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BEAT-WAVE ACCELERATOR STUDIES AT THE RUTHERFORD APPLETON LABORATORY

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ABSTRACT

The study carried out in 1982-83 at the Rutherford Appleton Laboratory to examine how one might use the beat-wave principle to construct a useful high energy accelerator is reviewed, and comments are made on later developments. A number of problems are evident to which solutions cannot at present be foreseen.

1 INTRODUCTION

Towards the end of 1982, following the ECFA-RAL meeting 'The Challenge of Ultra-High Energies' held in Oxford in October[1] it was decided to form a part-time study group based at the Rutherford Appleton Laboratory (RAL) for the purpose of further studying the beat-wave accelerator concept of Tajima and Dawson. The results were presented as a laboratory report in June 1983[2]. In the present report a summary is made of the findings, with comments in the light of more recent developments. Further background information on the study itself, with names of participants, are given in the acknowledgements at the end of this paper.

The idea of the beat-wave accelerator (BWA) was first described by Tajima and Dawson[3], and further papers had been given both at the Los Alamos meeting in February 1982[4] and at Oxford[1]. A study by Ruth and Chao[5] (published in ref 4 but not presented at the meeting) tackled the problem of finding a set of consistent parameters for a 5 TeV machine, based on a simplified linearized model for creating the beat-wave. The aim of the RAL study was to look in more detail at the Ruth-Chao design, and examine problems such as plasma formation, staging, gas scattering and beam focusing which had not yet been studied.

In the next section the assumptions made and parameters chosen in the Ruth-Chao study are outlined. Then follows a description of the RAL Study, and comments in the light of subsequent developments which suggest that better choices could have been made for some of the parameters. Difficulties were found that gave rise to problems for which a solution could not be forseen. Finally, some comments on the present outlook are presented.

2 THE RUTH-CHAO MODEL

There are two basic relations for the BWA, between the accelerating field strength and the beat-wave density, and between the phase velocity of the wave and the ratio of plasma to laser frequencies. In terms of the fundamental constants and the laser and plasma frequencies ω and ω_p, the accelerating field and phase velocity of the wave are

$$E_z = \alpha m_o c \omega_p / e \qquad (1)$$

$$\beta_z c = \left[1 - (\omega_p/\omega)^2\right]^{\frac{1}{2}} \simeq 1 - \omega_p^2/2\omega^2 \qquad (2)$$

where α is a constant of order but less than unity. It is immediately seen that large ω_p favours a high accelerating field but implies that for a relativistic particle there will be appreciable phase-slip between particle and wave, giving rise to the need for staging in very high energy machines. In addition to phase slip, laser power depletion sets a limit, which turns out to be of the same order of magnitude.

Ruth and Chao assumed Gaussian optics, with beam profiles in the 'under-dense' plasma the same as in vacuum. Although arguments for the validity of this assumption can be made, it may be that self-focusing is, in fact, a significant effect. This was, however, assumed not to occur in either the Ruth-Chao or the RAL studies. If Gaussian optics is assumed, then a short stage length allows a narrow beam, and therefore less laser power is needed to produce a given E_z. On the other hand more stages are required, implying the need for more lasers to produce the beat-waves. For stages of length limited by phase-slip or energy depletion it is found that these factors balance.

Using a simple linearized model for the build-up of the beat-wave, it is found that the laser energy required to produce a wave of given amplitude does not depend on the laser pulse length; in a real situation a very short pulse is desirable, to help combat essential non-linearities and avoid trouble from competing processes.

One of the interesting features to emerge from the Ruth-Chao analysis was that so many parameters are functions just of ω_p and ω_p/ω. This quantity can also be designated as γ_p, corresponding to the normalized total energy of a particle moving with the same velocity as the phase velocity of the wave. Some of these dependences are shown in Table 1; the symbol $\simeq$ denotes that quantities of order unity that depend on detailed assumptions are omitted. Basic assumptions are that the optics is Gaussian, particle energies are very high ($\gamma >> \gamma_p$) and that stage lengths are limited by phase-slip.

A list of parameters proposed by Ruth and Chao for a 5 TeV machine is given in Table 2. Figures in the RAL studies were the same, except where shown in brackets. The value of α in equation (1), which can be shown to be equivalent to the square of

Table I Parameter Dependences in Ruth-Chao Analysis

Accelerating field	$E_z \simeq m_o c\omega_p/e$
Stage length, depletion length	$L \simeq c\gamma_p^2/\omega_p$
Waist area	$\sigma_o^2 \simeq c^2\gamma_p/\omega_p^2$
Energy in laser pulse	$W\tau \simeq (m_o^2c^5/e^2)\gamma_p^3/\omega_p$
Beat wavelength/waist radius	$\lambda_p/\sigma_o \simeq \gamma_p^{-\frac{1}{2}}$

the ratio of the transverse oscillatory velocity of the plasma electrons to that of light, was taken as 0.5. This was derived as the criterion that electrons in the originally cold plasma should not be trapped in the wave. Injection of already relativistic particles into the accelerator was assumed.

Table II	Parameters for 5 TeV accelerator
Accelerating field	5 GeV/m
Stage length	10m (5m)
No of stages	100 (200)
Laser wavelength	1.06μ
Beat wavelength	260μ
Waist radius	1.3mm
Injection energy	>10 GeV
Laser energy per stage	17 KJ(8.5KJ)
Particles per pulse	< 5×10^{10}
Pulse length	140 psec (100 psec)

3 TOPICS CONSIDERED IN RAL STUDY

In the following sections brief accounts are given of the various topics considered in the RAL study, and the conclusions reached. Parameters different from those of Ruth and Chao are shown in brackets in Table II. Comments are made on these conclusions in the light of later developments.

a) Required Accelerator Parameters

It was assumed that 'conventional' machines, implying linac colliders for electron and synchrotron based storage rings for protons, might reach 0.4 and 20 TeV respectively, and that the beat-wave accelerator should aim higher than this. To be specific, an energy goal of 5 TeV for electrons was chosen. More recently there is increased confidence that 100 MeV/metre can be obtained in conventional machines, so that even 5 TeV only implies 50 km/linac; this is long, but by no means inconceivable.

For luminosity, the values considered as possibly acceptable, namely less than 10^{30} cm^{-2} sec^{-1} for protons at 10 + 10 TeV and 10^{30} for electrons at 1 TeV would now be considered too low by several orders of magnitude.

b) The Plasma Column

The central feature of the accelerator is a plasma column, the density of which must remain constant during the build-up of the beat-wave, and be everywhere uniform so that resonant conditions are maintained. For reasons given in section 3f a stage length of 5m, rather than 10m used by Ruth and Chao, was chosen; this implies twice as many stages, with 8.5 kJ laser power per stage. The tolerances depend on the build-up time of the beat-wave, which was taken as 100 psec, representing 115 cycles of the beat frequency. This value is now thought to be too long, because of the problems of relativistic detuning and the build-up of competing processes; it was chosen because the laser technology becomes more difficult as the pulse is shortened. Even with a very short pulse it is necessary to retain uniformity, and the suggested solution was to start with cold hydrogen gas. At the power levels of interest complete ionization occurs at the front end of the laser pulse, which might typically be several millimetres in length in place of the 3cm assumed earlier.

The transverse dimensions of the plasma channel depend on those of the laser pulse; it was assumed that the gas would be ionized where the power density exceeds 3 x 10^{13}watts cm^{-2}. This occurs within a radius σ_o of order 1.5σ, where the transverse power distribution is assumed to vary as exp $(-r^2/2\sigma^2)$. The column width is not constant; the stage length is assumed to be twice the Rayleigh length, and this implies that the transverse area at the waist is half that at the ends.

One obvious problem in a practical system is that of maintaining the gas column at a uniform density, and a vacuum in the region between the column and the mirror systems. No window can stand the energy deposition that would occur.

c Laser and Associated Optics

No comment was made on the laser to be used, although λ = 1.06 microns was assumed for the wavelength; the standard Nd glass laser clearly will not have adequate repetition rate.

A simple paraboloid was chosen to form the Gaussian waist. Taking 10 joules/cm^2 as the tolerable power density on the mirror surface, this requires a 16cm radius mirror with focal length about 600 metres to produce the required long and narrow waist. This distance might perhaps be considerably shortened by using more sophisticated optics with grazing incidence[6].

The mechanical arrangement of the mirrors and plasma columns requires careful consideration. It seems difficult to make the stages collinear; if they are not, then magnets are needed to deflect the particle beams. The next two paragraphs are quoted from ref 2.

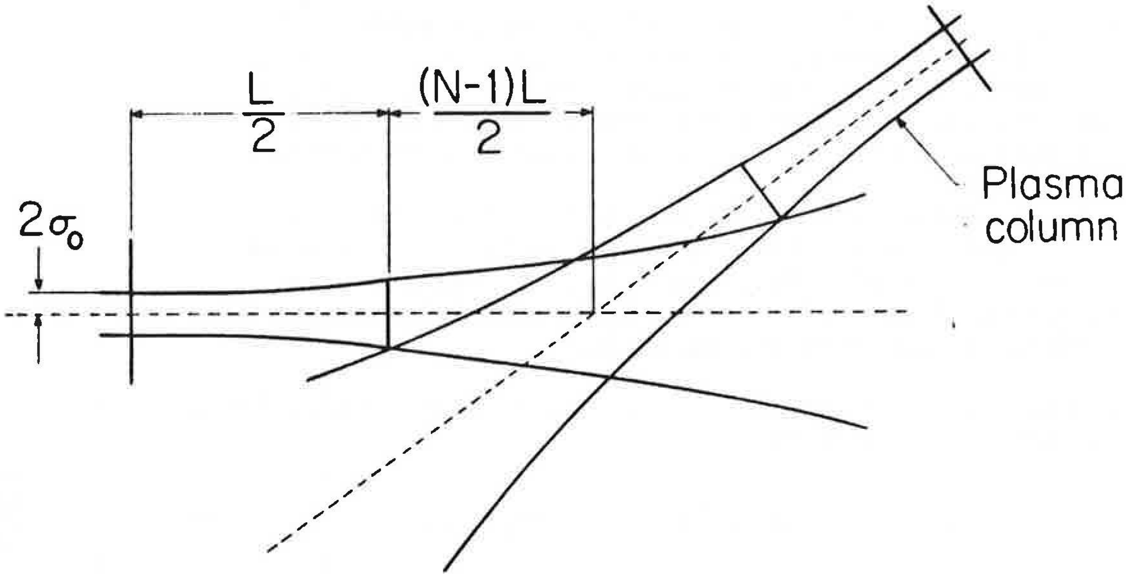

Fig 1 Beam configuration between stages

"The minimum requirement is that the mirrors should not intercept beams from other mirrors, and that only one beam should be present in each stage. Assuming mirrors with co-planar axes the deflection angle must certainly exceed the convergence angle $0.16/600 = 2.7\times10^{-4}$ radian. The ends of the stages must be separated to allow the insertion of a bending magnet system, and this further increases the minimum angle. The geometry is shown in Fig 1; it is assumed that the beams within a radius of 2σ must not intersect within the plasma region. As explained above, the stage length L=2R where R is the Rayleigh length. The beam radius σ at a distance NR from the centre of the plasma column is $2\sigma_0(1 + N^2)^{\frac{1}{2}}$. If, then, the spacing between the ends of the plasma columns is 2(N-1)R, the angle θ is $2\sigma_0/(N-1)R$. Substituting for R and σ

$$\theta = 4\sigma_0(1+N^2)^{\frac{1}{2}} / (N-1)L \qquad (3)$$

If the spacing is equal to L = 2R, then N = 2 and $\theta = 4\sqrt{5}\sigma_0/L = 1.8 \times 10^{-3}$ radians. For larger values of N, θ decreases to a lower limit of $4\sigma_0/L$.

In a practical system, in the presence of a vessel to contain the gas, supports, focusing and beam bending magnets etc. it is unlikely that it will be possible to attain such a small angle."

Despite further discussions, no credible alternative layout has been devised. This is a topic that needs further consideration.

d) Bending Magnet Requirements

There are two requirements that limit the bending angle θ given in equation (3). In the first place it must be possible to design as achromatic bending system, and second, energy loss arising from synchrotron radiation must not be large. It is evident from the form of equation (3) that with the layout assumed θ cannot be decreased indefinitely by increasing the spacing between stages.

It is not possible to consider the design of the bending magnets before the beam quality, described by the emittance and $\Delta p/p$, have been specified. These questions are discussed in section 3e below. Nevertheless some simple calculations to indicate orders of magnitude are possible.

First, the fractional energy loss from synchrotron radiation in a magnet of length 5 metres is

$$\Delta\gamma/\gamma = 1.27 \times 10^{-6} B^2 WS \quad , \quad (T, GeV, m) \qquad (4)$$

Not only does the synchrotron radiation introduce loss, but there is also an energy spread of order $\Delta\gamma/\gamma$. Since this is cumulative, the permitted value of $\Delta\gamma/\gamma$ is clearly very small, especially when there are many stages.

The minimum permissible magnet length may be found from the total angle of bend

$$\theta = 0.3BS/W \qquad (5)$$

From equations (3), (4) and (5), eliminating B and θ, (set at the minimum value of $4\sigma_o/L$), we find that

$$S = 2.2 \times 10^{-4} W^3 \sigma_o^2 / L^2 (\Delta\gamma/\gamma) \qquad (6)$$

Setting σ_o=1.3mm,L=5m, $\Delta\gamma/\gamma \simeq 10^{-3}$ yields S=15m at 1 TeV. At 5 TeV this is increased by at least a factor 125, probably more since $\Delta\gamma/\gamma$ would need to be less.

This illustrative calculation demonstrates the difficulty of the staging problem. A different approach is needed.

e) Beam Quality

The required beam quality can be determined from a knowledge of the luminosity required and the power that can be afforded. These considerations have been outlined by Richter[7] and Wilson[8]. In the RAL study this topic was not adequately treated, and an emittance of 10^{-5}/ m-rad, compatible with expected performance of a SLAC type injector was assumed. The energy spread was taken as 10%, a value probably too large to allow satisfactory focusing in the collision region.

Whilst the emittance is determined by the injector, the energy spread depends on the variation of phase experienced by the particles during acceleration. Particles injected at different phases of the beat-wave acquire different energies. Phases π to 2π are decelerating, and, as shown in the next section, 0 to $\pi/2$ are radially defocusing. Only the range $\pi/2$ to π is therefore usable, as shown in Fig 2.

For a short bunch initially extending from $\phi = \pi/2$ to $\phi = \pi/2+\phi_b$, the energy spread when the bunch has slipped through an angle ϕ_s gives rise to an energy spread normalized to the mean energy gain per stage

$$\Delta\gamma/\gamma_s = \phi_b \cos(\pi/2 + \phi_s) / \left[1 - \sin(\pi/2 + \phi_s)\right] \quad (7)$$

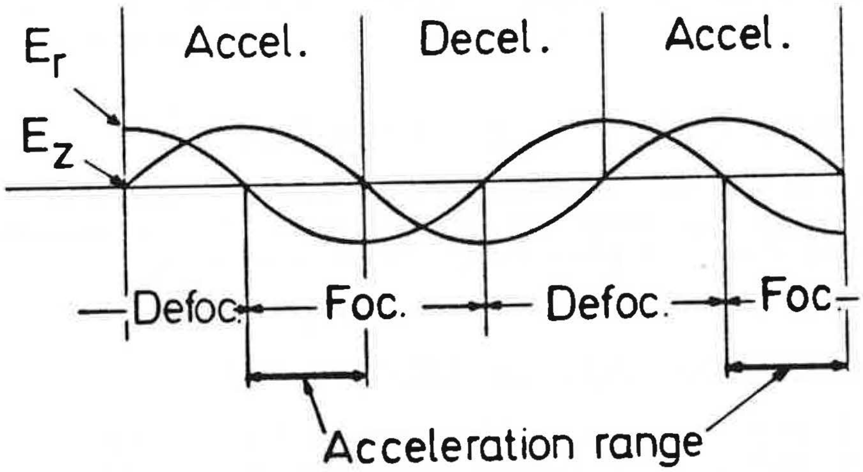

Fig 2 Accelerating and focusing fields, illustrating the phase range available for acceleration.

For $\phi = 13\pi/16$ assumed in ref 2 this is about $\Delta\gamma/\gamma_s \simeq \phi_s/2$. Thus, a 6° phase spread, corresponding to a bunch to space ratio of only 3%, corresponds to an energy spread of 5%. This is likely to be unacceptably large in the light of the requirements for final focusing.

No consideration was given in the RAL study of how these narrow bunches might be achieved.

f) Focusing of the Particle Beam in the Plasma Channel

The assumptions concerning the form of the plasma column were discussed in section 3b. The column width was determined from the breakdown strength in the gas, and was of order 2mm. Outside the column E_z is zero, so that across the column $\partial E_z/\partial r$, and hence $\partial E_r/\partial z$, is finite. Assuming a quadratic dependence of characteristic length r_o near the axis, it is readily shown that

$$|E_r| = \frac{\lambda r}{\pi r_o^2} |E_{zo}| \tag{8}$$

This represents a strong field that is focusing or defocusing according to the phase, as shown in Fig 2. Only phase angles between $\pi/2$ and π can be used for acceleration. It is straightforward to calculate the focusing strength, and hence the betatron wavelength, Λ, from equation (8). For the parameters assumed in the RAL study $\Lambda|\cos\phi|$ varies from about 2-25 metres as γ increases from 35 GeV at the end of the first stage to 5 TeV. At the assumed emittance the beam diameter would be much less than that of the plasma channel.

No detailed consideration was given to the problem of achromatic focusing between stages. Rough calculations indicated that even without allowing for synchrotron radiation it would be extremely difficult.

g) Multiple Scattering of Accelerated Beam by Plasma

The effects of multiple scattering were briefly discussed in ref 2, and, although considered worthy of investigation no detailed calculations were done. It has subsequently been shown by Montague[9] that the deterioration in beam quality is negligible.

h) Beam Intensity, Pulse Length, and Repetition Rate

All these factors are important in assessing the potential of any accelerating scheme. The number of particles that can be accelerated, provided that they can be produced at the required density initially, depends on the beam loading, and the fundamental limits here are expected to be as in any other accelerator. We might hopefully assume 5% transfer of energy from laser light to accelerated particles.

The pulse length depends on the time for which the beat-wave remains coherent after the laser pulse has passed. Since energy spread is to be avoided, the amplitude must not 'droop'. Not enough is yet known about the beat-wave process to give other than hopeful guesses.

The repetition rate depends on developments in laser technology. The limitation here is probably an economic one.

i) Luminosity

With so many unknowns, it is hardly profitable at present to estimate the luminosity that might be achievable. Values consistent with those quoted by Richter[7] and Rubbia[10] are not within sight. This question was considered in ref 2. The results need re-assessment and will not be discussed here.

One point emerged, however, that is worthy of note. Owing to the fact that the bunch is split into microbunches spaced by the beat wavelength, the beamstrahlung effect is worse by a factor equal to the ratio of the pulse spacing to pulse width. The disruption parameter, on the other hand, is unaffected. This arises because of the respective squared and linear dependences of these effects on the magnetic field strength.

4 CONCLUSIONS

Not enough is known about the detailed physics of the beat wave process, nor about how one might overcome the difficulties uncovered in the RAL studies, to make any meaningful assessment of what could be achieved in practice. Some of the assumptions made in the study, such that the laser beam shape is determined by free-space Gaussian optics, may not be correct. If self-focusing is important, for example[11], much smaller laser powers might be required. There may be approaches, perhaps related to the 'surfatron', which alter the constraints arising from phase slip[12]. While there may be scope for improvement in some respects, many other factors which will cause problems have not been investigated.

5 ACKNOWLEDGEMENTS

The Rutherford Appleton Study was started in October 1982, encouraged by the European Committee for Future Accelerators (EFCA) through its chairman (Dr J H Mulvey) and by Professor A Salam. This was a part-time activity, and an attempt was made to bring together participants in the fields of High Energy Physics, Particle Accelerators, Lasers, and Plasma Physics. These are listed below in alphabetical order.

Participants	Field of Interest
J E Allen*	Plasma Physics
R Bingham	Plasma Physics
J Butterworth	Particle Beam Transport
F E Close	High Energy Physics
R G Evans	Plasma Physics and Lasers
J D Lawson	Accelerators
G H Rees	Accelerators
R D Ruth+	Accelerators

* University of Oxford

\+ Stanford Linear Accelerator Centre. At CERN during period of study group.

The Study Group as such did not continue after the publication of ref 2, but some individual members continued with basic studies of beat-wave physics. This work has led to several of publications [13-15], and an experimental study in conjunction with Imperial College, London, has been approved [16].

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