The challenge of the solve phase of a multicore solver

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Sparse Direct Solvers

Solve $A\mathbf{x} = \mathbf{b}$ using a Cholesky factorization:

$$PAP^T = LL^T$$

Where A is...

- Large
- Sparse
- Positive Definite

Variants for more general matrices — LDL^T, LU.

Sparse Direct Solvers

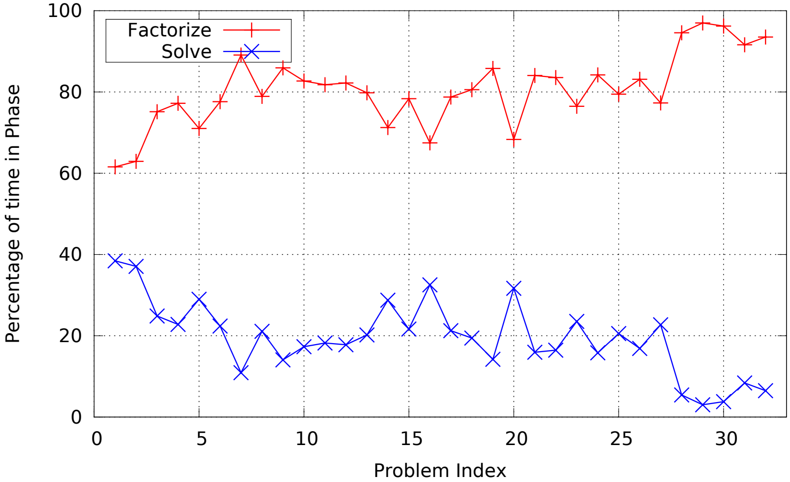
Four phases:

Ordering	Reduce fill-in (AMD, MeTiS, SCOTCH, ...)
Analyse	Construct data structures, plan computation
Factorize	Numerical computation
Solve	Triangular solves with L and L^T

Traditionally: Factorize phase by far the most time consuming:

$$1 \text{ Factorize} = 50\text{-}100 \text{ Solves}$$

Multicore. Oh dear.



Factorize parallel. Solve serial.

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So? Just parallelize the solve!

Lots of ways of doing this:

- Threaded BLAS/OpenMP do loops

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Regardless of how we did this...

...only a 2–3 times speedup

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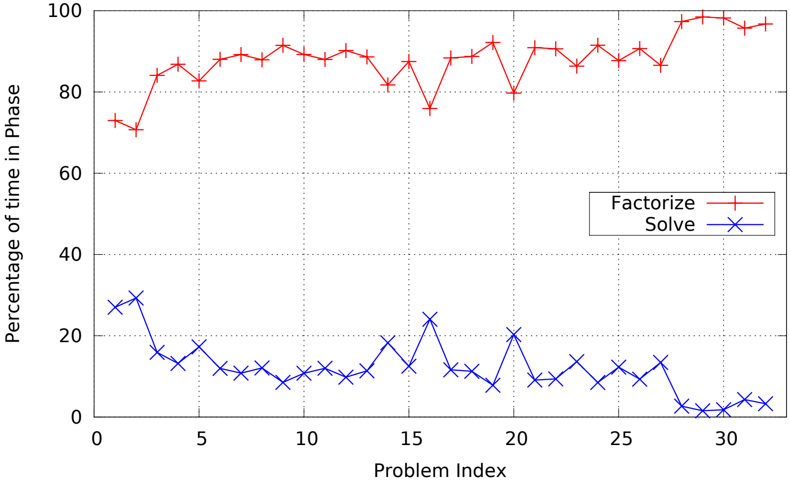
Not much better...



Factorize and solve parallel.

Why?

Typically:

	Flops	Cache Misses
Factor	$> 98\%$	60–80%
Solve	$< 2\%$	20–40%

Speedup on a single quad-core: < 1.30
Speedup on two quad-cores: < 3.00

Solve is memory bound.

Multicore $=$ more cores, same memory bandwidth.

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Interior-point Methods multiple correctors and often aggressive iterative refinement.

Can we fix it?

Need to exploit the cache.

or

Need to trade off more computation for less memory bandwidth.

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Compression?

Combined factor-solve

Forward substitution can be done at same time as factorization.
(data is already in cache!)

Well established in out-of-core solvers.

Halves time for first solve

But... doesn't help on subsequent solves.

Compression

Two *theoretical* limits:

Compression ratio How much bandwidth do we save?

Decompression rate How many extra cycles does this take?

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Exploit two levels of compression:

Algorithm specific We have deliberately introduced extra zeros to make the factorize go faster, does removing them help?

Generic Use a generic compression algorithm such as gzip, bzip2, LZO...

Compression ratios

	nemin = 32	LZO nemin = 32		nemin = 1		LZO nemin = 1	
	size	size	**CR**	size	**CR**	size	**CR**
1.	579	333	**1.74**	283	**2.04**	269	**2.14**
2.	981	567	**1.73**	489	**2.01**	463	**2.12**
3.	315	283	**1.11**	292	**1.08**	269	**1.17**
4.	407	328	**1.24**	342	**1.19**	303	**1.34**
5.	10829	10310	**1.05**	10304	**1.05**	10009	**1.08**

(size is storage for L in Megabytes)

1. CEMW/tmt_sym
2. Schmid/thermal2
3. GHS_psdef/crankseg_1
4. DNVS/shipsec1
5. GHS_psdef/audikw_1

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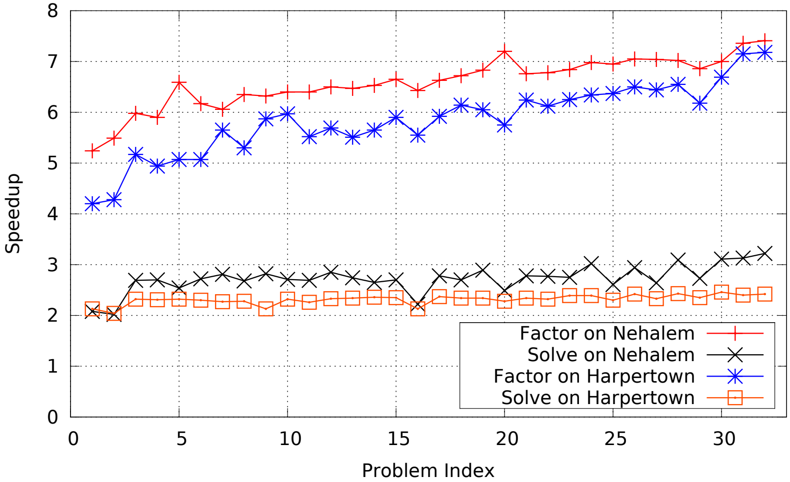
In practice...

Not even getting close to theoretical maxima.

	4 cores			8 cores			**CR**
	without	with	**ratio**	without	with	**ratio**	
1.	0.44	0.38	**1.16**	0.30	0.27	**1.11**	**1.74**
2.	0.76	0.67	**1.13**	0.53	0.48	**1.10**	**1.73**
3.	0.19	0.19	**1.00**	0.11	0.11	**1.00**	**1.11**
4.	0.25	0.24	**1.04**	0.15	0.14	**1.07**	**1.24**
5.	6.90	6.92	**1.00**	3.61	3.59	**1.01**	**1.05**

(times in seconds)

Are architectures improving?



Conclusions

- Multicore is not SMP
- Memory-bound operations are a problem
- Solve phase of direct methods now significant
- No easy fix

Any questions?
Any solutions?

Time distribution, all phases.