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4 G.e.v. Electron Accelerator

Resonant Magnet Network

Some notes on the analysis of the Pulse Power Supply Performance

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1.0 Summary

These notes describe the preliminary analysis of the pulse power supply for the accelerator guide-field magnets and form the basis from which an analogue computer study will be made to confirm the pulse-circuit operating characteristics and:-

(a) Optimise the frequencies ω_F, ω_P and hence the circuit component ratings for minimum magnet network disturbance

(b) Determine the effect of non-linearity in L_F and L_P and the inherent voltage harmonics in the rectified source voltage V_S.

(c) Investigate transient fault conditions:-

(i) Failure of ignitron to conduct

(ii) Failure of ignitron to extinguish

(iii) Advance and retarding of ignitron firing signal

(iv) Capacitor fault.

2.0 Required conditions of operation

It is required that the excitation of the guide-field magnets[1] shall have a constant frequency and amplitude, for a given accelerator energy level, during an operation period of some hours.

Since the resonant magnet L-C network is designed to maintain its natural frequency within extremely close limits and independant of temperature changes in capacitor dielectric, stability of frequency can be achieved by supplying the network AC losses in time-phase with the magnet excitation cycle. Similarly, apart from the bias-current which is supplied and controlled independantly, the required accuracy of magnet excitation amplitude will be achieved if the supply of AC power to the network equals the cyclic AC loss power.

Provision of this loss-power to the network by continuous excitation from an external source (i.e. motor-alternator set) would require an elaborate speed control system in order to avoid force-resonating the magnet network during the random "mains" frequency variations.

The preferred method is to isolate the resonant network from the "mains" and supply the cyclic AC losses as an impulse. This impulse of energy is applied during the descending portion of the magnet current waveform and the disturbance introduced must be completely attenuated before the next particle accelerating period (rising portion of the magnet current waveform).

[1] See EL/S-1 "Specification of scale-model energy storage choke" for a full description of the resonant magnet network.

This pulse-power supply comprises an energy storage capacitor, an associated charging circuit and a pulse discharge circuit triggered from the resonant magnet network.

The derivation of the appropriate equations and boundary conditions will now be described, based on a piece-wise linear analysis in which no saturation occurs.

3.0 Analysis

The guide-field magnets are excited with a fully biased sinusoidal waveform to give a magnet current of the form:-

$$i_m = I_{DC} - I_{AC} \sin \omega_a t$$

where $\omega_a = 2\pi$ times accelerator frequency

$$= 2\pi\, 50$$

$$= \sqrt{\frac{L_{ch} + L_m}{L_{ch} . L_m (C_{ch} + C_m)}}$$

The magnet voltage is written as:-

$$v_m = V_m \cos \omega_a t$$

and the choke current is given by:-

$$i_{ch} = I_{DC} + I'_{AC} \sin \omega_a t$$

where $I'_{AC} = 0.5\, I_{AC}$

since L_{ch} is chosen to equal $2 L_m$

3.1 Assumptions

(a) The primary and secondary windings of the energy storage choke L_{ch} are closely coupled and the leakage inductance is negligible.

(b) The circuit resistances are negligible except in the magnet network which is considered to have an ohmic loss, in shunt, equal to a magnet network Q of 100.

(c) The pulse current causes no disturbance in the magnet network.

(d) The half-cycle discharge of pulse current is sinusoidal and timed to occur symmetrically about the positive peak of the magnet voltage.

(e) The DC bias current is constant.

(f) The energy storage choke turns ratio is unity.

(g) The supply voltage V_S is constant and contains no harmonics.

(h) The magnet voltage v_m is constant during the current pulse.

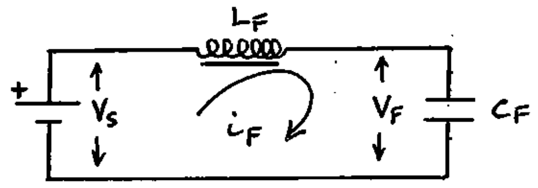
3.2 Boundary Conditions

Consider the pulse circuit shown in fig. 2. It is assumed that the half-cycle of pulse current i_p occurs symmetrically around the peak of v_m. Hence, since this voltage referred to the choke primary is in opposition to that developed across C_F at the peak of the charge then for full discharge of capacitor C_F (economically necessary), V_F must equal $2V_m$

This could be achieved by making $V_s = 2V_m$

However, this would lead to an excessive RMS/peak ratio of current i_F, hence a larger charging unit. More important, firing of the ignitron would effectively short-circuit the supply, drastically distorting and lengthening the current pulse i_p resulting, for certain values of L_p in failure of ignitron to extinguish.

Consequently some degree of decoupling between charge and discharge circuits must be achieved by the insertion of a filter inductance L_F.



Now $$L_F \frac{di_F}{dt} + \frac{Q_F}{C_F} = V_s \qquad (1)$$

also $$Q_F = \int i \, dt \qquad (2)$$

and $$i_F = C_F \frac{dv_F}{dt} \qquad (3)$$

hence $$L_F C_F \frac{d^2 v_F}{dt^2} + v_F = V_s$$

Giving a solution of the form:-

$$v_F = V_s + A \cos \omega_F t + B \sin \omega_F t \qquad (4)$$

where $\omega_F = \frac{1}{\sqrt{L_F C_F}}$

substituting for v_F

$$i_F = -A\omega_F C_F \sin \omega_F t + B\omega_F C_F \cos \omega_F t \qquad (5)$$

Assume that the boundary conditions are:-

$$i_F = 0, \quad v_F = 0 \quad \text{at} \quad t = 0$$

Hence $A = -V_s$ and $B = 0$

Therefore $$v_F = V_s \left[1 - \cos \omega_F t\right]$$

$$= 2V_s \quad \text{when} \quad t = \frac{\pi}{\omega_F}$$

<u>Physical argument.</u> Now $V_F = 2V_S$ (i.e. $= 2V_m$) is the required voltage condition for complete discharge of C_F when opposed, at the instant of discharge, by voltage υ_m. However, the above equations are not a solution for cyclic operation since although for the first charge cycle $i_F = 0$ at $t = 0$ and $t = \pi/\omega_F$, $i_F \neq 0$ during the pulse and hence the succeeding charging cycle commences with a finite value of i_F

Consequently, for true cyclic conditions i_F must have the same value at the commencement of each succeeding charging cycle and must also have identical values at the end of each charging cycle.

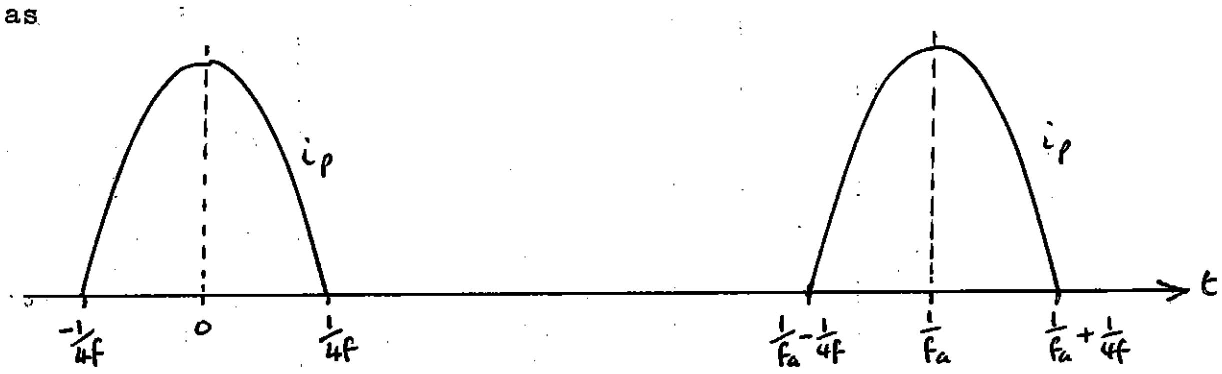
Therefore, let:-

i_{F_o} = the initial value of filter circuit current at the <u>commencement</u> of the charging period,

i_F = the value of filter current <u>during</u> the charging period,

i_F' = the final value of filter current at the <u>end</u> of the charging period,

and let us define the boundary conditions for the complete circuit as



$V_F = 2V_s$ at $t = -\frac{1}{4f}$, $\frac{1}{f_a} - \frac{1}{4f}$ ——— (BC.1)

$V_F = 0$ " " $= \frac{1}{4f}$ ——— (BC.2)

$i_p = 0$ " " $= -\frac{1}{4f}$ ——— (BC.3)

$i_F = i_{F_o}$ " " $= \frac{1}{4f}$ ——— (BC.4)

[illegible] " " = [illegible] ——— ([illegible])

$\frac{1}{f_a} = \frac{2\pi}{\omega_a}$ cyclic period of accelerator

$\omega_F = \frac{1}{\sqrt{L_F C_F}}$ angular frequency of filter circuit

$\omega_p = \frac{1}{\sqrt{L_p C_F}}$ angular frequency of pulse circuit

$\frac{1}{f} = \frac{2\pi}{\omega}$ cyclic period of the pulse discharge $\left(\doteq \frac{1}{f_p}\right)$

3.3 Charging Period $\frac{1}{4f} \leq t \leq \frac{1}{f_a} - \frac{1}{4f}$

Equations (4) and (5) give the general solution form of v_F and i_F :-

$$v_F = V_s + A\cos\omega_F t + B\sin\omega_F t \quad \text{———} (4)$$

$$i_F = -\omega_F C_F A\sin\omega_F t + \omega_F C_F B\cos\omega_F t \quad \text{———} (5)$$

where A and B are constants

From (BC.4)

$$i_{F_0} = -\omega_F C_F A\sin\frac{\pi\omega_F}{2\omega} + \omega_F C_F B\cos\frac{\pi\omega_F}{2\omega} \quad \text{———} (6)$$

where $\frac{1}{4f} = \frac{\pi}{2\omega}$

From (BC.2)

$$0 = V_s + A\cos\frac{\pi\omega_F}{2\omega} + B\sin\frac{\pi\omega_F}{2\omega} \quad \text{———} (7)$$

Solve (6) and (7) for A and B

$$i_{F_0}\cos\frac{\pi\omega_F}{2\omega} - V_s\omega_F C_F\sin\frac{\pi\omega_F}{2\omega} = B\omega_F C_F\left(\cos^2\frac{\pi\omega_F}{2\omega} + \sin^2\frac{\pi\omega_F}{2\omega}\right)$$

Therefore

$$B = \frac{i_{F_0}}{C_F\omega_F}\cos\frac{\pi\omega_F}{2\omega} - V_s\sin\frac{\pi\omega_F}{2\omega}$$

Substitute in (7)

$$A\cos\frac{\pi\omega_F}{2\omega} = -V_s - \frac{i_{F_0}}{C_F\omega_F}\cos\frac{\pi\omega_F}{2\omega}\sin\frac{\pi\omega_F}{2\omega} + V_s\sin^2\frac{\pi\omega_F}{2\omega}$$

Therefore

$$A = -V_s\cos\frac{\pi\omega_F}{2\omega} - \frac{i_{F_0}}{C_F\omega_F}\sin\frac{\pi\omega_F}{2\omega}$$

Substituting for A and B in equation (4) we obtain:-

$$v_F = V_s - \cos\omega_F t\left[V_s\cos\frac{\pi\omega_F}{2\omega} + \frac{i_{F_0}}{C_F\omega_F}\sin\frac{\pi\omega_F}{2\omega}\right] + \sin\omega_F t\left[\frac{i_{F_0}}{C_F\omega_F}\cos\frac{\pi\omega_F}{2\omega} - V_s\sin\frac{\pi\omega_F}{2\omega}\right]$$

$$= V_s - V_s\left[\cos\omega_F t\cos\frac{\pi\omega_F}{2\omega} + \sin\omega_F t\sin\frac{\pi\omega_F}{2\omega}\right] + \frac{i_{F_0}}{C_F\omega_F}\left[\sin\omega_F t\cos\frac{\pi\omega_F}{2\omega} - \cos\omega_F t\sin\frac{\pi\omega_F}{2\omega}\right]$$

$$\boxed{v_F = V_s\left[1 - \cos\left(\omega_F t - \frac{\pi\omega_F}{2\omega}\right)\right] + \frac{i_{F_0}}{C_F\omega_F}\sin\left(\omega_F t - \frac{\pi\omega_F}{2\omega}\right)} \quad \text{———} (8)$$

Using equation (3) and differentiating:-

$$\boxed{i_F = V_s C_F\omega_F\sin\left(\omega_F t - \frac{\pi\omega_F}{2\omega}\right) + i_{F_0}\cos\left(\omega_F t - \frac{\pi\omega_F}{2\omega}\right)} \quad \text{———} (9)$$

We know from (BC.1) that $V_F = 2V_s$ when $t = \frac{1}{f_a} - \frac{1}{4f} = \frac{2\pi}{\omega_a} - \frac{\pi}{2\omega}$
If we substitute this into equation (8), it enables us to calculate i_{F_0}.

$$2V_s = V_s - V_s\cos\left[\omega_F\left(\frac{2\pi}{\omega_a} - \frac{\pi}{2\omega}\right) - \frac{\pi\omega_F}{2\omega}\right] + \frac{i_{F_0}}{C_F\omega_F}\sin\left[\omega_F\left(\frac{2\pi}{\omega_a} - \frac{\pi}{2\omega}\right) - \frac{\pi\omega_F}{2\omega}\right]$$

$$V_s = -V_s\cos\left(\frac{2\pi\omega_F}{\omega_a} - \frac{\pi\omega_F}{\omega}\right) + \frac{i_{F_0}}{C_F\omega_F}\sin\left(\frac{2\pi\omega_F}{\omega_a} - \frac{\pi\omega_F}{\omega}\right)$$

Therefore

$$i_{F_0} = \frac{V_s C_F \omega_F (1+\cos\alpha)}{\sin\alpha}$$

where $\alpha = \frac{2\pi\omega_F}{\omega_a} - \frac{\pi\omega_F}{\omega}$

$$\frac{1+\cos\alpha}{\sin\alpha} = \frac{2\cos^2 \alpha/2}{2\cos\alpha/2 \sin\alpha/2} = \cot\frac{\alpha}{2}$$

Giving

$$\boxed{i_{F_0} = V_s C_F \omega_F \cot\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega}\right)} \qquad (10)$$

We can now write i_F in alternative form by substituting equation (10) in (9):-

$$i_F = V_s C_F \omega_F \left[\sin\left(\omega_F t - \frac{\pi\omega_F}{2\omega}\right) + \frac{\cos\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega}\right)\cos\left(\omega_F t - \frac{\pi\omega_F}{2\omega}\right)}{\sin\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega}\right)}\right]$$

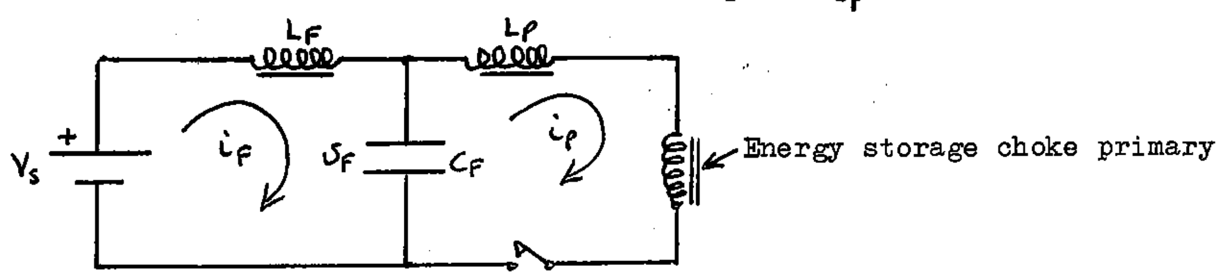
$$= V_s C_F \omega_F \frac{\cos\left(\omega_F t - \frac{\pi\omega_F}{2\omega} - \frac{\pi\omega_F}{\omega_a} + \frac{\pi\omega_F}{2\omega}\right)}{\sin\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega}\right)}$$

Therefore

$$\boxed{i_F = V_s C_F \omega_F \cdot \frac{\cos\left(\omega_F t - \frac{\pi\omega_F}{\omega_a}\right)}{\sin\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega}\right)}} \qquad (11)$$

3.4 Pulsing circuit $\quad -\frac{1}{4f} \leqslant t \leqslant \frac{1}{4f}$

We will now consider the circuit characteristics during the discharge of C_F. The ignitron switch conducts at $t = -1/4f$ and ceases to conduct when the current pulse i_p reaches zero.



The fundamental differential equations of the circuit are:-

$$L_F \frac{di_F}{dt} = V_s - v_F \qquad (12)$$

$$L_p \frac{di_p}{dt} = V_F - V_s \qquad (13)$$

$$i_p = i_F - C_F \frac{dv_F}{dt} \qquad (14)$$

We eliminate i_p and i_F and obtain a differential equation in v_F :

Substitute from equation (14) into (13):

$$L_p\left[\frac{di_F}{dt} - C_F\frac{d^2v_F}{dt^2}\right] = V_F - V_s$$

Substitute for $\frac{di_F}{dt}$ from equation (12):

$$L_p\left[\frac{V_s - V_F}{L_F} - C_F\frac{d^2v_F}{dt^2}\right] = V_F - V_s$$

Therefore

$$L_pC_F\frac{d^2v_F}{dt^2} + \left(1+\frac{L_p}{L_F}\right)v_F = \left(1+\frac{L_p}{L_F}\right)V_s$$

and

$$\frac{d^2v_F}{dt^2} + \omega_p^2\left(1+\frac{L_p}{L_F}\right)v_F = \omega_p^2\left(1+\frac{L_p}{L_F}\right)V_s$$

since

$$\omega_p^2 = \frac{1}{L_pC_F}$$

Putting

$$\omega^2 = \omega_p^2\left(1+\frac{L_p}{L_F}\right) \qquad (15)$$

$$\frac{d^2v_F}{dt^2} + \omega^2V_F = \omega^2V_s$$

so

$$v_F = V_s + A\cos\omega t + B\sin\omega t \qquad (16)$$

where A and B are arbitrary constants.

Integrating equation (13):

$$L_pi_p = C_1 + \frac{A}{\omega}\sin\omega t - \frac{B}{\omega}\cos\omega t \qquad (17)$$

Integrating equation (14):

$$L_Fi_F = C_2 - \frac{A}{\omega}\sin\omega t + \frac{B}{\omega}\cos\omega t \qquad (18)$$

where C_1 and C_2 are arbitrary constants

Since, in solving equations (12) - (14), equation (14) was differentiated, it is possible that C_1 and C_2 are related. So we will now substitute equations (17) and (18) back into equation (14) and check.

$$i_p - i_F = \frac{C_1}{L_p} - \frac{C_2}{L_p} + \frac{A}{\omega}\sin\omega t\left(\frac{1}{L_p}+\frac{1}{L_F}\right) - \frac{B}{\omega}\cos\omega t\left(\frac{1}{L_p}+\frac{1}{L_F}\right) \qquad (19)$$

We require this to be equal to:-

$$-C_F\frac{dv_F}{dt} = C_F\omega A\sin\omega t - C_F\omega B\cos\omega t \qquad (20)$$

Compare coefficients in (19) and (20)

Coefficient of $\sin\omega t$ in (19) $= \frac{A}{\omega L_p}\left(1+\frac{L_p}{L_F}\right)$

Coefficient of $\sin\omega t$ in (20) $= \frac{A}{\omega L_p}\left(\frac{\omega^2}{\omega_p^2}\right) = \frac{A}{\omega L_p}\left(1+\frac{L_p}{L_F}\right)$ using (15)

Similarly coefficients of $\cos\omega t$ are equal.

For the constant terms to be equal, we must have:-

$$\frac{c_1}{L_p} = \frac{c_2}{L_F} \quad \text{in (19)} \tag{21}$$

Hence the most general possible solutions are:-

$$i_p = \frac{c_1}{L_p} + \frac{1}{L_p\omega}\left(A\sin\omega t - B\cos\omega t\right) \tag{22}$$

$$i_F = \frac{c_1}{L_p} - \frac{1}{L_F\omega}\left(A\sin\omega t - B\cos\omega t\right) \tag{23}$$

$$v_F = V_s + A\cos\omega t + B\sin\omega t \tag{24}$$

where c_1, A and B are to be determined from the boundary conditions.

From (BC.1) $V_F = 2V_s$ at $t = -\frac{1}{4f}$
$= -\frac{\pi}{2\omega}$

so

$$B = -V_s \tag{25}$$

Using (BC.4)

$$i_{F_0} = \frac{c_1}{L_p} - \frac{A}{L_F\omega} \tag{26}$$

and $i_p = 0$ at the beginning of the pulse (BC.3), so:-

$$0 = \frac{c_1}{L_p} - \frac{A}{L_p\omega} \tag{27}$$

therefore, solving (26) and (27) for A and c_1 :-

$$i_{F_0} = \frac{A}{\omega}\left(\frac{1}{L_p} - \frac{1}{L_F}\right)$$

Hence $A = i_{F_0}\omega\left[\left(\frac{1}{L_p}\right)\left(1-\frac{L_p}{L_F}\right)\right]^{-1}$

i.e.

$$A = \frac{i_{F_0}\omega L_p}{\left(1-\frac{L_p}{L_F}\right)} \tag{28}$$

From equation (27) and substituting for A given in (28)

$$c_1 = \frac{A}{\omega} = \frac{i_{F_0} L_p}{\left(1-\frac{L_p}{L_F}\right)} \tag{29}$$

Thus, the solutions for i_p, i_F and v_F over the time $-\frac{\pi}{2\omega} \leqslant t \leqslant \frac{\pi}{2\omega}$ are:-

$$\boxed{i_p = \frac{i_{Fo}}{1 - \frac{L_p}{L_F}}\left(1 + \sin\omega t\right) + \frac{V_s}{L_p\omega}\cos\omega t} \qquad (30)$$

$$\boxed{i_F = \frac{i_{Fo}}{1 - \frac{L_p}{L_F}}\left(1 - \frac{L_p}{L_F}\sin\omega t\right) - \frac{V_s}{L_F\omega}\cos\omega t} \qquad (31)$$

$$\boxed{v_F = V_s\left(1 - \sin\omega t\right) + \frac{L_p\omega\, i_{Fo}}{\left(1 - \frac{L_p}{L_F}\right)}\cos\omega t} \qquad (32)$$

3.4.1 Comments

The following comments need to be made regarding the analysis so far:-

(a) From equation (9):

$$i_F\Big|_{t = \frac{1}{f_a} - \frac{1}{4f}} = V_s C_F \omega_F \sin\alpha + i_{Fo}\cos\alpha$$

where $\alpha = \frac{2\pi\omega_F}{\omega_a} - \frac{\pi\omega_F}{\omega}$

Substitute for i_{Fo} from equation (10)

$$i_F\Big|_{t = \frac{1}{f_a} - \frac{1}{4f}} = V_s C_F \omega_F\left(\sin\alpha + \cot\frac{\alpha}{2}\cos\alpha\right)$$

$$= V_s C_F \omega_F \cot\frac{\alpha}{2} = i_{Fo}$$

Thus from section 3.3 we expect $i_F = i_{Fo}$ at both ends of the charging period, and so also at both ends of the pulse period. However, the solutions of section 3.4 indicate that:-

$$i_F = i_{Fo}\left(\frac{1 + \frac{L_p}{L_F}}{1 - \frac{L_p}{L_F}}\right)$$

at the beginning of the pulse (i.e. end of charging sequence). But since $L_p/L_F << 1$, the discrepancy is small.

(b) At $t = 1/4f$

$$i_p = \frac{2 i_{Fo}}{1 - L_p/L_F} \qquad \text{from equation (30)}$$

and we should expect it to be zero (end of pulse). We note however that the dominant term of equation (30) is the last one, and although this is zero at $t = +1/4f$, it is negative for $t > 1/4f$ and so i_p is probably zero fractionally later.

(c) Note that since the current i_F is not zero during the pulse period then this component of i_F contributes to the supply of AC loss power (P_{AC}) to the resonant magnet network. Consequently, the magnet network energy loss, $\frac{2\pi}{\omega_a} \cdot P_{AC}$ in joules/cycle, is equal to the sum of the capacitor stored energy, $\frac{1}{2} C_F V_F^2$ plus

$$\frac{\pi}{\omega} V_s \left\{ \left[\int_{-\frac{1}{4f}}^{\frac{1}{4f}} i_F^2 \, dt + \int_{\frac{1}{4f}}^{\frac{1}{f_a} - \frac{1}{4f}} i_F^2 \, dt \right] \frac{\omega_a}{2\pi} \right\}^{\frac{1}{2}}$$

3.5 Peak, average and RMS values of i_P and i_F

We will now derive the peak, average and RMS values of the quantities i_P and i_F, and, in order to simplify these derivations we will use the approximation

$$\omega_P \doteq \omega$$

This is justified since L_F is much greater than L_P for the range of circuit parameters that will be used and hence equation (15).

$$\omega^2 = \omega_P^2 \left(1 + \frac{L_P}{L_F}\right) \text{ tends to } \omega^2 = \omega_P^2$$

i.e. $\left(1 + \frac{L_P}{L_F}\right) \doteq 1$

3.5.1 Peak value of current i_P

Using the above approximation then equation (30) gives:-

$$i_P = i_{F_0}\left(1 + \sin \omega_P t\right) + \frac{V_s}{L_P \omega_P} \cos \omega_P t \qquad (30a)$$

We note that from equation (10)

$$i_{F_0} = V_s C_F \omega_F \cot \beta$$

where

$$\beta = \frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega_P}\right) \qquad (33)$$

hence

$$i_{F_0} = \frac{V_s}{L_F \omega_F} \cot \beta \qquad (34)$$

Therefore, using $\omega_F^2 = \frac{1}{L_F C_F}$

$$i_P = \frac{V_s}{L_F \omega_F} \cot \beta \left(1 + \sin \omega_P t\right) + \frac{V_s}{L_F} \frac{\omega_P}{\omega_F^2} \cos \omega_P t$$

and using $L_P \omega_P^2 = L_F \omega_F^2$

$$i_P = \frac{V_s}{L_F \omega_F} \left[\cot \beta \left(1 + \sin \omega_P t\right) + \frac{\omega_P}{\omega_F} \cos \omega_P t \right] \qquad (35)$$

The peak value of i_p is reached when $\frac{di_p}{dt} = 0$, i.e. when:-

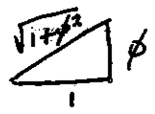
$$\cot\beta \, . \, \omega_p \cos\omega_p t - \frac{\omega_p^2}{\omega_F}\sin\omega_p t = 0$$

i.e. $t = \frac{1}{\omega_p}\tan^{-1}\phi$

where $\phi = \frac{\omega_F}{\omega_p}\cot\beta$

Substitute back into equation (35):-

$$\hat{i}_p = \frac{V_s}{L_F\omega_F}\left[\cot\beta\left(1+\sin.\tan^{-1}\phi\right)+\frac{\omega_p}{\omega_F}\cos\tan^{-1}\phi\right]$$



Therefore

$$\hat{i}_p = \frac{V_s}{L_F\omega_F}\left[\cot\beta\left(1+\frac{\phi}{\sqrt{1+\phi^2}}\right)+\frac{\omega_p}{\omega_F}\frac{1}{\sqrt{1+\phi^2}}\right]$$

now $\phi = \frac{\omega_F}{\omega_p}\cot\beta \; ; \; 1+\phi^2 = 1+\frac{\omega_F^2}{\omega_p^2}\cot^2\beta$

so $$\hat{i}_p = \frac{V_s}{L_F\omega_F}\left[\cot\beta\left(1+\frac{\omega_F}{\omega_p}\frac{\cot\beta}{(1+\frac{\omega_F^2}{\omega_p^2}\cot^2\beta)^{\frac{1}{2}}}\right)+\frac{\omega_p}{\omega_F}\frac{1}{(1+\frac{\omega_F^2}{\omega_p^2}\cot^2\beta)^{\frac{1}{2}}}\right]$$

$$= \frac{V_s}{L_F\omega_F}\left[\cot\beta+\frac{\cot\beta}{(\frac{\omega_p^2}{\omega_F^2}+\cot^2\beta)^{\frac{1}{2}}}+\frac{\omega_p^2}{\omega_F^2}\frac{1}{(\frac{\omega_p^2}{\omega_F^2}+\cot^2\beta)}\right]$$

Hence:-

$$\boxed{\hat{i}_p = \frac{V_s}{L_F\omega_F}\left[\cot\beta+\sqrt{\frac{\omega_p^2}{\omega_F^2}+\cot^2\beta}\right]} \qquad (36)$$

3.5.2 Average value of current i_p

We define the average to be:-

$$i_{p(av)} = \int_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} i_p \, dt \Big/ \frac{2\pi}{\omega_a}$$

Now $2\pi/\omega_a$ is the cycle time of the accelerator, but the integral is taken over the pulse time π/ω_p since i_p is zero during the charging of period of C_F.

So, using equation (35) in section 3.5.1, we can write i_p as:-

$$i_p = \frac{V_s}{L_F\omega_F}\cot\beta\left(1+\sin\omega_p t\right)+\frac{V_s}{L_F\omega_F}\cdot\frac{\omega_p}{\omega_F}\cos\omega_p t \qquad (35)$$

so $$i_{p(av)} = \int i_p \, dt \cdot \frac{\omega_a}{2\pi}$$

$$= \int \left[\cot\beta\left(1+\sin\omega_p t\right) + \frac{\omega_p}{\omega_F}\cos\omega_p t\right]\frac{V_s}{L_F\omega_F}\, dt \cdot \frac{\omega_a}{2\pi}$$

$$= \frac{\omega_a}{2\pi}\cdot\frac{V_s}{L_F\omega_F}\left[\cot\beta\left(t - \frac{1}{\omega_p}\cos\omega_p t\right) + \frac{1}{\omega_F}\sin\omega_p t\right]_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}}$$

$$= \frac{\omega_a}{2\pi}\cdot\frac{V_s}{L_F\omega_F}\left[\cot\beta \cdot \frac{\pi}{\omega_p} + \frac{2}{\omega_F}\right]$$

Therefore

$$\boxed{i_{p(av)} = \frac{V_s}{L_F\omega_F}\left[\frac{1}{2}\frac{\omega_a}{\omega_p}\cot\beta + \frac{1}{\pi}\frac{\omega_a}{\omega_F}\right]} \qquad (37)$$

3.5.3 RMS value of current i_p

We define the RMS value to be

$$i_{p(rms)} = \left[\frac{\omega_a}{2\pi}\int i_p^2 \, dt\right]^{\frac{1}{2}}$$

Using equation (35)

$$i_p^2 = \left(\frac{V_s}{L_F\omega_F}\right)^2\left\{\left[\cot^2\beta\left(1+2\sin\omega_p t+\sin^2\omega_p t\right)\right] + \frac{\omega_p^2}{\omega_F^2}\cos^2\omega_p t + 2\cot\beta\left(1+\sin\omega_p t\right)\frac{\omega_p}{\omega_F}\cos\omega_p t\right\}$$

using $\cos 2\phi = \begin{cases} 2\cos^2\phi - 1 \\ 1 - 2\sin^2\phi \end{cases}$; $\sin 2\phi = 2\sin\phi\cos\phi$

$$i_p^2 = \left(\frac{V_s}{L_F\omega_F}\right)^2\left\{\cot^2\beta\left[1+2\sin\omega_p t + \tfrac{1}{2}\left(1-\cos 2\omega_p t\right)\right]\right.$$
$$\left. + 2\cot\beta\left(\cos\omega_p t + \tfrac{1}{2}\sin 2\omega_p t\right)\frac{\omega_p}{\omega_F} + \frac{\omega_p^2}{2\omega_F^2}\left(1+\cos 2\omega_p t\right)\right\}$$

so

$$i_p^2 = \left(\frac{V_s}{L_F\omega_F}\right)^2\left\{\frac{3}{2}\cot^2\beta + \frac{1}{2}\frac{\omega_p^2}{\omega_F^2} + 2\cot^2\beta\sin\omega_p t + 2\frac{\omega_p}{\omega_F}\cot\beta\cos\omega_p t + \frac{\omega_p}{\omega_F}\cot\beta\sin 2\omega_p t\right.$$
$$\left. + \left(\frac{\omega_p^2}{2\omega_F^2} - \frac{1}{2}\right)\cos 2\omega_p t\right\}$$

We require $\int_{-\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}} i_p^2\, dt$, and only the first, second and fourth terms inside $\{\ \}$ contribute to the integral, therefore:-

$$i_{p(RMS)} = \left\{ \frac{\omega_a}{2\pi}\left(\frac{V_s}{L_F\omega_F}\right)^2 \left[\frac{3}{2}\cot^2\beta t + \frac{1}{2}\frac{\omega_p^2}{\omega_F^2} t + \frac{\omega_p}{\omega_F}.\frac{2\cot\beta}{\omega_p}\sin\omega_p t\right]_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} \right\}^{\frac{1}{2}}$$

$$= \frac{V_s}{L_F\omega_F}\left[\frac{\omega_a}{2\pi}\left(\frac{3}{2}\cot^2\beta.\frac{\pi}{\omega_p} + \frac{1}{2}\pi\frac{\omega_p}{\omega_F^2} + \frac{\omega_p}{\omega_F}.4\frac{\cot\beta}{\omega_p}\right)\right]^{\frac{1}{2}}$$

Hence

$$\boxed{i_{p(RMS)} = \frac{V_s}{L_F\omega_F}\left[\frac{3}{4}\frac{\omega_a}{\omega_p}\cot^2\beta + \frac{1}{4}\frac{\omega_a\omega_p}{\omega_F^2} + \frac{2}{\pi}\frac{\omega_a}{\omega_F}\cot\beta\right]^{\frac{1}{2}}} \qquad (38)$$

3.5.4 Peak value of current i_F

The expressions for i_F during the charging and pulse periods are given in equations (11) and (31). Making the same approximations as used in sections 3.5.1 - 3.5.3, i.e. $\omega \doteq \omega_p$, $1+\frac{L_p}{L_F} \doteq 1$; we may rewrite (11) and (31) as:-

$$i_F = \frac{V_s}{L_F\omega_F} . \frac{\cos\left(\omega_F t - \frac{\pi\omega_F}{\omega_a}\right)}{\sin\beta} \qquad (11a)$$

for $\frac{1}{4f_p} \leqslant t \leqslant \frac{1}{f_a} - \frac{1}{4f_p}$

and

$$i_F = i_{F_0} - \frac{V}{L_F\omega_p}\cos\omega_p t \qquad (31a)$$

for $-\frac{1}{4f} \leqslant t \leqslant \frac{1}{4f}$

substituting for i_{F_0} (equation 34) in equation (31a) gives

$$i_F = \frac{V_s}{L_F\omega_F}\left[\cot\beta - \frac{\omega_F}{\omega_p}\cos\omega_p t\right] \qquad (31b)$$

During charging

using equation (11a):-

$$\hat{i}_{F(charge)} = \frac{V_s}{L_F\omega_F}.\frac{1}{\sin\beta}\left[\cos\left(\omega_F t - \frac{\pi\omega_F}{\omega_a}\right)\right]_{MAX.}$$

$$= \frac{V_s}{L_F\omega_F}.\frac{1}{\sin\beta} \qquad (39)$$

the maximum occurring at $t = \frac{\pi}{\omega_a}$

During the pulse

using equation (31b):-

$$\hat{i}_{F\,(\text{pulse})} = \frac{V_s}{L_F\omega_F}\left[\cot\beta - \frac{\omega_F}{\omega_p}\cos\omega_p t\right]_{MAX}$$

$$= \frac{V_s}{L_F\omega_F}\left[\cot\beta + \frac{\omega_F}{\omega_p}\right] \qquad (40)$$

the maximum occurring at $\omega_p t = -\pi$. Now this is outside the range of the pulse, and we can therefore conclude that the peak during the pulse is less than this. Consequently since equation (40) is less than equation (39), then (39) gives the peak value required, i.e.:-

$$\boxed{\hat{i}_F = \frac{V_s}{L_F\omega_F}\cdot\frac{1}{\sin\beta}} \qquad (39)$$

where $\beta = \frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega_p}\right)$ from equation (33)

3.5.5 Average value of current i_F

$$i_{F\,(av)} = \frac{\omega_a}{2\pi}\int i_F\,dt$$

$$= \frac{\omega_a}{2\pi}\left\{\int_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} i_{F\,(\text{pulse})}\,dt + \int_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}} i_{F\,(\text{charge})}\,dt.\right\}$$

Using equation (31b)

$$\int_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} i_{F(\text{pulse})}\,dt = \frac{V_s}{L_F\omega_F}\left[\cot\beta . t - \frac{\omega_F}{\omega_p^2}\sin\omega_p t\right]_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}}$$

$$= \frac{V_s}{L_F\omega_F}\left[\frac{\pi}{\omega_p}\cot\beta - \frac{2\omega_F}{\omega_p^2}\right] \qquad (41)$$

Using equation (11a)

$$\int_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}} i_{F\,(\text{charge})}\,dt. = \frac{V_s}{L_F\omega_F}\left[\frac{1}{\omega_F\sin\beta}\cdot\sin\left(\omega_F t - \frac{\pi\omega_F}{\omega_a}\right)\right]_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}}$$

at the upper limit: $\omega_F t - \pi\frac{\omega_F}{\omega_a} = \frac{\pi\omega_F}{\omega_a} - \frac{\omega_F}{2\omega_p}$

$$= \beta$$

at the lower limit: $\omega_F t - \pi\frac{\omega_F}{\omega_a} = -\beta$

so
$$\int_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}} i_{F(charge)}\,dt = \frac{V_s}{L_F\omega_F^2 \sin\beta}\cdot 2\sin\beta$$

$$= \frac{2V_s}{L_F\omega_F^2} \qquad (42)$$

Substituting back to $i_{F(av)}$ from equations (41) and (42)

$$i_{F(av)} = \frac{\omega_a}{2\pi}\cdot\frac{V_s}{L_F\omega_F}\left[\frac{\pi}{\omega_p}\cot\beta - \frac{2\omega_F}{\omega_p^2} + \frac{2}{\omega_F}\right]$$

Therefore :-

$$\boxed{i_{F(av)} = \frac{V_s}{L_F\omega_F}\left[\frac{1}{2}\frac{\omega_a}{\omega_p}\cot\beta + \frac{1}{\pi}\frac{\omega_a}{\omega_F} - \frac{1}{\pi}\frac{\omega_a\omega_F}{\omega_p^2}\right]} \qquad (43)$$

3.5.6 RMS value of current i_F

$$i_{F(rms)} = \left\{\frac{\omega_a}{2\pi}\int_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} i^2_{F(pulse)}\,dt + \frac{\omega_a}{2\pi}\int_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}} i_F^2{}_{(charge)}\,dt\right\}^{\frac{1}{2}}$$

Using equation (31b)

$$i^2_{F(pulse)} = \left(\frac{V_s}{L_F\omega_F}\right)^2\left[\cot^2\beta - 2\cot\beta\cdot\frac{\omega_F}{\omega_p}\cos\omega_p t + \frac{\omega_F^2}{\omega_p^2}\cos^2\omega_p t\right]$$

$$= \left(\frac{V_s}{L_F\omega_F}\right)^2\left[\cot^2\beta + \frac{1}{2}\frac{\omega_F^2}{\omega_p^2} - 2\cot\beta\cdot\frac{\omega_F}{\omega_p}\cos\omega_p t + \frac{1}{2}\frac{\omega_F^2}{\omega_p^2}\cos 2\omega_p t\right]$$

Therefore

$$\int_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} i_F^2(\text{pulse})\,dt = \frac{V_s^2}{L_F^2\omega_F^2}\left[\cot^2\beta t + \frac{1}{2}\frac{\omega_F^2}{\omega_p^2}t - 2\cot\beta.\frac{\omega_F}{\omega_p^2}\sin\omega_p t + \frac{1}{4}\frac{\omega_F^2}{\omega_p^3}\sin 2\omega_p t\right]_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}}$$

$$= \frac{V_s^2}{L_F^2\omega_F^2}\left[\frac{\pi}{\omega_p}\cot^2\beta + \frac{\pi}{2}\frac{\omega_F^2}{\omega_p^3} - 4\cot\beta\,\frac{\omega_F}{\omega_p^2}\right] \qquad (44)$$

Using equation (11a)

$$i_F^2(\text{charge}) = \frac{V_s^2}{L_F^2\omega_F^2}\,\frac{\cos^2\left(\omega_F t - \pi\frac{\omega_F}{\omega_a}\right)}{\sin^2\beta}$$

$$= \frac{V_s^2}{L_F^2\omega_F^2}\cdot\frac{1}{2}\left[\frac{1+\cos\left(2\omega_F t - \frac{2\pi\omega_F}{\omega_a}\right)}{\sin^2\beta}\right]$$

Therefore

$$\int_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}} i_F^2(\text{charge}).\,dt = \frac{V_s^2}{2L_F^2\omega_F^2}\left[\frac{t}{\sin^2\beta} + \frac{1}{2\omega_F\sin^2\beta}\cdot\sin\left(2\omega_F t - \frac{2\pi\omega_F}{\omega_a}\right)\right]_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}}$$

at the upper limit:

$$2\omega_F t - \frac{2\pi\omega_F}{\omega_a} = \frac{4\pi\omega_F}{\omega_a} - \frac{\pi\omega_F}{\omega_p} - \frac{2\pi\omega_F}{\omega_a}$$

$$= \frac{2\pi\omega_F}{\omega_a} - \frac{\pi\omega_F}{\omega_p}$$

$$= 2\beta$$

at the lower limit:

$$= \frac{\pi\omega_F}{\omega_p} - \frac{2\pi\omega_F}{\omega_a}$$

$$= -2\beta$$

so

$$\int_{\frac{\pi}{2\omega_p}}^{\frac{2\pi}{\omega_a}-\frac{\pi}{2\omega_p}} i_F^2(\text{charge})\,dt = \frac{V_s^2}{2L_F^2\omega_F^2}\left[\frac{\frac{2\pi}{\omega_a}-\frac{\pi}{\omega_p}}{\sin^2\beta} + \frac{1}{\omega_F\sin^2\beta}\cdot\sin^2\beta\right]$$

$$= \frac{V_s^2}{L_F^2\omega_F^2}\left[\frac{\frac{\pi}{\omega_a}-\frac{1}{2}\frac{\pi}{\omega_p}}{\sin^2\beta} + \frac{1}{\omega_F}\cot\beta\right] \qquad (45)$$

Therefore, adding the integrals together (equations 44 and 45):-

$$i_{F\,(RMS)} = \left\{ \frac{\omega_a}{2\pi}\left[\frac{V_s}{L_F\omega_F}\right]^2 \left[\frac{\pi}{\omega_p}\cot^2\beta + \frac{\pi}{2}\frac{\omega_F^2}{\omega_p^3} - 4\cot\beta\cdot\frac{\omega_F}{\omega_p^2} + \frac{1}{\sin^2\beta}\left(\frac{\pi}{\omega_a} - \frac{1}{2}\frac{\pi}{\omega_p}\right) + \frac{1}{\omega_F}\cot\beta \right] \right\}^{\frac{1}{2}}$$

$$= \frac{V_s}{L_F\omega_F}\left\{ \frac{1}{2}\frac{\omega_a}{\omega_p}\cot^2\beta + \frac{1}{4}\frac{\omega_a\omega_F^2}{\omega_p^3} - \frac{2\omega_a\omega_F}{\pi\omega_p^2}\cot\beta + \frac{1}{2\sin^2\beta} - \frac{1}{4}\frac{\omega_a}{\omega_p\sin^2\beta} + \frac{1}{2\pi}\frac{\omega_a}{\omega_F}\cot\beta \right\}^{\frac{1}{2}}$$

Hence:-

$$\boxed{i_{F\,(RMS)} = \frac{V_s}{L_F\omega_F}\left[\frac{1}{\sin^2\beta}\left(\frac{1}{2} - \frac{1}{4}\frac{\omega_a}{\omega_p}\right) + \frac{1}{2}\frac{\omega_a}{\omega_p}\cot^2\beta + \left(\frac{1}{2\pi}\frac{\omega_a}{\omega_F} - \frac{2}{\pi}\frac{\omega_a\omega_F}{\omega_F^2}\right)\cot\beta + \frac{1}{4}\frac{\omega_a\omega_F^2}{\omega_p^3} \right]} \qquad (46)$$

3.5.7 Comment

On the basis of energy conservation $i_{p\,(av)}$ should equal $i_{F\,(av)}$. However equations (37) and (43) show that $i_{F\,(av)} \neq i_{p\,(av)}$ by the terms $-\frac{V_s}{L_F\omega_F}\cdot\frac{1}{\pi}\cdot\frac{\omega_a\omega_F}{\omega_p^2} = -\frac{V_s}{\pi}\cdot\frac{1}{L_F}\cdot\frac{\omega_a}{\omega_p^2}$

The discrepancy probably results from the approximation

$$1 \pm \frac{L_p}{L_F} \simeq 1$$

3.6 Circuit voltages

The voltages developed across the circuit components during the charging and pulse periods will now be considered, using the approximations detailed in section 3.5.

3.6.1 Capacitor C_F

The capacitor voltage v_F is given by

$$v_F = \frac{q_F}{C_F} = \frac{1}{C_F}\int i_F\,dt$$

During the charging period $\frac{1}{4f_p} \leq t \leq \frac{1}{f_a} - \frac{1}{4f_p}$ and using equation (11) for i_F we obtain:-

$$v_F(t) = \frac{1}{C_F}\int_{\frac{\pi}{2\omega_p}}^{t} V_s C_F \omega_F \frac{\cos\left(\omega_F t' - \pi\frac{\omega_F}{\omega_a}\right)}{\sin\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega_p}\right)} . dt'$$

$$= \frac{1}{C_F}\,\frac{V_s C_F \omega_F}{\sin\beta}\left[\frac{1}{\omega_F}\sin\left(\omega_F t' - \pi\frac{\omega_F}{\omega_a}\right)\right]_{\frac{\pi}{2\omega_p}}^{t}$$

$$= \frac{V_s \omega_F}{\sin\beta}\left[\frac{1}{\omega_F}\sin\left(\omega_F t - \pi\frac{\omega_F}{\omega_a}\right) + \frac{1}{\omega_F}\sin\beta\right]$$

$$\boxed{v_F = V_s\left[1 + \frac{\sin\left(\omega_F t - \frac{\pi\omega_F}{\omega_a}\right)}{\sin\beta}\right]} \qquad (47)$$

and during the discharge period $-\frac{1}{4f_p} \leq t \leq \frac{1}{4f_p}$ (i.e. pulse period) v_F is given by equation (32)

$$\boxed{v_F = V_s\left(1 - \sin\omega_p t\right) + L_p\omega_p i_{F_0}\cos\omega_p t} \qquad (32)$$

3.6.2 Filter choke L_F

The filter choke voltage v_{L_F} is given by:-

$$L_F\frac{di_F}{dt}$$

During the charging period, $\frac{1}{4f_p} \leq t \leq \frac{1}{f_a} - \frac{1}{4f_p}$, and using equation (11) for i_F we obtain:-

$$v_{L_F} = L_F\frac{d}{dt}\left[V_s C_F \omega_F \frac{\cos\left(\omega_F t - \pi\frac{\omega_F}{\omega_a}\right)}{\sin\frac{\pi}{2}\left(\frac{2\omega_F}{\omega_a} - \frac{\omega_F}{\omega_p}\right)}\right]$$

$$\boxed{v_{L_F} = -V_s\frac{\sin\left(\omega_F t - \pi\frac{\omega_F}{\omega_a}\right)}{\sin\beta}} \qquad (48)$$

where $L_F = \frac{1}{C_F\omega_F^2}$

and during the pulse period, $-\frac{1}{4f_p} \leqslant t \leqslant \frac{1}{4f_p}$, and using equation (31) this voltage is:-

$$v_{L_F} \simeq L_F \frac{d}{dt}\left[-\frac{V_s}{L_F\omega_p}\cos\omega_p t\right]$$

$$\left(\text{since} \quad 1 - \frac{L_p}{L_F}\sin\omega_p t \text{ is small}\right)$$

$$\boxed{v_{L_F} = V_s \sin\omega_p t} \qquad (49)$$

3.6.3 Pulse Choke L_p

The pulse choke voltage v_{L_p} is given by:-

$$L \frac{di_p}{dt}$$

therefore during the pulse period, $-\frac{1}{4f_p} \leqslant t \leqslant \frac{1}{4f}$ we have:-

$$v_{L_p} = L_p \frac{d}{dt}\left[i_{F_0}(1+\sin\omega_p t) + \frac{V_s}{L_p\omega_p}\cos\omega_p t\right]$$

$$\boxed{v_{L_p} = i_{F_0}\cos\omega_p t - \frac{V_s}{L_p\omega_p}\sin\omega_p t} \qquad (50)$$

3.6.4 Ignitron

The voltage developed across the ignitron during the charging period $\frac{1}{4f_p} \leqslant t \leqslant \frac{1}{f_a} - \frac{1}{4f_p}$ is given by:-

$$v_{ig} = v_F - v_m$$

using equation (47) and noting that $v_m = V_m \cos\omega_a t$ and $V_m = V_s$ we obtain :-

$$\boxed{v_{ig} = v_F - v_m = V_s\left[1 + \frac{\sin\left(\omega_F t - \frac{\pi\omega_F}{\omega_a}\right)}{\sin\beta} - \cos\omega_a t\right]} \qquad (51)$$

During the pulse period the voltage v_{ig} equals the ignitron arc-drop voltage.

3.6.5 Average primary voltage during pulse period

The energy storage choke secondary voltage V_m referred to the primary is:-

$$v_m' = \sqrt{2}\,\frac{N_1}{N_2}\,V_m\cos\omega_a t$$

where $\frac{N_1}{N_2}$ = primary/secondary turns ratio.

Hence, the average voltage during current pulse i_p is given by:-

$$v'_{m(av)} = \int_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}} \sqrt{2}\,\frac{N_1}{N_2}\, V_m \cos \omega_a t.\, dt$$

$$= \frac{\omega_p}{\pi}\left[\sqrt{2}\,\frac{N_1}{N_2}\, V_m \frac{1}{\omega_a} \sin \omega_a t\right]_{-\frac{\pi}{2\omega_p}}^{\frac{\pi}{2\omega_p}}$$

$$\boxed{v'_{m(av)} = \sqrt{2}\,\frac{N_1}{N_2} \cdot \frac{\omega_p}{\omega_a \pi} \cdot 2 \sin \frac{\pi \omega_a}{2\omega_p}} \qquad (52)$$

3.7 Pulse power supply values

The circuit parameters are dependant on the values chosen for ω_F and ω_p and these frequencies will be optimised, by an analogue computer study on the basis of:-

(a) minimum introduced disturbance to the magnet network during normal operation,

(b) economic circuit component ratings,

(c) acceptable transient fault values.

However, as an indication of the general current and voltage relationships in the pulse power supply circuit under normal operating conditions, the following values are tabulated (assuming unity turns ratio on the energy storage choke) for:-

$$\omega_a = 2\pi 50 \quad \text{radians/second}$$

$$\omega_F = \frac{\omega_a}{4}$$

$$\omega_p = 3\omega_a = 12\omega_F$$

$$i_{F(av)} = \frac{P_{AC}}{V_S}$$ neglecting pulse power supply losses

also $$i_{F(av)} = \frac{1.499 V_S}{L_F \omega_F}$$ from equation (43)

$$i_{F_0} = \frac{1.303 V_S}{L_F \omega_F} = 0.87\, i_{F(av)}$$ from equation (10)

$$\hat{i}_F = \frac{1.645 V_S}{L_F \omega_F} = 1.1\, i_{F(av)}$$ from equation (39)

$$i_{F(RMS)} = \frac{1.54 V_S}{L_F \omega_F} = 1.028\, i_{F(av)}$$ from equation (46)

$$i_{p(av)} = \frac{1.491\, V_s}{L_F w_F} \doteq i_{F(av)}$$ from equation (37)

$$\hat{i}_p = \frac{13.373\, V_s}{L_F w_F} = 8.93\, i_{F(av)}$$ from equation (36)

$$i_{p(RMS)} = \frac{3.96\, V_s}{L_F w_F} = 2.64\, i_{F(av)}$$ from equation (38)

$$L_F = \frac{1.499 V_s}{w_F\, i_{F(av)}}$$

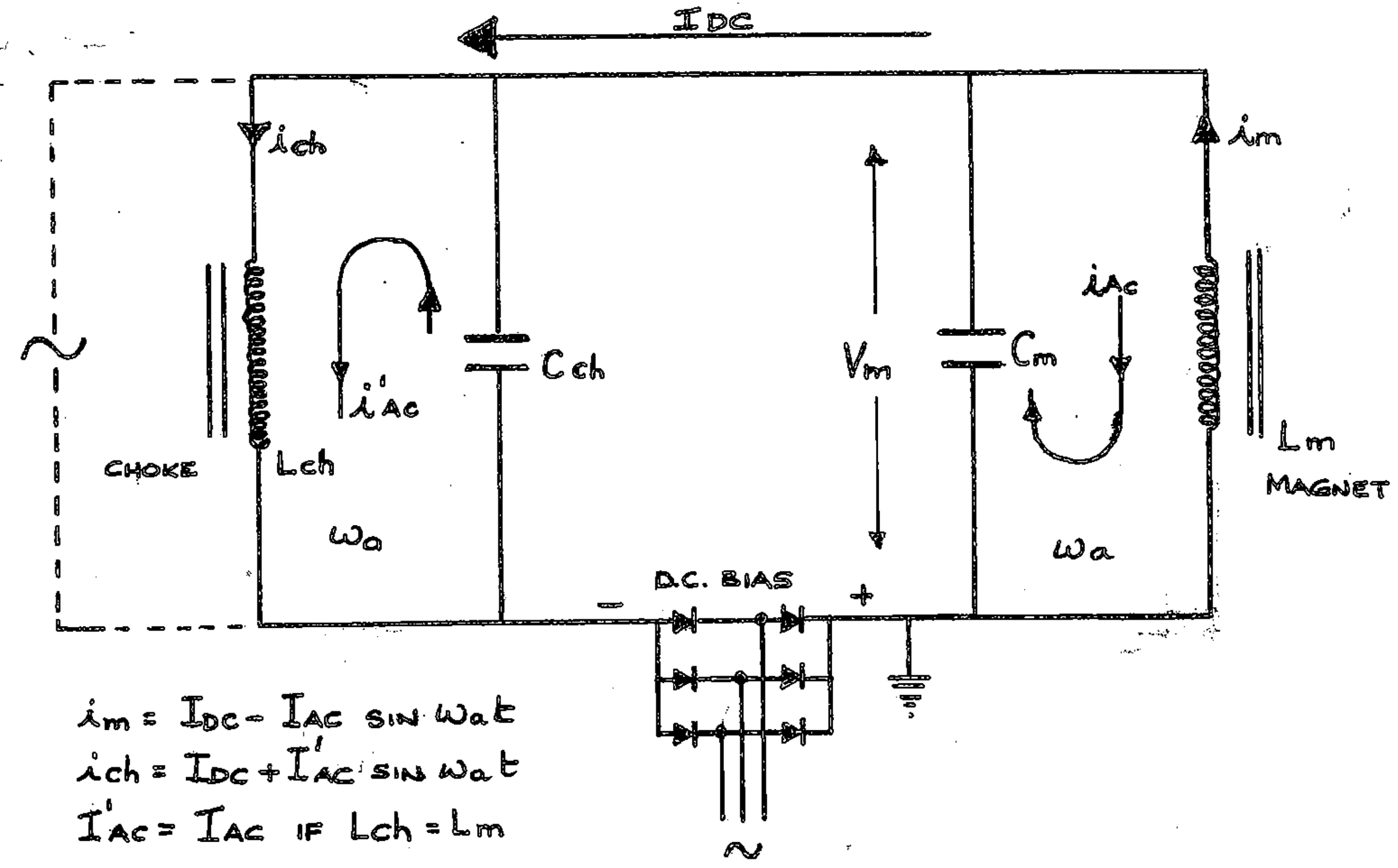
$$C_F = \frac{1}{L_F w_F^2}$$

$$L_p = \frac{1}{C_F w_p^2}$$

$$v'_{m(av)} = 0.955\left(\sqrt{2}\frac{N_1}{N_2} V_m\right)$$ during pulse, from equation (52)

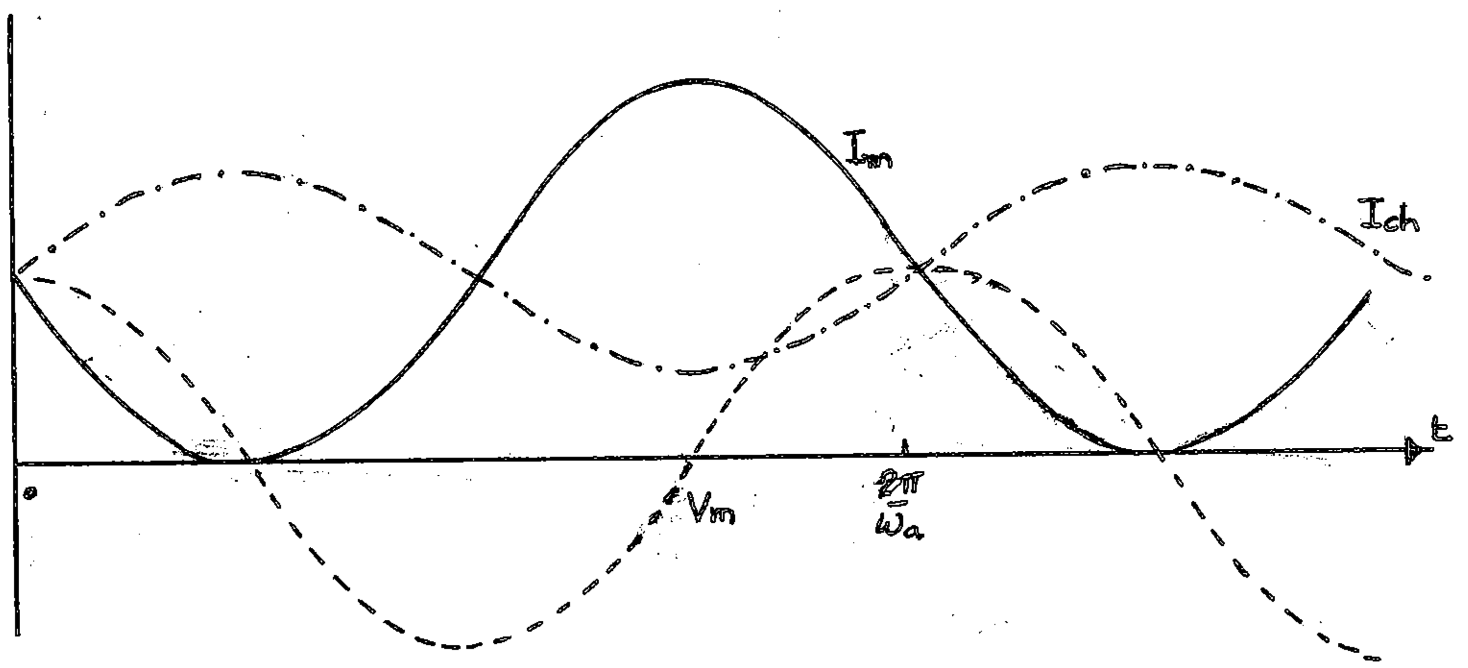
N.b. The above values compare closely with those given in Cambridge Electron Accelerator report CEA-68.

OCTOBER 1962.



$i_m = I_{DC} - I_{AC} \sin \omega_a t$
$i_{ch} = I_{DC} + I'_{AC} \sin \omega_a t$
$I'_{AC} = I_{AC}$ IF $L_{ch} = L_m$

FIG 1. SIMPLIFIED CIRCUIT :- SINGLE MAGNET SECTOR.



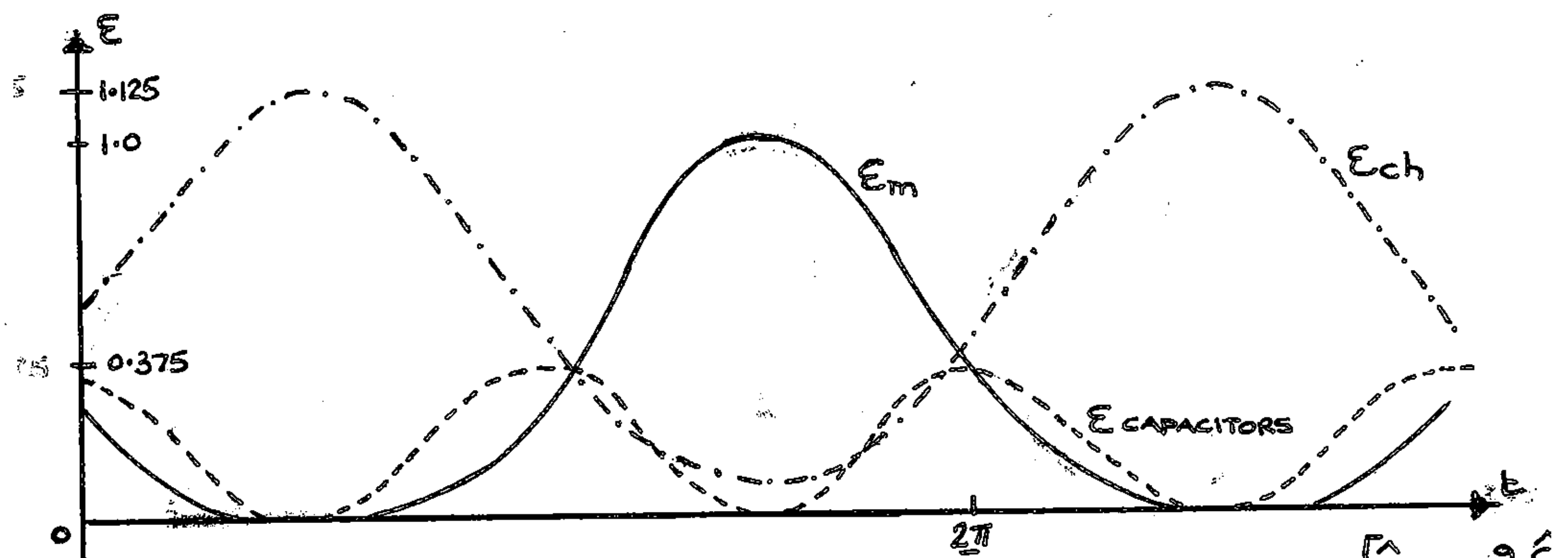


FIG 2 VOLTAGE, CURRENT & ENERGY CHARACTERISTICS

$\left[\hat{\mathcal{E}}_{ch} = \frac{9}{8}\hat{\mathcal{E}}\right.$

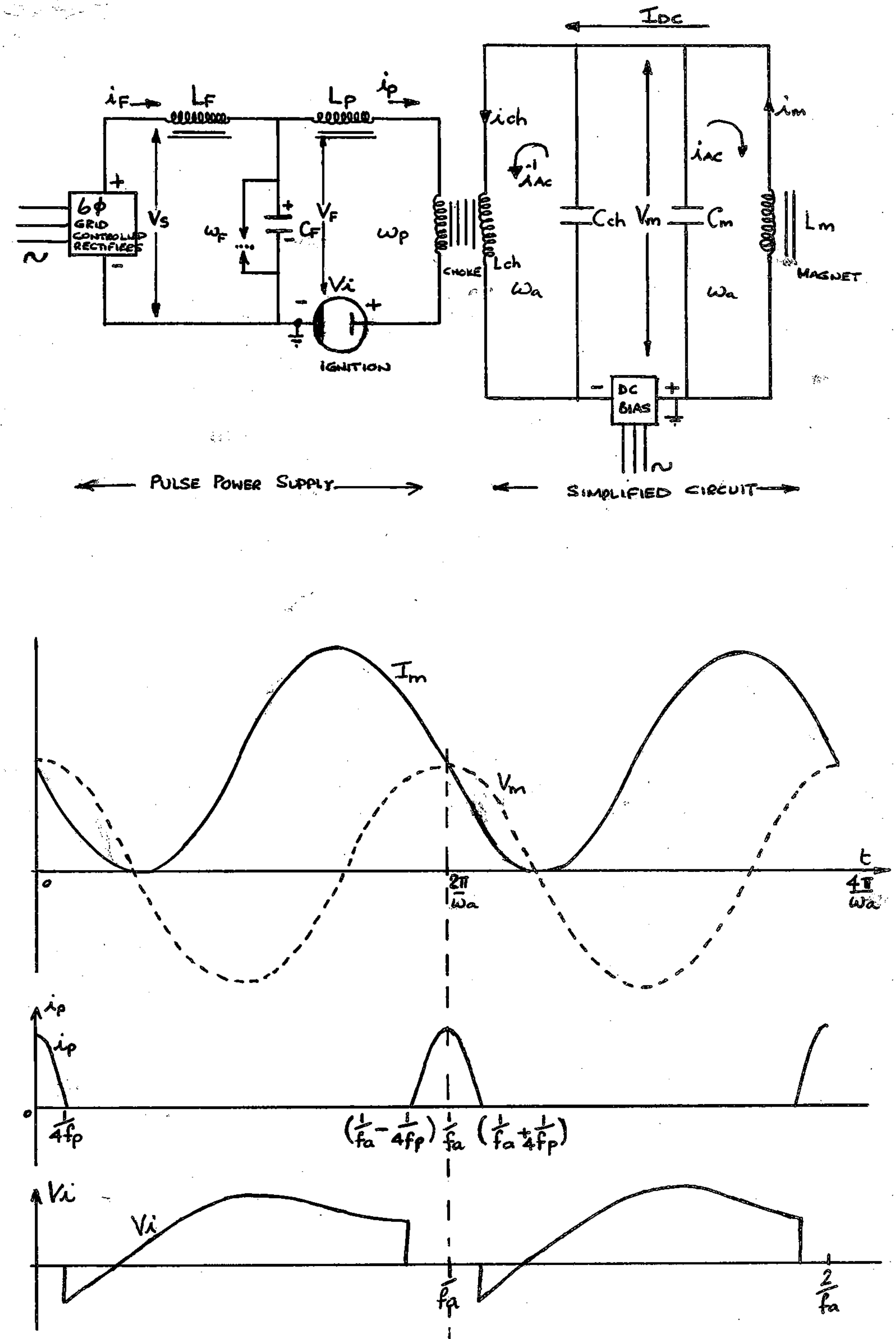


FIG 4 - PULSE POWER SUPPLY & CIRCUIT WAVEFORM.