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Spin-dependent non-singlet structure functions in next-to-leading order

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Abstract

We study in detail the flavor-non-singlet component of polarized structure functions in the framework of a consistent and complete next-to-leading order ($\mathcal{O}(\alpha_s)$) analysis. In this context, we discuss some important features of the calculation of the next-to-leading order corrections. Particular emphasis is put on the Q^2-evolution of sum-rules for the first moments of the non-singlet structure functions which, as we show, could serve to explore $SU(2)$ and $SU(3)$ breaking effects in relations between baryonic β-decay matrix elements and in the proton's polarized sea. Furthermore we make predictions for polarized non-singlet structure functions possibly measurable in a conceivable $\vec{e}\vec{p}$ collider mode of HERA.

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1 Introduction

The spin-dependent structure functions of protons, neutrons, and deuterons have received much attention both experimentally and theoretically in the past years. Since the advent of the EMC result [1] on the proton's $g_1^p(x, \langle Q^2 \rangle = 10.7\text{GeV}^2)$, most theoretical studies have been focussed on the singlet component of this structure function in order to explain its unexpected experimental smallness, hereby assuming that the non-singlet (NS) component is rather well understood. First experimental evidence for this latter assumption was provided recently by the confirmation [2, 3] of the Bjørken sum-rule [4] which relates the integrals (first moments) of g_1^p and g_1^n. However, this sum-rule, which depends merely on the fundamental $SU(2)$ isospin ($u \leftrightarrow d$) symmetry between matrix elements of charged and neutral axial currents and is therefore expected to hold, does not entirely fix the first moment of the NS component, $g_{1,NS}^p$, of g_1^p, since the latter can be written (in leading order (LO)) as

$$g_{1,NS}^p = \frac{1}{12}\Delta A_3(1) + \frac{1}{36}\Delta A_8(1) \quad , \tag{1}$$

where in terms of the first moments of the polarized (anti)quark densities $\Delta \overset{(-)}{q}(x, Q^2)$ we have

$$\begin{aligned} \Delta A_3(1) &= \int_0^1 \left(\Delta u + \Delta\bar{u} - \Delta d - \Delta\bar{d}\right)(x, Q^2)dx \quad , \\ \Delta A_8(1) &= \int_0^1 \left(\Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} - 2(\Delta s + \Delta\bar{s})\right)(x, Q^2)dx \quad . \end{aligned} \tag{2}$$

The Bjørken sum-rule [4] is equivalent to

$$\Delta A_3(1) = F + D = g_A = 1.2573 \pm 0.0028 \quad , \tag{3}$$

but information on ΔA_8 can only be obtained from hyperon β-decays. Assuming full $SU(3)$ symmetry between hyperon decay matrix elements of the flavor-changing weak axial currents and the neutral ones, one finds (with F, D taken from ref. [5])

$$\Delta A_8(1) = 3F - D = 0.579 \pm 0.025 \quad . \tag{4}$$

This approach has been seriously questioned in ref. [6], where the suggestion was made that $SU(3)_f$ symmetry is broken in such a way that only the *valence* $\Delta q_v = \Delta q - \Delta\bar{q}$ content of $\Delta A_3(1)$, $\Delta A_8(1)$, rather than the full combinations $\Delta A_3(1)$, $\Delta A_8(1)$, enters eqs.

(3),(4). In view of these uncertainties and of the fact that the baryonic β-decays cannot tell us anything about $g_{1,NS}^p$ except for the first moment, it is interesting to examine the NS sector of polarized structure functions in more detail in order to find other possible experimental clues to ΔA_3, ΔA_8, and the polarized valence densities Δu_v, Δd_v, hereby improving our present understanding of the relation between the first moments of these quantities and the F and D values. In this respect, it is necessary to consider not only the NS piece of the electromagnetic (e.m.) structure functions $g_1^{p,n} \equiv g_1^{ep,en}$, but also the polarized electroweak structure functions g_3, g_4 studied in refs. [7-14], which partly are pure NS quantities. Since possible measurements of such structure functions are likely to be performed at Q^2 much higher than those relevant for eqs. (3),(4), it is important to theoretically understand the Q^2-evolution of spin-dependent NS structure functions as well as possible. For this purpose, it is necessary to improve the theoretical predictions in the NS sector, by performing a complete and consistent next-to-leading order (NLO) analysis. All ingredients for this are available as we will see below, and there are some features of the NLO corrections which are interesting in themselves. Also, the theoretical predictions are much more reliable for the NS sector since it is not plagued by the anomaly contribution as is the case for the singlet contributions to polarized structure functions [15].

The remainder of this paper is organized as follows: In Section 2 we review the main LO results on spin-dependent NS structure functions. Section 3 gives a detailed account of the determination of the NLO corrections. Section 4 contains the numerical evaluation of our results, and in Section 5 we summarize our findings.

2 Spin-dependent non-singlet structure functions in leading order

The structure functions g_1, g_3, and g_4 appear in the hadronic tensor as (see, e.g., [12])

$$W_{\mu\nu} = -i\epsilon_{\mu\nu\rho\sigma}\frac{q^\rho p^\sigma}{p\cdot q}g_1 + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) g_3 + \frac{1}{p\cdot q}\left(p_\mu - \frac{p\cdot q}{q^2}q_\mu\right)\left(p_\nu - \frac{p\cdot q}{q^2}q_\nu\right) g_4 \quad , \tag{5}$$

where we have already replaced $s^\mu \to p^\mu$ for the spin vector of a longitudinally polarized nucleon with momentum p. In eq. (5), q denotes the momentum of the virtual boson probing the hadron. As is well-known, g_3 and g_4 do not contribute to purely e.m. scattering, but appear in (parity-violating) electroweak neutral current (NC) or charged current (CC) lepton-nucleon interactions. Therefore, their experimental accessibility may seem remote presently. However, they could be measured in $\overset{(-)}{\nu}$-scattering off a polarized target, and they would certainly play a role in deep-inelastic scattering (DIS) experiments at HERA if also the 820 GeV proton beam could be longitudinally polarized [16]. All relevant cross section formulas for $\overset{(-)}{\nu}\vec{p}$, $e^{\pm}\vec{p}$ interactions in terms of g_1, g_3, and g_4 can be found, e.g., in refs. [12, 13] and need not be repeated here. As was shown in [12], the LO expressions for the structure functions can be cast into the forms

$$\begin{aligned} g_1(n,Q^2) &= \frac{1}{2}\sum_q S_q\left(\Delta q(n,Q^2)+\Delta\bar{q}(n,Q^2)\right) \\ g_3(n,Q^2) &= \frac{1}{2}\sum_q R_q\left(\Delta q(n,Q^2)-\Delta\bar{q}(n,Q^2)\right) \\ g_4(n-1,Q^2) &= \sum_q R_q\left(\Delta q(n,Q^2)-\Delta\bar{q}(n,Q^2)\right)=2g_3(n,Q^2) \quad , \end{aligned} \tag{6}$$

where, as usual, the Mellin-n moments of a Bjørken-x-dependent function $g(x)$ are defined as $g(n)=\int_0^1 x^{n-1}g(x)dx$. The coefficients S_q and R_q in eq. (6) depend on the exchanged boson, γ^*, Z^0, $W^{\pm}$, in the DIS process and can be found in [12]. Obviously, for $W^{\pm}$-exchange (CC interactions), only the quark *or* the antiquark of a given flavor contributes, depending on the charge of the W. To become more specific, we write the various conceivable structure functions in LO in terms of the NS quark combinations (for eqs. (7-20) below we drop the obvious argument (n,Q^2) from all quantities)

$$\begin{aligned} &\Delta u_v=\Delta u-\Delta\bar{u} \ , \quad \Delta d_v=\Delta d-\Delta\bar{d} \ , \\ &\Delta A_3=\Delta u+\Delta\bar{u}-\Delta d-\Delta\bar{d} \ , \\ &\Delta A_8=\Delta u+\Delta\bar{u}+\Delta d+\Delta\bar{d}-2(\Delta s+\Delta\bar{s}) \end{aligned} \tag{7}$$

and the singlet

$$\Delta\Sigma=\Delta u+\Delta\bar{u}+\Delta d+\Delta\bar{d}+\Delta s+\Delta\bar{s} \tag{8}$$

(for $f=3$ flavors) as [7]

$$g_1^{ep\,(e.m.)} = \frac{1}{12}\Delta A_3+\frac{1}{36}\Delta A_8+\frac{1}{9}\Delta\Sigma \tag{9}$$

$$g_3^{ep,NC} = \frac{1}{4}\left(\frac{2}{3}\Delta u_v+\frac{1}{3}\Delta d_v\right) \tag{10}$$

for $\vec{e}\vec{p}$ NC interactions, where for g_1^{ep} we have only written the purely electromagnetic contribution (which dominates [12] if polarized electrons are used) since the other NC contributions do not easily lead to NS quantities. $g_3^{ep,NC}$ has been written only for the dominant [12] contribution from γZ^0 interference[1]. For CC structure functions ($\overset{(-)}{\nu} p \to e^{\pm} X$ or $e^{\pm} p \to \overset{(-)}{\nu} X$ scattering) one has [12]

$$
\begin{aligned}
g_1^{\nu p} &= \Delta d + \Delta\bar{u} + f_1(\lambda)\Delta s = \frac{1}{2}(\Delta d_v - \Delta u_v + \Delta\Sigma) - \frac{1}{6}(1 - f_1(\lambda))(\Delta\Sigma - \Delta A_8) \quad (11)\\
g_1^{\bar{\nu} p} &= \Delta u + \Delta\bar{d} + f_1(\lambda)\Delta\bar{s} = \frac{1}{2}(\Delta u_v - \Delta d_v + \Delta\Sigma) - \frac{1}{6}(1 - f_1(\lambda))(\Delta\Sigma - \Delta A_8) \quad (12)\\
g_3^{\nu p} &= -\Delta d + \Delta\bar{u} - f_3(\lambda)\Delta s = \frac{1}{2}(\Delta A_3 - \Delta u_v - \Delta d_v) - \frac{f_3(\lambda)}{6}(\Delta\Sigma - \Delta A_8) \quad (13)\\
g_3^{\bar{\nu} p} &= -\Delta u + \Delta\bar{d} + f_3(\lambda)\Delta\bar{s} = -\frac{1}{2}(\Delta A_3 + \Delta u_v + \Delta d_v) + \frac{f_3(\lambda)}{6}(\Delta\Sigma - \Delta A_8)\ , \quad (14)
\end{aligned}
$$

where we have introduced functions $f_i(\lambda = Q^2/(Q^2 + m_c^2))$ with $f_i(1) = 1$ (massless limit) which take fully into account the effects of the charm mass m_c in the $s \to c$ transition. In the LO considered in eqs. (11-14), the $f_i(\lambda)$ are simply given by the 'slow-rescaling' prescription [17] which yields $x_{Bj} = Q^2/2pq \to x_{Bj}/\lambda$ and therefore $f_1(\lambda) = f_3(\lambda) \equiv f(\lambda) = \lambda^n$ for the nth moment in eqs. (11-14). The expressions for the nth moment of the structure function $g_4/2x$, $g_4(n-1, Q^2)/2$, are the same [7, 8, 11, 12] as the right-hand-sides (rhs) of eqs. (10),(13),(14) with, however [12], $f_4(\lambda) = f(\lambda)/\lambda$ in the CC case. In this way one finds a (slight) violation of the Callan-Gross-like relation $g_4(n-1, Q^2) = 2g_3(n, Q^2)$ by terms of $\mathcal{O}(m_c^2/Q^2)$ due to the CC $s \to c$ transitions. Note that eqs. (11-14) can be easily seen to receive only corrections of $\mathcal{O}(\sin^2\theta_c m_c^2/Q^2)$ when taking into account the effects of Cabibbo-mixing; we can safely neglect these small terms. The structure functions for DIS scattering off neutron-targets can be easily obtained by changing the signs of the ΔA_3 terms and interchanging $\Delta u_v \leftrightarrow \Delta d_v$ in eqs. (9-14). From eqs. (9-14) we can, e.g., construct the following NS combinations [7]:

$$
\begin{aligned}
g_1^{ep\,(e.m.)} - g_1^{en\,(e.m.)} &= \frac{1}{6}\Delta A_3 \quad (15)\\
g_1^{\bar{\nu} p} - g_1^{\nu p} &= \Delta u_v - \Delta d_v \quad (16)\\
g_3^{\nu p} - g_3^{\nu n} &= \Delta A_3 \quad (17)\\
g_3^{\bar{\nu} p} + g_3^{\nu p} &= -(\Delta u_v + \Delta d_v) \quad (18)
\end{aligned}
$$

[1]As compared to ref. [12] we have dropped a factor $\left(4\sin^2\Theta_W\cos^2\Theta_W(Q^2 + M_Z^2)/Q^2\right)^{-1}$ from the normalization of $g_3^{ep,NC}$, where Θ_W is the Weinberg angle and M_Z the Z^0 mass.

$$9\left(g_1^{ep\,(e.m.)}+g_1^{en(e.m.)}\right) \;-\; \frac{6}{2+f(\lambda)}\left(g_1^{\nu p}+g_1^{\nu n}\right)=\frac{1}{2}\frac{5f(\lambda)-2}{f(\lambda)+2}\Delta A_8 \quad (19)$$

$$g_1^{\bar{\nu}p}+g_1^{\nu p}-\frac{2+f(\lambda)}{f(\lambda)}\left(g_3^{\bar{\nu}p}-g_3^{\nu n}\right) \;=\; \Delta A_8 \;, \quad (20)$$

etc. Besides these relations, the NC $g_3^{ep,NC}(n,Q^2)$ in eq. (10) is obviously also an entire NS quantity.

3 Next-to-leading order corrections

In order to study the evolution of the polarized NS structure functions in NLO it is necessary to recall the well-known solution (see, e.g., [18]) of the NS renormalization group equation relating the Mellin-n moments of a polarized NS structure function $\tilde{g}_{i,NS}=g_1$, g_3, $g_4/2x$ at the input scale Q_0^2 and at $Q^2>Q_0^2$:

$$\begin{aligned}\tilde{g}_{i,NS}(n,Q^2) &= \left(1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\left[2\Delta C_q^i(n)+\frac{\Delta\gamma_{NS}^1(n)}{2\beta_0}-\frac{\beta_1\gamma_{NS}^0(n)}{2\beta_0^2}\right]\right)\\ &\quad\times\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{NS}^0(n)/2\beta_0}\tilde{g}_{i,NS}(n,Q_0^2) \qquad (21)\\ &= \left(1+\frac{\alpha_s(Q^2)}{2\pi}\Delta C_q^i(n)\right)\Delta q_{NS}^i(n,Q^2)\;, \qquad (22)\end{aligned}$$

where

$$\frac{\alpha_s(Q^2)}{4\pi}\simeq\frac{1}{\beta_0\ln Q^2/\Lambda_{\overline{MS}}^2}-\frac{\beta_1}{\beta_0^3}\frac{\ln\ln Q^2/\Lambda_{\overline{MS}}^2}{\left(\ln Q^2/\Lambda_{\overline{MS}}^2\right)^2} \qquad (23)$$

with the QCD scale parameter $\Lambda_{\overline{MS}}$, $\beta_0=11-2f/3$, $\beta_1=102-38f/3$, and

$$\begin{aligned}\Delta q_{NS}^i(n,Q^2) &= \left(1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\left[\frac{\Delta\gamma_{NS}^1(n)}{2\beta_0}-\frac{\beta_1\gamma_{NS}^0(n)}{2\beta_0^2}\right]\right)\\ &\quad\times\left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{NS}^0(n)/2\beta_0}\Delta q_{NS}^i(n,Q_0^2) \qquad (24)\end{aligned}$$

is the suitable NS combination of polarized quark densities evolved from Q_0^2 to Q^2 via the LO (one-loop) and NLO (two-loop) NS anomalous dimensions[2] $\gamma_{NS}^0(n)$ and $\Delta\gamma_{NS}^1(n)$. The precise form of the NLO pieces (ΔC_q^i, $\Delta\gamma_{NS}^1$) in eqs. (21), (22) and (24) depends on the factorization scheme convention adopted for the relation (22) between the NS

[2]For the perturbative expansions of the QCD-β-function and the anomalous dimensions we use the expansion parameter $\alpha_s/4\pi$ (see, e.g., [18] for detailed expressions). Note also that we have omitted the 'Δ' for the (polarized) $\gamma_{NS}^0(n)$ since this quantity is trivially equal to its unpolarized counterpart [9, 19].

structure function and the relevant NS quark densities beyond the leading order. Since the $\tilde{g}_{i,NS}$ are physical, i.e. measurable, quantities and the $\gamma^0_{NS}(n)$ [9, 19] is convention-independent, it becomes evident from eq. (21) that the scheme dependences of $\Delta C^i_q(n)$ and $\Delta\gamma^1_{NS}(n)$ cancel each other such that the combination $2\Delta C^i_q(n) + \Delta\gamma^1_{NS}(n)/2\beta_0$ is scheme *independent* [18]. Needless to say that removing all NLO quantities (ΔC^i_q, $\Delta\gamma^1_{NS}$, β_1) in eqs. (21-24), we recover the LO results of eqs. (9-14) with the quark density combinations evolving according to the LO (γ^0_{NS} [9, 19]) NS evolution equation.

Both the essential ingredients for the NLO calculation, $\Delta C^i_q(n)$ and $\Delta\gamma^1_{NS}(n)$, can be found in the literature. To facilitate the further discussion, let us first turn to the first moments, $n = 1$, which are of particular interest in the polarized case. As was discussed in ref. [20] in the framework of the OPE, the operator corresponding to the first moments $\Delta A_3(1, Q^2)$ and $\Delta A_8(1, Q^2)$ is nothing but the NS axial current which is a conserved quantity and thus has vanishing anomalous dimensions to all orders, which in particular means $\Delta\gamma^1_{NS}(1) = 0$. Furthermore, the value of the first moment of the Wilson coefficient $\Delta C^1_q(n)$ for $g_1^{ep\,(e.m.)}$ was found in [20, 21] to be $\Delta C^1_q(1) = -3C_F/2$, giving $-3C_F$ for the scheme independent combination $2\Delta C^i_q(1) + \Delta\gamma^1_{NS}(1)/2\beta_0$ and, according to eq. (22), leading to the factor $(1 - \alpha_s/\pi)$ in the NS sector of g_1. Of course, both $\Delta C^1_q(n)$ and $\Delta\gamma^1_{NS}(n)$ depend on the regularization scheme adopted in their calculation[3], and different schemes will in principle give different answers even for the first moments $\Delta C^1_q(1)$, $\Delta\gamma^1_{NS}(1)$, though still respecting the condition $2\Delta C^1_q(1) + \Delta\gamma^1_{NS}(1)/2\beta_0 = -3C_F$. However, the conservation of the NS axial current *dictates* the vanishing of $\Delta\gamma^1_{NS}(1)$, and hence the value $\Delta C^1_q(1) = -3C_F/2$, which means that a scheme transformation has to be performed if these results are not automatically respected by the regularization scheme used.

Let us briefly list the results obtained for $\Delta C^1_q(1)$ (to be calculated in the process $\vec{\gamma}^*\vec{q} \to q(g)$ to $\mathcal{O}(\alpha_s)$) using various regulators also used previously in the corresponding calculations in the unpolarized case. The result of ref. [20], $\Delta C^1_q(1) = -3C_F/2$, was found using massless but off-shell incoming quarks and on-shell outgoing gluons. The same result for $\Delta C^1_q(1)$ is obtained [22] if one uses massless on-shell quarks, but off-shell gluons ($k^2 > 0$). Turning to dimensional regularization in $\overline{\mathrm{MS}}$, the result depends on how

[3]Needless to say that this has to be the same in the calculations of these quantities.

the Dirac matrix γ_5, appearing due to the projector on the quark's helicity, is treated in $n \neq 4$ dimensions. The prescription of a totally anticommuting γ_5 by Chanowitz et al. [23] yields [24, 25] again[4] $\Delta C_q^1(1) = -3C_F/2$. The same result is obtained [26] in the closely related γ_5 scheme of ref. [27], taking the $\gamma^* q$ vertex as the 'reading point' to be defined in that scheme. However, when using the original scheme of 't Hooft and Veltman [28] and Breitenlohner and Maison [29] (HVBM), or the equivalent prescription of refs. [30, 31], one obtains [25, 32]

$$\Delta C_q^1(1) = -\frac{7}{2} C_F \quad , \tag{25}$$

(naively) corresponding to a correction $(1 - 7\alpha_s/3\pi)$ in the NS sector of g_1 and to a non-zero value for the anomalous dimension, $\Delta\gamma_{NS}^1(1)$, in contradiction to the conservation of the NS axial vector current. Finally, the same result, $\Delta C_q^1(1) = -7C_F/2$, is obtained for massive on-shell quarks ($m_q \neq 0$) in the process $\vec{\gamma}^* \vec{q} \to q(g)$. For completeness, we list all the results for $\Delta C_q^1(n)$ for arbitrary Mellin-n in the Appendix. Comparing with the corresponding results [33-37] for the Wilson coefficient $C_q^2(n)$ for the unpolarized structure function $F_2/2x$ in the various regularizations, one finds that *all* the $\Delta C_q^1(n)$ with the property $\Delta C_q^1(1) = -3C_F/2$ satisfy

$$C_q^2(n) - \Delta C_q^1(n) = C_F \left(\frac{1}{n} + \frac{1}{n+1} \right) \tag{26}$$

(corresponding to $C_q^2(z) - \Delta C_q^1(z) = C_F(1+z)$ in Bjørken-x space). This implied regularization scheme independence of $C_q^2(n) - \Delta C_q^1(n)$ can be understood as follows: As a consequence of the factorization theorem, the difference $C_q^{DY}(n) - 2C_q^2(n)$, where $C_q^{DY}(n)$ are the n-moments of the $\mathcal{O}(\alpha_s)$ (NS) quark corrections to the unpolarized Drell-Yan process $q\bar{q} \to \gamma^*(g)$, has to be the same in *any* scheme (see, for example, [33, 34]). The same is true (see [24, 25]) for the difference $\Delta C_q^{DY}(n) - 2\Delta C_q^1(n)$ with the $\mathcal{O}(\alpha_s)$ quark corrections $-\Delta C_q^{DY}(n)$ to the *polarized* Drell-Yan process $\vec{q}\vec{\bar{q}} \to \gamma^*(g)$. On the other hand, the annihilating quark lines in this process trivially give $\Delta C_q^{DY}(n) = C_q^{DY}(n)$ *if* the regularization scheme used in the calculation of $\Delta C_q^{DY}(n)$ respects chirality conservation. It then automatically follows that $C_q^2(n) - \Delta C_q^1(n)$ is also the same in all such schemes[5].

[4]The same result in dimensional regularization was found earlier in [21] without specifying the γ_5-prescription.

[5]Alternatively, one can see the expected scheme invariance of $C_q^2(n) - \Delta C_q^1(n)$ from the fact that [33, 34] $C_q^2(n) - C_q^3(n) = C_F\left(\frac{1}{n} + \frac{1}{n+1}\right)$ is scheme invariant (where $C_q^3(n)$ is the coefficient function of the unpolarized structure function F_3) and from the similar appearance of F_3 and g_1 in the hadronic tensor.

In contrast to eq. (26), we have for the calculation in the HVBM scheme and for the $m_q \neq 0$ calculation (which gave $\Delta C_q^1(1) = -7C_F/2$)

$$C_q^2(n) - \Delta C_q^1(n) = C_F\left(\frac{1}{n} + \frac{1}{n+1}\right) + \left\{ \begin{array}{ll} 4C_F\left(\frac{1}{n} - \frac{1}{n+1}\right) & \text{HVBM} \\ 2C_F\frac{1}{n} & (m_q \neq 0) \end{array} \right. . \tag{27}$$

According to our previous observations, these regulators then necessarily break the relation $\Delta C_q^{DY}(n) = C_q^{DY}(n)$, i.e., break chirality. In fact, it was shown in ref. [25] that

$$\Delta C_{q,HVBM}^{DY}(n) = C_q^{DY}(n) - 8C_F\left(\frac{1}{n} - \frac{1}{n+1}\right) \tag{28}$$

in the HVBM scheme, showing how the terms $\sim (1/n - 1/(n+1))$ in eqs. (27),(28) cancel out in the difference $\Delta C_q^{DY}(n) - 2\Delta C_q^1(n)$, but individually break the relations $\Delta C_q^{DY}(n) = C_q^{DY}(n)$ and $C_q^2(n) - \Delta C_q^1(n) = C_F(1/n + 1/(n+1))$. Furthermore, the terms $\sim (1/n - 1/(n+1))$ in the HVBM scheme originate [25] from a configuration where the incoming and the outgoing particles in the process $\gamma^* q \to qg$ become collinear, and thus should rather be understood as part of the polarized (NLO) quark densities. Finally, as far as the massive calculation is concerned, the term $2C_F/n$ after the curly bracket in eq. (27) can be traced back to have its origin in a chirality breaking term $\sim m_q^2$ which survives the eventual limit $m_q \to 0$ since it happens to be multiplied by a double-pole term. Having found the origin of the additional terms in eq. (27) which lead to $\Delta C_q^1(1) = -7C_F/2$, we expect that similar terms would be present in the $\Delta\gamma_{NS}^1(n)$ when calculated in the HVBM or the massive scheme, such that scheme transformations, by means of $2\Delta C_q^1(n) + \Delta\gamma_{NS}^1(n)/2\beta_0 =$ invariant, could be performed to eliminate these terms from both ΔC_q^1 and $\Delta\gamma_{NS}^1(n)$. Hereby one would obtain the correct values $\Delta C_q^1(1) = -3C_F/2$ and $\Delta\gamma_{NS}^1(1) = 0$ (as dictated [20] by the the conservation of the NS axial current), and the relation $C_q^2(n) - \Delta C_q^1(n) = C_F(1/n + 1/(n+1))$ would be restored in each case.

To complete the discussion of the Wilson coefficients ΔC_q^i, let us specify our final choices for the coefficients for g_1, g_3, and g_4. Since, as we will see below, the anomalous dimension $\Delta\gamma_{NS}^1(n)$ is known within dimensional regularization in the $\overline{\text{MS}}$-scheme, we have to choose the Wilson coefficients accordingly. This means that the coefficient in (A.4) [21, 24, 26] (or the one in (A.5) after elimination of the chirality breaking term $\sim (1/n - 1/(n+1))$) is the relevant one for $g_1^{(e.m.)}$. It turns out [32, 38] that the coefficient is the same if electroweak contributions to g_1 (e.g. $g_1^{\nu p}$) from transitions between massless

($\lambda = 0$) quarks $q \to q'$ are considered. For the corrections to the structure functions g_3, $g_4/2x$ one finds[6] [32, 38]

$$\begin{aligned}
\Delta C_q^3(n) &= \Delta C_q^1(n) + C_F\left(\frac{1}{n} - \frac{1}{n+1}\right) \\
\Delta C_q^4(n) &= \Delta C_q^3(n) + 2C_F\frac{1}{n+1} \quad .
\end{aligned} \tag{29}$$

One notes the striking similarity to the relations between the quark corrections to the unpolarized structure functions F_3, F_1, $F_2/2x$ which is readily explained by the similarity of the corresponding hadronic tensors. Equation (29) shows that the Callan-Gross-type relation $g_4 = 2xg_3$ mentioned earlier is broken even for massless quarks beyond the LO. However, unlike its unpolarized analogue, $F_L \equiv F_2 - 2xF_1$, which (in the singlet case) also receives contributions from *gluon*-induced $\mathcal{O}(\alpha_s)$ corrections, $g_4 - 2xg_3 = 0$ is broken only by quark-induced corrections (eq. (29)) even in the singlet case, since corrections from incoming gluons cancel out for massless produced quarks [12].

In the case of the CC transition $s \to c$ we again have to take into account the mass of the charm quark which has an influence on the coefficient functions. For this purpose we have calculated the contribution of the process $W^+ s \to c(g)$ with $m_c \neq 0$ to g_1, g_3, g_4 in $\overline{\text{MS}}$ dimensional regularization, following the techniques developed in [39]. The results of our calculation can be found in the Appendix. It should be noted that the expressions have a smooth limit $m_c^2/Q^2 \to 0$ ($\lambda \to 1$), in which they reproduce eqs. (A.4),(29). From eqs. (A.6-A.9) we immediately read off the $\mathcal{O}(\alpha_s)$ corrections to the functions $f_i(\lambda, n)$ introduced in eqs. (11-14). For the first moment, $n = 1$, these functions then read in NLO:

$$\begin{aligned}
f_1(\lambda,1) &= \lambda\left(1 - 3\frac{\alpha_s}{2\pi}C_F\frac{(1-\lambda)}{\lambda}\ln(1-\lambda)\right) \\
f_3(\lambda,1) &= \lambda\left(1 + \frac{\alpha_s}{2\pi}C_F\frac{1-\lambda}{\lambda^2}\left[1 - \frac{\lambda}{2} + \left(\frac{1}{\lambda} - 1 - 3\lambda\right)\ln(1-\lambda)\right]\right) \\
f_4(\lambda,1) &= 1 \quad .
\end{aligned} \tag{30}$$

The last result that $f_4(\lambda, 1)$ receives no $\mathcal{O}(\alpha_s)$ corrections also holds for the corresponding function for the unpolarized structure function F_2 [40], where it is in accordance with the Adler sum-rule [41]. We emphasize that, similar to the unpolarized case [42, 43], our

[6] Eqs. (29) are actually independent of the regularization scheme chosen even in schemes where $\Delta C_q^1(1) \neq -3C_F/2$.

results (A.6-A.9) for the contribution of the transition $s \to c$ to the spin-dependent structure functions would enable a determination of the proton's $\overline{\text{MS}}$ polarized strange quark distribution via a detection of charmed final states in polarized CC DIS.

The (de facto) regularization scheme independence of the relation $C_q^2(n) - \Delta C_q^1(n) = C_F(1/n+1/(n+1))$ immediately implies that the scheme-dependent parts of the polarized and the unpolarized NS anomalous dimensions $\Delta\gamma_{NS}^1(n)$, $\gamma_{NS}^1(n)$ equal each other in all schemes. Even more, as was first observed in [20] and recently established in more detail in [26], the *full* expressions for $\Delta\gamma_{NS}^1(n)$ and $\gamma_{NS}^1(n)$ are exactly identical. This statement is correct in all regularization schemes, provided one has taken care to warrant $\Delta C_q^1(1) = -3C_F/2$ in the scheme used, eliminating, if present, chirality breaking terms as explained above.

There is, however, another subtlety involved in the equality of $\Delta\gamma_{NS}^1(n)$ and $\gamma_{NS}^1(n)$: As is well-known [44-46], there is no analytical continuation of the unpolarized $\gamma_{NS}^1(n)$ to arbitrary n, needed for the transformation from Mellin-n space into Bjørken-x space, that reproduces the results for $\gamma_{NS}^1(n)$ for *all* integer values of n. This is not unexpected since the OPE, first used to derive $\gamma_{NS}^1(n)$ in $\overline{\text{MS}}$ dimensional regularization [46], gives only an answer for *even* n if the moments of the NS contribution to the e.m. structure function F_2/x are considered, hereby artificially excluding odd values of n. Therefore, the analytic continuation of $\gamma_{NS}^1(n)$ only has to correctly reproduce the results for even values of n. On the other hand, as was shown in [47], *odd* values of n are relevant in the OPE for the NS combination $F_2^{\bar{\nu}p}/x - F_2^{\nu p}/x$ or for $F_3^{\bar{\nu}p} + F_3^{\nu p}$, meaning that in this case the analytic continuation of $\gamma_{NS}^1(n)$ has to reproduce the results at these values. These observations fit nicely and consistently to parton model considerations, where the NS quark combinations $q - q'$ and $q_v = q - \bar{q}$ can be easily seen to evolve [44, 48] with $P_+ \equiv P_{qq} + P_{q\bar{q}}$ and $P_- \equiv P_{qq} - P_{q\bar{q}}$, respectively, which are different beyond the LO, where P_{qq}, $P_{q\bar{q}}$ are the $q \to q$ and $\bar{q} \to q$ NLO NS splitting functions with flavor-non-diagonal contributions subtracted [44, 48]. The explicit calculation of P_{qq}, $P_{q\bar{q}}$ [44] shows that their Mellin-n moments satisfy[7]

$$\gamma_{NS}^1(n) = P_{qq}(n) + (-1)^n P_{q\bar{q}}(n) \quad , \tag{31}$$

[7]For simplicity we have normalized the $P_\pm$ relative to γ_{NS}^1.

which means that the analytic continuation of $\gamma^1_{NS}(n)$ which reproduces the values of $\gamma^1_{NS}(n)$ for *even* n equals the combination $P_+(n)$ for *arbitrary* n, whereas the other analytic continuation of $\gamma^1_{NS}(n)$, which is correct for *odd* n, corresponds to $P_-(n)$. In this way, the parton results of [44] provide the rule for the analytic continuation of the OPE results. The essence of all this is that the moments of the combination $F_2^{ep}/x - F_2^{en}/x$, or, more in general, the unpolarized $A_3(n,Q^2)$, $A_8(n,Q^2)$ (defined in analogy to eq. (7)) evolve with $P_+(n)$, whereas, e.g., the moments of $F_2^{\bar{\nu}p}/x - F_2^{\nu p}/x$, $F_3^{\bar{\nu}p} + F_3^{\nu p}$ (which consist of pure valence, $u_v(n,Q^2)$, $d_v(n,Q^2)$), evolve with $P_-(n)$.

The important difference in the polarized case is that the relevance of even and odd n in the OPE and for the analytic continuation is reversed here. As was shown in [7, 9], *odd* n contribute in the OPE analysis to the combinations $(g_1^{ep\,(e.m.)} - g_1^{en\,(e.m.)})(n,Q^2)$, $(g_1^{\bar{\nu}p} + g_1^{\nu p})(n,Q^2)$, $(g_3^{\nu p} - g_3^{\nu n})(n,Q^2)$, $(g_4^{\nu p} - g_4^{\nu n})(n-1,Q^2)$, whereas *even* n are relevant, e.g., for $(g_1^{\bar{\nu}p} - g_1^{\nu p})(n,Q^2)$, $(g_3^{\bar{\nu}p} + g_3^{\nu p})(n,Q^2)$, $(g_4^{\bar{\nu}p} + g_4^{\nu p})(n-1,Q^2)$. In terms of the polarized NS quark distribution combinations this means that $\Delta A_3(n,Q^2)$, $\Delta A_8(n,Q^2)$ (as defined in (7)) evolve with $P_-(n)$ and the polarized valence densities $\Delta u_v(n,Q^2)$, $\Delta d_v(n,Q^2)$ with $P_+(n)$ [49]. This situation is summarized by the relations $\Delta P_{qq} = P_{qq}$, $\Delta P_{q\bar{q}} = -P_{q\bar{q}}$ for the polarized analogues, $\Delta P_{q\stackrel{(-)}{q}}$, of $P_{q\stackrel{(-)}{q}}$.

4 Numerical results

We are now equipped with all ingredients for a consistent NLO analysis of the spin-dependent NS structure functions. Let us consider the first moment of the NS combinations in eqs. (10,15-20). To begin with, we recall that the first moment $P_-(1) = \gamma^1_{NS}(1) = \Delta\gamma^1_{NS}(1)$ vanishes [44]. In the unpolarized case this is in accordance with the Adler sum-rule [41] and the conservation of the number of valence quarks. In the polarized case it means that $\Delta A_3(n,Q^2)$ and $\Delta A_8(n,Q^2)$ for $n=1$ correctly do not evolve with Q^2, as required by the conservation of the NS axial current (see above):

$$\Delta A_{3,8}(1,Q^2) = \Delta A_{3,8}(1,Q_0^2) \quad . \tag{32}$$

In contrast to this, the first moment of P_+ is non-zero, which means that the first moment of the polarized valence densities evolves with Q^2 beyond the LO:

$$\Delta q_v(1,Q^2) = \left(1 + \frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\frac{P_+(1)}{2\beta_0}\right)\Delta q_v(1,Q_0^2) \quad , \tag{33}$$

with [44] $P_+(1) = 4C_F(C_F - C_A/2)(-13+12\zeta(2)-8\zeta(3)) \approx 2.5576$, and where we have used eq. (24) and the fact that the first moment of the LO NS anomalous dimension vanishes, $\gamma^0_{NS}(1) = 0$ [9, 19]. This yields the following sum-rules for the first moments of the polarized NS structure functions to $\mathcal{O}(\alpha_s)$:

$$\left(g_1^{ep\,(e.m.)} - g_1^{en\,(e.m.)}\right)(1,Q^2) = \frac{1}{6}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\Delta A_3(1,Q_0^2) \tag{34}$$

$$g_3^{ep,NC}(1,Q^2) = \frac{1}{4}\left(1-\frac{2\alpha_s(Q^2)}{3\pi}+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\frac{P_+(1)}{2\beta_0}\right)\left(\frac{2}{3}\Delta u_v+\frac{1}{3}\Delta d_v\right)(1,Q_0^2) \tag{35}$$

$$\frac{1}{2}g_4^{ep,NC}(0,Q^2) = \frac{1}{4}\left(1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\frac{P_+(1)}{2\beta_0}\right)\left(\frac{2}{3}\Delta u_v+\frac{1}{3}\Delta d_v\right)(1,Q_0^2) \tag{36}$$

$$\left(g_1^{\bar{\nu}p} - g_1^{\nu p}\right)(1,Q^2) = \left(1-\frac{\alpha_s(Q^2)}{\pi}+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\frac{P_+(1)}{2\beta_0}\right)(\Delta u_v-\Delta d_v)(1,Q_0^2) \tag{37}$$

$$\left(g_3^{\nu p} - g_3^{\nu n}\right)(1,Q^2) = \left(1-\frac{2\alpha_s(Q^2)}{3\pi}\right)\Delta A_3(1,Q_0^2) \tag{38}$$

$$\frac{1}{2}\left(g_4^{\nu p} - g_4^{\nu n}\right)(0,Q^2) = \Delta A_3(1,Q_0^2) \tag{39}$$

$$\left(g_3^{\bar{\nu}p} + g_3^{\nu p}\right)(1,Q^2) = -\left(1-\frac{2\alpha_s(Q^2)}{3\pi}+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\frac{P_+(1)}{2\beta_0}\right)(\Delta u_v+\Delta d_v)(1,Q_0^2) \tag{40}$$

$$\frac{1}{2}\left(g_4^{\bar{\nu}p} + g_4^{\nu p}\right)(0,Q^2) = -\left(1+\frac{\alpha_s(Q^2)-\alpha_s(Q_0^2)}{4\pi}\frac{P_+(1)}{2\beta_0}\right)(\Delta u_v+\Delta d_v)(1,Q_0^2) \tag{41}$$

$$\left[9\left(g_1^{ep\,(e.m.)}+g_1^{en(e.m.)}\right)-\frac{6}{2+f_1(\lambda,1)}\left(g_1^{\nu p}+g_1^{\nu n}\right)\right](1,Q^2) = \frac{1}{2}\,\frac{5f_1(\lambda,1)-2}{f_1(\lambda,1)+2}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right)\Delta A_8(1,Q_0^2) \quad , \tag{42}$$

etc. It should be noted that eq. (20) receives singlet contributions beyond the leading order, therefore we have not written down this equation any more. Eqs. (34-42) show how in principle measurements of the first moments of polarized NS structure functions even at large Q^2 can serve to independently determine the combinations $(\Delta u_v \pm \Delta d_v)(1,Q_0^2)$, $\Delta A_3(1,Q_0^2)$ and $\Delta A_8(1,Q_0^2)$. This is particularly interesting considering the question raised earlier of which combination of polarized parton distributions can be related to the F, D values measured in baryonic β-decays. To simplify the discussion, we follow the recent NLO analysis [49] to assume that at the low input scale $Q_0^2 = 0.34$ GeV^2 ($\equiv \mu_{NLO}^2$ [49]) we can neglect any effects of $SU(2)$ isospin breaking in relating β-decay matrix elements of charged and neutral currents as well as $SU(2)_f$ breaking in the proton's polarized sea. We then have $\Delta A_3(1, Q_0^2) = (\Delta u_v - \Delta d_v)(1, Q_0^2) = F + D$, and the rhs of eqs. (34,37-39) are completely specified, leading to unique predictions for the combinations of structure functions on the lhs in NLO of QCD. The first of these is of course the well-known Bjørken sum-rule [4] to $\mathcal{O}(\alpha_s)$ [20][8]. The results for eqs. (34,37-39) are displayed in Fig. 1 as functions of Q^2, where we have used [49] $\Lambda_{\overline{\rm MS}}^{(f=4)} = 200$ MeV. To account for $SU(3)$ breaking effects we parametrize the input quantities appearing on the rhs of eqs. (40-42) in the following way:

$$\Delta A_8(1, Q_0^2) = (3F - D)(1 - \epsilon_1) \qquad (43)$$

$$(\Delta u_v + \Delta d_v)(1, Q_0^2) = \frac{1}{1 - \epsilon_2}(3F - D) \ , \qquad (44)$$

which yields $(3F - D)/2(1 - \epsilon_2) + (F + D)/6$ for the combination $(2\Delta u_v/3 + \Delta d_v/3)(1, Q_0^2)$ in eqs. (35,36). Eqs. (43,44) are general enough to take into account all possible sources of $SU(3)$ breaking: ϵ_1 determines the deviation of the first moment $\Delta A_8(1, Q_0^2)$ from the value $3F - D$ obtained from hyperon β-decays. Such a deviation will occur if the use of $SU(3)$ symmetry for relating the matrix elements of charged and neutral axial currents is not justified. In this case, ϵ_1 could be significantly different from zero, even such that only the *valence* quarks contribute to $3F - D$ [6]. This possibility is taken into account by the parameter ϵ_2 which would vanish in the latter case. From the definition of ΔA_8 one furthermore sees that ϵ_1 and ϵ_2 together determine the amount of $SU(3)_f$ breaking in the proton's polarized sea:

$$2\frac{\Delta\bar{u} + \Delta\bar{d} - 2\Delta\bar{s}}{\Delta u_v + \Delta d_v}(1, Q_0^2) = (1 - \epsilon_1)(1 - \epsilon_2) - 1 \ . \qquad (45)$$

Fig. 1 shows our predictions for the NS structure functions of eqs. (35,36,40-42) for the conceivable choices [49] $\epsilon_1 = 0$, $\epsilon_2 = 0.105$ and $\epsilon_1 = 0.40$, $\epsilon_2 = 0$. It becomes obvious that the effects of changes in the ϵ_i are larger than the present experimental 4%-uncertainty [5] in the value for $3F - D$ (see eq. (4)) and that therefore a measurement of the quantities shown would help to decide about the amount of $SU(3)$ breaking. In particular, the parameter ϵ_2 could possibly be determined in NC,CC experiments with polarized beams

[8]Note that actually the corrections to $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s^3)$ to this sum-rule are known [31].

at HERA [16] via a measurement of $g_4^{ep,NC}(0,Q^2)$ or $(g_4^{\bar{\nu}p} + g_4^{\nu p})(0,Q^2) \equiv (g_4^{e^-p,CC} + g_4^{e^+p,CC})(0,Q^2)$ (or their g_3-analogues). Using the full Mellin-n-dependent expression for $\Delta C_q^4(n)$ from eqs. (29),(A.4) in eq. (22), we can obtain NLO predictions for the Bjørken-x dependence of the latter structure functions:

$$\left.\begin{array}{l} g_4^{ep,NC}(n-1,Q^2) \\ (g_4^{e^-p,CC} + g_4^{e^+p,CC})(n-1,Q^2) \end{array}\right\} = \left(1 + \frac{\alpha_s(Q^2)}{2\pi}\Delta C_q^4(n)\right)\left\{\begin{array}{l} \frac{1}{2}\left(\frac{2}{3}\Delta u_v + \frac{1}{3}\Delta d_v\right)(n,Q^2) \\ (-2)\left(\Delta u_v + \Delta d_v\right)(n,Q^2) \end{array}\right. , \tag{46}$$

where, again, the polarized valence quark densities are to be evolved according to eq. (24) with the correct analytic continuation $P_+(n)$ of the $\gamma_{NS}^1(n)$ found in [44, 46, 50]. The results for $g_4^{ep,NC}(x,Q^2)$ and $(g_4^{e^-p,CC} + g_4^{e^+p,CC})(x,Q^2)$ at $Q^2 = 1000$ GeV^2, found after Mellin-inverting eq. (46), are shown in Fig. 2, where for the polarized input valence densities $\Delta q_v(x,Q_0^2)$ at $Q_0^2 = 0.34$ GeV^2 we have used the two sets determined in ref. [49]. Both sets give a very good description of all existing data on deep-inelastic spin asymmetries in the valence region $x \gtrsim 0.2$, but they differ in the assumptions made about the role of $SU(3)_f$ symmetry breaking effects and therefore have different first moments [49], corresponding to the ϵ_1, ϵ_2 values used in Fig. 1. Thus the variation in the results shown in Fig. 2 for the different sets of polarized valence input densities reflects the present theoretical uncertainty in the predictions. Conversely, Fig. 2 shows that also a measurement of $g_4^{ep,NC}(x,Q^2)$ and $(g_4^{e^-p,CC} + g_4^{e^+p,CC})(x,Q^2)$ for $x \leq 0.2$ at HERA, if experimentally achievable, could help to shed light on the importance of $SU(3)_f$ symmetry breaking.

The different evolution of the polarized valence quark densities and the combination ΔA_3 beyond the LO induces a dynamical breaking of the $SU(2)_f$ symmetry in the proton's polarized sea[9]. Eq. (24) and our considerations concerning the analytic continuation of $\gamma_{NS}^1(n)$ predict

$$\begin{aligned} 2\left(\Delta\bar{u} - \Delta\bar{d}\right)(n,Q^2) &= \frac{(\alpha_s(Q^2) - \alpha_s(Q_0^2))}{4\pi}\,\frac{(P_-(n) - P_+(n))}{2\beta_0} \\ &\times \left(\frac{\alpha_s(Q^2)}{\alpha_s(Q_0^2)}\right)^{\gamma_{NS}^0(n)/2\beta_0} \left(\Delta u_v - \Delta d_v\right)(n,Q_0^2) \quad , \end{aligned} \tag{47}$$

where we have again assumed the nonperturbative input at Q_0^2 to be $SU(2)_f$-symmetric, $\Delta\bar{u}(n,Q_0^2) = \Delta\bar{d}(n,Q_0^2)$. Using again the polarized input valence distributions of ref. [49]

[9]There is also a dynamical breaking of $SU(3)_f$ symmetry in the sea induced by ΔA_8. We do not pursue this effect since it is most probably dominated by the $SU(3)_f$ breaking in the nonperturbative input (see eq. (41)) due to the larger strange mass.

at $Q_0^2 = 0.34$ GeV^2, we obtain a prediction for $(\Delta\bar{u}-\Delta\bar{d})(x, Q^2 = 10\text{GeV}^2)$ which is shown in Fig. 3. Both sets of polarized input valence densities considered in [49] lead to entirely indistinguishable results, since only the input combination $(\Delta u_v - \Delta d_v)(x, Q_0^2)$ is needed here whose first moment is fixed by the value $F + D$ (eq. (3)) in both cases. It should be noted that such a dynamical breaking of $SU(2)_f$ symmetry in the sea induced by two-loop evolution was considered in the unpolarized case in ref. [45], where it was found to be very small. The results in the polarized case differ in sign (due to the interchange $P_+ \leftrightarrow P_-$) and slightly in magnitude (due to the polarized valence input instead of the unpolarized one) from the unpolarized results. However, the *relative* effect of $SU(2)_f$ breaking is much larger in the polarized case since the polarized sea densities are probably much smaller than their unpolarized counterparts. Even so, when taking the first moment, one obtains[10]

$$\begin{aligned} 2\left(\Delta\bar{u} - \Delta\bar{d}\right)(1, Q^2 = 10\text{GeV}^2) &= -\frac{(\alpha_s(Q^2) - \alpha_s(Q_0^2))}{4\pi}\frac{P_+(1)}{2\beta_0}(F + D) \\ &\approx 0.006 \;, \end{aligned} \tag{48}$$

which means that unless the input at Q_0^2 strongly breaks $SU(2)_f$ symmetry the effect of the breaking is probably small compared to the size of $|\Delta\bar{u}(1, Q^2)|$, $|\Delta\bar{d}(1, Q^2)|$ which might well be of the order $\gtrsim 0.05$ [49]. It is straightforward to introduce parameters δ_1, δ_2 in analogy to ϵ_1, ϵ_2, which would parametrize genuine $SU(2)$ breaking effects in the first moment of the polarized sea and in the relation between charged and neutral axial current β-decay matrix elements. Measurements of the first moment of the structure functions in eqs. (34,37-39) (see also Fig. 1) would then allow to determine these parameters and to pin down $SU(2)$ breaking effects.

5 Summary and conclusions

We have performed a detailed study of spin-dependent non-singlet structure functions in the framework of a complete and consistent NLO QCD calculation. Our analysis is based on a careful discussion of the calculation of the $\mathcal{O}(\alpha_s)$ corrections to the structure functions, in which we have examined the regularization scheme dependence of the NS coefficient function ΔC_q^1 for g_1 with respect to the constraints imposed by axial current

[10]This number depends quite crucially on the value chosen for the input scale Q_0. Taking, e.g., $Q_0^2 = 1$ GeV^2 the result in eq. (48) is reduced by a factor 3.

conservation. We have also shown how to correctly take into account the two-loop evolution of polarized NS quark combinations. A further ingredient of our study is the full inclusion of the charm mass effects in the charged current $s \to c$ contributions to polarized electroweak structure functions.

Our numerical analysis has revealed that conceivable measurements of spin-dependent NS structure functions at HERA or in $\overset{(-)}{\nu}$ scattering experiments off polarized nucleon targets would serve to improve our understanding of the relations between the first moment of NS combinations of polarized quark densities and the F, D values extracted from hyperon-β-decays, and would also shed light on $SU(2)_f$, $SU(3)_f$ breaking in the nucleon's polarized sea. Finally, we have also shown that the latter symmetries are dynamically broken by NLO evolution in the NS sector.

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Appendix

In this Appendix we list the results for the polarized coefficient function $\Delta C_q^1(n)$ using various regulators in its calculation from the process $\vec{\gamma}^* \vec{q} \to q(g)$. In all cases we have chosen to just subtract the collinear pole contribution, which is then factorized into the (bare) quark distributions. The singular terms are of the forms $\gamma_{NS}^0(n) \ln(Q^2/|m^2|)$ (if some mass or off-shellness $\sqrt{|m^2|}$ is used as the regulator) or $\gamma_{NS}^0(n)(-1/\hat{\epsilon})$ (in dimensional regularization in the $\overline{\mathrm{MS}}$-scheme). Since the first moment of $\gamma_{NS}^0(n)$ vanishes [9, 19], the pole contribution drops out from the more important first moment anyway. The results in Mellin-n space below can be easily transformed into Bjørken-x space with the help of the Appendix of ref. [34].

Off-shell massless quarks, on-shell gluons: This calculation corresponds to the one of ref. [20], but our results slightly differ by a term $C_F(-1+2/(n+1))$ (which vanishes for $n=1$) due to the specific operator normalization chosen in [20] (see the Appendix A.2 of ref. [20] for details). For the result in the unpolarized case (F_2) see [33].

$$\Delta C_q^1(n) = C_F\left[-\frac{3}{2n}+\frac{2}{n+1}+\frac{2}{n^2}-\frac{2}{(n+1)^2}+\frac{3}{2}\sum_{j=1}^{n}\frac{1}{j}-4\sum_{j=1}^{n}\frac{1}{j^2}\right] \quad . \tag{A.1}$$

On-shell massless quarks, off-shell gluons: The $\Delta C_q^1(n)$ for this calculation can be obtained from [22]. For the unpolarized case see [35].

$$\begin{aligned}\Delta C_q^1(n) &= C_F\left[-\frac{9}{4}-\frac{3}{2n}+\frac{3}{n+1}+\frac{2}{n^2}-\frac{1}{(n+1)^2}\right.\\ &\quad\left.+\left(\frac{3}{2}-\frac{1}{n(n+1)}\right)\sum_{j=1}^{n}\frac{1}{j}-4\sum_{j=1}^{n}\frac{1}{j^2}+2\sum_{j=1}^{n}\frac{1}{j}\sum_{k=1}^{j}\frac{1}{k}\right] \quad . \end{aligned}\tag{A.2}$$

On-shell massive quarks, off-shell gluons: In this regularization we obtain:

$$\begin{aligned}\Delta C_q^1(n) &= C_F\left[-\frac{5}{2}-\frac{5}{2n}+\frac{2}{n+1}+\frac{1}{n^2}-\frac{2}{(n+1)^2}\right.\\ &\quad\left.+\left(\frac{7}{2}+\frac{1}{n(n+1)}\right)\sum_{j=1}^{n}\frac{1}{j}-2\sum_{j=1}^{n}\frac{1}{j^2}-2\sum_{j=1}^{n}\frac{1}{j}\sum_{k=1}^{j}\frac{1}{k}\right] \quad . \end{aligned}\tag{A.3}$$

Note that $\Delta C_q^1(1) = -7C_F/2$ in this scheme. See refs. [36, 20] for the corresponding unpolarized result.

Dimensional regularization: Using the γ_5-prescription of [23] (or its more systematic and consistent generalization [27]) one obtains in the $\overline{\text{MS}}$-scheme [24, 26]:

$$\begin{aligned}\Delta C_q^1(n) &= C_F\left[-\frac{9}{2}+\frac{1}{2n}+\frac{1}{n+1}+\frac{1}{n^2}\right.\\ &\quad\left.+\left(\frac{3}{2}-\frac{1}{n(n+1)}\right)\sum_{j=1}^{n}\frac{1}{j}-2\sum_{j=1}^{n}\frac{1}{j^2}+2\sum_{j=1}^{n}\frac{1}{j}\sum_{k=1}^{j}\frac{1}{k}\right] \quad . \end{aligned}\tag{A.4}$$

The same result in dimensional regularization was found earlier in [21] without specifying the γ_5-prescription. However, using the original scheme of 't Hooft and Veltman [28] and Breitenlohner and Maison [29] (or the equivalent one of refs. [30, 31]), one finds [25, 32] an additional term

$$\Delta C_q^1(n)^{\text{HVBM}} = \Delta C_q^1(n)^{(\text{A.4})} - 4C_F\left(\frac{1}{n}-\frac{1}{n+1}\right) \quad , \tag{A.5}$$

which leads to $\Delta C_q^1(1) = -7C_F/2$. For the unpolarized case see [34, 37].

We finally present our results for the coefficient functions $\Delta\tilde{C}_q^i$ for g_1, g_3, $g_4/2x$ for the transition $s \to c$, fully taking into account the effects due to the charm quark mass. The calculation was performed in $\overline{\rm MS}$ dimensional regularization in the γ_5-scheme of ref. [27], choosing the axial vertex as the reading point. Our Bjørken-x space results for $\Delta\tilde{C}_q^1$, $\Delta\tilde{C}_q^3$, $\Delta\tilde{C}_q^4$ fully agree with those of ref. [39] for the unpolarized $h_{3,q}$ $h_{1,q}$, $h_{2,q}$ (for F_3, F_1, $F_2/2x$), respectively, after eliminating an error in the coefficient A_2 in that paper which should read K_A instead of $K_A/2$. The differences $\Delta\tilde{C}_q^4 - \Delta\tilde{C}_q^1$, $\Delta\tilde{C}_q^4 - \Delta\tilde{C}_q^3$ which are regularization scheme independent, are in agreement with the results of [40] for the corresponding differences in the unpolarized case. We note that the results of ref. [42] seem in slight disagreement with both [39] (even after correction of the above mentioned error) and [40] and also with our calculation in this respect. Here we present the Mellin-n moments of our results. For this purpose it is convenient to present the moments for the differences $\Delta\tilde{C}_q^i(n,\lambda) - \Delta C_q^i(n)$, where the $\Delta C_q^i(n)$ are the (usual) massless coefficient functions given in eqs. (29), (A.4), and $\lambda = Q^2/(Q^2+m_c^2)$. Defining the sum

$$\mathcal{S}_\alpha(n,\lambda) \equiv \lambda^{-\alpha n}\left[\ln(1-\lambda) + \sum_{j=1}^{n}\frac{\lambda^{\alpha j}}{j}\right] \quad ,$$

and [39]

$$K_A(\lambda) \equiv \frac{1-\lambda}{\lambda}\ln(1-\lambda) \quad ,$$

we find:

$$\begin{aligned}
\Delta\tilde{C}_q^1(n,\lambda) - \Delta C_q^1(n) = C_F \Bigg[& -K_A(\lambda) + 2\sum_{i=1}^{n}\frac{1}{i}\left(\mathcal{S}_0(i,\lambda) - \mathcal{S}_1(i,\lambda)\right) \\
& + \frac{(n-1)(n+2)}{2n(n+1)}\left(\mathcal{S}_0(n,\lambda) - \mathcal{S}_1(n,\lambda)\right) - \frac{n(n-1)}{2(n+1)}\frac{1-\lambda}{\lambda}\mathcal{S}_1(n,\lambda) \\
& - \left(\frac{3}{2} + \frac{1}{n+1} - 2\sum_{j=1}^{n}\frac{1}{j}\right)\ln\lambda \Bigg] \qquad (A.6)
\end{aligned}$$

$$\left(\Delta\tilde{C}_q^3 - \Delta\tilde{C}_q^1\right)(n,\lambda) - \left(\Delta C_q^3 - \Delta C_q^1\right)(n) = C_F\left[\frac{1-\lambda}{\lambda}\frac{1}{n+1} + \frac{(1-\lambda)^2}{\lambda^2}\mathcal{S}_1(n,\lambda)\right] \qquad (A.7)$$

$$\begin{aligned}
\left(\Delta\tilde{C}_q^4 - \Delta\tilde{C}_q^3\right)(n,\lambda) - \left(\Delta C_q^4 - \Delta C_q^3\right)(n) = C_F\Bigg[& K_A(\lambda) - \frac{(1-\lambda^2)(1-2\lambda)}{\lambda^2}\mathcal{S}_1(n,\lambda) \\
& -(1-\lambda)\left(\frac{2}{n} + \frac{1}{\lambda(n+1)}\right)\Bigg] \quad . \qquad (A.8)
\end{aligned}$$

The last term in eq. (A.6) which contains the LO $\gamma_{NS}^0(n)$ [9, 19] is introduced if one chooses the scale Q^2 as the factorization scale [39]. It should be noted that, like in the LO (see eqs. (11-14)), an additional factor of λ^n (λ^{n-1}) is needed to calculate the contribution to the structure functions $g_1(n, Q^2)$, $g_3(n, Q^2)$ ($g_4(n-1, Q^2)/2$).

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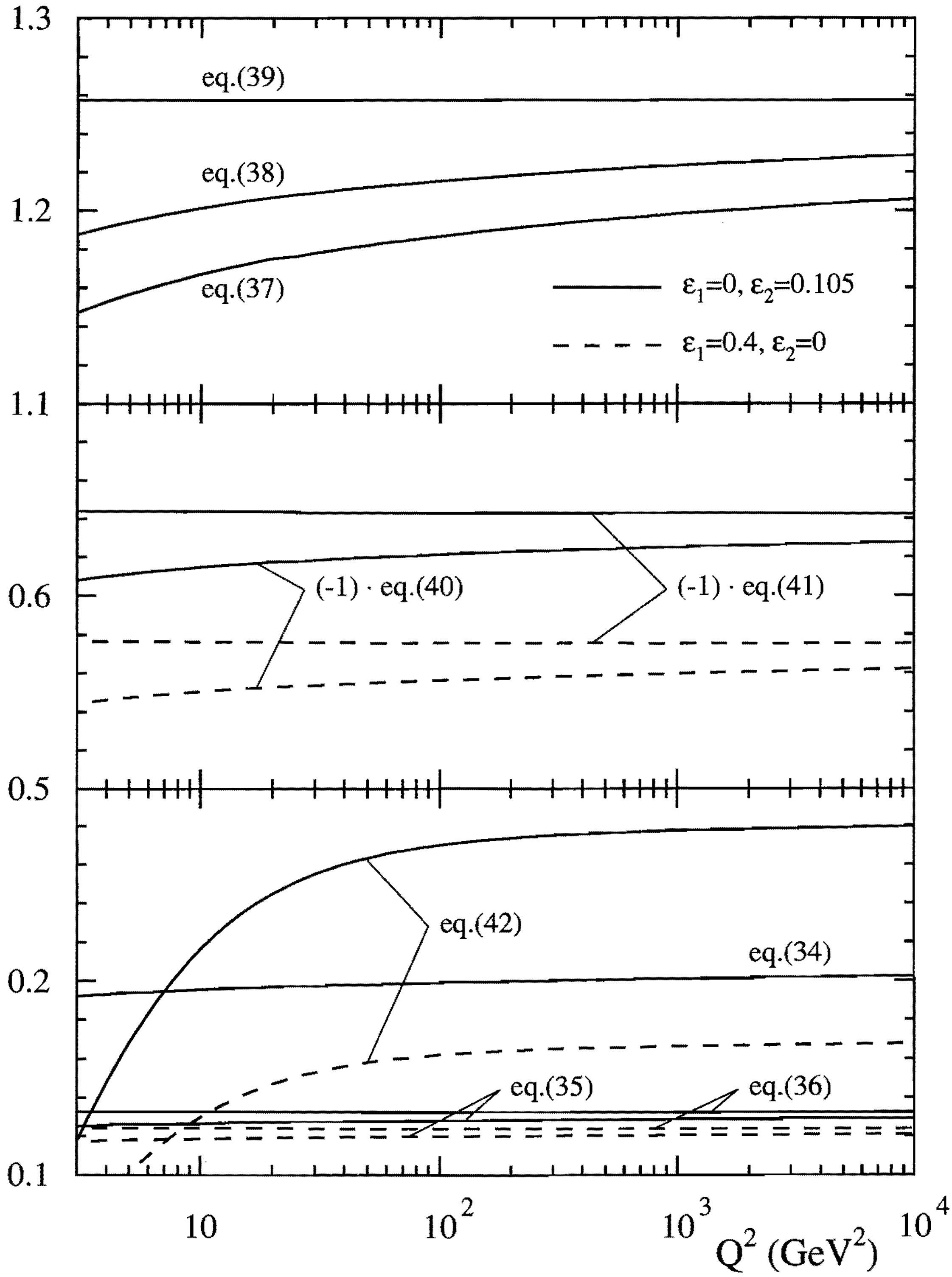
Figure Captions

Fig. 1 Predictions for the Q^2-evolution of the first moments of the various NS combinations of polarized structure functions as given in eqs. (34)-(42) for two conceivable choices of $SU(3)_f$ breaking parameters ϵ_1, ϵ_2 in eqs. (43),(44). The input scale for the evolution, $Q_0^2 = 0.34$ GeV^2, was chosen according to ref. [49], and $\alpha_s(Q^2)$ was calculated from eq. (23) with $\Lambda_{\overline{\mathrm{MS}}}$ from [49].

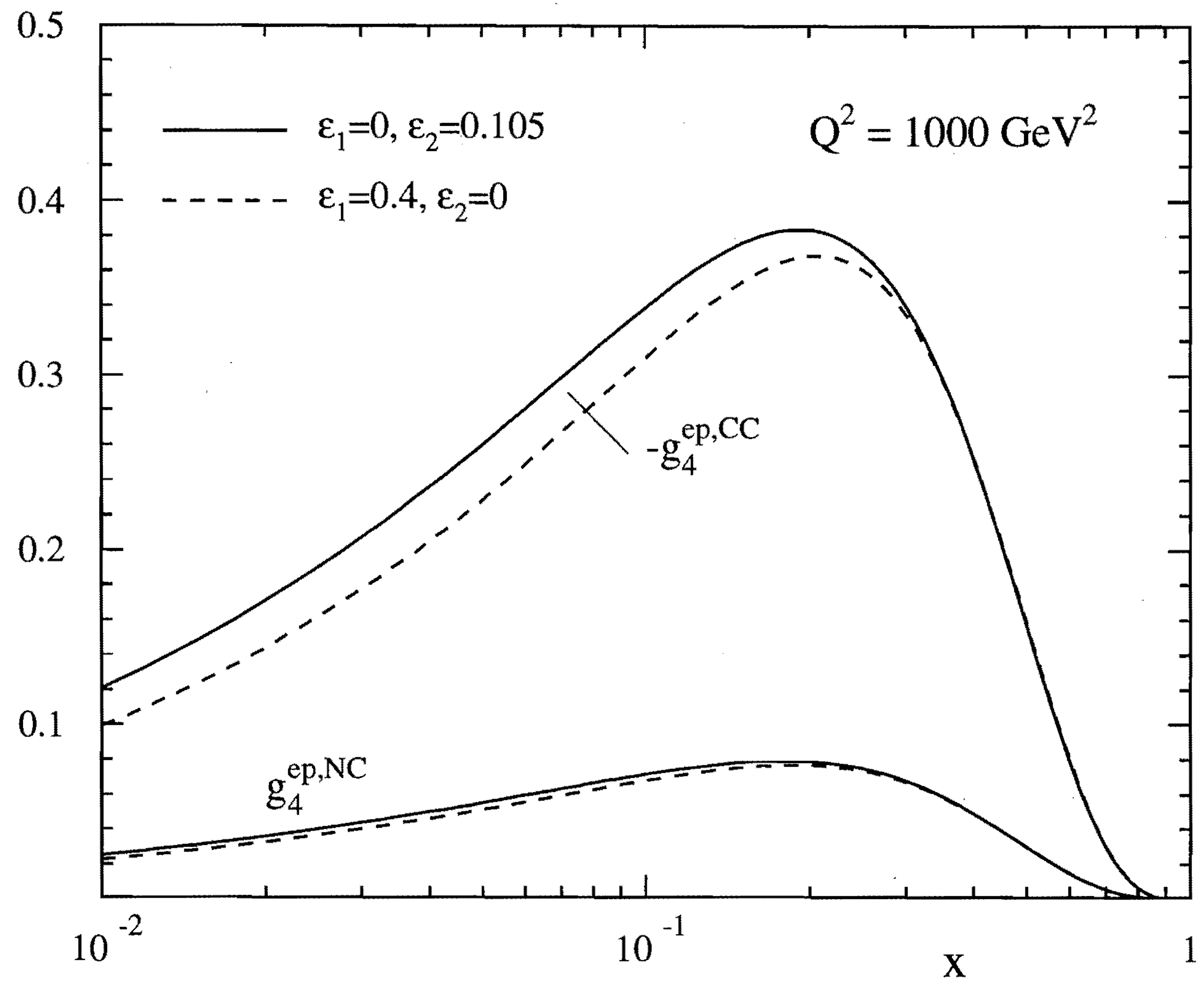
Fig. 2 Predictions for the NC and CC non-singlet structure functions (cf. eq. (46)) $g_4^{ep,NC}(x,Q^2)$ and $g_4^{ep,CC}(x,Q^2) \equiv \left(g_4^{e^-p,CC} + g_4^{e^+p,CC}\right)(x,Q^2)$, respectively, as measurable in a future polarized e^-p/e^+p collider mode of HERA [16]. For the predictions we have used the two sets of polarized input valence densities suggested in [49] which correspond to the $SU(3)_f$ breaking parameters introduced in Fig. 1.

Fig. 3 Prediction for the dynamical $SU(2)_f$ breaking of the proton's polarized sea $\left(\Delta\bar{u} - \Delta\bar{d}\right)(x,Q^2)$ at $Q^2 = 10$ GeV^2 according to eq. (47). The nonperturbative valence input $(\Delta u_v - \Delta d_v)(x,Q_0^2)$ at $Q_0^2 = 0.34$ GeV^2 was taken from the analysis in ref. [49]. For comparison the dashed line shows the averaged sea density $-\Delta\bar{q}(x,Q^2) \equiv -\left(\Delta\bar{u}(x,Q^2) + \Delta\bar{d}(x,Q^2)\right)/2$ determined within the 'standard scenario' of ref. [49].



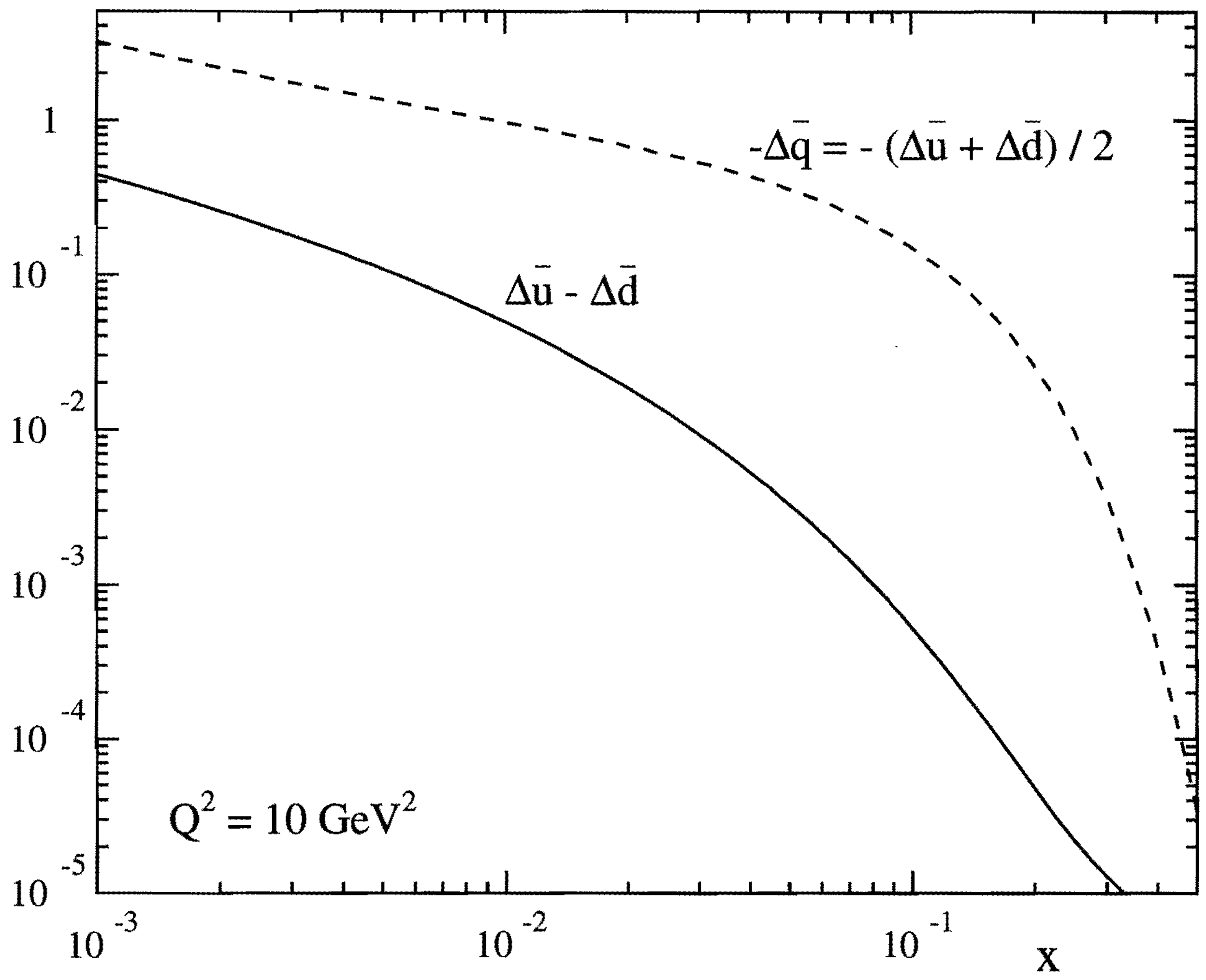
1.3
1.2
1.1
0.6
0.5
0.2
0.1
eq.(39)
eq.(38)
eq.(37)
ε1=0, ε2=0.105
ε1=0.4, ε2=0
(-1) · eq.(40)
(-1) · eq.(41)
eq.(42)
eq.(34)
eq.(35)
eq.(36)
10
10^2
10^3
10^4
Q^2 (GeV^2)

Fig. 1



0.5
0.4
0.3
0.2
0.1
ε_1=0, ε_2=0.105
ε_1=0.4, ε_2=0
Q^2 = 1000 GeV^2
$-g_4^{ep,CC}$
$g_4^{ep,NC}$
10^{-2}
10^{-1}
1
x

Fig. 2



$-\Delta\bar{q} = -(\Delta\bar{u} + \Delta\bar{d})/2$
$\Delta\bar{u} - \Delta\bar{d}$
$Q^2 = 10\ GeV^2$
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-3}
10^{-2}
10^{-1}
x

Fig. 3