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DUAL PROPERTY OF DIFFRACTIVE RESONANCES FROM SEMILOCAL FACTORISATION
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## ABSTRACT

By semilocal factorisation the ratio of diffractive resonance
production cross-sections on $\pi$ and $K$ should equal the ratio of $\pi$ and $K$ couplings to $P(f)$ if the resonances are dual to Pomeron (normal meson) exchange. Experimental data favours the former.

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The normal two-component scheme of duality in which resonances and background in the direct channel are respectively dual to Regge and Pomeron exchange in the crossed channel is well established for the case of twobody scattering. According to Mueller's optical theorem we can use data On inclusive reactions to get information on Reggeon-particle elastic scattering and the question then arises whether the usual duality scheme carries over for the Reggeon-particle case. Most of the dual models for inclusive reactions agree that normal duality should hold for the case when the Reggeon is a meson trajectory and experimental data seem to be consistent with this view ${ }^{1}$. However when the Reggeon is the Pomeron the situation is controversial. According to Einhorn et al ${ }^{2}$, we should expect an abnormal situation in which the cross-section for diffractive resonances ${ }^{3}$ is described, on the average, by the triple-Pomeron coupling - ie resonances dual to Pomeron exchange. On the other hand, several authors have suggested the normal duality scheme to apply also to diffractive resonances.

In this note we examine data on diffractive resonance production and, assuming that both semi-local duality and factorisation are valid in this case, test whether such resonances favour the normal or abnormal duality schemes.

Consider the process $\pi^{-} p \rightarrow p X$ via the exchange of the Pomeron in the t-channel, as shown in fig. la). if $M^{2}$ is the missing mass of $X$ then the large $M^{2}$ behaviour is given by the triple-Regge graph of fig. $1 b$ ), and we can write the first-moment finite mass sum rule ${ }^{4}$, at fixed $t$, in the form:-

$$
\begin{equation*}
\int_{0}^{N} v d v \frac{d^{2} \sigma}{d t d v}=\beta_{\pi}^{k}+\pi^{k}-g_{p p}^{k}(t) s \alpha_{p}^{2 \alpha_{p}(t)-2} \frac{N_{k}-2 \alpha_{p}(t)+2}{\alpha_{k}-2 \alpha_{p}(t)+2} \tag{1}
\end{equation*}
$$

where $v=\frac{1}{2}\left(M^{2}-t-m_{\pi}^{2}\right)$. Equation (1) is simply the analogue of the usual FESR but for the Pomeron-particle'amplitude. The odd moment ensures the correct crossing property for this amplitude. If, on the left hand side,
we consider only the contribution to the cross-section coming from the resonances in the missing mass then we expect $k$ on the right hand side to be meson Regge exchange for the case of normal duality or the Pomeron in the abnormal case.

If we now consider $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{p}+$ Resonances via Pomeron exchange we obtain a similar expression to eqn. (1) and taking the ratio of the two we get*

$$
\begin{equation*}
\frac{\sum v_{\text {RES }} \frac{d \sigma}{d t}\left(\pi^{-} p \rightarrow p+\text { Res. }\right)}{\sum v_{\text {RES }} \frac{d \sigma}{d t}\left(K^{+} p \rightarrow p+\text { Res. }\right)}=\frac{\beta^{k} \pi^{+} \pi^{-}}{\beta_{K^{+} K^{-}}^{k}}=r \tag{2}
\end{equation*}
$$

If $k$ is the Pomeron then we expect $r$ to be close to 1 (corresponding ratio of total cross-sections is 1.2 ). If $k=$ meson, we expect $r$ to be close to 2 , the ratio of the $\pi \pi$ and the $K \bar{K}$ couplings of the leading meson trajectory f. For the lowest resonances, of course, the ratio may be less than 2, due to the $\mathrm{f}^{\prime}$ contribution to the K -induced process. However, one expects the $\mathrm{f}^{\prime}$ contribution to die away rapidly and the ratio to approach 2 as one moves up in the resonance mass.

Thus we see that the measurement of the ratio $r$ can give an indication of which type of duality is valid for diffractive resonances. r close to 1 would favour abnormal duality while r close to 2 would favour the usual two component scheme as in two-body scattering.

While the principle of the method is essentially straightforward the actual determination of $r$ from data is rather less so. The basic difficulty is that of defining what a particular diffractive resonance is. There are several, widely used, criteria for estimating the cross-sections of $A_{1}, A_{3}$, Q, L corresponding to different separations of each resonance from the background.

[^0]We have in fact attempted to use every well-known definition of a diffractive resonance and evaluated $r$ in each case. From four different methods we find values for $r$.all of which are close to 1 . Thus it is fair to claim that the result indicates strong support for the abnormal scheme of duality.

We now describe the details of the analysis. the resonances we include are $A_{1}, A_{3}, Q$ and $L$ and also the contributions from elastic scattering ie $\pi$, K. The elastic cross-sections do not overwhelm the resonance contributions since the weighting factor $v$ severely damps down the magnitude of the elastic contribution. We choose to work in the $t$ range $0.1 \leqslant|t| \leqslant 0.3$. Our information on $A_{1}$ and $A_{3}$ production comes mainly from $\pi^{-} p \rightarrow \pi^{-} \pi^{+} \pi^{-}{ }^{-5}$, and since we are dealing with an "inclusive" situation we must correct for all decay modes of the diffractive resonances. Similarly the $Q$ and $L$ cross
 only correction is for the $\pi^{\circ} \pi^{\circ} \pi^{-}$mode and hence the factor is 2 . Similarly the $A_{3}$ has to be corrected for this mode but since the $f$ has $20 \%$ inelasticity the resulting $A_{3}$ correction factor $i s 1.88$. The $Q$ has to be corrected for $K^{+} \pi^{\circ} \pi^{\circ}$ and $K^{\circ} \pi^{\circ} \pi^{+}$and since the $K_{\rho} / K^{*} \pi$ ratio for the $Q$ is 0.2 the final factor is 2.38. For the $L$ we assume only $K^{* \hbar} \pi$ decay modes, the correction for $K^{+} \pi^{0} \pi^{0}$ and $K^{0} \pi^{0} \pi^{+}$giving a factor 2.25. However the $K^{k \pi}$ is known to decay equally into $K \pi$ and $K \pi \pi$ so $L$ decay into $K^{+} \pi^{+}{ }^{+}{ }^{-}$has to be eventually corrected by a factor 4.5 .

The four different criteria used for defining the resonances were Method 1

The diffractive resonance cross-sections were defined on the basis of a simple mass-cut in the $3 \pi$ or $K \pi \pi$ mass distribution
$A_{j}^{-}$was taken as the whole crosi-section in $\pi^{-} \pi^{+} \pi^{-}$from 1.0 to 1.2 GeV

| $\mathrm{A}_{3}$ | 11 | 11 | 11 | 11 | 11 | '1 | 11 | 11 | 11 | 11 | 1.6 to 1 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $Q^{+}$ | י | ' | 11 | 11 | 11 | 11 | 11 | 11 | $\mathrm{K}^{+}{ }^{+}{ }^{-}$ | " | 1.15 to | 1.35 |
| L+ | 11 | ' | 1 | 11 | 11 | 11 | 11 | 11 | 1 | " | 1.65 to | 1.95 |

The resulting cross-sections using the data of references ${ }^{5}$ ) and ${ }^{6}$ ) are shown in table 1 . The value of $r$ determined by this method is $0.77 \pm 0.05$.

## Method 2

Here we attempted to make a resonance-background separation. The most realistic background we took to be that described by a one-pion exchange Deck model. When this curve is subtracted from the $3 \pi$ spectrum we are left with two bumps, one which is the sum of $A_{1}$ and $A_{2}$, the other being the $A_{3}$. To remove the $A_{2}$, we made a mass cut from 1.26 to 1.36 GeV . Similarly by making a mass cut in $\mathrm{K} \pi \pi$ spectrum between 1.37 and $1.47, \mathrm{~K}^{* *}$ was removed. The cross-sections corresponding to the removal of Deck background are also shown in table 1 . The corresponding value of $r$ is $0.94 \pm 0.05$.

## Method 3

This is the same as method 2 but the " $A_{2}$ cross-section" is now included in the $A_{1}$ and " $K$ "* cross-section" with the $Q$. The value of $r$ was then $0.92 \pm 0.05$.

## Method 4

This was perhaps the most indirect definition of a diffractive resonance. The lllinois partial wave analysis takes the $A_{1}$ and $A_{3}$ to be the entire $J^{P}=1^{+}$and $2^{-}$parts of the $3 \pi$ system between 1.0 and 1.2 GeV and between 1.5 and 1.8 GeV respectively. Similarly the $Q$ and $L$ cross-sections correspond to the $1^{+}$and $2^{-}$waves of the $K \pi \pi$ system between 1.2 and 1.4 GeV and between 1.7 and 1.9 GeV . It should be noted however that the phase shifts in these partial waves do not show resonance-like behaviour. No background is subtracted in these waves. Using this definition, r turns
out to be $1.29 \pm 0.03$.

The analysis was carried out $p_{1 a b} \simeq 13 \mathrm{GeV} / \mathrm{c}$ and we assume that at such energies the t-channel is dominated by Pomeron exchange for these channels. In any case the inclusion of meson-exchange contributions would tend to bias the resulting value of $r$ towards 2 since normal duality is believed to be valid for meson-Reggeons. The fact that we obtain a value of $r$ so near to $l$ which is also common to several definitions for the diffractive resonances allows us to conclude that the experimental data favours the abnormal duality scheme, where these resonances are dual to the Pomeron.

Note that in methods 1, 2 and 3, $r$ shows no sign of increasing with the resonance mass. The only exception is the method 4, based on partial wave separation, where $r$ shows a drastic increase. The partial wave data on $Q$ and $L$, however, are still very preliminary; and if correct, they would account for only a small fraction of the $Q$ and $L$ bumps. Therefore we would not like to draw any firm conclusion for the case of resonances defined by the partial wave separation criterion.

The same conclusion on the duality behaviour of diffractive resonances was reached earlier ${ }^{7}$ from the scaling behaviour of the diffractive proton peak in $\pi^{-} p \rightarrow p X$. The scaling behaviour suggests that the PPM contribution is too small to describe the diffractive resonances, so that the latter must be described by the PPP term.

It is worth stressing that semilocal factorisation provides many useful relations for the normal (non diffractive) resonances, as well. For example the set of reactions $\pi^{-} p \rightarrow \rho^{\circ} n, f n, g n$ can be simply related, in turn, to the reactions $\gamma p \rightarrow \rho^{+} n, A_{2}^{+} n, g^{+} n$ where in each case we pick out the dominant $\pi$ exchange contribution. Here normal duality for the Reggeon-particle amplitude is applicable and semi-local factorisation
then gives, for example,
$\frac{\frac{d \sigma_{\pi}}{d t}(\pi-p+f n)}{\frac{d \sigma_{\pi}}{d t}\left(Y P+A_{2}^{+} n\right)}=\frac{2 \beta_{f \pi \pi}}{\beta_{f Y Y}}$
From Regge analysis of the total cross-sections, the ratio on the right hand side is 430. Data on $\pi^{-} p \rightarrow f n$ at $7 \mathrm{GeV}^{8}$ and $\gamma p \rightarrow A_{2}^{+}$at $5.3 \mathrm{GeV}^{9}$ for $0 \leqslant\left|t^{\prime}\right| \leqslant 0.1$ give the ratio on the left hand side as $527 \pm 300$. Despite the large experimental uncertainty we can regard this as consistent with the semi-local factorisation prediction. It will be particularly interesting to test the prediction for $\rho$ when the charged $\rho$ photoproduction data becomes available. In any case it is remarkable that $\pi$ induced reactions can be related to photo-induced process in this rather simple way. ACKNOWLEDGEMENTS

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The cross-sections tabulated correspond to the values of

$$
\int_{-0.3}^{-0.1} v_{R E S} \frac{d \sigma}{d t}\left(K_{K}^{\pi} p \rightarrow p+R E S\right) d t \text { in. mb. } G e V^{2}
$$

evaluated at $p_{1 a b} \simeq 10 \rightarrow 14 \mathrm{GeV} / \mathrm{c}$ using data of references 5 and 6 and the four methods described in the text, and corrected for unseen decay modes.

|  | Method 1 | Method 2 | Method 3 | Method 4 |
| :--- | :---: | :---: | :---: | :---: |
| $\pi$ | $0.245 \pm 0.009$ | $0.245 \pm 0.009$ | $0.245 \pm 0.009$ | $0.245 \pm 0.009$ |
| $A_{1}$ | $0.126 \pm 0.011$ | $0.062 \pm 0.006$ | $0.082 \pm 0.007$ | $0.120 \pm 0.020$ |
| $A_{3}$ | $0.249 \pm 0.020$ | $0.079 \pm 0.006$ | $0.079 \pm 0.006$ | $0.037 \pm 0.008$ |
| K | $0.219 \pm 0.009$ | $0.219 \pm 0.009$ | $0.219 \pm 0.009$ | $0.219 \pm 0.009$ |
| Q | $0.152 \pm 0.010$ | $0.089 \pm 0.006$ | $0.118 \pm 0.008$ | $0.065 \pm 0.006$ |
| L | $0.432 \pm 0.023$ | $0.103 \pm 0.009$ | $0.103 \pm 0.007$ | $0.005 \pm 0.002$ |
|  |  | $0.77 \pm 0.05$ | $0.94 \pm 0.05$ | $0.92 \pm 0.05$ |

## REFERENCES

1. P. Hoyer, R. G. Roberts, D. P. Roy, Nucl. Phys. B56, 173 (1973)
2. M. B. Einhorn, M. B. Green, M. A. Virasoro, Phys. Lett. 37B, 292 (1971); Phys. Rev. D6, 1675 (1972), Phys. Rev. D7, 102 (1973).
3. P. H. Frampton, Phys. Lett. 36B, 591 (1971); J. M. Wang, L. L. Wang, Phys. Rev. Lett. 26, 1287 (1971), R. L. Brower, R. E. Waltz, CERN TH-1335 (1971).
4. J. Kwiecinski, Lett. Nuo. Cim., 3, 619 (1972)
M. B. Einhorn, J. Ellis, J. Finkelstein, Phys. Rev. DS, 2063 (1972).
5. G. Ascoli et al Phys. Rev. D7, 669 (1973); Phys. Lett. 26, 929 (1972).
6. H. H. Bingham et al (CERN-Brussels collaboration) Nucl. Phys. B48, 589 (1972). R. Barloutand et al. (Saclay-Ecole Polytechnic-Rutherford collaboration), Nucl. Phys. B59, 374 (1973).
7. Chan Hong-Mo, H. I. Miettinen, R. G. Roberts, Nucl. Phys. B54, 411 (1973).
8. B. Y. Oh et al, Phys. Rev. Dl, 2494 (1970).
9. Y. Eisenberg et al Phys. Rev. Lett. 23, 1322 (1969).

(a)

(b)

FIG. 1


[^0]:    *Strictly speaking, semi-local factorisation implies that the ratio of the cross-sections of each pair of resonances, taken in turn, should be equal to r.

