LENDING COPY

102519 - 1002

B

DNPL EL/TM/12

Stability of Beam-Cavity System

By

M. Donald

National Institute for
Research in Nuclear Science
Electron Laboratory.

December, 1963.

Stability of Beam-Cavity System

The stability of a Beam-Cavity System has been investigated by K. W. Robinson and a 4th order algebraic equation derived, from which the complex resonant frequencies of the system may be found.

The equation has been evaluated for the parameters of NINA with different conditions of R. F. tuning.

The equation, with a slight change in nomenclature is as follows:

$$\gamma^4 + \gamma^3 . 2\alpha + \gamma^2 . \left[\Omega^2 + \alpha^2 + \alpha^2 \tan^2\psi\right] + \text{----}$$

$$\gamma . 2\alpha\, \Omega^2 + \left[\alpha^2 + \alpha^2 \tan^2\psi - \alpha^2 \frac{V_L}{V} \frac{\tan\psi}{\cos\phi}\right] \Omega^2 = 0$$

where $\Omega = \left[\dfrac{2\pi f^2 \alpha' \; V\beta \cos\phi}{E_s}\right]^{\frac{1}{2}}$ the synchronous angular frequency of the individual electrons in the synchrotorn.

$\alpha = \dfrac{\omega_o}{2Q_L}$ the natural rate of response of the tuned circuit

ω_o = the natural angular resonant frequency of the tuned circuit

Q_L = loaded Q factor of the tuned circuit

ψ = angular detuning of the tuned circuit, given by the admittance $Y = G(1+j \tan\psi)$

f = RF. drive frequency

α' = orbit compaction factor.

V = peak cavity voltage.

$V_L = I_L R\beta$

β = transit time factor of the cavity

E_s = synchronous energy of electrons in the synchrotron.

ϕ = Phase angle of beam current w.r.t. cavity voltage as shown in figure (2)

ΔV $\Delta\phi$ etc are assumed to be changing as $A_e{}^{\gamma t}$

1. CEA-11 R. F. Acceleration II by K. W. Robinson pp 14 - 16

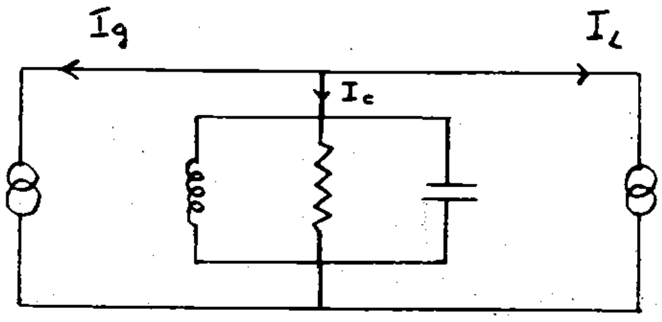


Fig (1)

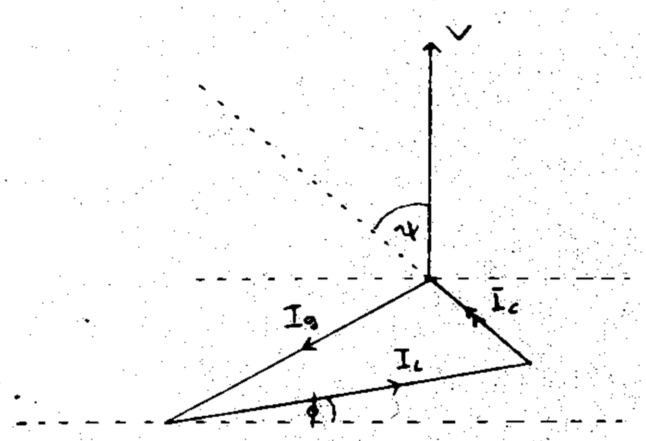


Fig (2)

I_g is the generator current

I_L is the 408 Mc/s component of the beam current. For a beam perfectly bunched at 408 Mc/s

$$I_L = 2\,\bar{I}_b$$

Where $\bar{I}_b$ is the mean D. C. circulating current.

For this condition $\phi = \phi_s$ the synchronous phase angle of the individual electrons.

If P_b is the power given to the beam to **keep** the electrons in synchronisation

$$P_b = V\bar{I}_b \sin\phi_s = \frac{V\,I_L}{2}\sin\phi$$

from "DC" beam condition **from vector diagram.**

Parameters

The parameters common to all the calculations were

$f = 408$ Mc/s

$\alpha' = .0453$

$\beta = 2/\pi$

$Q_u = 4 \times 10^4$ unloaded Q

G_c = cavity shunt conductance

G_o = waveguide shunt conductance as seen by the cavity.

giving $Q_L = \frac{Q_u}{6} = 6.667\ 10^3$

$$R = \frac{1}{G_o + G_c} = 6.667\ M\Omega$$

$\bar{I}_b = .272$ amps.

The parameters common to calculations at the time of injection were

$E_s = 4 \times 10^7$ eV

$\delta E_s = 7.4\ 10^4$ eV/turn, the increase in synchronous energy per turn.

$$\epsilon = .49 = \frac{I_L}{2\bar{I}_b}$$

The parameters common to calculation at full energy were

$E_s = 4 \times 10^9$ eV

$\delta E_s = 1.1 \times 10^6$ eV/turn

$\epsilon = 1.0$

The variable parameters were ϕ and ψ.

When ψ was not specified, conditions were worked out so that the input impedance of the cavity was purely resistive, this automatically fixing ψ and V.

When ψ was specified this procedure was not carried out, and ϕ was kept at a fixed value, this value being such that by appropriate choice of ψ the cavity could be totally matched to the waveguide.

Results.

Run (1)

Injection conditions. Reactive part of beam loading tuned out.

$\phi = 1^\circ$ to 32°

The cavity is perfectly matched when $\phi = 22^\circ$ $\psi = 58.83^\circ$

Graph (1A)

Real part of roots.

The real part of roots (1) and (2). For $\phi < 3^\circ$, real part of roots (1) and (2) $\simeq 1.1922\ 10^5$ changing very slowly as the beam loading increases.

The real part of roots (3) and (4) is also changing slowly.

If the real parts are designated α_1 α_2 α_3 α_4

then $-2\alpha = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = -\frac{\omega}{QL}$

when $\alpha_1 = \alpha_2$ and $\alpha_3 = \alpha_4$ then $-\alpha = \alpha_1 + \alpha_3 = -1.922\ 10^5$

When $\phi \simeq 3^\circ$ the first 2 roots become pure real, and when $\phi \simeq 27.3^\circ$ one of these roots reaches zero and then becomes positive. This condition corresponds to the condition when $\psi = \pi/2 - \phi$

Graph (1B) is an expanded version of (Graph A)

Graph (1C) Imaginary part of roots.

The imaginary part of roots (1) and (2) from $\phi = 0^\circ$ to $\phi = 3^\circ$ is very much less than the real part and is not significant.

The imaginary part of roots (3) and (4) $\simeq \Omega$ the synchrotron oscillation frequency. Even at a very high beam loading the deviation from Ω is small.

Run (2)

Full energy conditions. Reactive part of beam loading tuned out $\phi = 1^\circ$ to 60°. The cavity is perfectly tuned when $\phi = 45^\circ$ $\psi = 55.7^\circ$

Graph (2A)

Real Part of roots.

Graph (2B)

Imaginary part of roots.

The imaginary part of roots (1) and (2) is again not significant.

Run (3)

Injection conditions $\phi = 22^\circ$ $\psi = -40^\circ$ to $+ 60^\circ$

Graph (3A)

For cavity matched $\psi = 56.63^\circ$

Real Part of roots.

For $\psi < 0$ Real part of roots (3) and (4) is positive and the system is unstable.

Graph (3B)

Expanded version of 3A

Graph (3C)

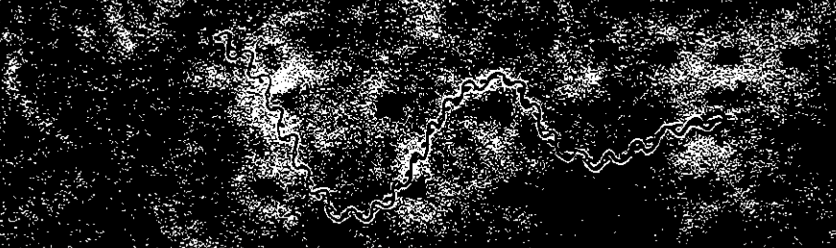
Imaginary part of roots.

The imaginary part of roots (1) and (2) can be larger than the real part and is therefore significant.

For matched conditions at injection $\psi = 56.63^\circ$

If ψ is increased to 65° roots (1) and (2) change rapidly.

The damping is increased from $.5 \times 10^5$ to $1.74\ 10^5$ and the motion is oscillatory fig (3), as opposed to exponential fig (4) for the case $\psi = 56.63^\circ$



----- Roots (1) & (2)
— Roots (3) & (4)

Fig (3).

----- Roots (1) & (2)
— Roots (3) & (4)

Fig (4)

The point of maximum stability seems to occur with ψ a little greater than 60°.

Fig (4). Full energy condition $\phi =$ [illegible] $\psi =$ [illegible] to [illegible]

Graph (4a)

Real Part of roots. For stability [illegible] $\psi = 33.7°$

The magnitudes of the roots are here seen to be comparable.

Graph (4b)

Imaginary part of roots.

When ψ is increased to [illegible]7° the imaginary part of roots (1) and (2) becomes equal to the real part. The value of ψ for maximum stability is seen to be [illegible]°.

Conclusions

By testing the equation for γ with the Routh-Hurwitz criterion the condition for no roots to have positive real parts is

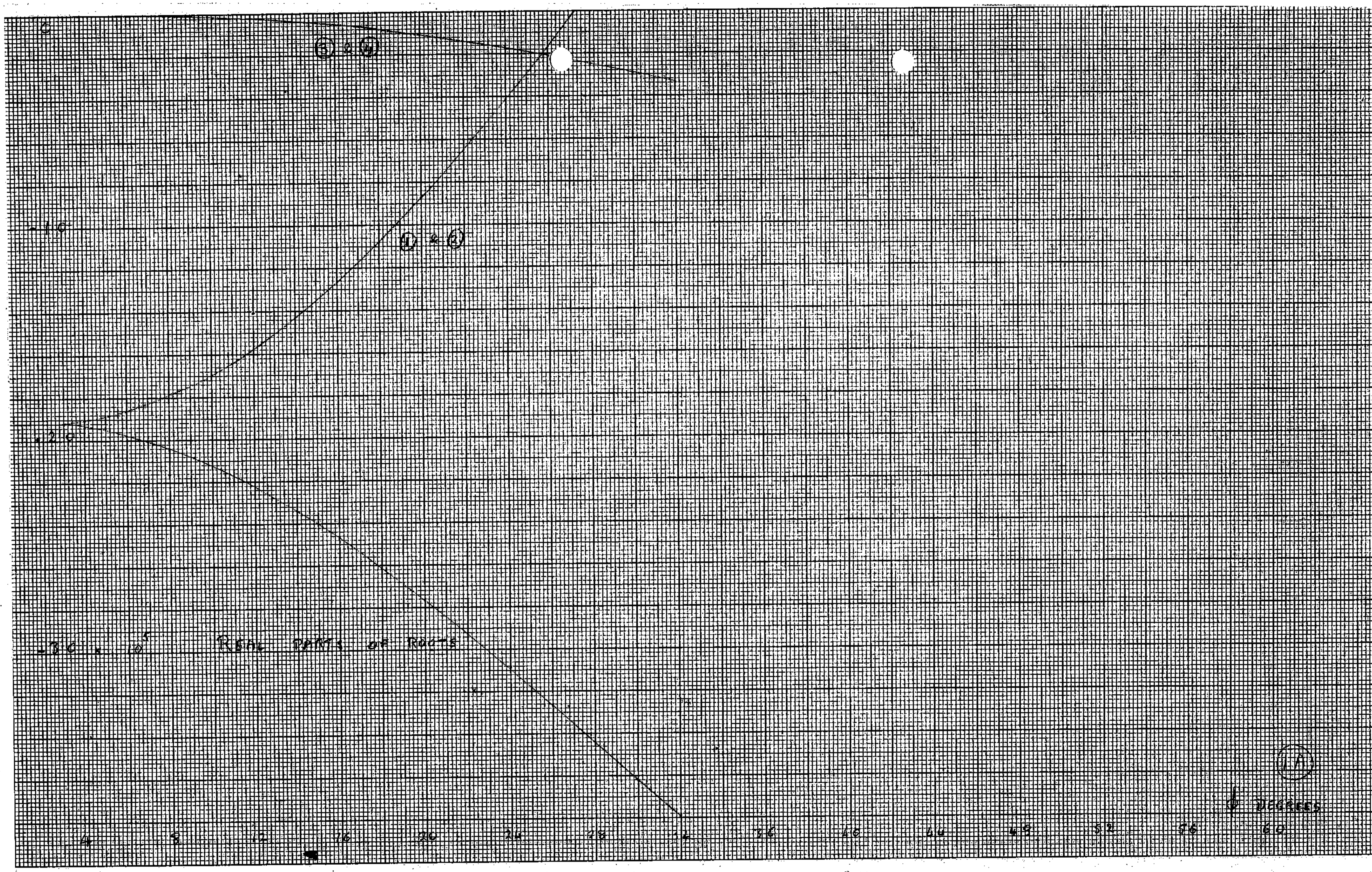
$$0 \leq \sin 2\psi \leq \frac{k}{V_1} \cos \phi_1$$

An examination of the values of the roots between these limits it is seen that the conditions for maximum stability require a slightly [illegible] [illegible] than that which is [illegible] for the best [illegible].

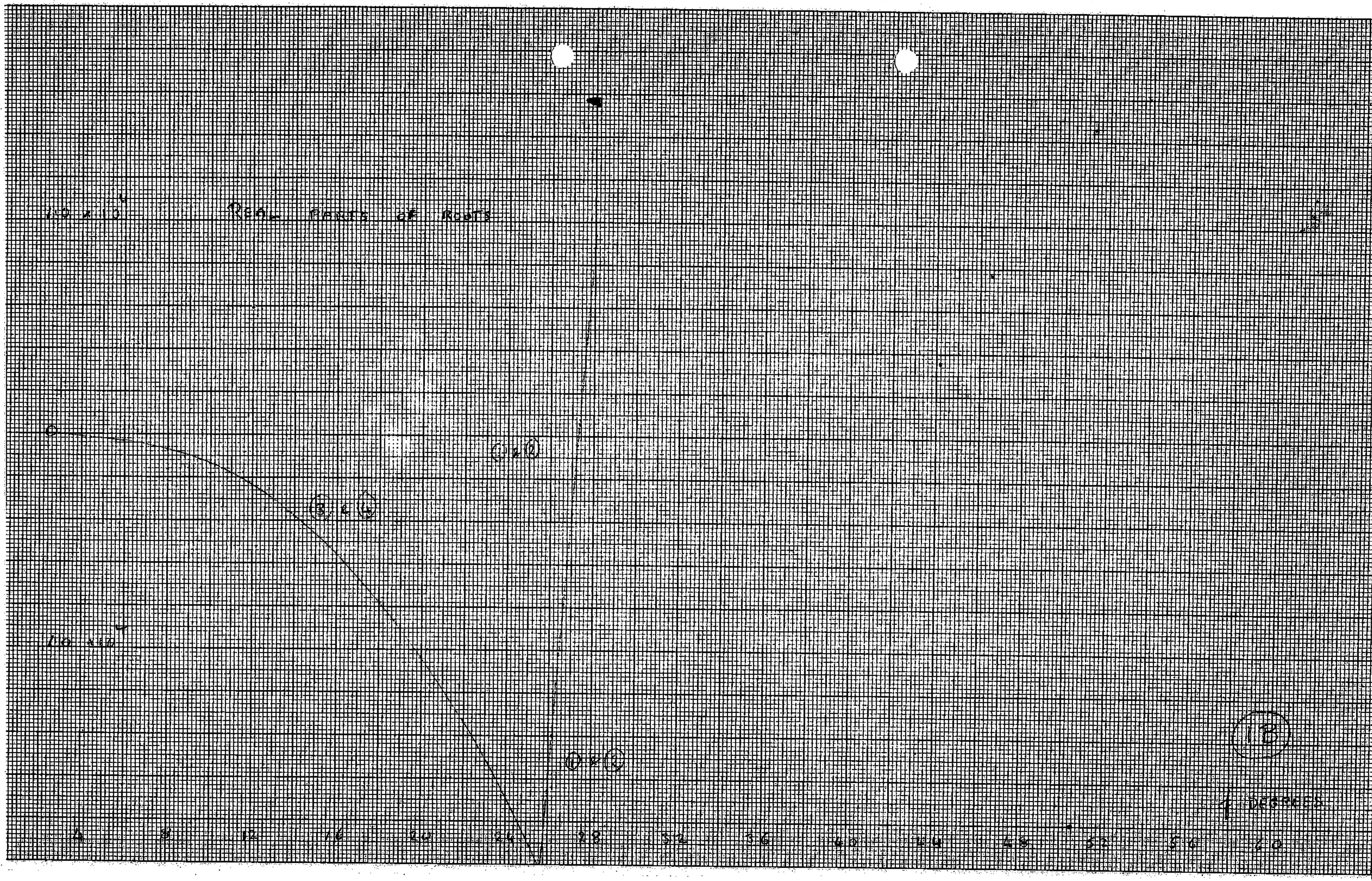
Note:

The theory is applicable only to small deviations from the equilibrium conditions. A large deviation from a stable equilibrium position may well result in instability, or conversely, a stable amplitude of oscillation may be possible about an unstable equilibrium condition.

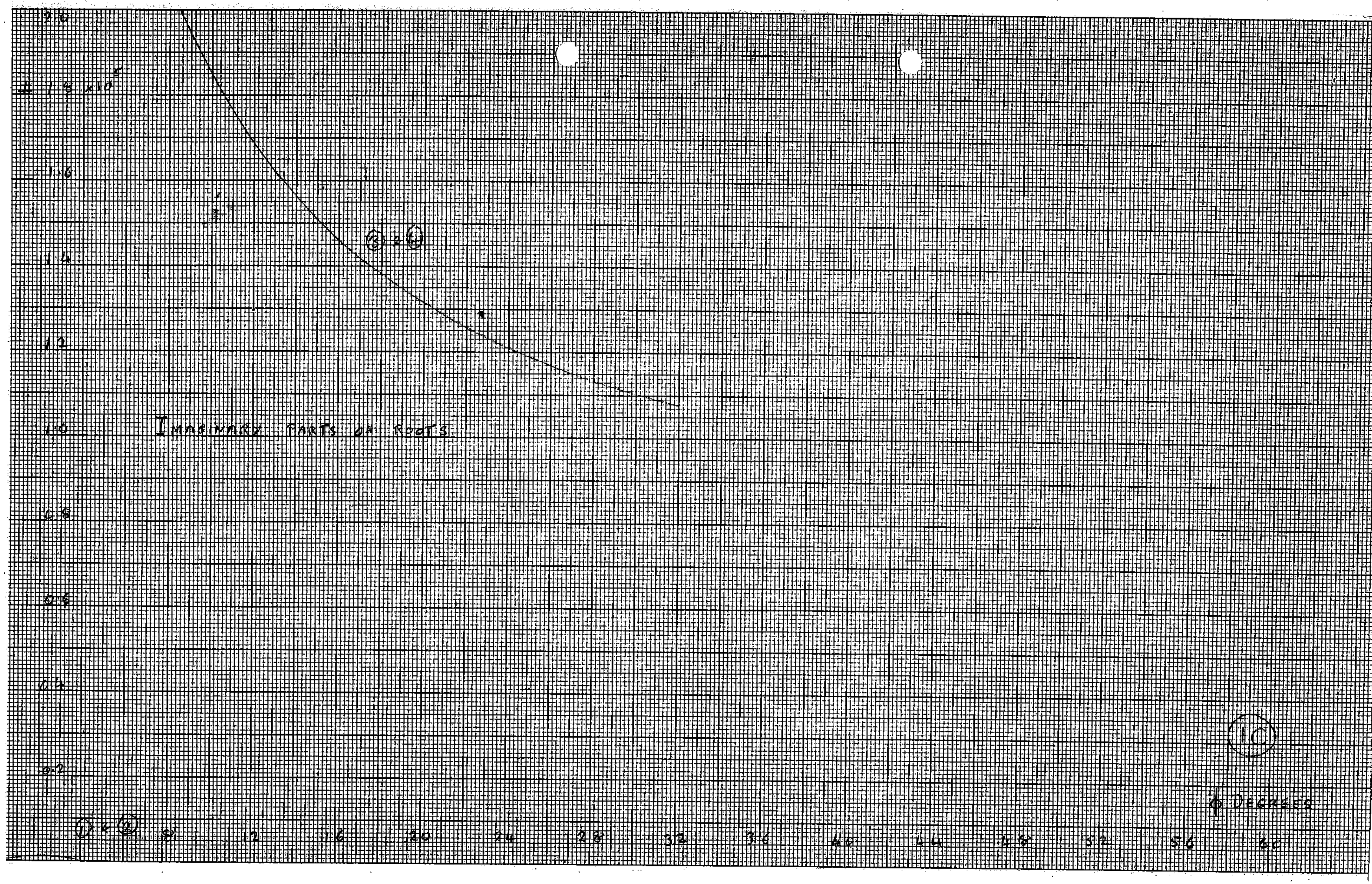
[illegible]

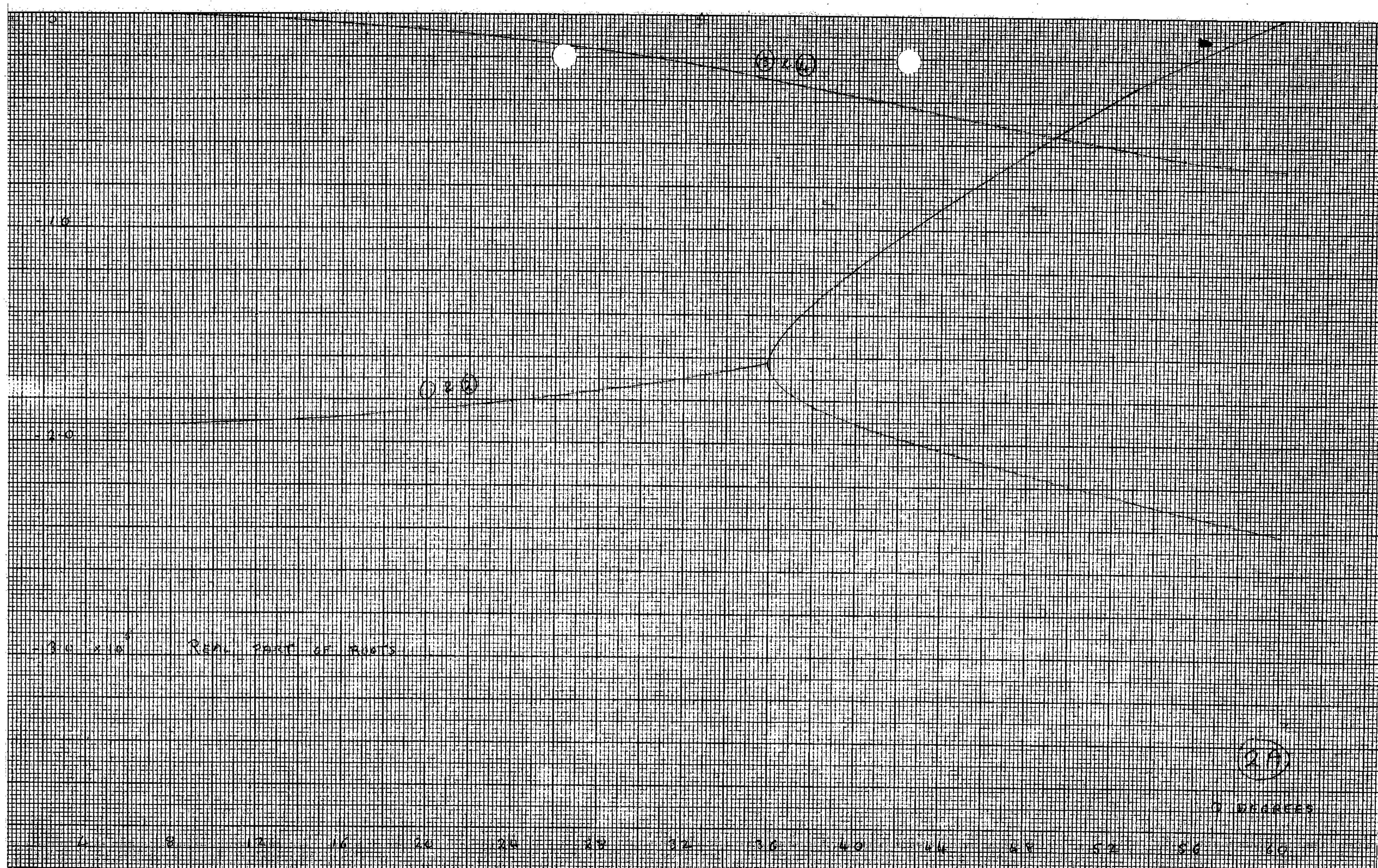


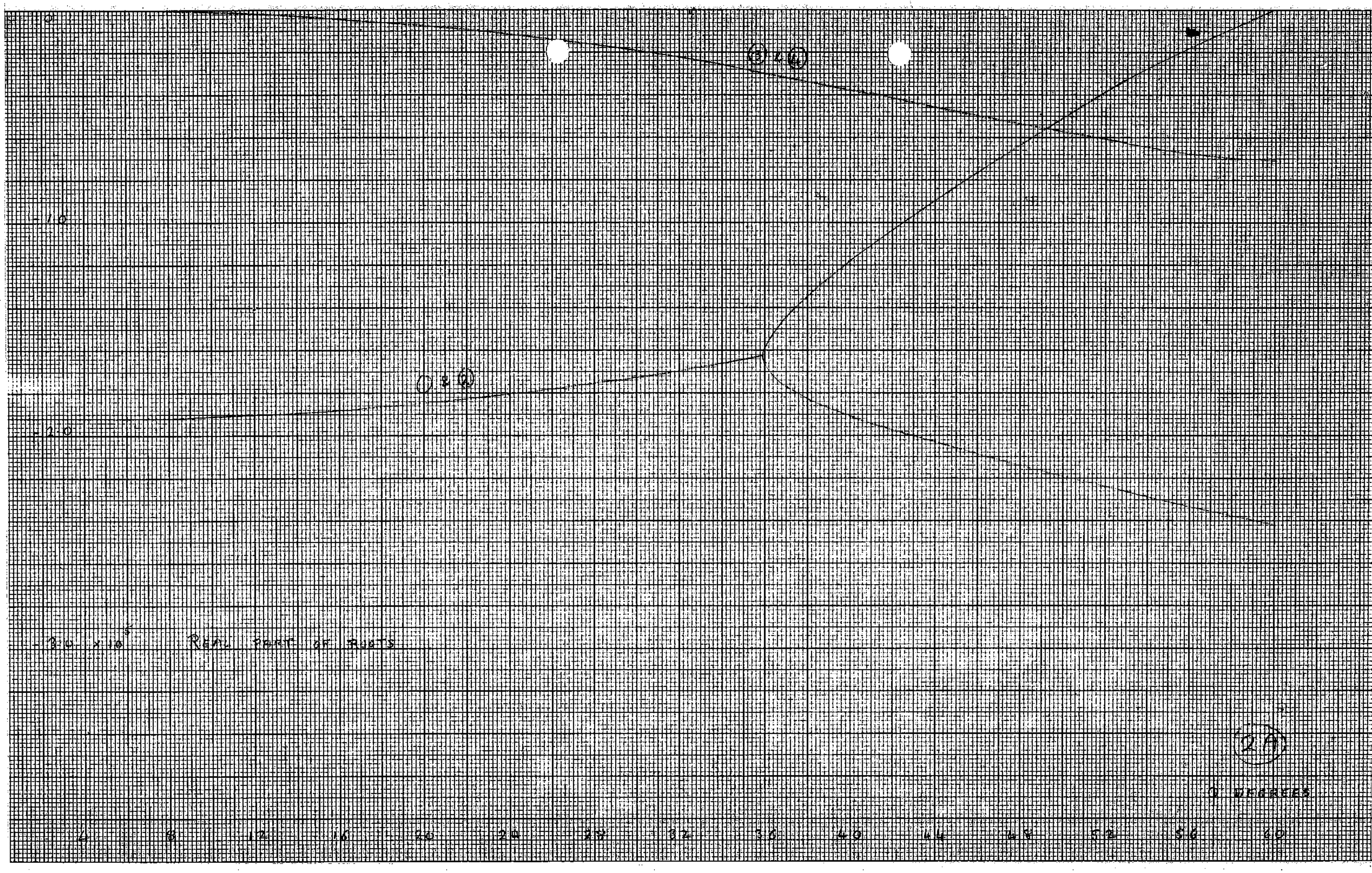
0
(3) & (4)
-1.0
(1) & (2)
-2.0
-3.0 × 10^5
REAL PARTS OF ROOTS
1A
φ DEGREES
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60



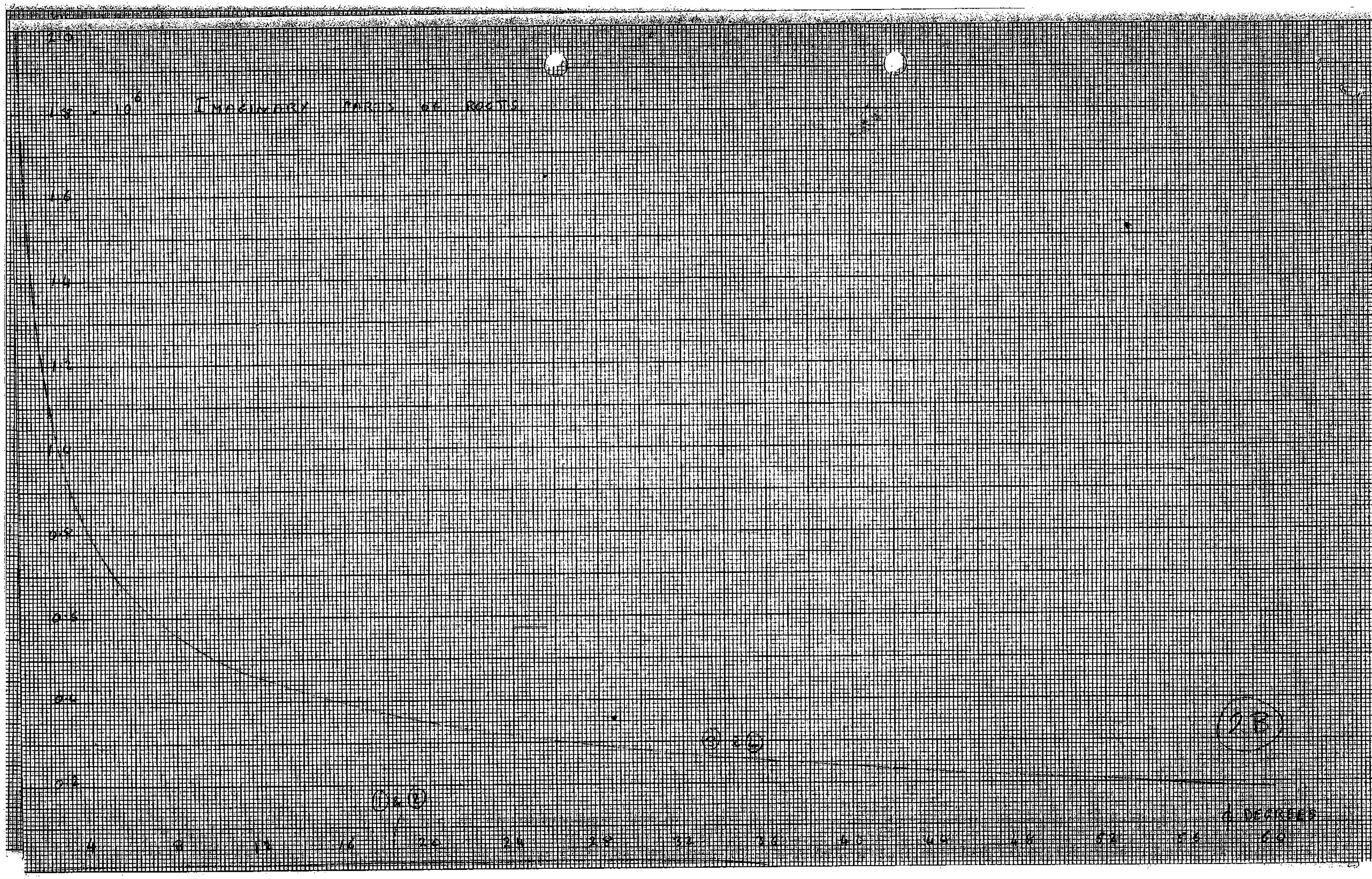
REAL PARTS OF ROOTS
①&②
③&④
①&②
18
γ DEGREES
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60

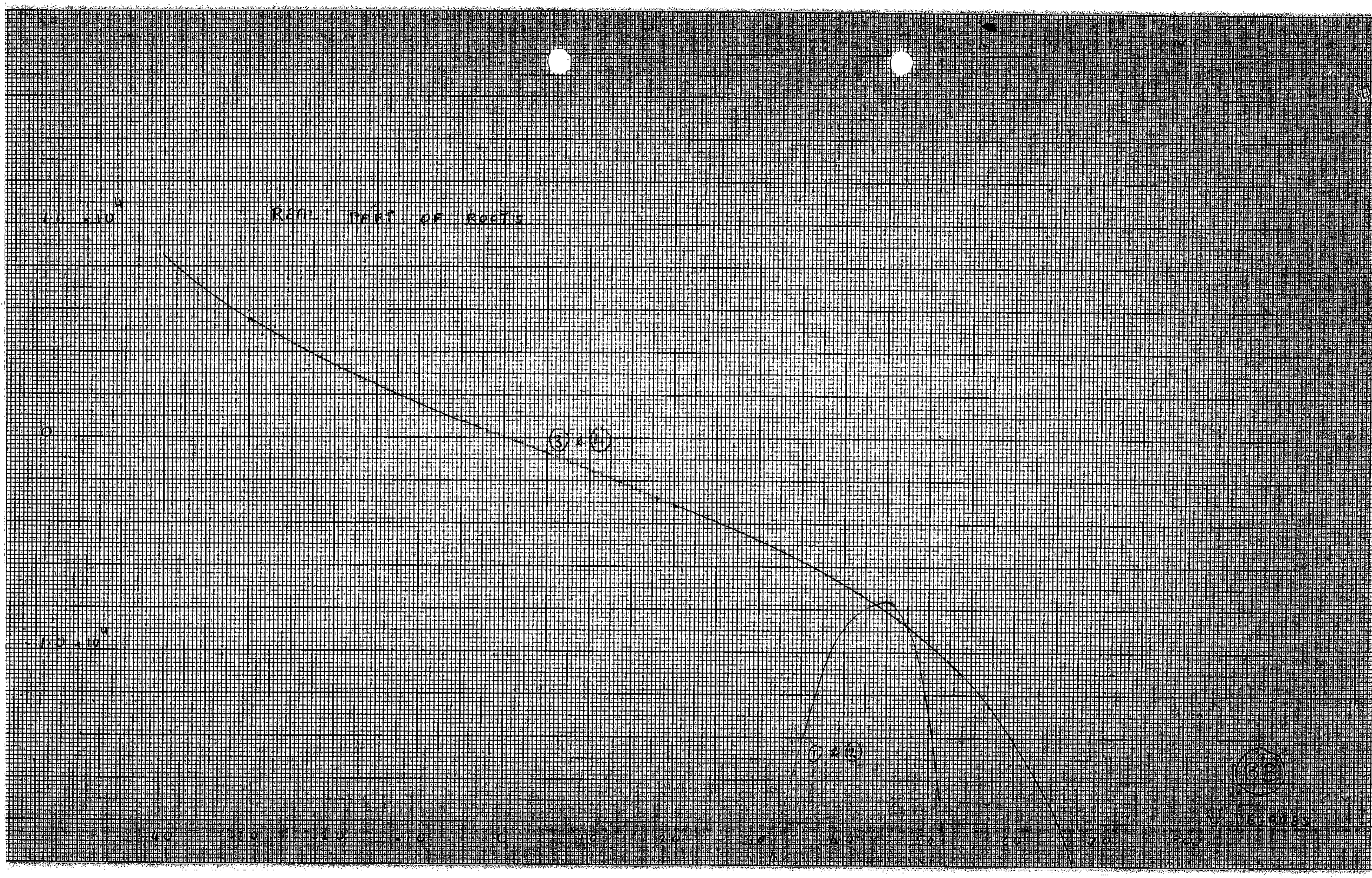

IMAGINARY PARTS OF ROOTS
1.8 x10
2.0
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
③ & ④
① & ②
8
12
16
20
24
28
32
36
40
44
48
52
56
60
ϕ DEGREES
10


0
-1.0
-2.0
-3.0 × 10^3 REAL PART OF ROOTS
① & ②
③ & ④
2A
θ DEGREES
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60

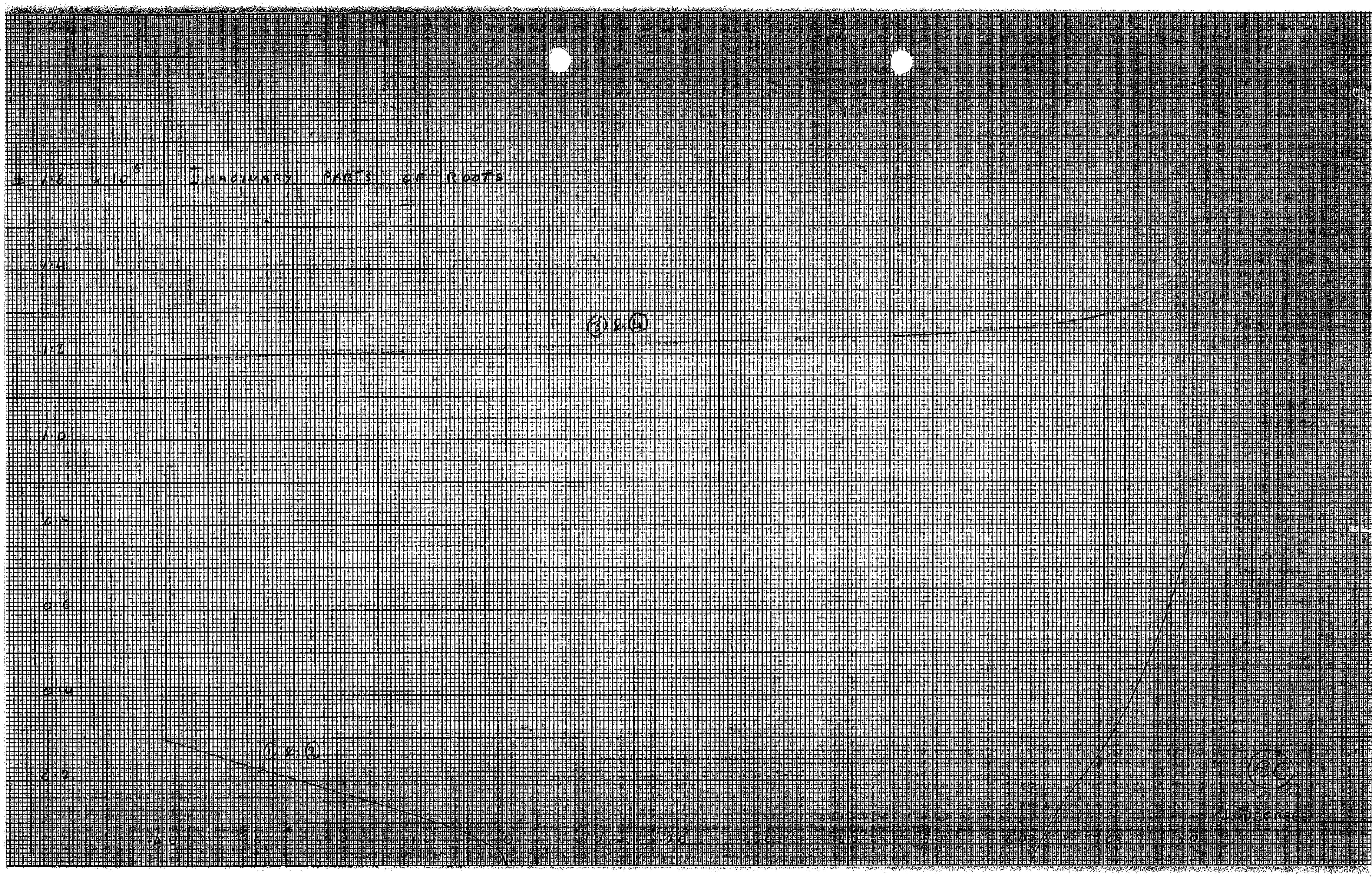


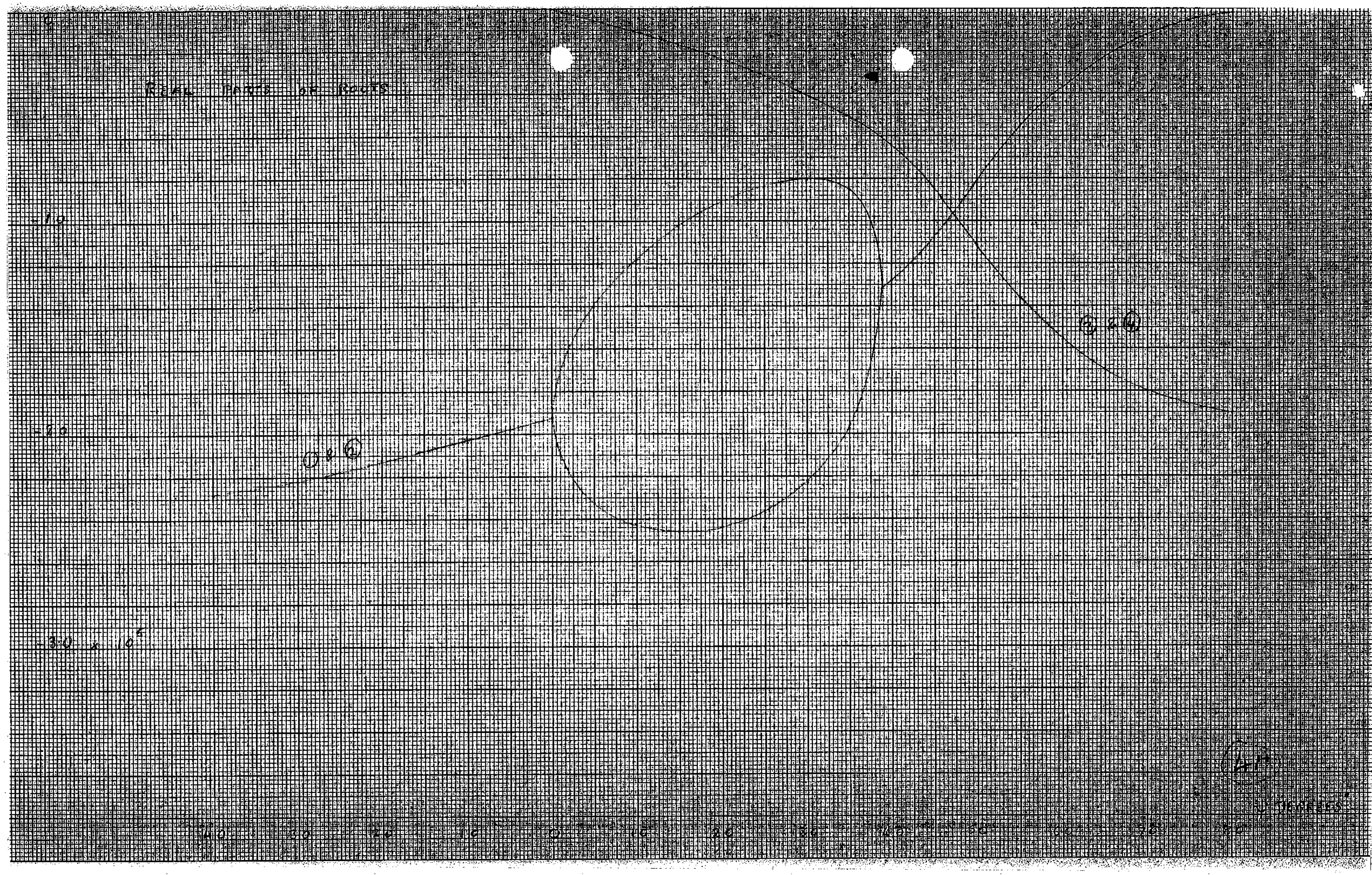
0
-1.0
-2.0
-3.0 x 10^5 REAL PART OF ROOTS
(1) & (2)
(3) & (4)
(2A)
θ DEGREES
4 8 12 16 20 24 28 32 36 40 44 48 52 56 60

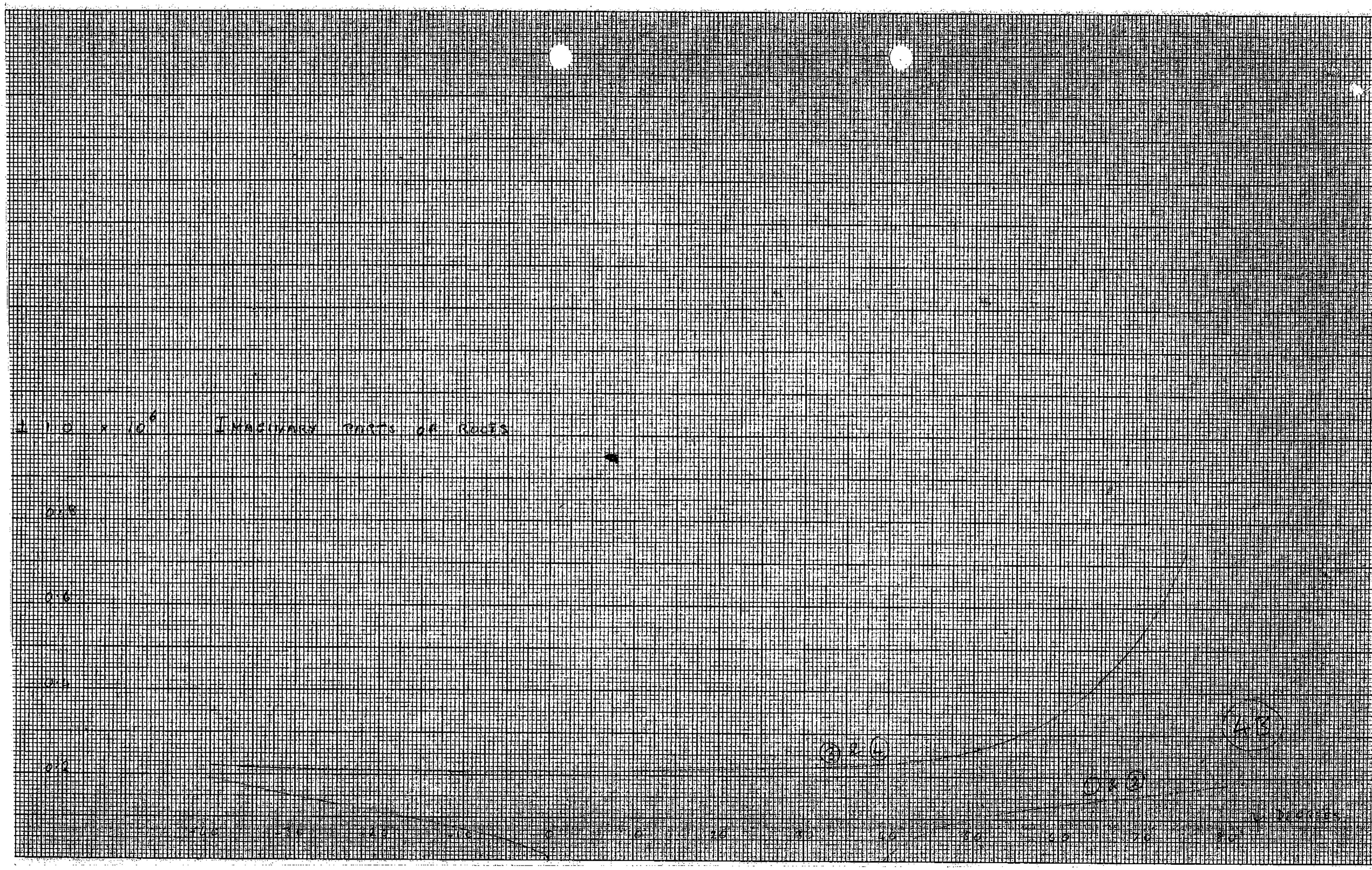

1.8 × 10^6 IMAGINARY PARTS OF ROOTS
2.0
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
1 & 2
3 & 4
2B
φ DEGREES
4
8
12
16
20
24
28
32
36
40
44
48
52
56
60



REAL PART OF ROOTS
3 & 4
1 & 2
0
V KNOTS


1·6 × 10⁶ Imaginary parts of roots
1·4
1·2
1·0
0·8
0·6
0·4
0·2
③ & ④
① & ②


REAL PARTS OF ROOTS
-1.0
-2.0
-3.0 x 10^6
1 & 2
3 & 4
DEGREES
14


1.0 × 10^6
IMAGINARY PARTS OF ROOTS
0.8
0.6
0.4
0.2
③ & ④
① & ②
4B
Degrees