' relation (see also [14]).

$$(\mu^2_1 - \mu^2_{L_i})(\mu^2_2 + \mu^2_{L_i}) = |\lambda_6|^2 \, . \tag{2.18}$$

This demonstrates nicely how, unlike the MSSM, the R-parity-breaking couplings ε_i from Eq. (2.2) naturally lead to non-zero vev's for the scalar neutrinos. Had we neglected the bilinear term $\varepsilon_{ab}\varepsilon_i \hat{L}^a_i \hat{H}^b_2$ in Eq. (2.2), as done usually, then there would have been no compelling reason to acquire non-zero vev's for the scalar neutrino fields even in the R-parity broken case. Moreover, if we conserve an individual lepton number, say L_i, in the Lagrangian/superpotential of the R-parity broken SUSY model —among other couplings, this implies that $\varepsilon_i = 0$—, this symmetry will not break spontaneously; so, we are free to choose the corresponding φ_i to have a vanishing vev. This follows again from Eqs. (2.17) and (2.18).

The spontaneous breaking of CP violation in the case of V_{Scalar}, Eq. (2.8), has been discussed in [6]. We supplement these considerations on CP properties by giving below all conditions necessary to restore CP conservation in $V^{2H} + V^L + V^{\not{L}}$. Denoting η_1, η_2 and η_{L_i} the CP phases of the fields ϕ_1, ϕ_2, and φ_i, respectively, we find from the requirement of CP conservation that (no summation convention below)

$$\lambda^*_6 \eta_1 \eta^*_2 = \lambda_6 \, ,$$
$$\kappa_{jk} = \kappa^*_{jk} \eta_{L_j} \eta^*_{L_k} \, ,$$
$$\kappa_{nmjk} = \kappa^*_{nmjk} \eta_{L_n} \eta_{L_m} \eta^*_{L_j} \eta^*_{L_j} \, ,$$
$$\kappa^*_i \eta^*_1 \eta^*_{L_i} = \kappa_i \, ,$$
$$\kappa'^*_j \eta^*_2 \eta^*_{L_j} = \kappa'_j \, ,$$
$$\kappa^*_{nmj} \eta_1 \eta^*_{L_j} \eta_{L_n} \eta_{L_m} = \kappa_{nmj} \, . \tag{2.19}$$

From Eq. (2.19), one can derive over twenty conditions for CP conservation which in contrast to (2.19) do not involve the CP phases η's. However, it is obvious that not all such conditions are independent. For instance, a set of independent conditions that follows from (2.19) is given by

$$\Im m(\lambda_6 \kappa_i \kappa'^*_j \kappa_{ij}) = 0 \, ,$$
$$\Im m(\kappa_i \kappa^*_j \kappa_{ij}) = 0 \, ,$$
$$\Im m(\kappa'_i \kappa'^*_j \kappa_{ij}) = 0 \, ,$$
$$\Im m(\kappa^*_{ni} \kappa^*_{mj} \kappa_{nmij}) = 0 \, ,$$
$$\Im m(\kappa^*_n \kappa_{mj} \kappa^*_{nmj}) = 0 \, , \tag{2.20}$$

where again no summation convention has been used. After spontaneous symmetry breaking, the first equation in the set (2.20) for $i = j$, i.e. $\Im m(\lambda_6 \kappa_i \kappa_i'^*) = 0$, translates into the following three equivalent conditions:

$$\Im m(\kappa_i w_i v_1) = 0 \,,$$
$$\Im m(\kappa_i' w_i v_2) = 0 \,,$$
$$\Im m(\lambda_6 v_1^* v_2) = 0 \,. \tag{2.21}$$

For the case at hand, it is, however, possible to have complex vev's such that CP gets broken spontaneously. For further details on this issue, the reader is referred to [6].

Some remarks on the vev's of sneutrinos, w_i, are in order. One should notice that through the kinetic term

$$\sum_i (D_\mu \mathcal{S}_i)^\dagger (D^\mu \mathcal{S}_i) \,,$$

where D_μ is the covariant derivative and $\mathcal{S}_i$ are the scalar fields in the theory, the sneutrino vev's contribute to the gauge bosons masses. In this way, the SM vev is obtained by

$$v \equiv \sqrt{v_1^2 + v_2^2 + \sum_i w_i^2} \; = \; \frac{2 M_W}{g} \,. \tag{2.22}$$

As a consequence, w_i and the angle β defined by

$$\tan \beta = \frac{v_1}{v_2} \,, \tag{2.23}$$

for real v_i, may be regarded as free parameters of the theory, while v_i are not free any longer, but determined by

$$v_1 = \sin \beta \sqrt{v^2 - \sum_i w_i^2} \,,$$
$$v_2 = \cos \beta \sqrt{v^2 - \sum_i w_i^2} \,. \tag{2.24}$$

Evidently, the vev's of the scalar neutrinos, w_i, cannot have arbitrarily large values, but they are bounded from above, as can be readily seen from Eqs. (2.22) and (2.24).

3 Mass matrices

In this section, we will present the mass matrices of the neutralino/neutrino as well as those of the chargino/charged lepton states. Since the lepton number is explicitly broken,

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the neutrinos will mix with the neutralinos to give the neutrinos mass. A natural seesaw mechanism emerges, in which μ, v_1, v_2, and the gaugino mass parameters M and M' act as the heavy scales, and the lepton-number-breaking couplings ε_i together with the sneutrino vev's w_i constitute the light Dirac components of the seesaw matrix. It will turn out that only one neutrino becomes massive through this mechanism at the tree level. These considerations are relevant for putting limits on the lepton-number-breaking parameters. In particular, one can already infer constraints on those parameters from the τ-neutrino mass. Furthermore, the neutralino–neutrino or chargino–charged lepton mixing will enter the interaction Lagrangians of W and Z, giving rise to non-SM processes, through which the new parameters can also be constrained.

3.1   Neutralino–neutrino mixing

In general, there are two mechanisms that can give rise to neutrino masses in the Born approximation. For example, one possibility is to give masses to the left-handed neutrinos through the vev of an exotic Higgs field which transform under $SU(2)_L$ as a triplet. The other mechanism requires, in general, the mixing of the left-handed neutrinos with other neutral fields of the theory. The latter are usually taken to be the right-handed neutrinos, introducing hereby additional fields in the theory. In our minimal R-parity broken SUSY model, in which right-handed neutrinos are absent, the rôle of the new neutral fields required for the afore-mentioned mixing will be assumed by the gauginos and higgsinos.

In two component notation, let Ψ' denote the column vector of neutrinos and neutralinos

$$\Psi_0^{'T} = (\psi_{L_1}^1, \ \psi_{L_2}^1, \ \psi_{L_3}^1, \ -i\lambda', \ -i\lambda_3, \ \psi_{H_1}^1, \ \psi_{H_2}^2), \tag{3.1}$$

where $\psi_{L_i}^1$ are the neutrino fields —the upper index indicates the component of the doublet—, $-i\lambda'$ and $-i\lambda_3$ are the unmixed photino and gaugino states, respectively, and the last two entries refer to the two higgsino fields. In the Weyl basis, the Lagrangian describing the neutralino/neutrino masses is then given by

$$\mathcal{L}_{mass}^{\chi^0} = -\frac{1}{2}\Psi_0^{'T}\mathcal{M}_0\Psi_0' + \text{H.c.}, \tag{3.2}$$

where the mass matrix has the general seesaw-type structure

$$\mathcal{M}_0 = \begin{pmatrix} 0 & m \\ m^T & M_4 \end{pmatrix}. \tag{3.3}$$

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Here, the sub-matrix m is the following 3×4 dimensional matrix:

$$m = \begin{pmatrix} -\frac{1}{2}g'w_e & \frac{1}{2}gw_e & 0 & -\varepsilon_e \\ -\frac{1}{2}g'w_\mu & \frac{1}{2}gw_\mu & 0 & -\varepsilon_\mu \\ -\frac{1}{2}g'w_\tau & \frac{1}{2}gw_\tau & 0 & -\varepsilon_\tau \end{pmatrix}. \tag{3.4}$$

In Eq. (3.3), M_4 is the usual 4×4 dimensional neutralino mass matrix of the MSSM, which has the form

$$M_4 = \begin{pmatrix} cM & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 \\ 0 & M & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 \\ -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu \\ \frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0 \end{pmatrix}, \tag{3.5}$$

where M is the common gaugino mass parameter and $c = \dfrac{5g'^2}{3g^2} \simeq 0.5$. The seesaw hierarchy is now evident, when constraints on neutralino masses and upper limits on lepton-number-violating couplings will be considered in Section 5. We will then find that $(M_4)_{ij} \gg m_{kl}$ in agreement with experimental constraints on neutralino and neutrino masses. We can utilize this posterior fact to calculate the diagonalization of $\mathcal{M}_0$ in an approximate way in terms of the small matrix-valued quantity defined as

$$\xi = mM_4^{-1}. \tag{3.6}$$

Parenthetically, we wish to draw the reader's attention to one exact result in connection with the diagonalization of $\mathcal{M}_0$. Because of the different hypercharge assignments of the two higgsinos and the absence of light-neutrino masses at the tree level, the first three lines together with the last line in $\mathcal{M}_0$ are not linearly independent. As an immediate consequence of the latter, two neutrino masses are exactly zero in the Born approximation [5].

Let us now define the mass eigenstates Ψ_0 by the rotation

$$\Psi_{0i} = \Xi_{ij}\Psi'_{0j},$$
$$\Xi^*\mathcal{M}_0\Xi^\dagger = \widehat{\mathcal{M}}_0, \tag{3.7}$$

where $\widehat{\mathcal{M}}_0$ is the diagonal matrix with neutrino/neutralino masses as elements. To leading order in ξ expansion, the approximate form of Ξ^* is readily estimated to be

$$\Xi^* = \begin{pmatrix} V_\nu^T & 0 \\ 0 & N^* \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{2}\xi\xi^\dagger & -\xi \\ \xi^\dagger & 1 - \frac{1}{2}\xi^\dagger\xi \end{pmatrix}, \tag{3.8}$$

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where the second matrix block-diagonalizes $\mathcal{M}_0$ to the form $\mathrm{diag}(m_{eff}, M_4)$ with

$$m_{eff} = -m\,M_4^{-1}\,m^T = \frac{cg^2 + g'^2}{D}\begin{pmatrix} \Lambda_e^2 & \Lambda_e\Lambda_\mu & \Lambda_e\Lambda_\tau \\ \Lambda_e\Lambda_\mu & \Lambda_\mu^2 & \Lambda_\mu\Lambda_\tau \\ \Lambda_e\Lambda_\tau & \Lambda_\mu\Lambda_\tau & \Lambda_\tau^2 \end{pmatrix}. \tag{3.9}$$

The quantities Λ_i and D newly introduced are defined as follows:

$$\vec{\Lambda} \equiv \mu\vec{w} - v_1\vec{\varepsilon}, \tag{3.10}$$

and

$$D \equiv 4\frac{det M_4}{M} = 2\mu\left[-2cM\mu + v_1 v_2\left(cg^2 + g'^2\right)\right]. \tag{3.11}$$

The sub-matrices N and V_ν in Eq. (3.8) diagonalize M_4 and m_{eff} in the following way:

$$N^* M_4 N^\dagger = \mathrm{diag}(m_{\tilde{\chi}_i^0}), \tag{3.12}$$

where $m_{\tilde{\chi}_i^0}$ are the heavy neutralino masses only. For the diagonalization of M_4, we have retained the notation and convention of Ref. [15]. For the neutrino case, we obtain

$$V_\nu^T\,m_{eff}\,V_\nu = \mathrm{diag}(0,\ 0,\ m_\nu), \tag{3.13}$$

where the only non-zero neutrino mass is given by

$$m_\nu = tr(m_{eff}) = \frac{cg^2 + g'^2}{D}\,|\vec{\Lambda}|^2. \tag{3.14}$$

Furthermore, an analytic calculation of the rotation matrix V_ν gives [5]

$$V_\nu = \begin{pmatrix} \cos\theta_{13} & 0 & -\sin\theta_{13} \\ \sin\theta_{23}\sin\theta_{13} & \cos\theta_{23} & \sin\theta_{23}\cos\theta_{13} \\ \sin\theta_{13} & \sin\theta_{23} & \cos\theta_{13}\cos\theta_{23} \end{pmatrix}, \tag{3.15}$$

where the mixing angles are expressed through the vector $\vec{\Lambda}$ as follows:

$$\tan\theta_{13} = -\frac{\Lambda_e}{\sqrt{\Lambda_\mu^2 + \Lambda_\tau^2}}, \qquad \tan\theta_{23} = \frac{\Lambda_\mu}{\Lambda_\tau}. \tag{3.16}$$

In [5], the baryogenesis constraint on all lepton-number-violating couplings were applied, which led to a solution to the solar neutrino puzzle through vacuum oscillations. In that case, the neutrino mass m_ν came out rather naturally of order 10^{-5} eV, while the mixing angle θ_{13} was predicted to be large, i.e. $\tan\theta_{13} \simeq -1/\sqrt{2}$. Since we can evade the constraints from BAU by conserving one individual lepton number, our scenario regarding the light neutrino mass, m_ν, is quite different.

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In Section 6, we will discuss some numerical examples of the neutrino mass as well as the resulting constraints on w_i, ε_i together with the constraints emerging from exotic processes. Here we note in passing that appreciable values for w_i and ε_i in the GeV range result in a tau-neutrino mass of $\mathcal{O}(\text{MeV})$ which is still allowed by laboratory constraints. Let us now demonstrate explicitly, by an example, how the ε_i terms can change some of the phenomenological implications. We choose the following set of parameters: $M = \mu = 2M_W$, $\tan\beta = 1$, $\varepsilon_\tau = w_\tau = 0$, $w_e = w_\mu \equiv w = 1$ GeV, $\varepsilon_e = \varepsilon_\mu \equiv \varepsilon$. If we now put $\varepsilon = 0$, then $m_\nu \simeq 3.5$ MeV. On the other hand, the same soft-SUSY parameters and vev's, but having now $\varepsilon = 4w$ instead, give $m_\nu \simeq 38.5$ MeV, which already exceeds the laboratory limit on the tau-neutrino mass.

3.2 Chargino–charged lepton mixing

Similar to the case of neutralino–neutrino mixing, the explicit violation of the lepton number allows also for chargino–charged lepton mixing. In two component notation, the mass term takes the form

$$\mathcal{L}_{mass}^{\chi^+} = -\zeta'^T \mathcal{M}_+ \omega' + \text{H.c.},\qquad(3.17)$$

where in the vector ζ', we gather the lower components of a charged Dirac spinor in the Weyl representation, *i.e.*

$$\zeta'^T = (\psi_{L_1}^2,\ \psi_{L_2}^2,\ \psi_{L_3}^2,\ -i\lambda_-,\ \psi_{H_1}^2),\qquad(3.18)$$

whereas ω' contains the upper components

$$\omega'^T = (\psi_{R_1},\ \psi_{R_2},\ \psi_{R_3},\ -i\lambda_+,\ \psi_{H_2}^1).\qquad(3.19)$$

In order to establish contact between the notation of the MSSM in [15] or that of our minimal $\not{R}$ model and the usual SM notation, we note that the charged leptons are represented by their charged conjugate fields, *i.e.*

$$l_i^C = \begin{pmatrix} \psi_{R_i} \\ \bar{\psi}_{L_i}^2 \end{pmatrix}.$$

In this basis, the chargino/charged-lepton mass matrix $\mathcal{M}_+$ appearing in Eq. (3.17) may be written down as

$$\mathcal{M}_+ = \begin{pmatrix} M_l & E \\ E' & S \end{pmatrix},\qquad(3.20)$$

where S is the usual MSSM chargino mass matrix given by

$$S = \begin{pmatrix} M & \frac{1}{\sqrt{2}} g v_2 \\ \frac{1}{\sqrt{2}} g v_1 & \mu \end{pmatrix}. \tag{3.21}$$

The sub-matrices E and E' which give rise to chargino–charged lepton mixing are defined as follows:

$$E = \begin{pmatrix} \frac{1}{\sqrt{2}} g w_e & \varepsilon_e \\ \frac{1}{\sqrt{2}} g w_\mu & \varepsilon_\mu \\ \frac{1}{\sqrt{2}} g w_\tau & \varepsilon_\tau \end{pmatrix}, \tag{3.22}$$

and

$$E' = \begin{pmatrix} 0 & 0 & 0 \\ \Upsilon_e & \Upsilon_\mu & \Upsilon_\tau \end{pmatrix}, \tag{3.23}$$

where $\Upsilon_l \sim \dfrac{m_l}{v_1} w_l$ and m_l are the lepton masses. For our numerical purposes, we will assume that M_l is a diagonal matrix whose elements can be identified, to a high accuracy, with the physical lepton masses m_i. In addition, we can neglect the elements of E' as compared to the other entries in Eq. (3.20). Therefore, we will be working in the approximation $E' = 0$.

Let us now express the mass eigenstates ζ and ω in terms of the states ζ' and ω' via the unitary transformations

$$\zeta_i = \Sigma_{ij} \zeta'_j, \quad \omega = \Omega_{ij} \omega'_j. \tag{3.24}$$

The bi-diagonalization leads then to the diagonal matrix $\widehat{\mathcal{M}}_+$ whose elements are the chargino and lepton masses

$$\Sigma^* \mathcal{M}_+ \Omega^\dagger = \widehat{\mathcal{M}}_+. \tag{3.25}$$

Proceeding now as in the case of the neutralino–neutrino mixing, we carry out an approximate diagonalization for $\mathcal{M}_+$. In this way, the expansion parameters are found to be

$$\begin{aligned} \xi_L^* &= E S^{-1}, \\ \xi_R^* &= M_l^\dagger E S^{-1} (S^{-1})^T = M_l^\dagger \xi_L^* (S^{-1})^T. \end{aligned} \tag{3.26}$$

Note that $\xi_R \sim \xi_L m_l / M$. To leading order in ξ_L and ξ_R, the rotation matrices are written down as

$$\Sigma^* = \begin{pmatrix} V_L & 0 \\ 0 & U^* \end{pmatrix} \begin{pmatrix} 1 - \frac{1}{2} \xi_L^* \xi_L^T & -\xi_L^* \\ \xi_L^T & 1 - \frac{1}{2} \xi_L^T \xi_L^* \end{pmatrix}, \tag{3.27}$$

and

$$\Omega^\dagger = \begin{pmatrix} 1 - \frac{1}{2} \xi_R^* \xi_R^T & \xi_R^* \\ -\xi_R^T & 1 - \frac{1}{2} \xi_R^T \xi_R^* \end{pmatrix} \begin{pmatrix} V_R^\dagger & 0 \\ 0 & V^\dagger \end{pmatrix}. \tag{3.28}$$

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Adopting the convention of [15] for the matrices, which also appear in the MSSM, we have

$$U^* S V^\dagger = \widehat{S} \,,$$
$$V_L M_l V_R^\dagger = \widehat{M}_l \,, \qquad\qquad (3.29)$$

where, as before, the hatted matrices are diagonal.

4   The W- and Z-boson interaction Lagrangians

In this section, we will derive the interaction Lagrangians of Z and W bosons with neutralinos/neutrinos and charginos/charged leptons. We will first present general expressions and, subsequently, use the analytic results of the approximate diagonalization of the mass matrices, given in the preceeding section, to calculate the mixing matrices in the first order approximation. Here and in the following, because of the mixing, we collectively call Ψ_0 all the neutralinos, with neutrinos being the light neutralinos, and ζ, ω all the charginos, where the charged leptons are the light charginos.

4.1   General expressions

Starting from two component notation and defining for convenience the matrix

$$T^Z = \mathrm{diag}(1,\ 1,\ 1,\ 0,\ 0,\ 1,\ -1) \,, \qquad\qquad (4.1)$$

the interaction Lagrangian of Z with neutralinos reads

$$\mathcal{L}_{int}^{Z\chi^0\chi^0} = -\frac{g}{2\cos\theta_w} Z^\mu \bar{\Psi}'_{0i} T^Z_{ij} \bar{\sigma}_\mu \Psi'_{0j} \,. \qquad\qquad (4.2)$$

After replacing the weak eigenstates Ψ'_{0i} by four component Majorana mass eigenstates χ_i^0 in Eq. (4.2), we obtain

$$\mathcal{L}_{int}^{Z\tilde{\chi}^0\chi^0} = -\frac{g}{4\cos\theta_w} Z^\mu \bar{\chi}_i^0 \gamma_\mu \left(i\Im m \tilde{C}_{ij} - \gamma_5 \Re e \tilde{C}_{ij} \right) \chi_j^0 \,, \qquad\qquad (4.3)$$

where

$$\tilde{C}_{ij} = (\tilde{C}^\dagger)_{ij} = (\Xi T^Z \Xi^\dagger)_{ij} \,. \qquad\qquad (4.4)$$

It is easy to check that Eq. (4.3) reproduces the Z-neutralino-neutralino interaction of the MSSM, when the leptonic $\not{R}$ admixture is neglected.

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To calculate the Z-chargino-chargino coupling, we again define two auxiliary matrices

$$T_R^Z = \text{diag}(0, 0, 0, 2, 1),$$
$$T_L^Z = \text{diag}(1, 1, 1, 2, 1),\qquad(4.5)$$

such that $\mathcal{L}_{int}^{Z\tilde{\chi}^-\chi^-}$ takes the form

$$\mathcal{L}_{int}^{Z\tilde{\chi}^-\chi^-} = \frac{g}{2\cos\theta_w} Z^\mu \Big[\bar{\zeta}_i'(T_L^Z)_{ij}\bar{\sigma}_\mu\zeta_j' - \bar{\omega}_i'(T_R^Z)_{ij}\bar{\sigma}_\mu\omega_j' $$
$$+ 2\sin^2\theta_w(\bar{\omega}_i'\bar{\sigma}_\mu\omega_i' - \bar{\zeta}_i'\bar{\sigma}_\mu\zeta_i') \Big].\qquad(4.6)$$

Denoting by χ_i^- the physical charginos in the four-component Dirac notation pertaining to the definition of Eq. (3.18), the above Lagrangian can be written down as

$$\mathcal{L}_{int}^{Z\tilde{\chi}^-\chi^-} = \frac{g}{2\cos\theta_w} Z^\mu \bar{\chi}_i^- \gamma_\mu \big(\tilde{A}_{ij}^L P_L + \tilde{A}_{ij}^R P_R \big)\chi_j^-,\qquad(4.7)$$

where $P_L(P_R) = [1 - (+)\gamma_5]/2$ and

$$\tilde{A}_{ij}^L = (\Sigma T_L^Z \Sigma^\dagger)_{ij} - 2\delta_{ij}\sin^2\theta_w,$$
$$\tilde{A}_{ij}^R = (\Omega^* T_R^Z \Omega^T)_{ij} - 2\delta_{ij}\sin^2\theta_w.\qquad(4.8)$$

As done above for the neutral-current interactions, for the charged-current case we first introduce two auxiliary matrices given by

$$T^L = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad T^R = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\sqrt{2} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}.\qquad(4.9)$$

In the Weyl weak eigenbasis, we can then write

$$\mathcal{L}_{int}^{W\chi^-\chi^0} = -\frac{g}{\sqrt{2}} \big(\bar{\zeta}_i' T_{ij}^L \bar{\sigma}^\mu \Psi_{0j}' + \bar{\Psi}_{0i}' T_{ij}^R \bar{\sigma}^\mu \omega_j' \big) W_\mu^- + \text{H.c.},\qquad(4.10)$$

and

$$\mathcal{L}_{int}^{W\chi^-\chi^0} = -\frac{g}{\sqrt{2}} W^{-\mu} \bar{\chi}_i^- \gamma_\mu \big(\tilde{B}_{ij}^L P_L + \tilde{B}_{ij}^R P_R \big)\chi_j^0 + \text{H.c.},\qquad(4.11)$$

in four-component mass eigenbasis notation. The mixing matrices are

$$\tilde{B}_{ij}^L = (\Sigma T^L \Xi^\dagger)_{ij},$$
$$\tilde{B}_{ij}^R = -[\Omega^*(T^R)^T \Xi^T]_{ij}.\qquad(4.12)$$

In the next section, we will give analytic approximate expressions for all mixing matrices defined above.

4.2 Mixing matrices

In Section 4.1, we have derived the analytic expressions of the interaction Lagrangians of the W and Z bosons with chargino and neutralino states in the R-parity-violating SUSY model. However, the mixings $\tilde{A}^L$, $\tilde{A}^R$, $\tilde{B}^L$, $\tilde{B}^R$, and $\tilde{C}$ that govern these interactions are high dimensional matrices, involving a large number of parameters. Therefore, it is more convenient to find approximative forms for the mixing matrices that will enable us to appreciate the strength of the Z- and W-boson couplings in the model under consideration.

To facilitate our presentation, we first introduce the following auxiliary matrices:

$$
\begin{aligned}
d &= \operatorname{diag}(2,1), \qquad t_z = \operatorname{diag}(0,0,1,-1), \\
t_L &= \begin{pmatrix} 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \\
t_R &= \begin{pmatrix} 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.
\end{aligned}
\tag{4.13}
$$

Substituting the unitary mixing matrices of Eqs. (3.8), (3.27), and (3.28) into Eqs. (4.4), (4.8), and (4.12), and neglecting terms of $\mathcal{O}(\xi_L^3, \xi^3)$ and higher, we obtain

$$
\tilde{A}^L = -2\sin^2\theta_w \mathbf{1} + \begin{pmatrix} 1 - \tilde{\xi}_L(1-d)\tilde{\xi}_L^\dagger & \tilde{\xi}_L(1-d)U^\dagger \\ U(1-d)\tilde{\xi}_L^\dagger & UdU^\dagger \end{pmatrix},
\tag{4.14}
$$

$$
\tilde{A}^R = -2\sin^2\theta_w \mathbf{1} + \begin{pmatrix} 0 & -\tilde{\xi}_R dV^T \\ -V^* d\tilde{\xi}_R^\dagger & V^* dV^T \end{pmatrix},
\tag{4.15}
$$

$$
\tilde{B}^L = \begin{pmatrix} V^l - \frac{1}{2}\tilde{\xi}_L\tilde{\xi}_L^\dagger V^l - \frac{1}{2}V^l\tilde{\xi}^*\tilde{\xi}^T + \tilde{\xi}_L t_L\tilde{\xi}^T & (V^l\tilde{\xi}^* - \tilde{\xi}_L t_L)N^\dagger \\ U(\tilde{\xi}_L^\dagger V^l - t_L\tilde{\xi}^T) & Ut_L N^\dagger \end{pmatrix},
\tag{4.16}
$$

$$
\tilde{B}^R = \begin{pmatrix} 0 & -\tilde{\xi}_R t_R N^T \\ -V^* t_R \tilde{\xi}^\dagger & V^* t_R N^T \end{pmatrix},
\tag{4.17}
$$

$$
\tilde{C} = \begin{pmatrix} 1 - \tilde{\xi}^*(1-t_z)\tilde{\xi}^T & \tilde{\xi}^*(1-t_z)N^\dagger \\ N(1-t_z)\tilde{\xi}^T & Nt_z N^\dagger \end{pmatrix}.
\tag{4.18}
$$

Here, we have defined $\tilde{\xi}_L = V_L^*\xi_L$, $\tilde{\xi}_R = \widehat{M}_l\tilde{\xi}_L(S^{-1})^\dagger$, $\tilde{\xi} = V_\nu^T\xi$, and $V^l = V_L^*V_\nu$. Furthermore, the unitary matrices V_L, V_R, V_ν, U, V, N, together with the mixing matrices ξ_L, ξ_R, and ξ are defined in Section 3. In the derivation of Eqs. (4.15) and (4.17), we have also used the fact that $\xi_R = \mathcal{O}(\xi_L m_l/M)$.

From Eqs. (4.14)–(4.18), it is now easy to see how the R-parity-violating couplings to ordinary leptons deviate from the SM vertices. To leading order in ξ_L and ξ, we find that

the interactions of the W and Z bosons with left-handed charged leptons and neutrinos are modified, whereas the corresponding couplings to right-handed charged leptons remain unaffected, having the SM form.

5 Laboratory and cosmological constraints

Our aim is to constrain the parameter space of this $\not{R}$ scenario, by taking laboratory and cosmological constraints into account. For this purpose, we will pay special attention to limits derived from low-energy processes and LEP data, such as charged lepton decays of the form $l^- \to l'^- l_1^- l_1^+$, flavour-changing Z-boson decays $Z \to l_i l_j$, the invisible width of the Z boson, charged-current universality in muon and tau decays, lepton universality at the Z peak, and charged-current universality in pion decays. In this vein, we will report some phenomenological implications of our minimal model that may be relevant to explain the intriguing anomaly found by the KARMEN collaboration [16]. In the last section, we will discuss the viability of our model when cosmological constraints are considered, such as the requirement of not washing out the primordial BAU and the absence of large disruptive reheating effects caused by an unstable τ neutrino with $m_{\nu_\tau} = \mathcal{O}(10)$ MeV.

5.1 $l^- \to l'^- l_1^- l_1^+$

As has been found in Lagrangian (4.7), the model predicts flavour-changing neutral current (FCNC) Zll' couplings at the tree level. These new $\not{R}$ interactions induce τ and μ decays into three lighter charged leptons. In this way, we obtain

$$B(l^- \to l'^- l_1^- l_1^+) = \frac{\alpha_w^2 m_l^4}{1536\pi M_W^4} \frac{m_l}{\Gamma_l} \left(\left| \tilde{A}_{ll'}^L \right|^2 + \left| \tilde{A}_{ll'}^R \right|^2 \right) \left(\left| \tilde{A}_{l_1 l_1}^L \right|^2 + \left| \tilde{A}_{l_1 l_1}^R \right|^2 \right), \tag{5.1}$$

where $\alpha_w = g^2/4\pi$ and Γ_l is the total width of the decaying charged lepton l.

The experimental upper limit on the branching ratio of $\mu^- \to e^- e^- e^+$ is given by [13]

$$B(\mu^- \to e^- e^- e^+) \leq 1.0 \cdot 10^{-12}, \tag{5.2}$$

at 90% confidence level (CL). Recently, CLEO collaboration [17] has considerably lowered experimental upper bounds on branching ratios of neutrinoless τ-lepton decays. They have found

$$B(\tau^- \to e^- e^+ e^-) \leq 3.3 \cdot 10^{-6},$$

$$B(\tau^- \to \mu^- e^+ e^-) \leq 3.4 \cdot 10^{-6},$$
$$B(\tau^- \to e^- \mu^+ \mu^-) \leq 3.6 \cdot 10^{-6},$$
$$B(\tau^- \to \mu^- \mu^+ \mu^-) \leq 4.3 \cdot 10^{-6}, \tag{5.3}$$

at 90% CL. Theoretical predictions obtained for the observables given in Eqs. (5.2) and (5.3) will be discussed in Section 6.

5.2 $Z \to l^- l'^+$ and $Z \to \nu\nu$

The presence of FCNC Zll' couplings at the tree level will also give rise to flavour-violating Z-boson decays at LEP. The theoretical prediction of their branching ratios is determined by

$$B(Z \to l^- l'^+ \text{ or } l^+ l'^-) = \frac{\alpha_w}{12 \cos^2 \theta_w} \frac{M_Z}{\Gamma_Z} \left(\left| \tilde{A}_{ll'}^L \right|^2 + \left| \tilde{A}_{ll'}^R \right|^2 \right), \tag{5.4}$$

where $\Gamma_Z = 2.49$ GeV is the total width of the Z boson measured experimentally [13]. Furthermore, an analysis of this kind of decays at LEP yields

$$B(Z \to e^- \mu^+ \text{ or } e^+ \mu^-) \leq 6.0 \cdot 10^{-6},$$
$$B(Z \to e^- \tau^+ \text{ or } e^+ \tau^-) \leq 1.3 \cdot 10^{-5},$$
$$B(Z \to \tau^- \mu^+ \text{ or } \tau^+ \mu^-) \leq 1.9 \cdot 10^{-5}, \tag{5.5}$$

at 95% CL. In addition, the Lagrangian (4.3) modifies the invisible width of the Z boson through the non-universal and flavour-dependent $Z\nu_i\nu_j$ tree-level couplings. It is then easy to obtain the branching ratio for the total invisible Z-boson width, which is assumed to be caused mainly by $Z \to \nu_i\nu_j$

$$B(Z \to \nu\bar{\nu}) = \frac{\alpha_w}{24 \cos^2 \theta_w} \frac{M_Z}{\Gamma_Z} \sum_{\nu_i,\nu_j} \left| \tilde{C}_{\nu_i\nu_j} \right|^2. \tag{5.6}$$

On the other hand, an experimental analysis on the Z pole gives [13]

$$1 - \frac{B(Z \to \nu\bar{\nu})}{B_{SM}(Z \to \text{invisible})} \leq 1.31 \cdot 10^{-2}, \tag{5.7}$$

where $B_{SM}(Z \to \text{invisible})$ is the SM prediction for the invisible width of the Z boson. In Section 6, we will analyze the phenomenological impact of the new-physics decay channels mentioned above on restricting our model.

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5.3 Universality violation at the Z peak

Interesting limits on R-parity breaking, nonuniversal, diagonal Zll couplings can be extracted from measurements of lepton universality on the Z-boson pole. In order to impose constraints, we will adopt the LEP observable based on leptonic Z-boson partial width differences studied in [18]

$$U_{br}^{(ll')} = \frac{\Gamma(Z \to l^+l^-) - \Gamma(Z \to l'^+l'^-)}{\Gamma(Z \to l^+l^-) + \Gamma(Z \to l'^+l'^-)} = \frac{|\tilde{A}_{ll}^L|^2 - |\tilde{A}_{l'l'}^L|^2}{|\tilde{A}_{ll}^L|^2 + |\tilde{A}_{l'l'}^L|^2}, \tag{5.8}$$

where $l \neq l'$. A combined experimental analysis for the observable U_{br} gives [13]

$$|U_{br}^{(ll')}| \leq 5.0 \cdot 10^{-3}, \tag{5.9}$$

at 1σ level, almost independent of the charged leptons l and l'. Another relevant observable involving leptonic asymmetries, which has been analyzed in [19], is

$$\Delta \mathcal{A}_{ll'} = \frac{\mathcal{A}_l - \mathcal{A}_{l'}}{\mathcal{A}_l + \mathcal{A}_{l'}} = \left(\frac{1}{\mathcal{A}_l^{(SM)}} - 1\right) U_{br}^{(ll')}, \tag{5.10}$$

where $\mathcal{A}_l^{(SM)} = 0.14$ is the leptonic asymmetry predicted theoretically in the SM. In the last step of Eqs. (5.8) and (5.10), we have used the fact that, to a good approximation, the tree-level coupling of the Z boson to right-handed charged leptons is universal in our minimal R-parity violating SUSY model, $i.e.$ $\tilde{A}_{ll}^R = \tilde{A}_{l'l'}^R$ as can be seen from Eq. (4.15). Considering the experimental upper bound on U_{br} given in Eq. (5.9), Eq. (5.10) furnishes the upper limit

$$\Delta \mathcal{A}_{ll'} \leq 3.0 \cdot 10^{-2}, \tag{5.11}$$

which is slightly below the present experimental sensitivity at LEP [20] $[\Delta \mathcal{A}_\tau^{LEP}/\mathcal{A}_l^{(SM)} = 0.07$, at $1\sigma]$ and Stanford Linear Collider (SLC) [21] $[\Delta \mathcal{A}_e^{SLC}/\mathcal{A}_l^{(SM)} = 0.04$, at $1\sigma]$. It is also interesting to notice that the apparent difference of $\Delta \mathcal{A}_{\tau e} \simeq -10\%$ between the measured leptonic asymmetries $\mathcal{A}_e^{SLC}$ and $\mathcal{A}_\tau^{LEP}$ cannot be predicted in our $\not{R}$ model, without invalidating the inequality (5.11) at the same time.

5.4 Decays $\mu \to e\nu\nu$ and $\tau \to e\nu\nu$

Useful constraints can be obtained from possible deviations of charged-current universality in τ-lepton decays. In fact, measures of such deviations can be defined and

straightforwardly be calculated as follows:

$$R_{\tau e} = \frac{\Gamma(\tau \to e\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})} = R_{\tau e}^{SM} \frac{\sum_{\nu_i} \left[|\tilde{B}_{\tau\nu_i}^L|^2 + |\tilde{B}_{\tau\nu_i}^R|^2 \right]}{\sum_{\nu_j} \left[|\tilde{B}_{\mu\nu_j}^L|^2 + |\tilde{B}_{\mu\nu_j}^R|^2 \right]}, \tag{5.12}$$

$$R_{\tau\mu} = \frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\mu \to e\nu\bar{\nu})} = R_{\tau\mu}^{SM} \frac{\sum_{\nu_i} \left[|\tilde{B}_{\tau\nu_i}^L|^2 + |\tilde{B}_{\tau\nu_i}^R|^2 \right]}{\sum_{\nu_j} \left[|\tilde{B}_{e\nu_j}^L|^2 + |\tilde{B}_{e\nu_j}^R|^2 \right]}. \tag{5.13}$$

In Eqs. (5.12) and (5.13), the SM contributions to the observables, $R_{\tau e}^{SM}$ and $R_{\tau\mu}^{SM}$, have been factored out. Of course, deviations from the SM values can also be induced by the λ-dependent interactions in Eq. (2.4). These observables are used to constrain the couplings λ_{ijk} as a function of the mass of the scalar right-handed leptons [9]. To avoid excessive complication, we assume that all $\lambda_{ijk} = 0$ and focus our study mainly on the phenomenological consequences originating from the ε_i terms in the superpotential. Furthermore, experimental limits related to the ratios $R_{\tau e}$ and $R_{\tau\mu}$ may be presented in the following way [22]:

$$1 - \frac{R_{\tau e}}{R_{\tau e}^{SM}} = 0.040 \pm 0.024, \tag{5.14}$$

$$1 - \frac{R_{\tau\mu}}{R_{\tau\mu}^{SM}} = 0.032 \pm 0.024, \tag{5.15}$$

at 1σ level. Constraints obtained from Eqs. (5.14) and (5.15) on the parameters of our $\not{R}$ model will be discussed in Section 6.

5.5 Charged-current universality in pion decays

Complementary to the physical quantities $R_{\tau e}$ and $R_{\tau\mu}$ are the constraints derived from the ratio $R_\pi = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ in the π^- decays. R_π is an observable that measures possible deviations from charged-current universality in the $e - \mu$ system. It is not difficult to obtain

$$R_\pi = R_\pi^{SM} \frac{\sum_{\nu_i} \left[|\tilde{B}_{e\nu_i}^L|^2 + |\tilde{B}_{e\nu_i}^R|^2 \right]}{\sum_{\nu_j} \left[|\tilde{B}_{\mu\nu_j}^L|^2 + |\tilde{B}_{\mu\nu_j}^R|^2 \right]}. \tag{5.16}$$

In addition, the 1σ experimental bound related to R_π is given by [22]

$$\frac{R_\pi}{R_\pi^{SM}} - 1 = 0.003 \pm 0.003. \tag{5.17}$$

It is again worth mentioning that similar deviations of $e - \mu$ universality can arise from the presence of λ'-dependent couplings through the interaction Lagrangian (2.5). In our analysis, we will assume that all $\lambda'_{ijk} = 0$. This may also be reflected by the fact that the current experimental lower bound on the half-lifetime of the ^{76}Ge $0\nu\beta\beta$ decay leads to the tight constraint [11]

$$\lambda'_{111} \leq 3.9 \cdot 10^{-4} \left(\frac{m_{\tilde{q}}}{100 \text{ GeV}} \right)^2 \left(\frac{m_{\tilde{g}}}{100 \text{ GeV}} \right)^{1/2}, \tag{5.18}$$

where $\tilde{q}$ ($\tilde{g}$) is the scalar quark (gluino).

5.6 KARMEN anomaly

Recently, the KARMEN collaboration, which operates at RAL, has reported an anomaly [16] in the time-dependence of decay spectra coming from stopped pions. To account for the KARMEN anomaly, one can make the plausible assumption that a new massive weakly-interacting particle, say x, is produced in the pion decays, *i.e.* $\pi^+ \rightarrow \mu^+ x$ [16,23]. The mass of this hypothetical particle should be $m_x \simeq 33.9$ MeV, since it should explain the apparent $\sim 2\sigma$ bump present in the time distribution of decaying muon events, which should normally fall off exponentially. This experimental peak occurs with a time delay of 3.6 μsec after all pulsed pions have promptly decayed.

A recent study [23] suggests that the x particle should have similar features with those of a neutrino, but it cannot be the ν_τ, because $m_{\nu_\tau} < 31$ MeV at 95% CL [13], or another predominantly-isodoublet neutrino, without affecting limits coming from the supernova 1987A. The authors in [23] further advocate that a mainly-sterile neutrino scenario could, in principle, be compatible with all constraints —both terrestrial and astrophysical—, since the production of x particles both in supernova and in the early universe could then be suppressed. Although in our $\not{R}$ model the coupling mixing matrices describing the charged- and neutral-current interactions differ crucially from usual singlet-neutrino scenarios [24], the above discussion is still valid and translates into the requirement that one neutralino state, *e.g.* χ, should be light, having a mass $m_\chi = m_x$. Assuming that the KARMEN anomaly gets resolved by the decay $\chi \rightarrow e^- e^+ \nu$, we have for the Majorana fermion χ [23]

$$|\tilde{B}^L_{ex}||\tilde{B}^L_{\mu x}| \simeq 0.6 \cdot 10^{-6},$$

$$|\tilde{B}^L_{ex}| \lesssim 2.5 \cdot 10^{-4},$$

$$|\tilde{B}^L_{\mu x}| \lesssim 4.5 \cdot 10^{-2}. \tag{5.19}$$

The bounds presented in Eq. (5.19) are obtained from a number of phenomenological requirements, such as the absence of a correction to the Michael ρ parameter in $\mu \rightarrow e\nu\nu$,

negligible decay events in neutrino beams, no anomalous contributions to $\pi \to e\chi$, limits from neutrinoless double-β decays, etc.

Because of the large number of parameters existing in our model, it appears not difficult to accommodate the upper limits and relations given in Eq. (5.19). However, the soft-SUSY breaking parameters in our model have to satisfy the following hierarchy scheme:

$$M\,(= 2M') \gtrsim 500 \text{ GeV},$$
$$\mu \lesssim 30 \text{ MeV},$$
$$\vec{\varepsilon} \sim \frac{\mu}{v_1}\vec{w}, \tag{5.20}$$

which is mainly prescribed by the fact that $m_\chi = 33.9$ MeV. From Eq. (5.20), we find that only SUSY models with a μ at the scale of 10 MeV have a chance to account for the KARMEN anomaly. Similar R-parity broken SUSY models were also discussed in Ref. [4]. Adapting the results of [4], one can estimate that for $\tan\beta = 1$ and $w_\tau < 60$ GeV, $B(Z \to \chi\chi) \simeq w_\tau^4/(3v^4) < 1.~10^{-3}$ in compliance with the LEP bound on invisible Z-boson decays in Eq. (5.7). However, such light-μ scenarios may encounter the known μ hierarchy problem, where $\mu \sim M_{Pl}$ as derived naively from supergravity. Even though one could invoke the Guidice–Masiero mechanism [25] to obtain a value of μ at the electroweak scale, the small value of $\mu = \mathcal{O}(10)$ MeV would, however, require an additional unnatural suppression of the gravitational couplings in the Kähler potential.

5.7 Cosmological and astrophysical constraints

The minimal SUSY model with explicit R nonconservation contains lepton-number violating interactions that can wash out any primordial BAU generated at the GUT scale via the $B + L$-violating sphaleron interactions [26–28], which are in thermal equilibrium above the critical temperature of the electroweak phase transition [29]. Sphalerons generally conserve the individual quantum numbers $B/3 - L_i$ [30,7]. In particular, it has been shown in [7] that if only one separate lepton number is preserved in thermal equilibrium ($e.g.$ L_i) and finite masses for the charged leptons are taken into account in the analysis of chemical potentials, this is then sufficient to protect any primordial excess in L_i, which can be converted later on, via sphalerons, into the observed BAU. For our purposes, we will assume that only one separate lepton number is conserved each time in the full Lagrangian, when low-energy experiments are considered. For definiteness, in our numerical analysis we will consider that either $w_e = \varepsilon_e = 0$ or $w_\tau = \varepsilon_\tau = 0$. Of course, one can use a complementary restriction and put $w_\mu = \varepsilon_\mu = 0$, which, however, will not alter our phenomenological constraints discussed in Sections 5.1–5.5 in an essential way.

There is a great number of bounds coming from astrophysics, such as those obtained from the dynamics of red giants and white dwarfs, or the absence of a distorted spectrum of the $2.73°$ K blackbody radiation background [31]. However, we find more worrying the severe limits derived from possible reheating effects of a decaying massive neutral relic with $m_\nu \simeq 10 - 40$ MeV and especially those obtained from the primordial nucleosynthesis [32,33]. In particular, τ-neutrino decays with a lifetime bigger than about 1 sec or so may increase the elemental ^{4}He abundance by making it incompatible with astrophysical observations. Imposing the latter constrain, we find

$$|\tilde{B}^L_{e\nu_\tau}|^2 \gtrsim 10^{-4} \left(\frac{30\,\text{MeV}}{m_{\nu_\tau}}\right)^5 , \qquad (5.21)$$

which is only applicable for $m_{\nu_\tau} \gtrsim 10 - 50$ MeV [32]. In fact, the bound of Eq. (5.21) is not so restrictive, since it simply constrains only the mixing-matrix element $V^l_{e\nu_\tau} > 10^{-2}$ in Eq. (4.16), which is not excluded from solar neutrino oscillation scenarios. In our analysis of laboratory observables, we sum up over all invisible light neutrinos, so the unitary matrix V^l becomes practically redundant. Moreover, the nonobservation of a γ ray burst from the Solar Maximum satellite after the supernova 1987A neutrinos were detected may point towards the fact that the τ neutrino mainly decays inside the supernova core. This leads again to ν_τ lifetimes compatible with the approximate inequality of Eq. (5.21). Even though the predicted supernova luminosity will increase in such a case, an allowed window of scenarios that maximally violate L_μ and L_τ may be present in the ~ 3 MeV neutrino-sphere [34].

As has also been pointed out by the authors in Ref. [34], there may exist viable cosmological models in which ν_τ is stable with a mass of $\mathcal{O}(10)$ MeV. Such a solution requires an alteration of the standard cosmological picture by, $e.g.$, reheating the universe even after inflation to only a few MeV and invoking low-temperature baryogenesis as well [35]. Then, the resulting ν_τ may not overclose the universe but it can even constitute the cold dark matter.

For neutrinos with $m_\nu \lesssim 0.1$ MeV, the cosmological bound regarding their lifetimes, τ_ν, is different. In fact, τ_ν should not be larger the age of the universe, $i.e.$ $\tau_\nu \gtrsim 10^{23}(\frac{m_\nu}{1\,\text{eV}})$ sec [33]. Furthermore, it is worth mentioning that radiative decays of massive neutrinos with $0.1 \lesssim m_\nu \lesssim 10$ keV have also found some applications in cosmology and astrophysics [36]. In this context, most noticeable is probably the Gunn-Peterson test [37], $i.e.$, the search of primordial elements in the intergalactic medium. There seems to be a deficiency of neutral hydrogen and helium in the intergalactic medium [37]. A source of photo-ionization of these elements might be a radiatively decaying neutrino. Here, we

simply comment on the fact that the mass range of neutrino required for such an explanation is different from what is suggested by the solar neutrino puzzle and the atmospheric neutrino problem. To ionize singly ionized helium, m_ν should be bigger than 109 eV, since the ionization potential is 54.4 eV. A recent investigation of this issue may be found in [38].

6 Numerical results

In this section, we will present numerical predictions as well as constraints on the basic parameters of our $\not R$ model, which have been discussed in Section 5. Although there is a large number of parameters that could vary independently, it is important to remark that there exists a strong correlation between new-physics observables and light neutrino masses. This seems to be a generic feature of most of the R-parity broken SUSY models considered in the literature [34,39]. However, a novel feature of our minimal $\not R$ scenario is that the size of the scalar-neutrino vev's and the ε_i terms can, in principle, be unconstrained. In fact, if $\vec{\Lambda} \simeq 0$ in Eq. (3.10), which is a form of alignment in the flavour space between the vev's of the sneutrinos, $\vec{w}$, and the $\not R$ terms, $\vec{\varepsilon}$, this condition alone is sufficient to evade upper limits on the tau-neutrino mass for any value of the SUSY parameters M, M', μ, and $\tan\beta$.

In order to understand how all new-physics interactions are proportional to Λ_i and hence depend on m_{eff} [or m_{ν_τ}] in Eq. (3.9) [Eq. (3.14)], we evaluate the mixing matrix ξ defined in Eq. (3.6). Thus, we have

$$
\begin{aligned}
\xi_{i1} &= \frac{g'M\mu}{2\det M_4}\Lambda_i\,, \\
\xi_{i2} &= -\frac{gcM\mu}{2\det M_4}\Lambda_i\,, \\
\xi_{i3} &= \frac{\varepsilon_i}{\mu} + \frac{(cg^2 + g'^2)Mv_2}{4\det M_4}\Lambda_i\,, \\
\xi_{i4} &= -\frac{(cg^2 + g'^2)Mv_1}{4\det M_4}\Lambda_i\,,
\end{aligned}
\tag{6.1}
$$

for $i = 1\,(e)$, $2\,(\mu)$, and $3\,(\tau)$. It is now easy to see that only the elements ξ_{i3} contain the dominant contributions characterized by being *not* proportional to Λ_i. However, these contributions vanish identically in the relevant expression

$$
\delta_{\nu_i\nu_j} - \tilde{C}_{\nu_i\nu_j} = [\tilde\xi^*(1 - t_z)\tilde\xi^T]_{\nu_i\nu_j}\,,
\tag{6.2}
$$

given in Eq. (4.18), since the element of the diagonal matrix $(1 - t_z)_{33} = 0$. Consequently, in the limit of vanishing τ-neutrino mass, the invisible Z-boson width predicted in our $\not R$ model will coincide with that found in the SM.

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Similar strong m_{ν_τ} dependence occurs in the non-SM part of the couplings $Zl_i l_j$ and $Wl_i \nu_j$ via the mixing matrix ξ_L, which is given by

$$
(\xi_L^*)_{i1} = \frac{g\,\Lambda_i}{\sqrt{2}(M\mu - \frac{1}{2}g^2 v_1 v_2)},
$$

$$
(\xi_L^*)_{i2} = \frac{\varepsilon_i}{\mu} - \frac{g^2 v_2\,\Lambda_i}{2\mu(M\mu - \frac{1}{2}g^2 v_1 v_2)}. \tag{6.3}
$$

One can readily see that the dominant terms in ξ_L are contained in the elements $(\xi_L)_{i2}$. However, in the $Zl_i l_j$ coupling, the new-physics contributions are determined by

$$
[\xi_L(1-d)\xi_L^\dagger]_{ij} = (\xi_L)_{i1}(\xi_L^*)_{j1}, \tag{6.4}
$$

and the elements $(\xi_L)_{i2}$ always get killed by the diagonal matrix $(1-d)$. Thus, leptonic FCNC Z-boson decays and associated universality-breaking effects are proportional to Λ_i and are absent if ν_τ is massless. Moreover, we find that the non-SM contributions present in the coupling $Wl\nu$ in Eq. (4.16) are proportional to

$$
\left(-\xi_L \xi_L^\dagger - \xi^* \xi^T + 2\xi_L t_L \xi^T\right)_{ij} = -(\xi_L)_{i2}[(\xi_L^*)_{j2} - \xi_{j3}] - \xi_{j3}[\xi_{i3}^* - (\xi_L)_{i2}]. \tag{6.5}
$$

Substituting Eqs. (6.1) and (6.3) into Eq. (6.5), it is easy to verify that new-physics effects in charged-current interactions are also very strongly correlated with the light neutrino mass m_ν.

For reasons mentioned above, we will work in the seesaw approximation by keeping the mass of ν_τ finite. For our illustrations, we will consider the following modest $\not{R}$ SUSY scenarios:

Scenario	(type of line)	$\tan\beta$	M [GeV]	μ [GeV]	ε_μ (or ε_τ) [GeV]
I	(solid)	1	50	500	0
II	(dashed)	1	50	−50	−0.5
III	(dotted)	1	100	200	1
IV	(dash–dotted)	4	200	400	2

(6.6)

where $M' = M/2$ and the type of line used in our plots is also indicated.

First, we will study possible limits on the $\not{R}$ models in Eq. (6.6) that may be derived by the nonobservation of a muon decay into three electrons. In Fig. 1(a), numerical predictions for $B(\mu^- \to e^- e^- e^+)$ as a function of m_{ν_τ} are displayed for $w_\mu/w_e = 1$. The horizontal dotted line indicates the present experimental limit. Fig. 1(a) also shows the strong quadratic dependence of $B(\mu^- \to e^- e^- e^+)$ on m_{ν_τ}. In particular, if w_μ and w_e are

comparable in size ($e.g.$, $w_\mu/w_e = 1$), this constraint is more severe. Qualitatively, we find that

$$\frac{w_e w_\mu}{w_e^2 + w_\mu^2} \frac{m_\nu}{M} \lesssim 10^{-6} \, . \tag{6.7}$$

Of course, this limit gets relaxed for large vev ratios w_μ/w_e. The bound derived from $B(\mu^- \to e^- e^- e^+)$ is more sensitive to the soft-SUSY gaugino mass M. To be more precise, our analysis yields the following upper limits on m_ν:

$$
\begin{array}{c|c}
\text{Scenario} & m_\nu \, [\text{MeV}] \\
\hline
\text{I} & 0.20 \\
\text{II} & 0.57 \\
\text{III} & 0.43 \\
\text{IV} & 0.89
\end{array}
\tag{6.8}
$$

at 90% CL. We also remark that τ-lepton number is assumed to be conserved so as to protect a primordial excess in L_τ from being erased by processes that are in thermal equilibrium. It is then obvious that for scenarios with $w_\mu/w_e = 1$ and $M = 200$ GeV, $m_{\nu_\tau} < 0.9$ MeV. From (6.8), we see that scenario I gives a stronger limit than the experimental one on the mass of ν_μ, which is currently $m_{\nu_\mu} < 0.27$ MeV at 90% CL [13]. As the non-SM couplings depend crucially on the τ-neutrino mass, the less than 1 MeV upper bound on a massive neutrino gives little chance to see new-physics effects in other observables. However, if $\Delta L_e = 0$ in the model, $i.e.$ $w_e = \varepsilon_e = 0$, inequality (6.7) is trivially fulfilled and the so-derived neutrino mass bound does not apply any longer.

In Fig. 1(b), numerical estimates reveal that non-SM contributions to the invisible Z-boson width are one order of magnitude smaller than the present experimental sensitivity. As a result, experimental searches for physics beyond the SM, based solely on neutrino counting at the Z peak, are bound to be inadequate to unravel the nature of our minimal $\not{R}$ model.

From Fig. 2(a), it can be seen that our minimal $\not{R}$ model may predict universality-breaking effects via the observable $U_{b\tau}$ in excess of 10^{-3}. Such new-physics phenomena might be seen at LEP, if all the experimental data accumulated in the year 1995 are analyzed.

Furthermore, in Fig. 2(b), we give theoretical predictions for the observable $R_\pi/R_\pi^{SM} - 1$ given in Eq. (5.17). Possible deviations from lepton universality in charged-current interactions turn out to be one order of magnitude smaller than those that can be accessed in experiment. Also, beyond the realm of detection are found to be possible violations of charged-current universality in the decays $\tau \to e\nu\nu$ and $\mu \to e\nu\nu$, which are measured by virtue of the physical quantities $R_{\tau e}$ and $R_{\tau\mu}$. Theoretically, similar is predicted to be the

situation for the size of the FCNC Z-boson mediated decays, such as $\tau^- \to \mu^- e^- e^+$ and $Z \to ll'$. More explicitly, it is estimated that

$$B(\tau^- \to \mu^- e^- e^+) \lesssim 1. 10^{-9},$$
$$B(Z \to l^- l'^+ \text{ or } l^+ l'^-) \lesssim 1. 10^{-8},$$
$$1 - \frac{R_{\tau e}}{R_{\tau e}^{SM}} \lesssim 1. 10^{-4},$$
$$1 - \frac{R_{\tau \mu}}{R_{\tau \mu}^{SM}} \lesssim 1. 10^{-4}. \tag{6.9}$$

There may also be other places where R-parity violation could manifest its presence. Of course, if neutralinos are lighter than the Z boson, one could search for distinctive signatures caused by decays of the form $Z \to \nu_\tau \chi^0$ or $\tau^\pm \chi^\mp$, where χ^0 and $\chi^\pm$ decay subsequently into two b-quark jets accompanied by a large amount of missing mass [34]. However, if the production threshold of heavy neutralinos and charginos is above the LEP centre of mass energy, one then has to rely on studies of possible indirect non-SM signals via sensitive observables devoid of ambiguities coming from the evaluation of hadronic matrix elements, as those discussed in Sections 5.1–5.4. In the same logic, R-parity violating effects may also be probed in the $\nu_\mu e$ scattering, even though experimental data do not impose very stringent constraints as compared to those resulting from $B(\mu \to e^- e^- e^+)$ [9,40]. Since our minimal $\not{R}$ model only modifies the leptonic sector, one may derive useful constraints from atomic parity violation measurements of the effective 'weak charge', Q_W, of a heavy nucleus. In the case of $^{133}_{55}$Cs, one has [40]

$$Q_W^{exp}(^{133}_{55}\text{Cs}) - Q_W^{SM}(^{133}_{55}\text{Cs}) = 73.5 \cdot [\tilde{\xi}_L(1-d)\tilde{\xi}_L^\dagger]_{11} \leq 3.74, \tag{6.10}$$

at 1σ. The above bound turns out to be rather weak when compared to that derived from $B(\mu \to eee)$. Finally, for reasons that have already been mentioned in Section 5.3, possible limits obtained directly from forward-backward-asymmetry observables similar to $\Delta \mathcal{A}_{ll'}$ are estimated to be much weaker than those determined by the universality-breaking parameter $U_{br}^{(ll')}$ in Eq. (5.8), and are therefore not taken into consideration here.

7 Conclusions

The minimal R-parity broken SUSY model contains bilinear lepton-number-violating terms (ε_i), which cannot in general be eliminated by a re-definition of the superfields provided soft-SUSY breaking parameters are simultaneously present in the superpotential.

The consideration of these ε_i mass terms, which involve the chiral multiplets of the left-handed leptons and the Higgs field with $Y = +1$, give rise naturally to non-vanishing vev's, w_i, of the scalar neutrinos after the spontaneous breaking of the gauge symmetry. In particular, if the vectors $\vec{w}$ and $\vec{\varepsilon}$, spanned in the flavour space, satisfy a kind of alignment relation, $\vec{\Lambda} = 0$, forced, e.g., by some horizontal symmetry, the afore-mentioned w_i and ε_i parameters are not restricted by limits on the τ-neutrino mass. Furthermore, constraints from primordial nucleosynthesis and the observed BAU have been considered. Specifically, to evade BAU constraints has been sufficient to impose that at least one separate leptonic number has to be conserved in our $\not{R}$ model, e.g. $w_\tau = \varepsilon_\tau = 0$ and $w_e = \varepsilon_e = 0$.

Our main interest has been to investigate the phenomenological implications of this novel $\not{R}$ model in the light of a number of terrestrial, astrophysical, and cosmological constraints. To be more concrete, we have considered a typical set of $\not{R}$ models as is stated in (6.6) and confronted it with results obtained from LEP, CLEO and other experiments. We have found that the resulting non-SM contributions to the couplings $Z\nu\nu$, Zll', and $Wl\nu$ show a strong correlation with the τ-neutrino mass and vanish in the massless limit. This direct correlation between the size of R-parity-violating phenomena and the magnitude of the neutrino mass appears to be a generic feature of most of the R-parity broken models considered in the literature [34,39]. In our analysis, the most severe constraint comes from $B(\mu \to eee)$ for $\not{R}$ scenarios, where $\Delta L_\tau = 0$, and L_e and L_μ are maximally violated. In this way, we have been able to set an upper bound on m_{ν_τ} by means of Eq. (6.7). For instance, for $M = \mu = 2M_W$ and $w_e = w_\mu$, we find that $m_{\nu_\tau} \stackrel{<}{\sim} 1$ MeV. Especially, for scenario I in Eq. (6.6), we have $m_\nu < 0.2$ MeV as has been given in Eq. (6.8), which is even tighter than the current experimental bound on the mass of the μ neutrino. The remaining observables leave the main bulk of the parameter space unconstrained. The most encouraging prediction is obtained for the universality-violating observable U_{br}, with $U_{br} \stackrel{<}{\sim} 2.10^{-3}$. Such phenomena might be seen at LEP, when the analysis of all the data of the year 1995 is completed.

For our purposes, we need not study the combined effect of the trilinear R-parity-violating couplings λ and λ', i.e. $\lambda_{ijk} = \lambda'_{ijk} = 0$. The reason is that the Yukawa couplings λ_{ijk} and λ'_{ijk} are not sufficient to explain possible new-physics phenomena that can be shown up in certain low-energy processes and LEP observables, such as $B(l^- \to l'^- l_1^- l_1^+)$, $B(Z \to ll')$, and U_{br}, discussed in Sections 5.1–5.3. In this context, we remark that the KARMEN anomaly can, in principle, be explained by assuming the presence of a fourth light neutralino, even though an unnaturally small value of $\mu = \mathcal{O}(10)$ MeV may be required.

A Appendix

For completeness, we present that part of the scalar potential (2.8) that contains the charged singlet fields E_i. This also consists of a lepton-number conserving contribution (V_+^L) and a lepton-number violating one $(V_+^{\not L})$. The former reads

$$
\begin{aligned}
V_+^L &= [\mu_{+ij}^2(E_i^*E_j) + \text{H.c.}] + [\mu_{ij}(\phi_2^\dagger\varphi_i)E_j + \text{H.c.}] + [\mu_{ij}'(\phi_1^\dagger\varphi_i)E_j + \text{H.c.}] \\
&\quad + \lambda(\sum_k E_k^*E_k)^2 + (\tilde\kappa_{jk} - \lambda\delta_{jk})(\phi_1^\dagger\phi_1)(E_j^*E_k) + \lambda(\phi_2^\dagger\phi_2)(E_k^*E_k) \\
&\quad + (\mu_{ijnm} - \lambda\delta_{in}\delta_{jm})(\varphi_i^\dagger\varphi_n)(E_j^*E_m) + 4\tilde\kappa_{nmij}(\varphi_n^\dagger\varphi_i)(E_m^*E_j)\,, \quad\quad\quad (A.1)
\end{aligned}
$$

where we have defined

$$
\begin{aligned}
\lambda &= \frac{1}{2}g'^2\,, \\
\mu_{ij} &= \mu^*h_{ij}\,, \\
\tilde\kappa_{jk} &= h_{ij}^*h_{ik} = \tilde\kappa_{kj}^*\,, \\
\mu_{ijnm} &= h_{ij}^*h_{nm} = \mu_{nmij}^*\,, \\
\tilde\kappa_{nmij} &= \lambda_{knm}^*\lambda_{kij} = \tilde\kappa_{ijnm}^*\,. \quad\quad\quad (A.2)
\end{aligned}
$$

In Eq. (A.2), h_{ij} and λ_{ijk} are couplings from the superpotential (2.2) and (2.3), respectively. μ_{+ij}^2 and μ_{ij}' are soft-SUSY breaking parameters from Eq. (2.7).

For the lepton-number-violating contribution, we obtain

$$
V_+^{\not L} = -2i\tilde\kappa_{ijk}(\phi_1^T\tau_2\varphi_j)(E_i^*E_k) + \kappa_{ijk}'(\varphi_i^T\tau_2\varphi_j)E_k + \text{H.c.}\,, \quad\quad\quad (A.3)
$$

where

$$
\tilde\kappa_{ijk} = h_{ni}^*\lambda_{njk}\,, \quad\quad\quad (A.4)
$$

and κ_{ijk}' is a soft-SUSY breaking parameter contained in Eq. (2.7).

By analogy with Eq. (2.19), from $V_+^L + V_+^{\not L}$, we can derive conditions for not having CP violation in this part of the potential. These conditions are listed below

$$
\begin{aligned}
\mu_{ijnm}^*\eta_{L_n}^*\eta_{L_i}\eta_{+m}^*\eta_{+j} &= \mu_{ijnm}\,, \\
\tilde\kappa_{jk}^*\eta_{+k}^*\eta_{+j} &= \tilde\kappa_{jk}\,, \\
\mu_{ij}^*\eta_2\eta_{L_i}^*\eta_{+j}^* &= \mu_{ij}\,, \\
\mu_{ij}'^*\eta_1\eta_{L_i}^*\eta_{+j}^* &= \mu_{ij}'\,, \\
\tilde\kappa_{nmij}^*\eta_{L_n}^*\eta_{L_i}\eta_{+m}^*\eta_{+j} &= \tilde\kappa_{nmij}\,, \\
\tilde\kappa_{ijk}^*\eta_1^*\eta_{L_j}^*\eta_{+i}\eta_{+k}^* &= \tilde\kappa_{ijk}\,, \\
\kappa_{ijk}'^*\eta_{L_i}^*\eta_{L_j}^*\eta_{+k}^* &= -\kappa_{ijk}'\,, \quad\quad\quad (A.5)
\end{aligned}
$$

where summation convention is not implied. In Eq. (A.5), η_{+j} are the CP phases of the scalar fields E_j, similar to the notation of Eq. (2.19). In general, both sets, (2.19) and (A.5), should not be viewed independently of one another. For instance, using the equalities in Eqs. (2.19) and (A.5), one can derive (no summation convention)

$$\Im m\left(\lambda_6 \mu_{ij} \mu_{nm}'^{*} \tilde{\kappa}_{mj} \kappa_{in}\right) = 0\,,$$
$$\Im m\left(\lambda_6 \mu_{ijnm}^{*} \mu_{nm} \mu_{ij}'^{*}\right) = 0\,,$$
$$\Im m\left(\mu_{ijnm}^{*} \kappa_{ni} \tilde{\kappa}_{mj}^{*}\right) = 0\,, \tag{A.6}$$

and many similar relations of this kind.

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Figure Captions

Fig. 1: $B(\mu^- \to e^- e^- e^+)$ and $1 - B(Z \to \nu\nu)/B_{SM}(Z \to \text{invisible})$ versus τ-neutrino mass, for the scenarios given in Eq. (6.6). We indicate the present phenomenological limit by an horizontal dotted line.

Fig. 2: The observables $U_{b\tau}$ and $R_\pi/R_\pi^{SM} - 1$ as a function of the τ-neutrino mass, for $\not{R}$ models stated in Eq. (6.6). Horizontal dotted lines denote the present experimental bounds.
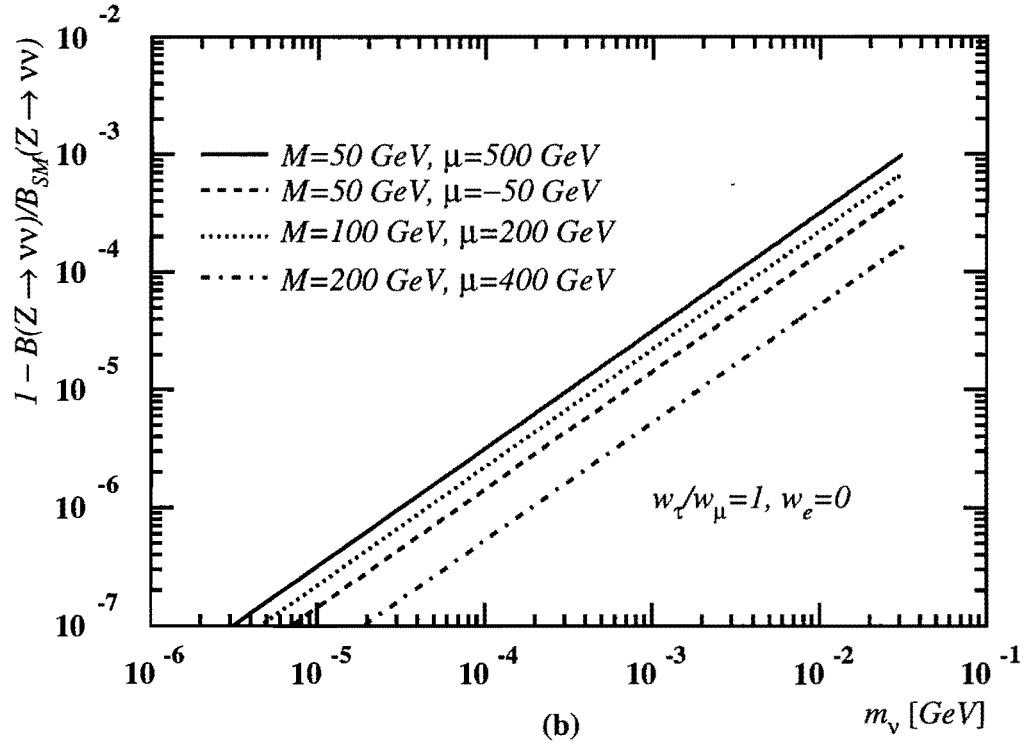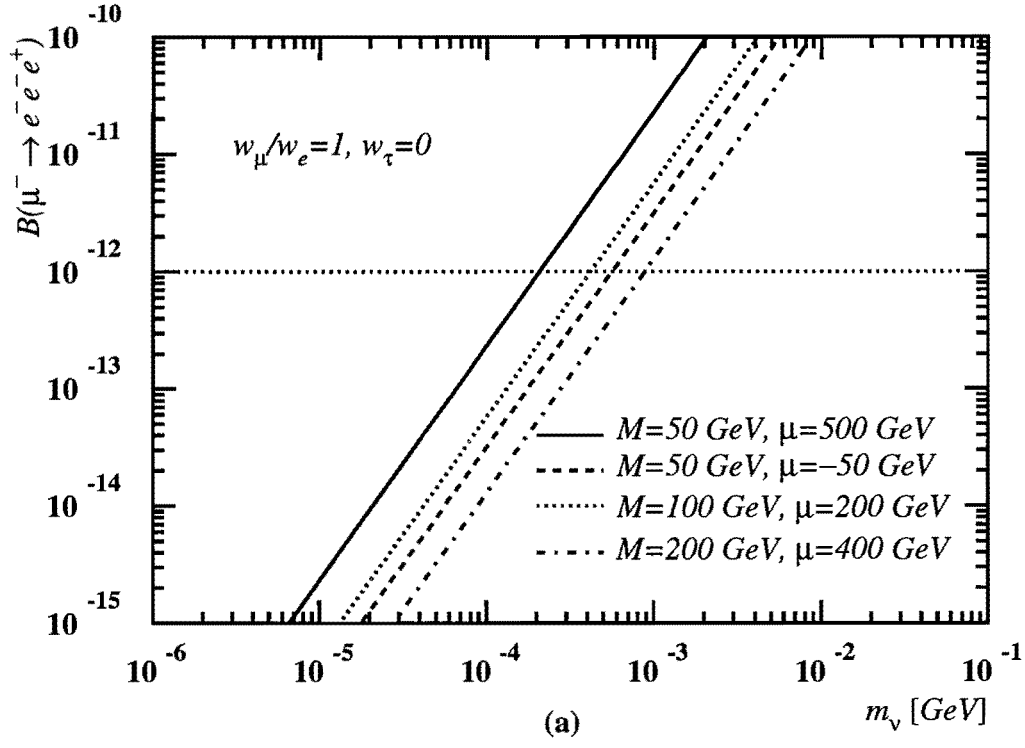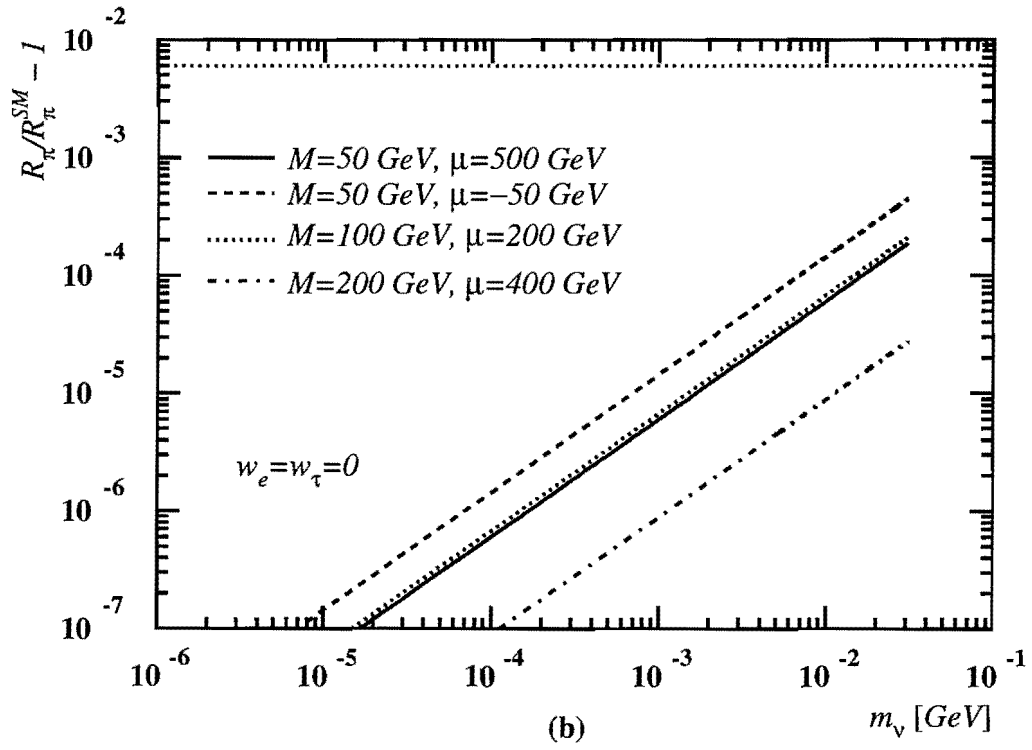
(a)

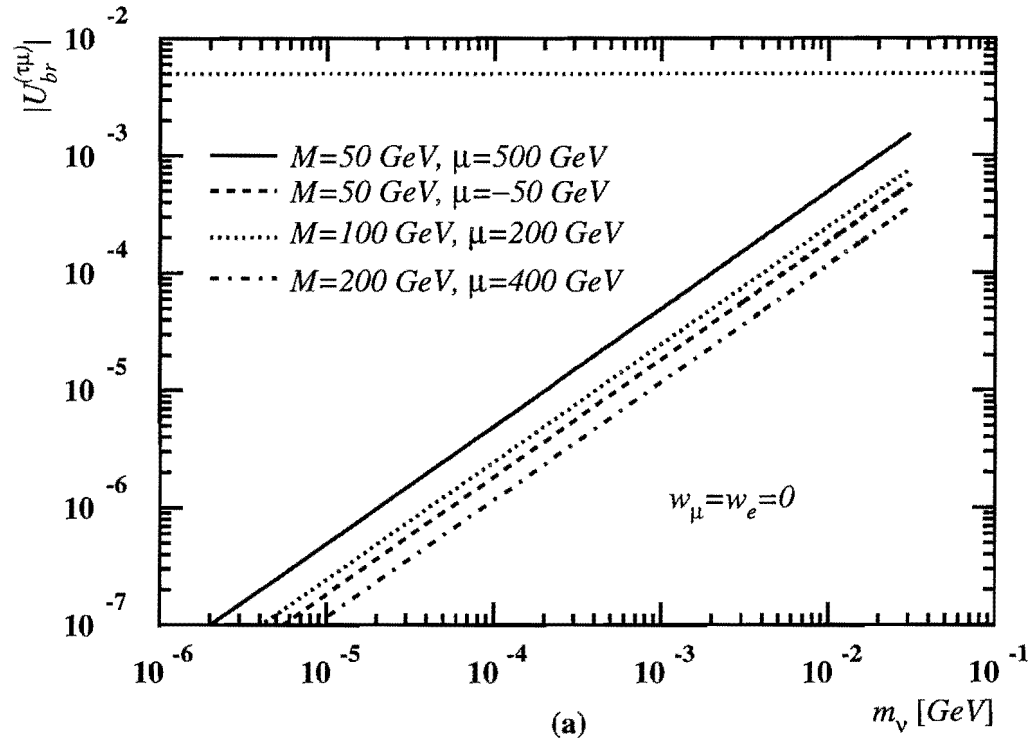(b)

Fig. 1

Fig. 2