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Abstract

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1 COMPOSITE SYSTEMS: FROM MOLECULES TO QUARKS

The first sign that there is an underlying structure at some level of matter is the existence of an excitation spectrum. Thus molecules exhibit a spectrum due to the quantised motions of their constituent atoms. In turn atomic spectra are due to their electrons, those of nuclei are due to their constituent protons and neutrons and, we now realise, those of the hadrons are due to their constituent quarks. Qualitatively these look similar but quantitatively they are radically different. Molecular excitations are on the scale of meV, atoms eV, nuclei MeV (note M versus m !) and the hadrons are hundreds of MeV. These represent, inter alia, the different length scales and binding energies of the respective structures. They also are related to the different momentum or energy scales needed for probes to resolve substructures directly.

The discovery of the atomic nucleus was achieved with probes (α particles) that are provided by natural radioactive sources. These resolved the atom and revealed its nucleus but saw the latter only as a point of charge; the inner structure was not then apparent. If the beams are, say, electrons that have been accelerated to energies of some hundreds of MeV, the protons and neutrons become visible as individuals. If the beams have energies of tens to hundreds of GeV, the inner structure of the nucleons is resolved and their quarks are directly seen.

I do not wish here to enter into a debate on the detailed relation between quarks as revealed in the latter "deep inelastic" experiments and those that drive spectroscop: it is the latter on which I shall concentrate. In particular I shall motivate the exciting possibility that new varieties of hadron are emerging which are associated with the degrees of freedom available to the force fields that bind quarks.

So first we need to ask what force holds hadrons together.

The quarks carry electrical charges but also carry an extra charge called "colour". There are three varieties, let's call them red blue green, and they attract and repel as do electrical charges: like colours repel and unlike attract (technically when in antisymmetric quantum states). Thus three different colours can mutually attract and form a baryon but a fourth is left neutral: attracted by two and repelled by the third. The restriction of attractions to the antisymmetric state causes the attraction and repulsion to counterbalance, (the importance of symmetry will be discussed in section 2).

The analogy between colour charge and electrical charge goes further.

By analogy with QED one can form QCD, quantum chromo(or colour) dynamics. Instead of photons as radiation and force carriers one has (coloured) gluons. The gluons are in general coloured because a quark that is coloured Red (say) can turn into a Green by radiating a gluon that is coloured "Red-Green"; (technically the 3 colours form the basis representation of an $SU(3)$ group and the gluons transform as the regular representation, the octet - see section 2). The fact that the gluons carry the colour, or charge, whereas photons do not carry any charge, causes gluons to propagate differently from photons. Whereas photons can voyage independently, gluons can mutually attract en route (they "shine in their own light"). Not only does this affect the long range behaviour of the

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forces but it also suggests that bound states of pure glue, known as glueballs, may exist. The existence of glueballs and other hadrons where glue is excited ("hybrids") will be the focus of these lectures. First I shall discuss the ways that colour manifests itself in more familiar systems such as baryons and nuclei.

To illustrate the similarities and differences between QED and QCD we can list particles and clusters according to whether they feel the force or not. Those that feel the force may do so because they manifestly carry the charge (such as electrons or ions in QED or quarks and gluons in QCD) or because the charge is hidden internally (such as atoms and molecules in QED or nucleons and nuclei in QCD). Contrast these systems with those states that do not feel the force directly as they neither carry nor contain the charge (such as neutrinos and photons in QED or leptons in QCD). Note in particular that the force carriers, the photon and gluon, are in different parts of this matrix; the gluons have manifest colour charge and feel the QCD forces whereas the photons do not have electrcial charge and do not directly feel the QED forces.

We can go further and examine the particular set of systems with "hidden" charge within them. There are three broad classes of these "consequential" forces. In QED the atoms and molecules feel covalent, van der Waals and ionic forces; the former pair being due essentially to constituent exchange and two-photon exchange between two separate pairs of constituents respectively. The analogues in QCD are quark exchange and two-gluon exchange; there is no analogue of ionic forces at long range due to the property of confinement of colour in QCD.

The confinement also breaks the naive similarity between QED and QCD forces in that the quark exchange ("covalent" force) involves clusters or bags of colourless combinations of quark and antiquark known as mesons (of which the light pion is the most obvious in nuclear forces). Nonetheless we can imagine nuclei as being the QCD analogues of molecules. The van der Waal, two gluon exchange, forces will also be affected by confinement and, presumably, will involve glueballs as effective exchange objects. However, as the lightest glueballs are not expected to exist below 1GeV, their effective range is very restricted and so they are unlikely to affect nuclear forces in any immediately observable way. The search for colour analogues of van der Waals forces is likely to be unfruitful in my opinion until someone comes up with a smart idea.

I shall first show the important role that colour plays for quarks in baryons and then I shall contrast baryons, made of three constituent quarks, with the nuclei ^3H and ^3He which are made of three nucleons. Then I shall concentrate on colour in mesons, extracting information about the colour forces from spectroscopy (section 3). I shall then discuss decays (section 4) in order to contrast with glueball decays (section 5) before considering a "realistic" picture where glueball and quarkonia mix (section 6). The production of glueballs is discussed in section 7, the phenomenology of hybrids in section 8 and some attempts to test the hybrid interpretation are in the final section.

2 COLOUR, THE PAULI PRINCIPLE AND SPIN-FLAVOUR CORRELATIONS

2.1 Colour

The Pauli principle forbids fermions to coexist in the same quantum state. Historically this created a paradox in that the baryon $\Omega^-(S^\uparrow S^\uparrow S^\uparrow)$ appeared manifestly to violate this.

If quarks possess a property called colour, any quark being able to carry any one of three colours (say red, green, blue), then the Ω^- (and any baryon) can be built from distinguishable quarks:

$$\Omega^-(S_R^\uparrow S_G^\uparrow S_B^\uparrow).$$

This was how the idea of threefold colour first entered particle physics. Subsequently the idea developed that colour is the source of a relativistic quantum field theory, QCD or quantum chromodynamics, and is the source of the strong forces that bind quarks in hadrons. I shall first discuss the idea of colour and how, when combined with the Pauli principle, it determines the properties of baryons. Then I shall develop the idea of it as source of the interquark forces.

If quarks carry colour but leptons do not, then it is natural to speculate that colour may be the property that is the source of the strong interquark forces - absent for leptons.

Electric charges obey the rule "like repel, unlike attract" and cluster to net uncharged systems. Colours obey a similar rule: "like colours repel, unlike (can) attract". If the three colours form the basis of an SU(3) group, then they cluster to form "white" systems - viz. the singlets of SU(3). Given a random soup of coloured quarks, the attractions gather them into white clusters, at which point the colour forces are saturated. Nuclear forces are then the residual forces among these clusters.

If quark (Q) and antiquark $(\bar{Q})$ are the $\underset{\sim}{3}$ and $\underset{\sim}{\bar{3}}$ of colour SU(3), then combining up to three together gives SU(3) multiplets of dimensions as follows (see e.g. ref.[1]):

$$QQ = \underset{\sim}{3} \times \underset{\sim}{3} = \underset{\sim}{6} + \underset{\sim}{\bar{3}}$$

$$Q\bar{Q} = \underset{\sim}{3} \times \underset{\sim}{\bar{3}} = \underset{\sim}{8} + \underset{\sim}{1}$$

The $Q\bar{Q}$ contains a singlet - the physical mesons. Coloured gluons belong to the $\underset{\sim}{8}$ representation and are confined. Combining QQ with a third Q gives

$$QQQ = \underset{\sim}{10} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{1}.$$

where the $\underset{\sim}{1}$ arose when QQ pairs were in $\underset{\sim}{\bar{3}}$.

Note the singlet in QQQ - the physical baryons.

For clusters of three or less, only $Q\bar{Q}$ and QQQ contain colour singlets and, moreover, these are the only states realized physically. Thus are we led to hypothesize that only colour singlets can exist free in the laboratory; in particular, the quarks will not exist as free particles.

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2.2 Symmetries and correlations in baryons

To have three quarks in colour singlet:

$$1 \equiv \frac{1}{\sqrt{6}}[(RB - BR)Y + (YR - RY)B + (BY - YB)R]$$

any pair is in the $\underset{\sim}{\bar{3}}$ and is antisymmetric. Note that $\underset{\sim}{3} \times \underset{\sim}{3} = \underset{\sim}{6} + \underset{\sim}{\bar{3}}$. These are explicitly

$$
\begin{array}{cc}
\underset{\sim anti}{\bar{3}} & \underset{\sim sym}{6} \\
RB - BR & RB + BR \\
RY - YR & RY + YR \\
BY - YB & BY + YB \\
 & RR \\
 & BB \\
 & YY
\end{array}
\tag{1}
$$

Note well: <u>Any Pair is Colour Antisymmetric</u>

The Pauli principle requires total antisymmetry and therefore any pair must be: <u>Symmetric in all else</u> ("else" means "apart from colour").

This is an important difference from nuclear clusters where the nucleons have no colour (hence are trivially <u>symmetric</u> in colour!). Hence for nucleons Pauli says

$$\underline{\text{Nucleons are Antisymmetric in Pairs}} \tag{2}$$

and for quarks

$$\underline{\text{Quarks are Symmetric in Pairs}} \tag{3}$$

(in all apart from colour).

If we forget about colour (colour has taken care of the antisymmetry and won't affect us again), then (i) Two quarks can couple their spins as follows

$$
\left\{
\begin{array}{ll}
S = 1: & \text{symmetric} \\
S = 0: & \text{antisymmetric}
\end{array}
\right\}
\tag{4}
$$

(ii) Two u, d quarks similarly form isospin states

$$
\left\{
\begin{array}{ll}
I = 1: & \text{symmetric} \\
I = 0: & \text{antisymmetric}
\end{array}
\right\}
\tag{5}
$$

(iii) In the ground state $L = 0$ for all quarks; hence the orbital state is trivially symmetric. Thus for pairs in $L = 0$, we have due to Pauli that

$$
\left\{
\begin{array}{lll}
S = 1 & \text{and} \quad I = 1 & \text{correlate} \\
S = 0 & \text{and} \quad I = 0 & \text{correlate}
\end{array}
\right\}
\tag{6}
$$

Thus the Σ^0 and Λ^0 which are distinguished by their u, d being $I = 1$ or 0 respectively also have the u, d pair in spin $= 1$ or 0 respectively:

$$\left\{ \begin{array}{ccc} \Sigma^0[S(u,d)_{I=1}] & \leftrightarrow & S(u,d)_{S=1} \\ \Lambda^0[S(u,d)_{I=0}] & \leftrightarrow & S(u,d)_{S=0} \end{array} \right\} \tag{7}$$

Thus, the spin of the Λ^0 is carried entirely by the strange quark.

2.3 Colour, the Pauli principle and magnetic moments

The electrical charge of a baryon is the sum of its constituent quark charges. The magnetic moment is an intimate probe of the correlations between the charges and spins of the constituents. Being wise, today we can say that the neutron magnetic moment was the first clue that the nucleons are not elementary particles. Conversely the facts that quarks appear to have $g \simeq 2$ suggests that they *are* elementary (or that new dynamics is at work if composite).

A very beautiful demonstration of symmetry at work is the magnetic moment of two similar sets of systems of three, viz.

$$\left\{ \begin{array}{cc} N; & P \\ ddu; & uud \end{array} \right\} \quad \mu_p/\mu_N = -3/2$$

and the nuclei

$$\left\{ \begin{array}{cc} H^3; & He^3 \\ NNP; & PPN \end{array} \right\} \quad \mu_{He}/\mu_H = -2/3$$

The Pauli principle for nucleons requires He^4 to have *no* magnetic moment:

$$\mu[He^4; P^\uparrow P^\downarrow N^\uparrow N^\downarrow] = 0.$$

Then

$$He^3 \equiv He^4 - N$$
$$H^3 \equiv He^4 - P$$

and so

$$\frac{\mu_{He^3}}{\mu_{H^3}} = \frac{\mu_N}{\mu_p}$$

To get at this result in a way that will bring best comparison with the nucleon three-quark example, let's study the He^3 directly.

$He^3 = ppn$: pp are flavour symmetric; hence Pauli requires that they be spin anti-symmetric; i.e., $S = 0$.

Thus

$$[He^3]^\uparrow \equiv (pp)_0 n^\uparrow \tag{8}$$

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and so the *pp* do not contribute to its magnetic moment. The magnetic moment (up to mass scale factors) is

$$\mu_{He^3} = 0 + \mu_n. \tag{9}$$

Similarly,

$$\mu_{H^3} = 0 + \mu_p. \tag{10}$$

which, of course, gives the result that we got before, as it must. But deriving it this way is instructive as we see when we study the nucleons in an analogous manner.

The proton contains u, u flavour symmetric and *colour antisymmetric*; thus the spin of the "like" pair is symmetric ($S = 1$) in contrast to the nuclear example where this pair had $S = 0$. Thus coupling spin 1 and spin 1/2 together, the Clebsches yield (where subscripts denote S_z)

$$p^\uparrow = \frac{1}{\sqrt{3}}(u, u)_0 d^\uparrow + \frac{2}{\sqrt{3}}(u, u)_1 d^\downarrow \tag{11}$$

(contrast Eq. (8)), and (up to mass factors)

$$\mu_p = \frac{1}{3}(0 + d) + \frac{2}{3}(2u - d). \tag{12}$$

Suppose that $\mu_{u,d} \propto e_{u,d}$, then

$$\mu_\mu = -2\mu_d \tag{13}$$

so

$$\frac{\mu_p}{\mu_N} = \frac{4u - d}{4d - u} = -\frac{3}{2} \tag{14}$$

(the neutron follows from proton by replacing $u \leftrightarrow d$).

I cannot overstress the crucial, hidden role that colour played here in getting the flavour-spin correlation right.

3 THE POTENTIAL AND THE FORCE

The following remarks are by no means rigorous and are intended only to abstract some general suggestive features about the dynamics from the spectroscopy of hadrons. They will also enable us to draw up some empirical guidelines for identifying the nature of light hadrons.

We all know what the spectrum of a Coulomb potential looks like, with the energy gap between the first two levels already being well on the way to ionisation energies. The spectrum of hadrons is not like this in that the gap between 1S and 2S is similar to (though slightly greater than) that between 2S and 3S, and so on to 3S and 4S etc. The P states are found slightly above the midway between the corresponding S states. This is similar to that of a linear potential (which is near enough to a harmonic that for many purposes the latter is often used for analytical calculations). A comparison is shown in fig 1.

It is instructive to use the particle data tables[2] and to place the $b\bar{b}$ states on this spectrum, noting the relative energy gaps between the 1S,2S, 3S and the 1P,2P states. Now do the same for $c\bar{c}$ but rescaled downwards by 6360MeV (so the $\psi(3097)$ and $\Upsilon(9460)$ start the 1S states at the same place). It is remarkable that where corresponding levels have been identified in the two spectroscopies, there is a rather similar pattern both qualitatively and even quantitatively (see figs 2a,b).

We shall consider the implications of this for light hadrons later but first we can abstract the message that the potential between heavy flavours is linear to a good approximation. This immediately tells us about the spatial dynamics of the force fields. Let me show you how.

In the case of a U(1) charge, as in electrostatics, the force fields spread out in space symmetrically in all three dimensions. Thus the intensity crossing a sphere at distance R dies as the surface area, hence as $\frac{1}{R^2}$. The potential is the integral of this, hence proportional to $\frac{1}{R}$, the Coulomb form. We see that the Coulomb potential is "natural" in a 3-D world.

Contrast this with the empirical message from the $Q\bar{Q}$ spectroscopy, where $V(R) \sim R$. Here the intensity $\sim \frac{dV}{dR} \sim constant$. The intensity does not spread at all; it is indeed "linear". From this empirical observation we have the picture that the gauge fields, the gluons, transmit the force as if in a tube of colour flux. This is also substantiated by computer simulations of QCD ("lattice QCD"). There is some limited transverse spread but to a first approximation one is encouraged towards models where a linear flux tube drives the dynamics.

This is what we find for the long range nature of the potential, where the gluons have mutually interacted while transmitting the force. At short range one expects there will be a significant perturbation arising from single gluons travelling between the quarks independently; this will be akin to the more familiar case of QED where independent photon exchange generates the $\frac{1}{R}$ behaviour discussed above. Hence our intuition is that the full potential in QCD will have a structure along the lines of

$$V(R) \sim \frac{\alpha_s}{R} + aR \tag{15}$$

where α_s is the strong coupling strength in QCD and a is a constant with dimensions of energy per unit length; this is in effect the tension in the flux tube and empirically is about 1GeV/fermi. This potential, when plotted on graph paper, looks similar to a log(R) at the distance scales of hadrons. It is for this reason that the absolute energy gap between 1S and 2S say is nearly independent of the constituent mass: the solutions to the Schrodinger equation for a log potential show that the energy gaps are independent of mass (for $\frac{1}{R}$ they grow $\sim M$ whereas for R they fall as $M^{-\frac{1}{3}}$ and the competition "accidentally" cancels.)

As an aside we can illustrate this.

Consider the Schrodinger equation

$$(\nabla^2 + 2m_1 R^N)\psi_1(R) = 2m_1 E_1 \psi_1(R)$$

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and similar form for m_2 with E_2 and $\psi_2(R)$. Let $R \to \lambda R$ where $\psi_2(R) \equiv \psi_1(\lambda R)$. Thus we compare

$$(\nabla^2 + 2m_1\lambda^{N+2}R^N)\psi_1(\lambda R) = 2m_1 E_1 \lambda^2 \psi_1(\lambda R)$$

with

$$(\nabla^2 + 2m_2 R^N)\psi_2(R) = 2m_2 E_2 \psi_2(R)$$

Recognising that $\psi_2(R) \equiv \psi_1(\lambda R)$, on the L.H.S. we have

$$\lambda^2 \equiv \left(\frac{m_2}{m_1}\right)^{2/(N+2)}$$

and on the R.H.S.

$$\frac{E_2}{E_1} \equiv \frac{m_1}{m_2}\lambda^2$$

Hence

$$\frac{E_2}{E_1} = \left(\frac{m_2}{m_1}\right)^{-\frac{N}{N+2}}$$

shows how the energy levels scale with constituent mass in a potential R^N.

We can make a further analogy between QED and QCD via the magnetic perturbations on the ground states. In hydrogen the magnetic interaction between electron and proton causes a hyperfine splitting between the $^3S_1$ and $^1S_0$ levels. This is inversely proportional to the constituent masses and proportional to the expectation of the wavefunction at the origin and to $\langle \vec{S}_1 \cdot \vec{S}_2 \rangle$. For mesons one finds a similar splitting where for $Q\bar{q}$ states the $^3S_1$ and $^1S_0$ levels are as follows

	K	D	D_s	B	B_s
$m(^3S_1)$	0.89	2.01	2.11	5.32	5.33
$m(^1S_0)$	0.49	1.87	1.97	5.27	5.28

(the vector is raised by 1 unit and the pseudoscalar reduced by three units relative to the unperturbed values; this follows from $\langle 2\vec{S}_1 \cdot \vec{S}_2 \rangle \equiv \langle (\vec{S}_1 + \vec{S}_2)^2 - 2\vec{S}_i^2 \rangle \equiv S(S+1) - \frac{3}{2}$). Qualitatively we see that the magnitude of the splitting is smaller as one proceeds to heavier flavours, in line with the inverse mass property of (chromo)magnetic interactions. Quantitavely the behaviour is interesting. For a potential $V(R) \sim R^N$ the wavefunction at the origin behaves as

$$\psi(0)^2 \sim \mu_{ij}^{\frac{3}{2+N}} \tag{16}$$

where μ is the reduced mass

$$\frac{1}{\mu_{ij}} = \frac{1}{m_i} + \frac{1}{m_j} \tag{17}$$

Now if we assume that

$$\frac{(m_V + m_P)_{ij}}{2} \equiv m_i + m_j \tag{18}$$

and note that, in hyperfine splitting

$$m_V - m_P \sim \frac{\psi(0)^2}{m_i m_j} \tag{19}$$

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where i, j are constituent quarks comprising Vector or Pseudoscalar mesons ($q_i \bar{q}_j$), we can find the best value of N in the potential. If one forms $m_V^2 - m_P^2$ you will see that this is flavour independent to a remarkable accuracy; then following the above hints you will immediately see that $N = 1$ is preferred; the wavefunctions of the linear potential are those that fit best in the perturbation expression.

The mean mass of the ground states is nearer to the vector (spin triplet) than the pseudoscalar (spin singlet). If we look at the mass gap between the $^3S_1$ 1S and the $^3P_2$ 1P levels, we find again a remarkable flavour independence, not just for the $b\bar{b}$ and $c\bar{c}$ already mentioned but for the strange and nonstrange too.

	$u\bar{d}$	$u\bar{s}$	$s\bar{s}$	$c\bar{u}$	$c\bar{c}$	$b\bar{b}$
$m(^3P_2)$	1320	1430	1525	2460	3550	9915
$m(^3S_1)$	770	892	1020	2010	3100	9460
gap	550	540	500	450	450	450

Thus although the splittings between $^3S_1$ and $^1S_0$ are strongly mass dependent, as expected in QCD, the $S - P$ mass gaps are to good approximation fairly similar across the flavours. Even though the light flavoured states are above threshold for decays into hadrons, the memory of the underlying potential remains and, at least empirically, we can produce an ouline skeleton for the spectroscopic pattern anticipated for all flavours. I illustrate this in fig 2. The absolute separations of 1S, 2S, 3S and those of 1P,2P have been taken from the known heavy flavours and rescaled slightly to make a best fit where the 1D,1F and even 1G are found by the high spin states in each of these levels. A numerical solution of the spectrum in a model where $Q\bar{Q}$ are connected by a linear flux-tube is shown in fig.3; this is indeed very similar to the data and empirical spectrum illustrated in fig 2c.

Unless certain J^{PC} have strong energy shifts through coupling to open channels, this should give a reliable guide to the energies of light hadron multiplets. When we combine the $q\bar{q}$ spins to singlet or triplet ($S = 0, 1$) and then combine in turn with the orbital angular momentum we can construct a set of $^{2S+1}L_J$ states. We shall be interested later in the possible discovery of a scalar glueball and so we shall also need to be aware that scalar mesons can be formed in the quark model as $^3P_0$ states. From the figure we anticipate these to lie in the region around $1.2(n\bar{n}) - 1.6(s\bar{s})$GeV.

A list of the low lying quarkonium multiplets is given below

	S=1 triplet	S=0 singlet
S	$^3S_1$ 1^{--}	$^1S_0$ 0^{-+}
P	^3P_J $0^{++}1^{++}2^{++}$	$^1P_1$ 1^{+-}
D	^3D_J $1^{--}2^{--}3^{--}$	$^1D_2$ 2^{-+}
F	^3F_J $2^{++}3^{++}4^{++}$	$^1F_3$ 3^{+-}

The spectroscopy of baryons and mesons is now rather well understood, at least in outline, to an extent that if there are "strangers" lurking among the conventional states, there is a strong likelihood that they can be smoked out.

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Such a hope is now becoming important as new states are appearing and may have a radical implication for our understanding of strong-QCD. The reason has to do with the nature of gluons. As gluons carry colour charge and can mutually attract, it is theoretically plausible that gluons can form clusters that are overall colourless (like conventional hadrons) but which contain only gluons. These are known as "glueballs" and would represent a new form of matter on the 1fm scale.

Glueballs are a missing link of the standard model. Whereas the gluon degrees of freedom expressed in L_{QCD} have been established beyond doubt in high momentum data, their dynamics in the strongly interacting limit epitomised by hadron spectroscopy are quite obscure. This may be about to change as a family of candidates for gluonic hadrons (glueballs and hybrids) is now emerging [3, 4, 5]. These contain both hybrids around 1.9GeV and a scalar glueball candidate at $f_0(1500)$.

In advance of the most recent data, theoretical arguments suggested that there may be gluonic activity manifested in the 1.5 GeV mass region. Lattice QCD is the best simulation of theory and predicts the lightest "primitive" (ie quenched approximation) glueball to be 0^{++} with mass 1.55 ± 0.05 GeV [6]. Recent lattice computations place the glueball slightly higher in mass at 1.74 ± 0.07 GeV [7] with an optimised value for phenomenology proposed by Teper[8] of 1.57 ± 0.09 GeV. That lattice QCD computations of the scalar glueball mass are now concerned with such fine details represents considerable advance in this field. Whatever the final concensus may be, these results suggest that scalar mesons in the 1.5 GeV region merit special attention. Complementing this has been the growing realisation that there are now too many 0^{++} mesons confirmed for them all to be $Q\overline{Q}$ states [2, 3, 4, 9].

I will introduce some of my own prejudices about glueballs and how to find them. I caution that we have no clear guide and so others may have different suggestions. At this stage any of us, or none of us, could be right. We have to do the best we can guided by experience. It is indeed ironical that the lattice predicts that the lightest glueball exists in the same region of mass as quarkonium states of the same $J^{PC} = 0^{++}$. If this is indeed the case in nature, the phenomenology of glueballs may well be more subtle than naive expectations currently predict.

We shall be interested later in the possible discovery of "hybrid" states, where the gluonic fields are dynamically excited in presence of quarks. Among these we shall be particularly interested in $0^{-+}, 1^{-+}, 2^{-+}, 1^{--}$ and possibly 1^{++}. Note that the 1^{-+} configuration does not occur for $Q\overline{Q}$ and so discovery of such a resonant state would be direct evidence for dynamics beyond the simple quark model. The other quantum numbers can be shared by hybrids and ordinary states. The mass of these lightest hybrids is predicted to be around 1.9GeV in a dynamical model where quarks are connected by a flux tube. The numerical solution of the dynamics is discussed in ref[10] and endorses the earlier estimates by Isgur and Paton[11].In fig.3 we see a comparison of the predicted hybrid spectroscopy and that of the conventional states. The mass of the 2^{-+} hybrid is predicted to be tantalisingly close to that of the conventional $^1D_2$ with which it shares the same overall J^{PC} quantum numbers. Comparison with fig.2 shows that this mass region is also near to that of $3S$ states which include 0^{-+} and 1^{--}, quantum numbers shared with the

lightest hybrids. Furthermore, the 1^{++} hybrid shares quantum numbers with the $2P(^3P_1)$ quarkonium and, following fig.2, we may anticipate that here too a similarlity in mass may ensue. Thus on mass grounds alone it may be hard to disentangle hybrids and glueballs from conventional states. It will be important to investigate both the production and decay patterns of these various objects.

As regards the decays, we need to study both the flavour dependence in a multiplet and also the spin and other intrinsic dynamical dependences that may help to distinguish conventional quarkonia from states where the gluonic degrees of freedom are excited. We shall therefore first look at the flavour dependence.

4 QUARKONIUM DECAY AMPLITUDES

Let's review some basics of the flavour dependence of two body decays for a $q\bar{q}$ state of arbitrary flavour. This will be helpful in assigning meosns to nonets and will also help us to understand some general features of glueball decays.

Consider a quarkonium state

$$|Q\overline{Q}\rangle = cos\alpha|n\bar{n}\rangle - sin\alpha|s\bar{s}\rangle \tag{20}$$

where

$$n\bar{n} \equiv (u\bar{u} + d\bar{d})/\sqrt{2}. \tag{21}$$

The mixing angle α is related to the usual nonet mixing angle θ [2] by the relation

$$\alpha = 54.7° + \theta. \tag{22}$$

For $\theta = 0$ the quarkonium state becomes pure SU(3)$_f$ octet, while for $\theta = \pm 90°$ it becomes pure singlet. Ideal mixing occurs for $\theta = 35.3°$ (-54.7°) for which the quarkonium state becomes pure $s\bar{s}$ ($\bar{n}n$).

In general we define

$$\eta = cos\phi|n\bar{n}\rangle - sin\phi|s\bar{s}\rangle \tag{23}$$

and

$$\eta' = sin\phi|n\bar{n}\rangle + cos\phi|s\bar{s}\rangle \tag{24}$$

with $\phi = 54.7° + \theta_{PS}$, where θ_{PS} is the usual octet-singlet mixing angle in SU(3)$_f$ basis where

$$\eta = cos(\theta_{PS})|\eta_8\rangle - sin(\theta_{PS})|\eta_1\rangle, \tag{25}$$

$$\eta' = sin(\theta_{PS})|\eta_8\rangle + cos(\theta_{PS})|\eta_1\rangle. \tag{26}$$

The decay of quarkonium into a pair of mesons $Q\overline{Q} \rightarrow M(Q\bar{q}_i)M(q_i\overline{Q})$ involves the creation of $q_i\bar{q}_i$ from the vacuum. If the ratio of the matrix elements for the creation of $s\bar{s}$ versus $u\bar{u}$ or $d\bar{d}$ is denoted by

$$\rho \equiv \frac{\langle 0|V|s\bar{s}\rangle}{\langle 0|V|d\bar{d}\rangle}, \tag{27}$$

then the decay amplitudes of an isoscalar 0^{++} (or 2^{++}) are proportional to

$$\begin{aligned}
\langle Q\overline{Q}|V|\pi\pi\rangle &= cos\alpha \\
\langle Q\overline{Q}|V|K\overline{K}\rangle &= cos\alpha(\rho - \sqrt{2}tan\alpha)/2 \\
\langle Q\overline{Q}|V|\eta\eta\rangle &= cos\alpha(1 - \rho\sqrt{2}tan\alpha)/2 \\
\langle Q\overline{Q}|V|\eta\eta'\rangle &= cos\alpha(1 + \rho\sqrt{2}tan\alpha)/2.
\end{aligned}$$

$$(28)$$

The corresponding decay amplitudes of the isovector are

$$\begin{aligned}
\langle Q\overline{Q}|V|K\overline{K}\rangle &= \rho/2 \\
\langle Q\overline{Q}|V|\pi\eta\rangle &= 1/\sqrt{2} \\
\langle Q\overline{Q}|V|\pi\eta'\rangle &= 1/\sqrt{2},
\end{aligned}$$

$$(29)$$

and those for K^* decay

$$\begin{aligned}
\langle Q\overline{Q}|V|K\pi\rangle &= \sqrt{3}/2 \\
\langle Q\overline{Q}|V|K\eta\rangle &= (\sqrt{2}\rho - 1)/\sqrt{8} \\
\langle Q\overline{Q}|V|K\eta'\rangle &= (\sqrt{2}\rho + 1)/\sqrt{8}.
\end{aligned}$$

$$(30)$$

For clarity of presentation we have presented eqn. 28,29 and 30 in the approximation where $\eta \equiv (n\bar{n} - s\bar{s})/\sqrt{2}$ and $\eta' \equiv (n\bar{n} + s\bar{s})/\sqrt{2}$, i.e. for a pseudoscalar mixing angle $\theta_{PS} \sim -10°$ ($\phi = 45°$). This is a useful mnemonic; the full expressions for arbitrary η, η' mixing angles θ_{PS} are given in ref.[4]. Exact $SU(3)_f$ flavour symmetry corresponds to $\rho = 1$; empirically $\rho \geq 0.8$ for well established nonets such as 1^{--} and 2^{++} [12, 13].

The partial width into a particular meson pair $M_i M_j$ may be written as

$$\Gamma_{ij} = c_{ij}|M_{ij}|^2 \times |F_{ij}(\vec{q})|^2 \times p.s.(\vec{q}) \equiv \gamma_{ij}^2 \times |F_{ij}(\vec{q})|^2 \times p.s.(\vec{q}) \qquad (31)$$

where $p.s.(\vec{q})$ denotes the phase-space, $F_{ij}(\vec{q})$ are model-dependent form factors which are discussed in detail in ref.[4], M_{ij} is the relevant amplitude (eqn. 28,29 or 30) and c_{ij} is a weighting factor arising from the sum over the various charge combinations, namely 4 for $K\overline{K}$, 3 for $\pi\pi$, 2 for $\eta\eta'$ and 1 for $\eta\eta$ for isoscalar decay (eqn. 28), 4 for $K\overline{K}$, 2 for $\pi\eta$ and 2 for $\pi\eta'$ for isovector decay (eqn. 29) and 2 for K^* decays (eqn. 30). The dependence of $\gamma_{ij}^2 = c_{ij}|M_{ij}|^2$ upon the mixing angle α is shown in fig. 4a for the isoscalar decay in the case of $SU(3)_f$ symmetry, $\rho = 1$.

The figure illustrates some general points.

An $s\bar{s}$ state corresponds to $\alpha = 90^0$ for which $\pi\pi$ vanishes. The $K\overline{K}$ vanishes when there is destructive interference between $s\bar{s}$ and $n\bar{n}$; notice that the $\eta\eta$ tends to vanish

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here also as it tends to be roughly $\frac{1}{4}$ of the $K\bar{K}$ independent of the mixing angle α (this would be an exact relation for the ideal η used in the text; the figure shows the results for realistic η flavour composition). This correlation between $\eta\eta$ and $K\bar{K}$ is expected for any quarkonium state and a violation in data will therefore be significant in helping identify "strangers".

This pattern of decays is expected to hold true for any meson that contains $q\bar{q}$ in its initial configuration. Thus it applies to conventional or to hybrid multiplets and distinguishing between them will depend on dynamical features associated with the gluonic excitation or the spin states of the quarks. The case of glueballs is qualitatively different in that there is no intrinsic flavour present initially and so the pattern of decays will depend, inter alia, on the dynamics of flavour creation.

The traditional assumption has been that as glueballs are flavour singlets, their decays should be analogous to those of a flavour singlet quarkonium. The case of a flavour singlet corresponds to $\alpha = -30^0$ (or 150^0). Here we see that $\eta\eta' \rightarrow 0$ and the other channels are populated in proportion to their charge weighting (namely 4:3:1 for $K\bar{K} : \pi\pi : \eta\eta$). A flavour singlet glueball would be expected to show these ratios too if it decays through a flavour singlet intermediate state.

We can now look into the decays of glueballs by finding examples of decays where gluons are already believed to play a role. The data are sparse and do show consistency with the flavour singlet idea; however, one must exercise caution before applying this too widely. I shall first illustrate the flavour singlet phenomenon as it manifests itself for gluonic systems at energies far from the mass scales of light-flavoured quarkonium. Then I shall investigate what modifications may be expected for glueballs at mass scales of 1-2GeV where quarkonium states with the same J^{PC} may contaminate the picture.

5 PRIMITIVE GLUEBALL DECAYS

The decays of $c\bar{c}$, in particular $\chi_{0,2}$, provide a direct window on G dynamics in the $0^{++}, 2^{++}$ channels insofar as the hadronic decays are triggered by $c\bar{c} \rightarrow gg \rightarrow Q\bar{Q}Q\bar{Q}$ (fig. 5a). It is necessary to keep in mind that these are in a different kinematic region to that appropriate to our main analysis but, nonetheless, they offer some insights into the gluon dynamics. Mixing between hard gluons and 0^{++}, 2^{++} $Q\bar{Q}$ states (fig. 5c) is improbable at these energies as the latter 1 - 1.5 GeV states will be far off their mass-shell. Furthermore,the narrow widths of $\chi_{0,2}$ are consistent with the hypothesis that the 3.5 GeV region is remote from the prominent $0^+, 2^+$ glueballs, G. Thus we expect that the dominant decay dynamics is triggered by hard gluons directly fragmenting into two independent $Q\bar{Q}$ pairs (fig. 5a) or showering into lower energy gluons (fig. 5b). We consider the former case now; mixing with $Q\bar{Q}$ (fig. 6c) and $G \rightarrow GG$ (fig. 6b) will be discussed in section 6.

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$G \rightarrow Q Q \bar{Q} \bar{Q}$

This was discussed in ref. [14] and the relative amplitudes for the process shown in fig. 5a read

$$
\begin{aligned}
\langle G|V|\pi\pi\rangle &= 1 \\
\langle G|V|K\overline{K}\rangle &= R \\
\langle G|V|\eta\eta\rangle &= (1 + R^2)/2 \\
\langle G|V|\eta\eta'\rangle &= (1 - R^2)/2,
\end{aligned}
\tag{32}
$$

with generalizations for arbitrary pseudoscalar mixing angles given in ref.[4] and where $R \equiv \langle g|V|s\bar{s}\rangle / \langle g|V|d\bar{d}\rangle$. SU(3)$_f$ symmetry corresponds to $R^2 = 1$. In this case the relative branching ratios (after weighting by the number of charge combinations) for the decays $\chi_{0,2} \rightarrow \pi\pi, \eta\eta, \eta\eta', K\overline{K}$ would be in the relative ratios $3 : 1 : 0 : 4$. Data for χ_0 are in accord with this where the branching ratios are (in parts per mil) [2]:

$$
\begin{aligned}
B(\pi^0\pi^0) &= 3.1 \pm 0.6 \\
\frac{1}{2}B(\pi^+\pi^-) &= 3.7 \pm 1.1 \\
\frac{1}{2}B(K^+K^-) &= 3.5 \pm 1.2 \\
B(\eta\eta) &= 2.5 \pm 1.1.
\end{aligned}
\tag{33}
$$

No signal has been reported for $\eta\eta'$. Flavour symmetry is manifested in the decays of χ_2 also:

$$
\begin{aligned}
B(\pi^0\pi^0) &= 1.1 \pm 0.3 \\
\frac{1}{2}B(\pi^+\pi^-) &= 0.95 \pm 0.50 \\
\frac{1}{2}B(K^+K^-) &= 0.75 \pm 0.55 \\
B(\eta\eta) &= 0.8 \pm 0.5,
\end{aligned}
\tag{34}
$$

again in parts per mil. The channel $\eta\eta'$ has not been observed either. These results are natural as they involve hard gluons away from the kinematic region where G bound states dominate the dynamics. If glueballs occur at lower energies and mix with nearby $Q\bar{Q}$ states, this will in general lead to a distortion of the branching ratios from the "ideal" equal weighting values above. (A pedagogical example will be igven in the next section). It will also cause significant mixing between $n\bar{n}$ and $s\bar{s}$ in the quarkonium eigenstates. Conversely, "ideal" nonets, where the quarkonium eigenstates are $n\bar{n}$ and $s\bar{s}$, are expected

to signal those J^{PC} channels where the masses of the prominent glueballs are remote from those of the quarkonia.

An example of this is the 2^{++} sector where the quarkonium members are "ideal" which suggests that G mixing is nugatory in this channel. These data collectively suggest that prominent 2^{++} glueballs are not in the $1.2 - 1.6$ GeV region which in turn is consistent with lattice calculations where the mass of the 2^{++} primitive glueball is predicted to be larger than 2 GeV. The sighting of a 2^{++} state in the glueball favoured central production, decaying into $\eta\eta$ with no significant $\pi\pi$[15] could be the first evidence for this state. There are also interesting signals from BES on a narrow state in this mass region seen in $\psi \to \gamma MM$ where MM refer to mesons pairs, $\pi\pi, K\bar{K}$ with branching ratios consistent with flavour symmetry[16].

6 $Q\bar{Q}$ AND GLUEBALL DECAYS IN STRONG COUPLING QCD

In the strong coupling ($g \to \infty$) lattice formulation of QCD, hadrons consist of quarks and flux links, or flux tubes, on the lattice. "Primitive" $Q\bar{Q}$ mesons consist of a quark and antiquark connected by a tube of coloured flux whereas primitive glueballs consist of a loop of flux (fig. 6a,b) [11]. For finite g these eigenstates remain a complete basis set for QCD but are perturbed by two types of interaction [17]:

1. V_1 which creates a Q and a $\bar{Q}$ at neighbouring lattice sites, together with an elementary flux-tube connecting them, as illustrated in fig. 6c,

2. V_2 which creates or destroys a unit of flux around any plaquette (where a plaquette is an elementary square with links on its edges), illustrated in fig. 6d.

The perturbation V_1 in leading order causes decays of $Q\bar{Q}$ (fig. 6e) and also induces mixing between the "primitive" glueball (G_0) and $Q\bar{Q}$ (fig. 6f). It is perturbation V_2 in leading order that causes glueball decays and leads to a final state consisting of $G_0 G_0$ (fig. 6g); decays into $Q\bar{Q}$ pairs occur at higher order, by application of the perturbation V_1 twice. This latter sequence effectively causes G_0 mixing with $Q\bar{Q}$ followed by its decay. Application of V_1^2 leads to a $Q^2\bar{Q}^2$ intermediate state which then turns into colour singlet mesons by quark rearrangement (fig. 5a); application of V_2 would lead to direct coupling to glue in η, η' or $V_2 \times V_1^2$ to their $Q\bar{Q}$ content (fig. 5b).

The absolute magnitudes of these various contributions require commitment to a detailed dynamics and are beyond the scope of this first survey. We concentrate here on their **relative** contributions to the various two body pseudoscalar meson final states available to 0^{++} meson decays.For $Q\bar{Q} \to Q\bar{q}q\bar{Q}$ decays induced by V_1, the relative branching ratios are given in eqn. 28 where one identifies

$$\rho \equiv \frac{\langle Q\bar{s}s\bar{Q}|V_1|Q\bar{Q}\rangle}{\langle Q\bar{d}d\bar{Q}|V_1|Q\bar{Q}\rangle}. \tag{35}$$

The magnitude of ρ and its dependence on J^{PC} is a challenge for the lattice. We turn now to consider the effect of V_1 on the initial "primitive" glueball G_0. Here too we allow for possible flavour dependence and define

$$R^2 \equiv \frac{\langle s\bar{s}|V_1|G_0\rangle}{\langle d\bar{d}|V_1|G_0\rangle}. \tag{36}$$

The lattice may eventually guide us on this magnitude and also on the ratio R^2/ρ. In the absence of this information we shall leave R as free parameter and set $\rho = 1$.

6.1 Glueball-$Q\overline{Q}$ mixing at $O(V_1)$

In this first orientation we shall consider mixing between G_0 (the primitive glueball state) and the quarkonia, $n\bar{n}$ and $s\bar{s}$, at leading order in V_1 but will ignore that between the two different quarkonia which is assumed to be higher order perturbation.

The mixed glueball state is then

$$G = |G_0\rangle + \frac{|n\bar{n}\rangle\langle n\bar{n}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}} + \frac{|s\bar{s}\rangle\langle s\bar{s}|V_1|G_0\rangle}{E_{G_0} - E_{s\bar{s}}} \tag{37}$$

which may be written as

$$G = |G_0\rangle + \frac{\langle n\bar{n}|V_1|G_0\rangle}{\sqrt{2}(E_{G_0} - E_{n\bar{n}})}\{\sqrt{2}|n\bar{n}\rangle + \omega R^2|s\bar{s}\rangle\} \tag{38}$$

where

$$\omega \equiv \frac{E_{G_0} - E_{n\bar{n}}}{E_{G_0} - E_{s\bar{s}}} \tag{39}$$

is the ratio of the energy denominators for the $n\bar{n}$ and $s\bar{s}$ intermediate states in old fashioned perturbation theory (fig. 5d).

Denoting the dimensionless mixing parameter by

$$\xi \equiv \frac{\langle d\bar{d}|V_1|G_0\rangle}{E_{G_0} - E_{n\bar{n}}}, \tag{40}$$

the eigenstate becomes, to leading order in the perturbation,

$$N_G|G\rangle = |G_0\rangle + \xi\{\sqrt{2}|n\bar{n}\rangle + \omega R^2|s\bar{s}\rangle\} \equiv |G_0\rangle + \sqrt{2}\xi|Q\overline{Q}\rangle \tag{41}$$

with the normalization

$$N_G = \sqrt{1 + \xi^2(2 + \omega^2 R^4)}, \tag{42}$$

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Recalling our definition of quarkonium mixing

$$|Q\overline{Q}\rangle = cos\alpha|n\bar{n}\rangle - sin\alpha|s\bar{s}\rangle \qquad (43)$$

we see that G_0 has mixed with an effective quarkonium of mixing angle α where $\sqrt{2}tan\alpha = -\omega R^2$ (eqn . 28). For example, if $\omega R^2 \equiv 1$, the $SU(3)_f$ flavour symmetry maps a glueball onto quarkonium where $tan\alpha = -1/\sqrt{2}$ hence $\theta = -90°$, leading to the familiar flavour singlet

$$|Q\overline{Q}\rangle = |u\bar{u} + d\bar{d} + s\bar{s}\rangle/\sqrt{3}. \qquad (44)$$

When the glueball is far removed in mass from the $Q\bar{Q}$, $\omega \to 1$ and flavour symmetry ensues; the $\chi_{0,2}$ decay and the 2^{++} analysis earlier are examples of this "ideal" situation. However, when $\omega \neq 1$, as will tend to be the case when G_0 is in the vicinity of the primitive $Q\bar{Q}$ nonet (the 0^{++} case of interest here), significant distortion from naive flavour singlet can arise.

In particular lattice QCD suggests that the "primitive" scalar glueball G_0 lies at or above 1500 MeV, hence above the $I = 1$ $Q\bar{Q}$ state $a_0(1450)$ and the (presumed) associated $n\bar{n}$ $f_0(1370)$. Hence $E_{G_0} - E_{n\bar{n}} > 0$ in the numerator of ω. The $\Delta m = m_{s\bar{s}} - m_{n\bar{n}} \approx 200 - 300$ MeV suggests that the primitive $s\bar{s}$ state is in the region 1600-1700 MeV. Hence it is quite possible that the primtive glueball is in the vicinity of the quarkonium nonet, maybe in the middle of it. Indeed, the suppression of $K\bar{K}$ in the $f_0(1500)$ decays suggests a destructive interference between $n\bar{n}$ and $s\bar{s}$ such that $\omega R^2 < 0$. This arises naturally if the primitive glueball mass is between those of $n\bar{n}$ and the primitive $s\bar{s}$. As the mass of $G_0 \to m_{n\bar{n}}$ or $m_{s\bar{s}}$, the $K\bar{K}$ remains suppressed though non-zero; thus eventual quantification of the $K\bar{K}$ signal will be important.

The decay into pairs of glueballs, or states such as η that appear to couple to gluons, is triggered by the perturbation V_2. This can drive decays into $\eta\eta$ and is discussed in ref.[4]. This breaks the connection between $\eta\eta$ and $K\bar{K}$ that is a signature for quarkonium as illustrated earlier. The phenomenology of the $f_0(1500)$ appears to have these features.

If the $f_0(1550 \pm 50)$ becomes accepted as a scalar glueball, consistent with the predictions of the lattice, then searches for the 0^{-+} and especially the 2^{++} at mass 2.22 ± 0.13 GeV [8] may become seminal for establishing the lattice as a successful calculational laboratory. There are tantalising indications of a state produced in $\psi \to \gamma 0^- 0^-$ at BES whose decays may be consistent with those of a flavour blind glueball (flavour blind as it is removed from the prominent quarkonia of the same quantum numbers)[16].

It also adds confidence to the predictions that gluonic degrees of freedom are excited in the 2GeV mass region when $q\bar{q}$ "seeds" are already present. Such states are known as hybrids and these too may be showing up (see later).

7 PRODUCTION RATES

There are two main phenomenological pillars on which glueball phenomenology now tend to agree. These are their mass spectroscopy (at least for the lightest few states), and their

optimised production mechanisms. We shall see that the "interesting" states appear to share these properties.

Meson spectroscopy has been studied for several decades and the spectrum of $q\bar{q}$ states has emerged. Why in all this time has it been so hard to identify glueballs and hybrids if they exist below 2GeV?

Some time ago I suggested [14] this to be due to the experimental concentration on a restricted class of production mechanisms and on final states with charged pions and kaons. We will consider each of these in turn.

Experiments historically have tended to use beams of quarks (contained within hadrons) hitting targets which are also quark favoured. The emergence of states made from quarks was thereby emphasised. To enhance any gluonic signal above the quark "noise" required one to destroy the quarks. Hence the focussing on three particular production mechanisms, [14] in each of which the candidate scalar glueball[4] has been seen.

1. Radiative J/ψ decay: $J/\psi \rightarrow \gamma + G$ [18]

2. Collisions in the central region away from quark beams and target: $pp \rightarrow p_f(G)p_s$ [19, 20].

3. Proton-antiproton annihilation where the destruction of quarks creates opportunity for gluons to be manifested. This is the Crystal Barrel [21]-[24] and E760 [25, 26] production mechanism in which detailed decay systematics of $f_0(1500)$ have been studied.

4. Tantalising further hints come from the claimed sighting [27] of the $f_0(1500)$ in decays of the hybrid meson candidate [5] $\pi(1800) \rightarrow \pi f_0(1500) \rightarrow \pi\eta\eta$.

The signals appear to be prominent in decay channels such as $\eta\eta$ and $\eta\eta'$ that are traditionally regarded as glueball signatures. This recent emphasis on neutral final states (involving π^0, η, η') was inspired by the possibiblity that η and η' are strongly coupled to glue and reinforced by the earlier concentrations on charged particles. This dedicated study of neutrals was a new direction pioneered by the GAMS Collaboration at CERN announcing new states decaying to $\eta\eta$ and $\eta\eta'$ [28]. Note from the decays of quarkonia, fig 4, the channels $\eta\eta$ and $K\bar{K}$ are strongly correlated for quarkonia. Thus observation of states that couple strongly to η are signatures for non-quarkonia and, to the extent that η couples to glue, may be a glueball signature.

These qualitative remarks are now becoming more quantitative following work on ψ radiative decays that is currently being extended [29, 30] By combining the known B.R. $(\psi \rightarrow \gamma R)$ for any resonance R with perturbative QCD calculation of $\psi \rightarrow \gamma(gg)_R$ where the two gluons are projected onto the J^{PC} of R, one may estimate the gluon branching ratio $B(R \rightarrow gg)$. One may expect that

$$
\begin{aligned}
B(R[Q\bar{Q}] \rightarrow gg) &= 0(\alpha_s^2) \simeq 0.1 \\
B(R[G] \rightarrow gg) &= \tfrac{1}{2} \text{ to } 1
\end{aligned}
\tag{45}
$$

Known $Q\bar{Q}$ resonances (such as $f_2(1270)$) satisfy the former; we seek examples of the latter.

For example, perturbative QCD gives

$$B(\psi \to \gamma^3 P_J) = \frac{128}{5} \frac{\alpha\alpha_s}{\pi} \frac{1}{(\pi^2 - 9)} \frac{|R_p'(0)|^2}{m^3 M^2} x|H_J|^2$$

where m, M are the resonance and ψ masses respectively, $R_p'(0)$ is the derivative of the P−state wavefunction at the origin and the J dependent quantity $x|H|^2$ is plotted in fig 7. One can manipulate the above formula into the form, for scalar mesons

$$10^3 B(\psi \to \gamma 0^{++}) = \left(\frac{m}{1.5\,GeV}\right)\left(\frac{\Gamma_{R \to gg}}{96\,MeV}\right)\frac{x|H|^2}{35}$$

The analysis of ref. [18] suggests $B(\psi \to \gamma f_0(1500) \simeq 10^{-3})$. Thus a very broad $Q\bar{Q}$ state (width ~ 500 MeV) could be present at this level, but for $f_0(1500)$ with $\Gamma_T = 100\text{-}150$ MeV, one infers $B(f_0 \to gg) = 0.6$ to 0.9 which is far from $Q\bar{Q}$. Such arguments need more careful study but do add to the interest in the $f_0(1500)$.

Thus the $f_0(1500)$ has the right mass and is produced in the right places to be a glueball and with a strength (in $\psi \to \gamma f_0$) consistent with a glueball. Its total width is out of line with expectations for a $Q\bar{Q}$[4]. Its branching ratios are interesting and may also signify a glueball that is mixed in with the neighbouring $Q\bar{Q}$ nonet. It is a state for which data are accumulating and will be worth watching.

8   THE HYBRID CANDIDATES

The origins of the masses of gluonic excitations on the lattice are known only to the computer. Those in the flux tube have some heuristic underpinning. The $Q\bar{Q}$ are connected by a colour flux with tension 1 GeV/fm which leads to a linear potential in accord with the conventional spectroscopy (section 3).

The simplest glue loop is based on four lattice points that are the corners of a square. As lattice spacing tends to zero one has a circle, the diameter is $\simeq 0.5$ fm, the circle of flux length is then $\simeq 1.5$ fm and, at 1 GeV/fm, the ballpark 1.5 GeV mass emerges. In the limit of lattice spacing vanishing, its 3-D realisation is a sphere, and hence it is natural that this is 0^{++}.

The next simplest configuration is based on an oblong, one link across and two links long. The total length of flux is $\simeq \frac{3}{2}$ larger than the square and the ensuing mass $\simeq \frac{3}{2} \times 1.5$ GeV $\simeq 2.2$ GeV. In the 3-D continuum limit this rotates into a rugby ball shape rather than a sphere. A decomposition in spherical harmonics contains $L \geq 0$, in particular 2^{++}. This is by no means rigorous (!) but may help to give a feeling for the origin of the glueball systematics in this picture, inspired by the lattice.

Finally one has the prediction that there exist states where the gluonic degrees of freedom are excited in the presence of $Q\bar{Q}$. With the 1 GeV/fm setting the scale, one finds [10, 11] that the lightest of these "hybrid" states have masses of order 1 GeV above

their conventional $q\bar{q}$ counterparts. Thus hybrid charmonium may exist at around 4 GeV, just above the $D\bar{D}$ pair production threshold. More immediately accessible are light quark hybrids that are expected in the 1.5 to 2 GeV range after spin dependent mass splittings are allowed for.

There are tantalising sightings of an emerging spectroscopy as I shall now review.

It is well known that hybrid mesons can have J^{PC} quantum numbers in combinations such as $0^{--}, 0^{+-}, 1^{-+}, 2^{+-}$ etc. which are unavailable to conventional mesons and as such provide a potentially sharp signature.

It was noted in ref.[31] and confirmed in ref.[32] that the best opportunity for isolating exotic hybrids appears to be in the 1^{-+} wave where, for the I=1 state with mass around 2 GeV, partial widths are typically

$$\pi b_1 : \pi f_1 : \pi \rho = 170 \; MeV : 60 \; MeV : 10 \; MeV \qquad (46)$$

The narrow $f_1(1285)$ provides a useful tag for the $1^{-+} \rightarrow \pi f_1$ and ref.[33] has recently reported a signal in $\pi^- p \rightarrow (\pi f_1)p$ at around 2.0 GeV that appears to have a resonant phase.

Note the prediction that the $\pi \rho$ channel is not negligible relative to the signal channel πf_1 thereby resolving the puzzle of the production mechanism that was commented upon in ref. [33]. This state may also have been sighted in photoproduction [34] with $M = 1750$ and may be the $X(1775)$ of the Data Tables, ref.[2].

A recent development is the realisation that even when hybrid and conventional mesons have the **same** J^{PC} quantum numbers, they may still be distinguished [32] due to their different internal structures which give rise to characteristic selection rules[35, 11, 32]. As an example consider the $\rho(1460)$.

(i) If $q\bar{q}$ in either hybrid or conventional mesons are in a net spin singlet configuration then the dynamics of the flux-tube forbids decay into final states consisting only of spin singlet mesons.

For $J^{PC} = 1^{--}$ this selection rule distinguishes between conventional vector mesons which are ${}^3S_1$ or ${}^3D_1$ states and hybrid vector mesons where the $Q\bar{Q}$ are coupled to a spin singlet. This implies that in the decays of hybrid ρ, the channel πh_1 is forbidden whereas πa_1 is allowed and that πb_1 is analogously suppressed for hybrid ω decays; this is quite opposite to the case of ${}^3L_1$ conventional mesons where the πa_1 channel is relatively suppressed and πh_1 or πb_1 are allowed[36, 17]. The extensive analysis of data in ref.[37] revealed the clear presence of $\rho(1460)$[2] with a strong πa_1 mode but no sign of πh_1, in accord with the hybrid situation. Furthermore, ref.[37] finds evidence for $\omega(1440)$ with no visible decays into πb_1 which again contrasts with conventional $q\bar{q}$ (${}^3S_1$ or ${}^3D_1$) initial states and in accord with the hybrid configuration.

(ii) The dynamics of the excited flux-tube in the **hybrid** state suppresses the decay to mesons where the $q\bar{q}$ are ${}^3S_1$ or ${}^1S_0$ "$L = 0$" states. The preferred decay channels are to $(L = 0) + (L = 1)$ pairs[11, 31]. Thus in the decays of hybrid $\rho \rightarrow 4\pi$ the πa_1 content is predicted to be dominant and the $\rho\rho$ to be absent. The analysis of ref.[37] finds such a pattern for $\rho(1460)$.

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(iii) The selection rule forbidding $(L = 0) + (L = 0)$ final states no longer operates if the internal structure or size of the two $(L = 0)$ states differ[11, 35]. Thus, for example, decays to $\pi + \rho$, $\pi + \omega$ or $K + K^*$ may be significant in some cases[32, 38], and it is possible that the **production** strength could be significant where an exchanged π, ρ or ω is involved, as the exchanged off mass-shell state may have different structure to the incident on-shell beam particle. This may be particularly pronounced in the case of **photoproduction** where couplings to $\rho\omega$ or $\rho\pi$ could be considerable when the ρ is effectively replaced by a photon and the ω or π is exchanged. This may explain the production of the candidate exotic $J^{PC} = 1^{-+}$ (ref.[33]) and a variety of anomalous signals in photoproduction.

The first calculation of the widths and branching ratios of hybrid mesons with conventional quantum numbers is in ref.[32]: the $0^{-+}, 2^{-+}$ and the 1^{--} are predicted to be potentially accessible. It is therefore interesting that each of these J^{PC} combinations shows rather clear signals with features characteristic of hybrid dynamics and which do not fit naturally into a tidy $Q\bar{Q}$ conventional classification.

We have already mentioned the 1^{--}. Turning to the 0^{-+} wave, the VES Collaboration at Protvino confirm their enigmatic and clear 0^{-+} signal in diffractive production with 37 GeV incident pions on beryllium [27]. Its mass and decays typify those expected for a hybrid: $M \approx 1790$ MeV, $\Gamma \approx 200$ MeV in the $(L = 0) + (L = 1)$ $\bar{q}q$ channels $\pi^- + f_0$; $K^- + K_0^*$, $K(K\pi)_S$ with no corresponding strong signal in the kinematically allowed $L = 0$ two body channels $\pi + \rho$; $K + K^*$. This confirms the earlier sighting by Bellini et al[39], listed in the Particle Data group[2] as $\pi(1770)$.

The resonance also appears to couple as strongly to the enigmatic $f_0(980)$ as it does to $f_0(1300)$, which was commented upon with some surprise in ref. [27]. This may be natural for a hybrid at this mass due to the predicted dominant KK_0^* channel which will feed the $(KK\pi)_S$ (as observed [27]) and hence the channel $\pi f_0(980)$ through the strong affinity of $K\bar{K} \to f_0(980)$. Thus the overall expectations for hybrid 0^{-+} are in line with the data of ref.[27]. Important tests are now that there should be a measureable decay to the $\pi\rho$ channel with only a small πf_2 or KK^* branching ratio. At the Hadron95 conference it was learned that in the $\pi\eta\eta$ final state the glueball candidate is seen: $\pi(1.8) \to \pi f_0(1500) \to \pi\eta\eta$.

This leaves us with the 2^{-+}. There are clear signals of unexplained activity in the 2^{-+} wave in several experiments for which a hybrid interpretation may offer advantages. These are discussed in ref.[32].

These various signals in the desired channels provide a potentially consistent picture. The challenge now is to test it. Dedicated high statistics experiments with the power of modern detection and analysis should re- examine these channels. Ref.[38] suggests that the hybrid couplings are especially favourable in *low-energy* photoproduction and as such offer a rich opportunity for the programme at an upgraded CEBAF or possibly even at HERA. If the results of ref.[40] are a guide, then photoproduction may be an important gateway at a range of energies and the channel $\gamma + N \to (b_1\pi) + N$ can discriminate hybrid 1^{--} and 2^{-+} from their conventional counterparts.

Thus to summarise, we suggest that data are consistent with the existence of low lying

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multiplets of hybrid mesons based on the mass spectroscopic predictions of ref.[11] and the production and decay dynamics of ref. [32]. Specifically the data include

$$
\begin{aligned}
0^{-+} & \quad (1790\ MeV; \Gamma = 200\ MeV) & \to & \quad \pi f_0; K\bar{K}\pi & \quad (47) \\
1^{-+} & \quad (\sim 2\ GeV; \Gamma \sim 300\ MeV) & \to & \quad \pi f_1; \pi b_1(?) \\
2^{-+} & \quad (\sim 1.8\ GeV; \Gamma \sim 200\ MeV) & \to & \quad \pi b_1; \pi f_2 \\
1^{--} & \quad (1460\ MeV; \Gamma \sim 300\ MeV) & \to & \quad \pi a_1
\end{aligned}
$$

Detailed studies of these and other relevant channels are called for together with analogous searches for their hybrid charmonium analogues, especially in photoproduction or $e^+ e^-$ annihilation.

9 RADIALOGY

If these states are not glueballs and hybrids, what are they? On masses alone they could be radial excitations as we have already noted. The decay patterns have been seen to fit well with gluonic excitations but we need to close the argument by considering the decays in the radial hypothesis. Only if the hybrid succeeds and radial fails can one be sure to have a convincing argument.

As an illustration consider the 0^{-+} (1800) which could be either the hybrid or the radial $3S({}^1S_0)$ quarkonium (denoted π_{RR}). In fig 8 we see the width in an S.H.O. calculation as a function of the oscillator strength β. The $\pi\rho$ channel is small near the preferred value of $\beta \simeq 0.35$ - 0.4 GeV and so both radial and hybrid share the property of suppressed $\pi\rho$ and, to some extent, KK^*. It is therefore encouraging that data show a clear absence of $\pi\rho$ in the 1800 region in contrast to πf_0 which shows two clear bumps at 1300 and 1800 MeV. Notice that for π_{RR} the πf_0 is small for all β in dramatic contrast to the hybrid, where this channel is predicted to dominate, and also apparently in contrast to data. The $\pi\rho_R$ (radial ρ) is predicted to be large for π_{RR} and hence one would expect a significant branching ratio $\pi_{RR} \to \pi\rho_R \to 5\pi$ which is not apparent in data though more study is warranted.

A discriminator between π_{RR} and hybrid π_H is the $\rho\omega$ channel. This is a dominant channel for π_{RR} whereas it is predicted to be absent for π_H [32].

Another example that distinguishes hybrid and radial is in the 1^{++} sector. There is a clear signal in πf_1 [33]; a_{1H} is forbidden to decay into πb_1 due to the spin selection rule [32] whereas $a_{1R} \to \pi b_1$ with a branching ratio equal to $a_{1R} \to \pi f_2$ over the full β range and moreover $\Gamma(a_{1R} \to \pi b_1) \gtrsim 2\Gamma(a_{1R} \to \pi f_1)$. (see fig 9) The πf_2 channel may be easier experimentally. In any event we see that there are characteristic differences in the branching ratios for radials and hybrids states that should enable a clear separation to be made.

After years of searching, at last we have some potential candidates for mesons where the gluonic degrees of freedom are excited. Furthermore there are some clear selection

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rules and other discriminators in their decay branching ratios that can help to verify their existence and thereby complete the strong QCD sector of the standard model.

Acknowledgements

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Figure Captions

Figure 1: Comparative spectroscopies of Coulomb, linear and oscillator potentials

Figure 2: Template for $q\bar{q}$ spectroscopy; (a) bottomium, (b) charmonium (c) u and d flavours

Figure 3: The lightest L=0-3 $q\bar{q}$ ($q = u, d$) and $_\Lambda L =_1 P$ hybrid masses from Monte Carlo (after ref.[10]). Square brackets denote masses used as input

Figure 4: γ_{ij}^2 as a function of α for $\rho = 1$ (a) and $\rho = 0.75$ or 1.25 (b) for quarkonium decay (up to a common multiplicative factor). Dotted line: $\pi\pi$; dash-dotted line: $K\overline{K}$; dashed line: $\eta\eta'$; solid line: $\eta\eta$.

Figure 5: Contributions to gluonium decay: $QQ\bar{Q}Q$ (a), GG (b), $Q\overline{Q}$ (c) and interpretation as $Q\bar{Q}$ mixing (d) involving the energy denominator $E_G - E_{Q\bar{Q}}$

Figure 6: Glueballs, quarkonia and perturbations: (a) primitive $Q\bar{Q}$ and (b) primitive glueball G_0 in flux tube simulation of lattice QCD; perturbation V_1 (c) and V_2 (d); the effect of V_1 on $Q\bar{Q}$ is shown in (e), and on G is shown in (f); the effect of V_2 on G is shown in (g).

Figure 7: $x|H|^2$ versus $x \equiv 1 - \frac{m^2}{M_\psi^2}$. Solid line is 0^{++}, dashed line is 2^{++}

Figure 8: Partial widths of a $\pi_{RR}(1800)$ second radial excitation; $^3P_0$ model normalised to $f_2(1270)$ width with wavefunction parameter β variable

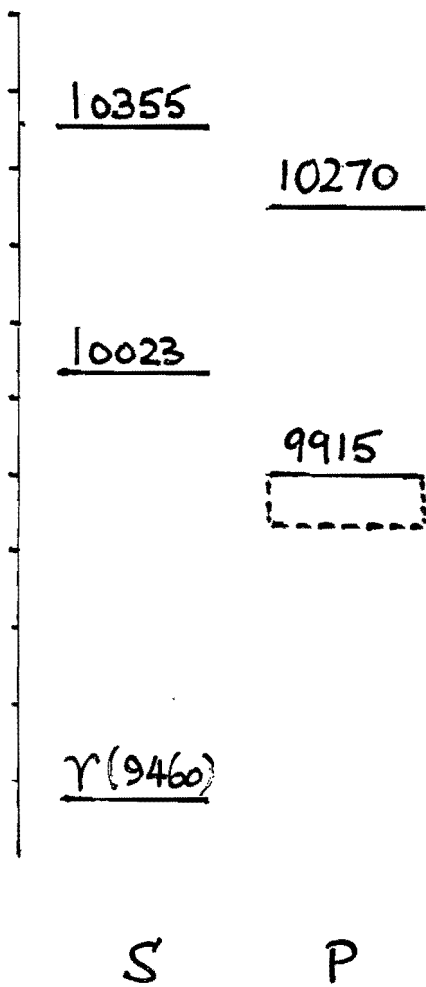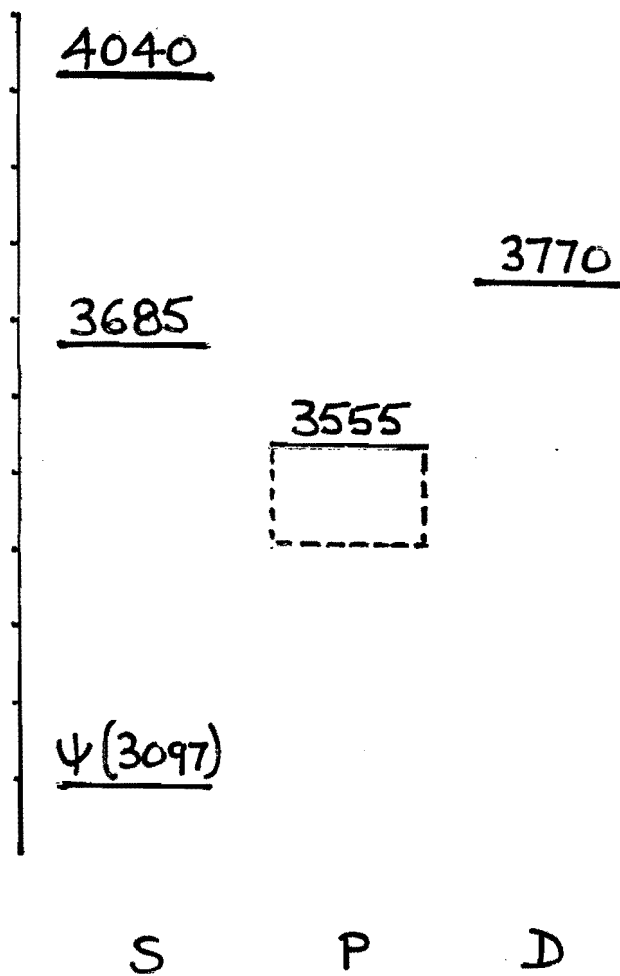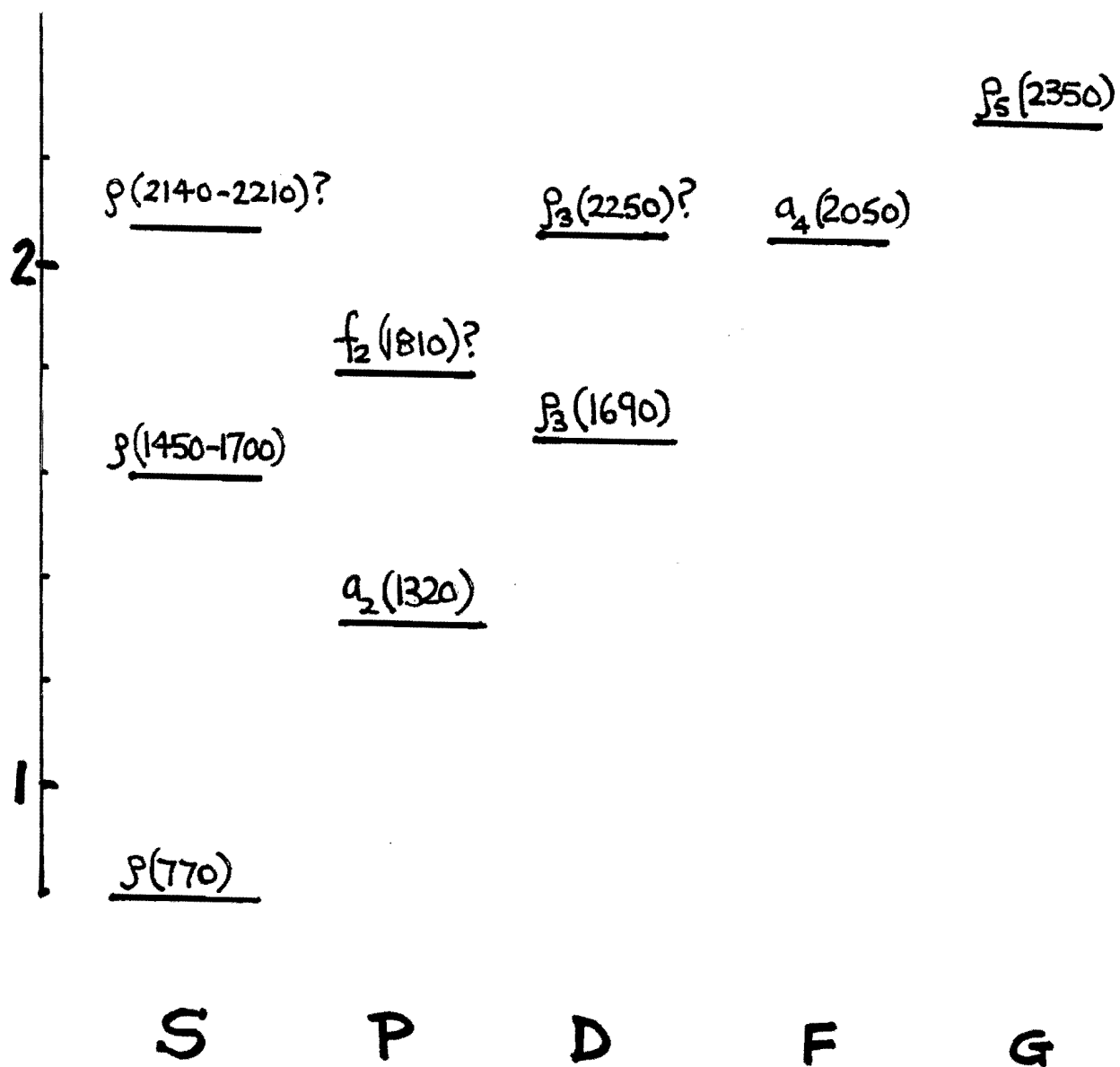Figure 9: As previous figure but for $a_{1R}(1700)$ radial excitation

Fig 1

10355

10270

10023

9915

Υ(9460)

S   P

(a)

4040

3685

3770

3555

Ψ(3097)

S   P   D

(b)

Fig 2

$\rho_5 (2350)$

$\rho (2140-2210)?$

$\rho_3 (2250)?$ $a_4 (2050)$

2—

$f_2 (1810)?$

$\rho_3 (1690)$

$\rho (1450-1700)$

$a_2 (1320)$

1—

$\rho (770)$

S   **P**   **D**   **F**   **G**

Fig 2 (c)

Fig 3

Fig. 4

Fig. 5

a)

b)

c)

d)
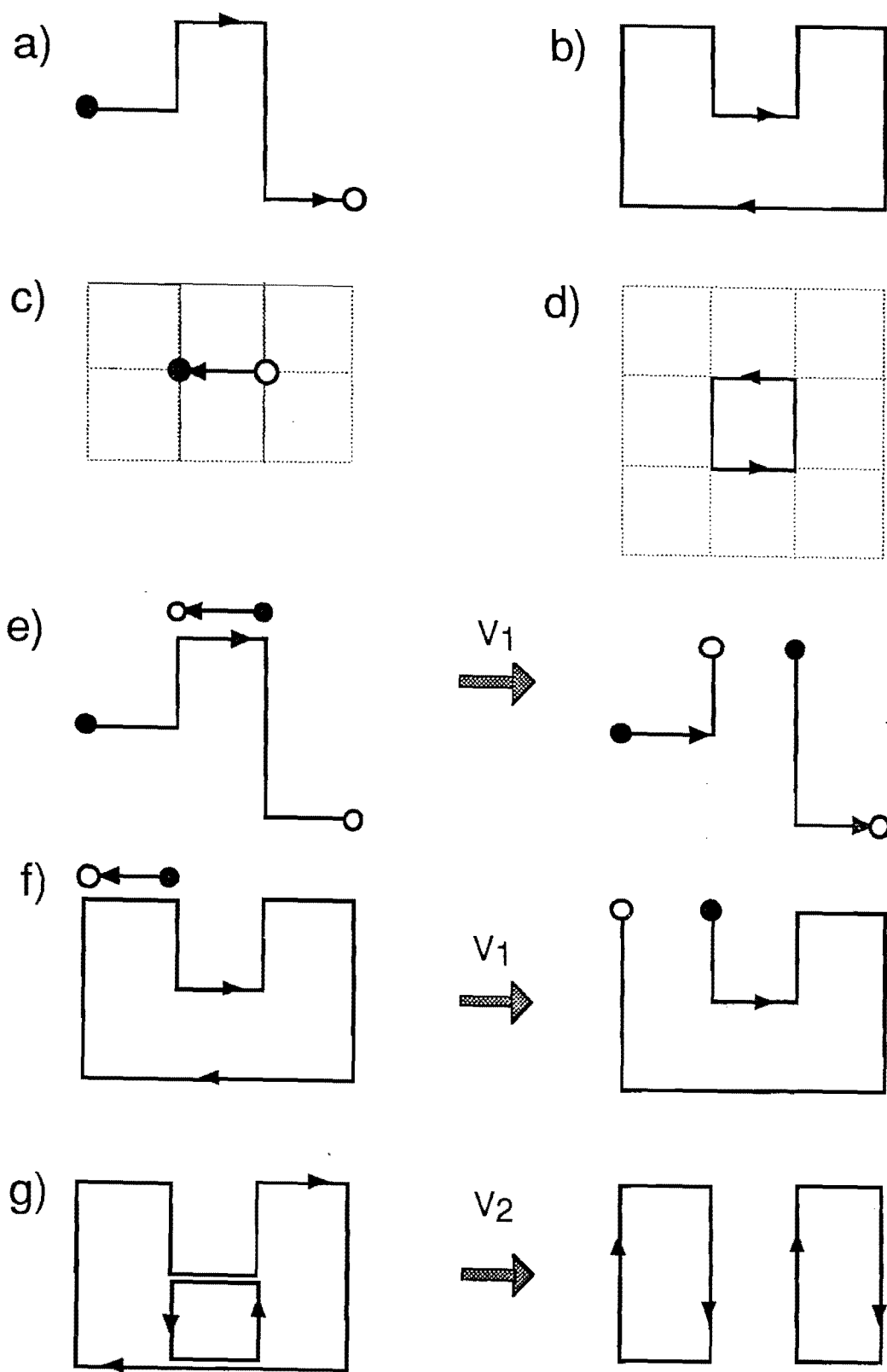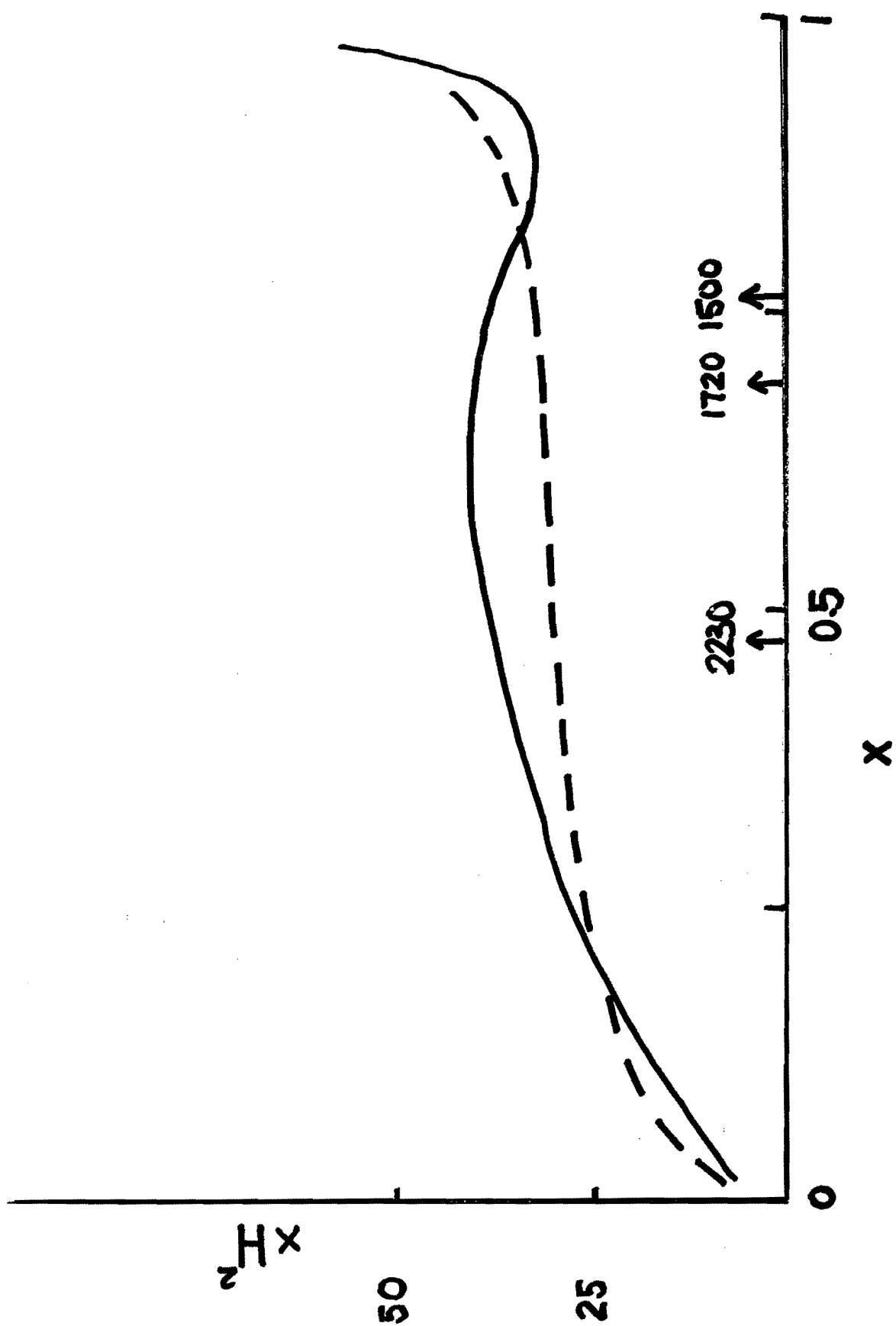
e) V_1

f) V_1

g) V_2

Fig.6

Fig 7

$3^1S_0 \pi (1800)$

Partial widths of a $3\,{}^1S_0\,\pi\,(1800)$ second radial excitation.

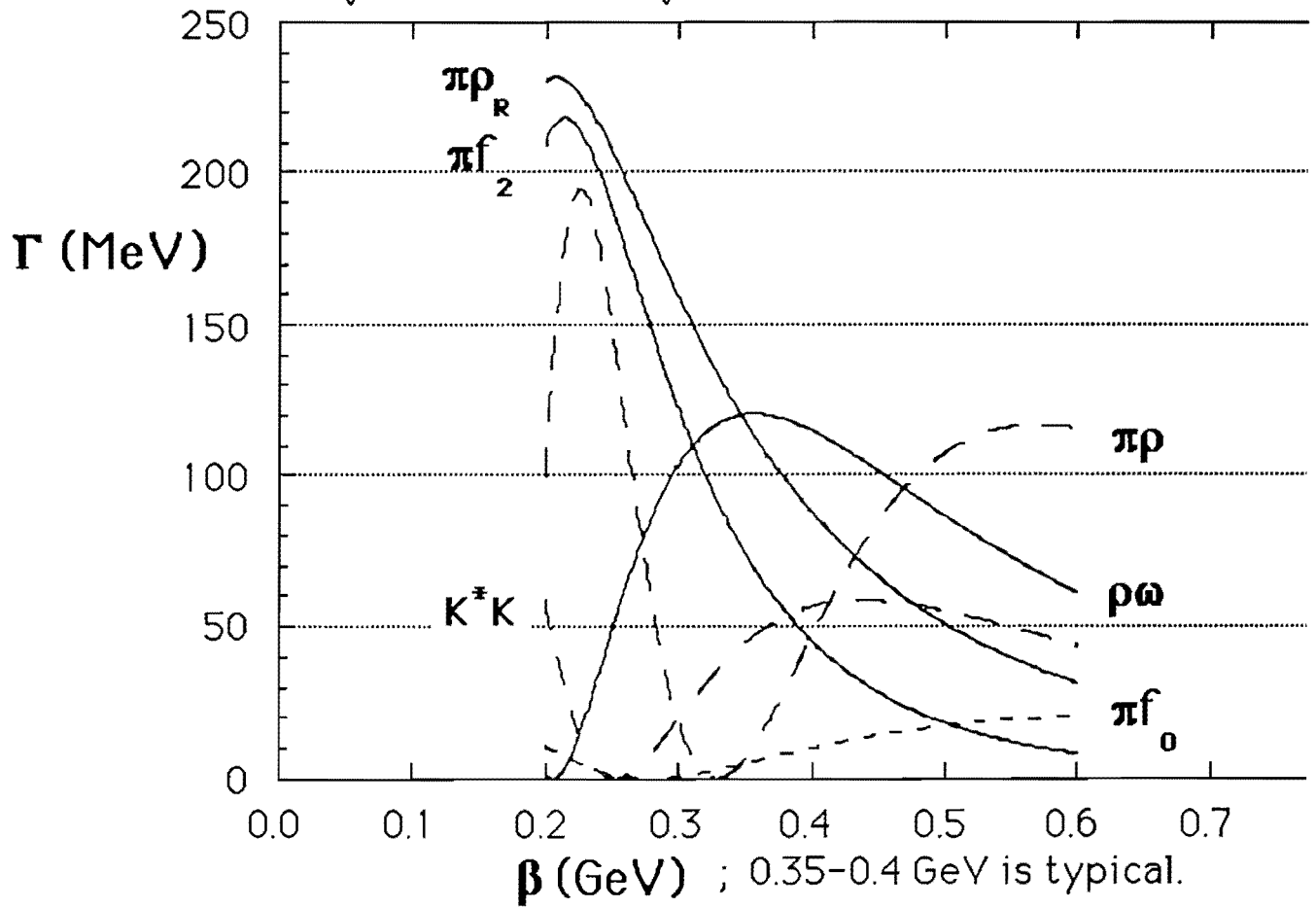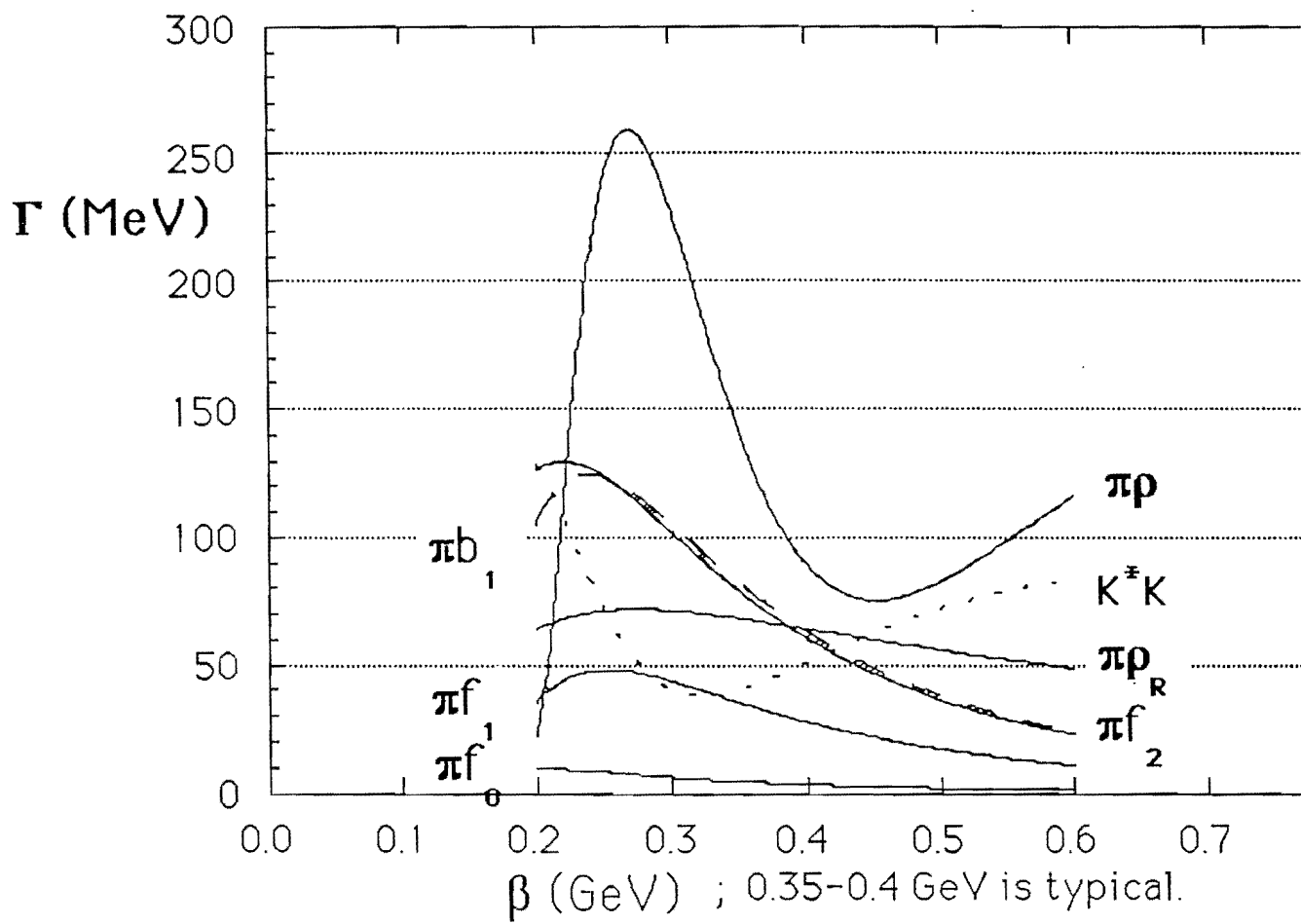($^3P_0$ decay model, with $\gamma_0 = 0.5$ and wavefunction parameter β variabl

Γ (MeV)

$\pi\rho_R$

πf_2

$\pi\rho$

$\rho\omega$

K^*K

πf_0

β (GeV) ; 0.35-0.4 GeV is typical.

Fig 8

$a_{1R}(1700)$

Γ (MeV)

300

250

200

150

100

50

0

$\pi\rho$

$K^{\pm}K$

$\pi\rho_R$

πf_2

πb_1

πf_1

πf_0

0.0   0.1   0.2   0.3   0.4   0.5   0.6   0.7

β (GeV) ; 0.35–0.4 GeV is typical.

Fig 9