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Smoothing Circuit for DC Plate Supply to R. F. Amplifier

By

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1. Introduction

In NINA, the RF power which has to be supplied to the accelerating structure and to the beam rises during the acceleration period to compensate for the increasing radiation by the electrons. The RF power is switched off during the decreasing half of the magnetic field cycle.

Using the latest available figures for power amplifier efficiency, and an estimate of the most stringent requirement during extraction, the DC power input to the RCA 2054 tube plate will be as shown in fig 1.

The DC component of plate current is obtained from this by dividing by the voltage, which can be assumed to droop during the acceleration period from 12.5 kv to 11 kv. This is shown in Fig. 2.

A good approximation to the current curve is a sawtooth superimposed on a square wave, and the idealised waveform also shown in Fig. 2. has been used in the calculations. This is pessimistic over most of the period as it assumes a rather lower efficiency than used in Fig. 1. The "on" and "off" periods have however been made equal to simplify the calculations. An open circuit voltage of 14 kv was used in all the calculations, which is about 10% high.

The problem then is to choose a smoothing circuit which will limit the voltage fluctuations at the output to a level satisfactory for the amplifier, and limit the fluctuation of current drawn from the power supply to a level satisfactory to the rectifiers and the electricity mains supply. In principle the parameters of the rectifier set should be known, but we shall lump these in the form of series resistance and inductance into the external circuit, which with large external smoothing components, should be sufficiently accurate.

2. Simple case without inductance: R - C smoothing.

The circuit used is shown in Fig 3a. V is the unknown load voltage. This gives the following equation:

$$\left.\begin{aligned} V &= V_B - I_B R \\ I_B &= I_C + I_L \\ I_C &= C\frac{dV}{dt} \end{aligned}\right\} \qquad (1)$$

whence

$$V + RC\frac{dV}{dt} - V_B = -RI_L \qquad (2)$$

I_L is given by : $I_L = i_1 + \frac{i_2}{T}t$ from $t = 0$ to $t = T$

$$I_L = 0 \quad \text{from } t = T \text{ to } t = 2T \qquad (3)$$

Boundary conditions are: V is continuous at $t = T$

$$V_{t=0} = V_{t=2T} \qquad (4)$$

Solving these equations, we find V is given by:

a) $t = 0$ to $t = T$

$$V = V_B - Ri_2\frac{t}{T} + Ae^{-\frac{t}{RC}} - Ri_1 + R^2C\frac{i_2}{T} \qquad (5)$$

and b) $t = T$ to $t = 2T$

$$V = A'e^{-\frac{t}{RC}} + V_B \qquad (6)$$

$$\text{where } A' = \frac{Ri_1 + Ri_2 - R^2C\frac{i_2}{T} - Ri_1 e^{-\frac{T}{RC}} + R^2C\frac{i_2}{T}e^{-\frac{T}{RC}}}{e^{-\frac{T}{RC}}\left(e^{-\frac{2T}{RC}} - 1\right)} \qquad (7)$$

and $A = A' + e^{\frac{T}{RC}}\left(Ri_1 + Ri_2 - R^2C\frac{i_2}{T}\right)$

equations (5) (6) and (7) can be used to plot V against t.
The voltage drop ΔV from time $t = 0$ to $t = T$ can be calculated directly from

$$\Delta V = \left[\left(1 - e^{-\frac{T}{RC}}\right)\left(Ri_1 - R^2C\frac{i_2}{T}\right) + Ri_2\right]\left(1 + e^{-\frac{T}{RC}}\right)^{-1} \qquad (8)$$

The rectifier current is given by

$$I_B = \frac{V_B - V}{R} \qquad (9)$$

The maximum value, $\hat{I}_B$ occurs at $t = T$ and is given by

$$\hat{I}_B = \frac{\Delta V}{R\left(1 - e^{-\frac{T}{RC}}\right)} \qquad (10)$$

3. Numerical Results for R-C Smoothing.

a) Fig 4 shows $\frac{\Delta V}{V_B}$ and Fig 5 shows $\frac{\hat{I}_B}{I_{B\,mean}}$ as functions of C

for various values of R.

$T = 0.01$ secs, $i_1 = 39$ amps $i_2 = 73$ amps. $V_B = 14$ KV.

b) Curves of $V_B - V$ are given for two particular cases in Fig 6.

(i) $R = 10$ ohms $C = 200\,\mu f$.

(ii) $R = 20$ ohms $C = 300\,\mu f$.

The other parameters are as in (a) above

N.B. The RMS value of the current is 55.5 amps, so the power dissipated in a resistor is 3.1 KW per ohm.

The rectifier current is given by (9) and is seen to be the same shape as $(V_B - V)$.

c) Summary: R-C smoothing (neglecting the problem of limiting surges at switch-on) will give adequately low values of ΔV, but leads to either large variations in I_B, for low values of R, or a large waste of power for large R, with greater mean voltage drop. In any case, any practical power unit will have sufficient inherent inductance for these calculations to be invalid.

4. Extension to include a smoothing choke or inductive source.

The circuit is shown in Fig 3(b). B now has an inductance L in series with it, so the equations (1) become

$$\left.\begin{aligned} V &= V_B - I_B R - L\frac{dI_B}{dt} \\ I_B &= I_C + I_L \\ I_C &= C\frac{dV}{dt} \end{aligned}\right\} \qquad (11)$$

whence $L C \ddot{V} + R C \dot{V} + V = V_B - R I_L - L \dot{I}_L$ (12)

and as before, $I_L = i_1 + \frac{i_2}{T} t \quad t = 0 \text{ to } t = T$
$I_L = 0 \quad t = T \text{ to } t = 2T$ (13)

Solving (12) for V as before, we find:

<u>For t = 0 to t = T</u>

$$V = A_1 e^{-\alpha t} \cos(\omega t + \phi_1) - K_1 t + D \quad (14)$$

<u>For t = T to t = 2T</u>

$$V = A_2 e^{-\alpha t} \cos(\omega t + \phi_2) + V_B \quad (15)$$

where $\alpha = \frac{R}{2L} \qquad \omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2} \qquad K_1 = \frac{R i_2}{T}$

$$D = V_B - R i_1 - L \frac{i_2}{T} + R^2 C \frac{i_2}{T}$$

A_1 A_2 ϕ_1 ϕ_2 are to be found from the boundary conditions, which are that both V and I_B must be continuous at t = 0 and t = T.

I_B is given by

<u>t = 0 to t = T</u> $\quad I_B = i_1 + \frac{i_2}{T} t - C \Big[A_1 \alpha e^{-\alpha t} \cos(\omega t + \phi_1) + A_1 \omega e^{-\alpha t} \sin(\omega t + \phi_1) + K_1 \Big]$ (16)

<u>t = T to t = 2T</u> $\quad I_B = - C \Big[A_2 \alpha e^{-\alpha t} \cos(\omega t + \phi_2) + A_2 \omega e^{-\alpha t} \sin(\omega t + \phi_2) \Big]$ (17)

The boundary conditions lead to:

$$A_1 e^{-\alpha T} \cos(\omega T + \phi_1) - K_1 T + D = A_2 e^{-\alpha T} \cos(\omega T + \phi_2) + V_B$$

$A_1 e^{-\alpha T} \big[\alpha \cos(\omega T + \phi_1) + \omega \sin(\omega T + \phi_1) \big] + K_1 - \frac{i_1 + i_2}{C} = A_2 e^{-\alpha T} \big[\alpha \cos(\omega T + \phi_2) + \omega \sin(\omega T + \phi_2) \big]$

$A_1 \cos \phi_1 + D = A_2 e^{-2\alpha T} \cos(2\omega T + \phi_2) + V_B$

$A_1 \big[\alpha \cos \phi_1 + \omega \sin \phi_1 \big] + K_1 - \frac{i_1}{C} = A_2 e^{-2\alpha T} \big[\alpha \cos(2\omega T + \phi_2) + \omega \sin(2\omega T + \phi_2) \big]$

Simplifying equations (18) for the constants, we get

$$A_1 e^{-\alpha T}\cos(\omega T + \phi_1) - A_2 e^{-\alpha T}\cos(\omega T + \phi_2) + D - V_B - K_1 T = 0$$

$$A_1 e^{-\alpha T} c \cos(\omega T + \phi_1 + \mathcal{E}) - A_2 e^{-\alpha T} c \cos(\omega T + \phi_2 + \mathcal{E}) + K_1 - \frac{1}{C}(i_1 + i_2) = 0$$

$$A_1 \cos\phi_1 - A_2 e^{-2\alpha T}\cos(2\omega T + \phi_2) + D - V_B = 0$$

$$A_1 c \cos(\phi_1 + \mathcal{E}) - A_2 e^{-2\alpha T} c \cos(2\omega T + \phi_2 + \mathcal{E}) + K_1 - \frac{i_1}{C} = 0$$

where $\cos\mathcal{E} = \frac{\alpha}{c}$ $\quad \sin\mathcal{E} = -\frac{\omega}{c}$ $\quad c^2 = \frac{1}{LC}$

Solution of the equations:

write them as

$$\begin{aligned}
-ap-bq+ax+by &= K\\
cp-dq-cx+dy &= K\\
p+ex-fy &= K\\
gp+hq+jx+ly &= K
\end{aligned}$$

where

$$\begin{aligned}
p &= A_1 \cos\phi_1\\
q &= A_1 \sin\phi_1\\
x &= A_2 \cos\phi_2\\
y &= A_2 \sin\phi_2\\
a &= -k_1\, a \cos\omega T\\
b &= k_1\, a \sin\omega T\\
c &= k_2\, a\, c \cos(\omega T + \mathcal{E})\\
d &= k_2\, c \sin(\omega T + \mathcal{E})\\
e &= -a^2 \cos(2\omega T)\\
f &= -a^2 \sin(2\omega T)\\
g &= k_3\, c \cos\mathcal{E}\\
h &= -k_3\, c \sin\mathcal{E}\\
j &= -k_3\, a^2 c \cos(2\omega T + \mathcal{E})\\
l &= k_3\, a^2 c \sin(2\omega T + \mathcal{E})
\end{aligned}$$

$$k_1 = \frac{V_B - D}{V_B - D + K_1 T} \qquad k_2 = \frac{(V_B - D)\,C}{i_1 + i_2 - K_1 C} \qquad k_3 = \frac{(V_B - D)\,C}{i_1 - K_1 C}$$

$$a = e^{-\alpha T} \qquad K = V_B - D = Ri_1 + L\,\frac{i_2}{T} - R^2 C\,\frac{i_2}{T}$$

whence

$$x(ah - bge + bj + aeh) + y(bh + bl + bgf - afh) = Kh + Kb - Kbg + Kah$$

$$x(-ch - dge + dj - ceh) + y(dh + dl + dgf + cfh) = Kh + Kd - Kdg - Kch$$

If the actual numerical values of a.....ℓ are computed and used in these two equations, x and y are easily found, and hence p and q. The constants A, A_2 ϕ, ϕ_2 are then known, and can be used in equations (14) - (17) to plot V and I_B

Note that to obtain results for a simple square wave for I_2, we merely put $i_2 = 0$, and i, the amplitude of the square wave.

5. Alternative Method of Analysis

a) Method An interesting and appropriate alternative method of finding the voltage waveform is given in the paper "Circuit Waveforms for Periodic Waves" by P. L. Waidelich in the IEEE Transactions on Communications and Electronics, March 1963 page 108. This paper gives in a Table the sum functions of a number of useful Fourier Series. The technique is to convert the applied waveform to a Fourier Series, either from the Table or by analysis, and to apply this Series to the circuit. The steady-state sinusoidal analysis is straight-forward. The resulting Fourier Series for the unknown voltage, of little use in itself, is converted back to a simple waveform function by use of the Table again.

b) Fourier Analysis of the Load Waveform

By the normal method, the waveform of Fig.2 is easily shown to be:

$$I_L = \frac{1}{2}\left(i_1 + \frac{1}{2} i_2\right) - \frac{2i_2}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2n-1)\omega t}{(2n-1)^2} + \frac{2i_1 + i_2}{\pi} \sum_{n=1}^{\infty} \frac{\sin(2n-1)\omega t}{2n-1} - \frac{i_2}{\pi} \sum_{n=1}^{\infty} \frac{\sin 2n\,\omega t}{2n} \qquad (19)$$

$$\text{where } \omega = \frac{\pi}{T}$$

c) Solution for V

As before, the equation to be solved is

$$LC\ddot{V} + RC\dot{V} + V = V_E - RI_L - L\dot{I}_L$$

this time with I_L given by (19). The solution is a steady-state solution, so there are no arbitrary constants.

The solution for this waveform is rather complicated, but is quoted here for completeness:

$$= V_F - R\frac{1}{2}\left(i_1 + \frac{1}{2} i_2\right) + A_1 \sum \frac{\cos(2n-1)\omega t}{(2n-1)^2} + A_a \sum \frac{\cos(2n-1)\omega t}{(2n-1)^2 + a^2} + A_b \sum \frac{\cos(2n-1)\omega t}{(2n-1)^2 + b^2}$$

$$+ B_a \sum \frac{(-1)^n \cos n\omega t}{n^2 + a^2} + B_b \sum \frac{(-1)^n \cos n\omega t}{n^2 + b^2}$$

$$+ C_1 \sum \frac{\sin(2n-1)\omega t}{2n-1} + C_a \sum \frac{(2n-1)\sin(2n-1)\omega t}{(2n-1)^2 + a^2} + C_b \sum \frac{(2n-1)\sin(2n-1)\omega t}{(2n-1)^2 + b^2}$$

$$+ D_1 \sum \frac{(-1)^n \sin n\omega t}{n} + D_a \sum \frac{(-1)^n n \sin n\omega t}{n^2 + a^2} + D_b \sum \frac{(-1)^n n \sin n\omega t}{n^2 + b^2}$$

Where: $A_1 = \dfrac{2Ri_2}{\pi^2}$ $\quad A_a = \dfrac{k_1}{a^2(a^2 - b^2)} + \dfrac{k_{2a}(a^2 - c^2)}{(a^2 - b^2)}$. $A_b = \dfrac{k_{2a}(c^2 - b^2)}{(a^2 - b^2)} - \dfrac{k_1}{b^2(a^2 - b^2)}$

$B_a = k_3 \dfrac{(a^2 - c^2)}{(a^2 - b^2)}$ $\quad B_b = k_3 \dfrac{(c^2 - b^2)}{(a^2 - b^2)}$ $\quad C_1 = \dfrac{c^2 k_1'}{a^2 b^2} + \dfrac{k_{2a}'}{a^2 b^2}$

$C_a = \dfrac{(c^2 - a^2)k_1' + k_{2a}'}{a^2(a^2 - b^2)}$ $\quad C_b = \dfrac{(b^2 - c^2)k_1' - k_{2a}'}{b^2(a^2 - b^2)}$

$D_1 = \dfrac{k_3'}{a^2 b^2}$ $\quad D_a = \dfrac{k_3'}{a^2(a^2 - b^2)}$ $\quad D_b = \dfrac{k_3'}{b^2(a^2 - b^2)}$

and $k_1 = \dfrac{2Ri_2}{\pi^2 \omega^4 L^2 C^2}$ $\quad k_{2a} = \dfrac{2i_1}{\pi\omega C}$ $\quad k_3 = -\dfrac{i_2}{\pi\omega C}$

$k_1' = \dfrac{2i_2}{\pi^2 \omega C}$ $\quad k_{2a}' = \dfrac{-2Ri_1}{\pi\omega^4 L^2 C^2}$ $\quad k_3' = \dfrac{Ri_2}{\pi\omega^4 L^2 C^2}$

$a^2 b^2 = \dfrac{1}{\omega^4 L^2 C^2}$ $\quad a^2 + b^2 = \dfrac{R^2}{\omega^2 L^2} - \dfrac{2}{\omega^2 LC}$ $\quad c^2 = \dfrac{R^2}{\omega^2 L^2} - \dfrac{1}{\omega^2 LC}$

The series can all be converted to algebraic functions by transforms given in the Paper or elsewhere, and a simple computer programme could produce the waveforms quite quickly.

d) <u>Solution for a Square-Wave load</u>

To obtain a useful algebraic result, without putting in numerical values at an early stage, we make the simplification that the load current is a squarewave (with the same average value as before). Then $i_2 = 0$, and the solution simplifies to:

$$V = V_B - R\frac{1}{2}i + k_{2a}\frac{(a^2-c^2)}{(a^2-b^2)}\sum\frac{\cos(2n-1)\omega t}{(2n-1)^2+a^2} + k_{2a}\frac{(c^2-b^2)}{(a^2-b^2)}\sum\frac{\cos(2n-1)\omega t}{(2n-1)^2+b^2}$$

$$+\frac{k_2'a}{a^2b^2}\sum\frac{\sin(2n-1)\omega t}{2n-1} + \frac{k_2'a}{a^2(a^2-b^2)}\sum\frac{(2n-1)\sin(2n-1)\omega t}{(2n-1)^2+a^2} - \frac{k_2'a}{b^2(a^2-b^2)}\times$$

$$\sum\frac{(2n-1)\sin(2n-1)\omega t}{(2n-1)^2+b^2}$$

whence by the table, we find

A) $\underline{0 < \omega t < \pi}$

$$V = V_B - \tfrac{1}{2}Ri + k_{2a}\frac{(a^2-c^2)}{(a^2-b^2)}\frac{\pi}{4a\sinh a\pi}\Big[\cosh a(\omega t-\pi) - \cosh a\omega t\Big]$$

$$+ k_{2a}\frac{(c^2-b^2)}{(a^2-b^2)}\frac{\pi}{4b\sinh b\pi}\Big[\cosh b(\omega t-\pi) - \cosh b\omega t\Big] + \frac{k_2'a}{a^2b^2}\frac{\pi}{4}$$

$$+\frac{k_2'a}{a^2(a^2-b^2)}\frac{\pi}{4\sinh a\pi}\Big[\sinh a\omega t - \sinh a(\omega t-\pi)\Big] - \frac{k_2'a}{b^2(a^2-b^2)}\frac{\pi}{4\sinh b\pi}\times$$

$$\Big[\sinh b\omega t - \sinh b(\omega t-\pi)\Big]$$

B) $\underline{\pi < \omega t < 2\pi}$

$$V = V_B - \frac{1}{2}Ri - k_{2a}\frac{(a^2-c^2)}{(a^2-b^2)}\frac{\pi}{4a\sinh a\pi}\Big[\cosh a(\omega t-2\pi) - \cosh a(\omega t-\pi)\Big]$$

$$- k_{2a}\frac{(c^2-b^2)}{(a^2-b^2)}\frac{\pi}{4b\sinh b\pi}\Big[\cosh b(\omega t-2\pi) - \cosh b(\omega t-\pi)\Big] - \frac{k_2'a}{a^2b^2}\frac{\pi}{4}$$

$$-\frac{k_{2a}'}{a^2(a^2-b^2)}\frac{\pi}{4\sinh a\pi}\Big[\sinh a(\omega t-\pi) - \sinh a(\omega t-2\pi)\Big] + \frac{k_2'a}{b^2(a^2-b^2)}\frac{\pi}{4\sinh b\pi}\times$$

$$\Big[\sinh b(\omega t-\pi) - \sinh b(\omega t-2\pi)\Big]$$

However, they do yield a simple and useful equation for ΔV,

i.e. $V(t=0) - V(t=T)$

$$\frac{\Delta V}{i} = \frac{\left(\frac{2}{C} - \frac{R^2}{L}\right)}{\left(\frac{4}{LC} - \frac{R^2}{L^2}\right)^{\frac{1}{2}}} \left(\frac{\sin \pi y}{\cosh \pi x + \cos \pi y}\right) + R \frac{\sinh \pi x}{(\cosh \pi x + \cos \pi y)} \tag{20}$$

where $\pi x = \frac{RT}{2L}$ and $\pi^2 y^2 = \frac{T^2}{LC} - \frac{R^2 T^2}{4 L^2}$

6. Infinite Inductance

The value of the voltage droop for infinite inductance (i.e. constant I_B) is given by

$$\Delta V = \frac{I_{av} T}{C}$$

This is plotted in Fig 7. for I_{av} = 38 amps and T = .01 sec.

(note I_{av} is the average over time 2T.)

This applies to any waveform of period 2T and unity "mark-to-space" ratio.

7. Results.

We use T = .01 sec. i_1 = 39 amps i_2 = 73 amps.

or for a square waveform i = 76 amps.

V_B = 14 KV throughout.

Figs 8 to 13 show the voltage and rectifier current waveforms for various R,L,C values. Most are given for a square waveform for easier computation and comparison with the Fourier method. The important features are tabulated in the Table below.

$\frac{I_B \max}{I_B \min}$	Fig.	R. ohm.	L henry	C μf	Waveform	$\frac{V_o - V_T}{V_B}$ %	$\frac{V_{max} - V_{min}}{V_B}$ %	$\frac{V_o - V_T}{V_B}$ (Fourier) %	$\frac{V_{max} - V_{min}}{V_B}$ (L=0) %
∞	8	10	.03	200	Square	5.85	12.3	5.86	13.6
∞	9	10	.1	200	Square		21.7	21.75	13.6
10.8	10	10	.25	100	Square		41.6	41.6	27.2
1.39	11	10	.75	120	Square		25.0	25.1	23.0
1.41	12	10	.75	120	Correct		25.0	-	23.0
1.17	13	[illegible]	.75	250	Correct		[illegible]	-	[illegible]

8. <u>Conclusions</u>

In order to prevent large voltage fluctuations (i.e.ringing) it is necessary to avoid having the filter resonant frequency coinciding with the 50 c/s operating frequency. If a higher resonant frequency is used, we find very large current variations, with the current cutting off for part of each cycle. This would cause severe overloading of certain rectifiers in the power supply. There is then no alternative, (except possibly a more complex circuit such as having a capacitor in parallel with the inductor), to providing a large inductor and a large capacitor. It is shown in the table above that an inductor of .75 henry is almost equivalent to one of infinite inductance for this purpose. The capacitor can then be chosen simply according to the voltage ratio tolerable. 250 μf appears to be about the smallest possible.

With this arrangement, the voltage variation is $\pm$ 6%, and the rectifier current fluctuation $\pm$ 8%.

The choice of circuit also has repercussions on the crow-bar system. A large inductance means that the rectifier set will hardly "see" the short-circuit before the AC contactor is operated. The stored energy to be removed from the capacitor will be high however, for instance with 250 μf at 13 KV it will be 21 KJ.

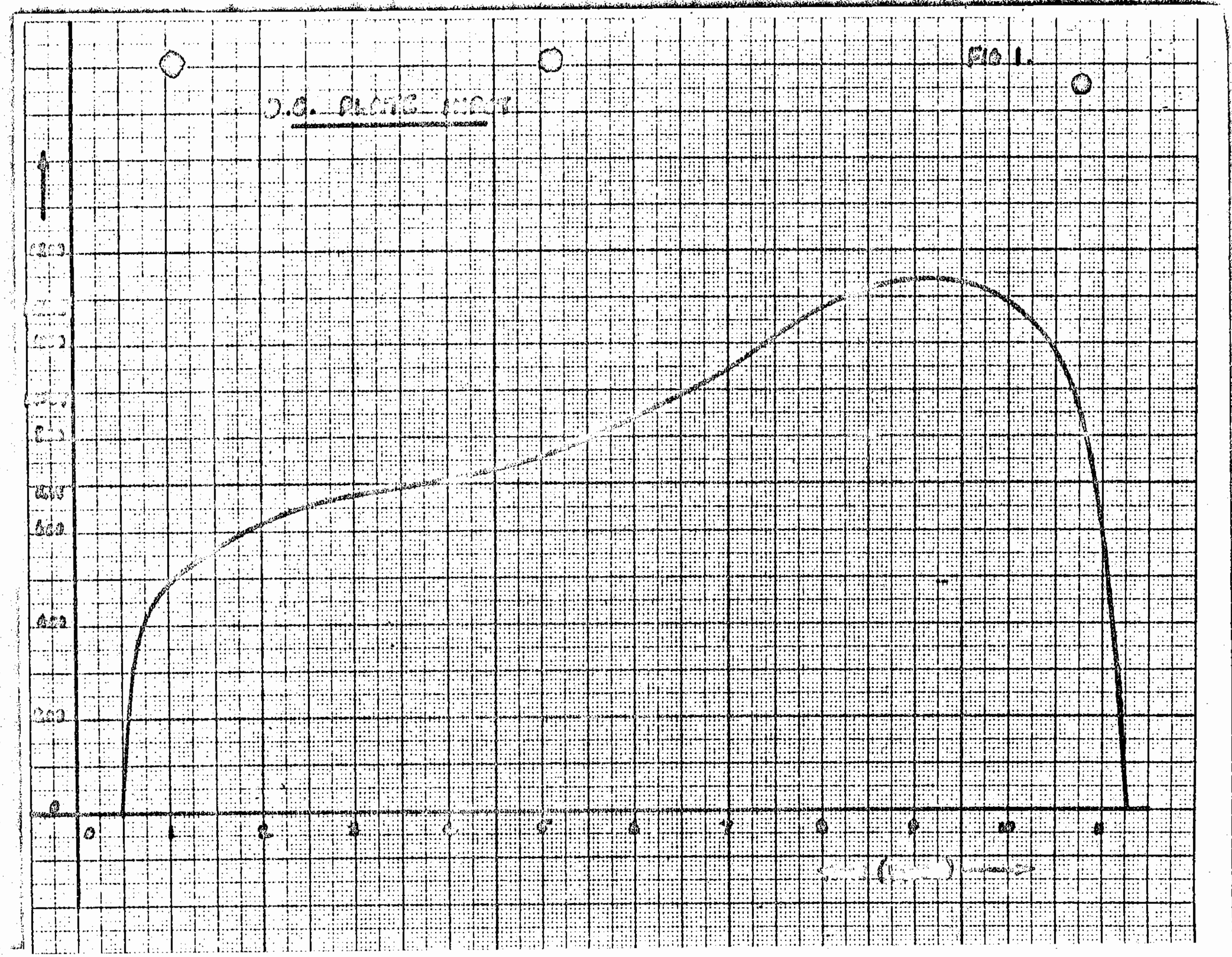


FIG 1.
0
200
400
600
1200
0
1
2
10

FIG. 2.

D.C. PLANT CURRENT.

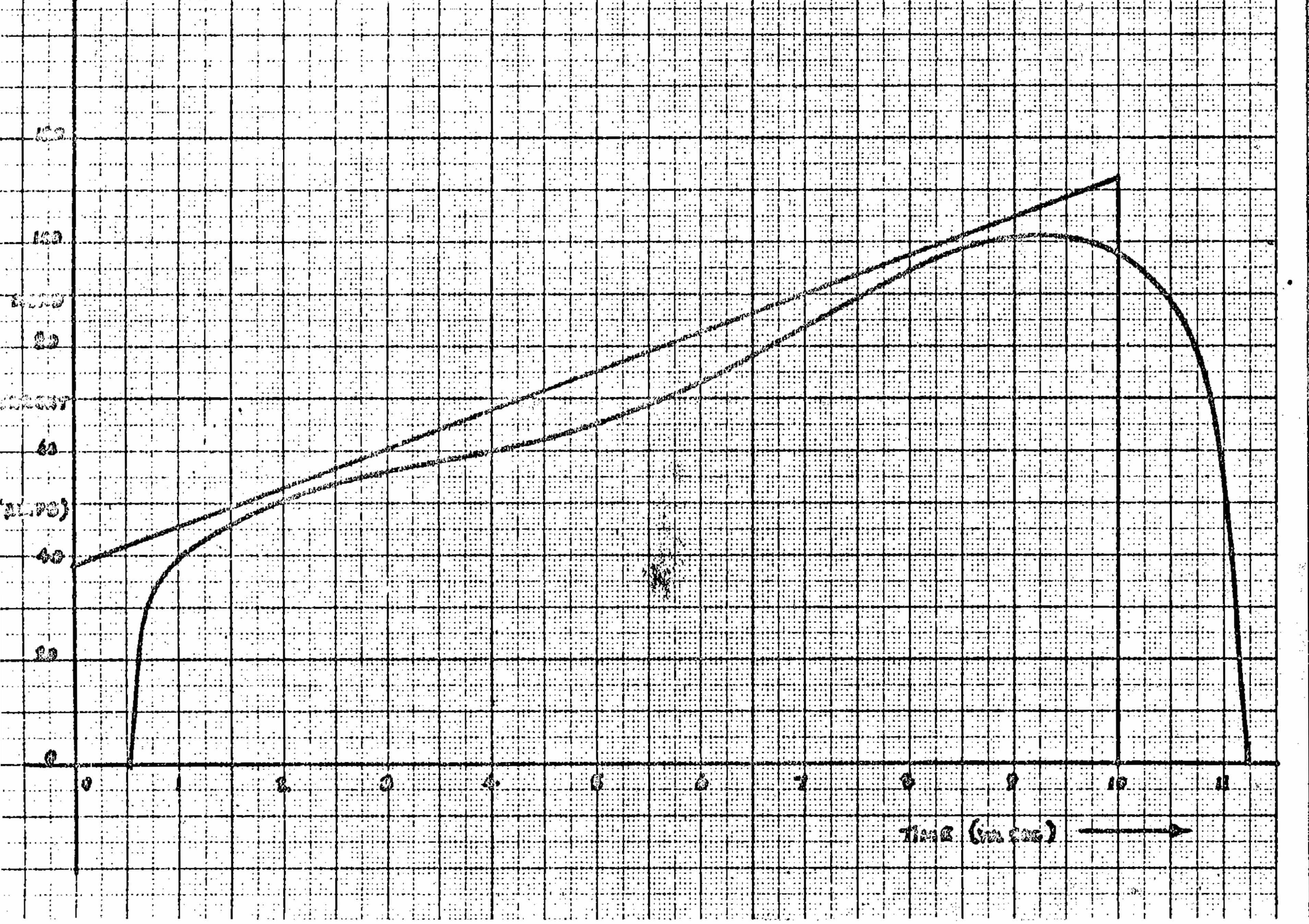
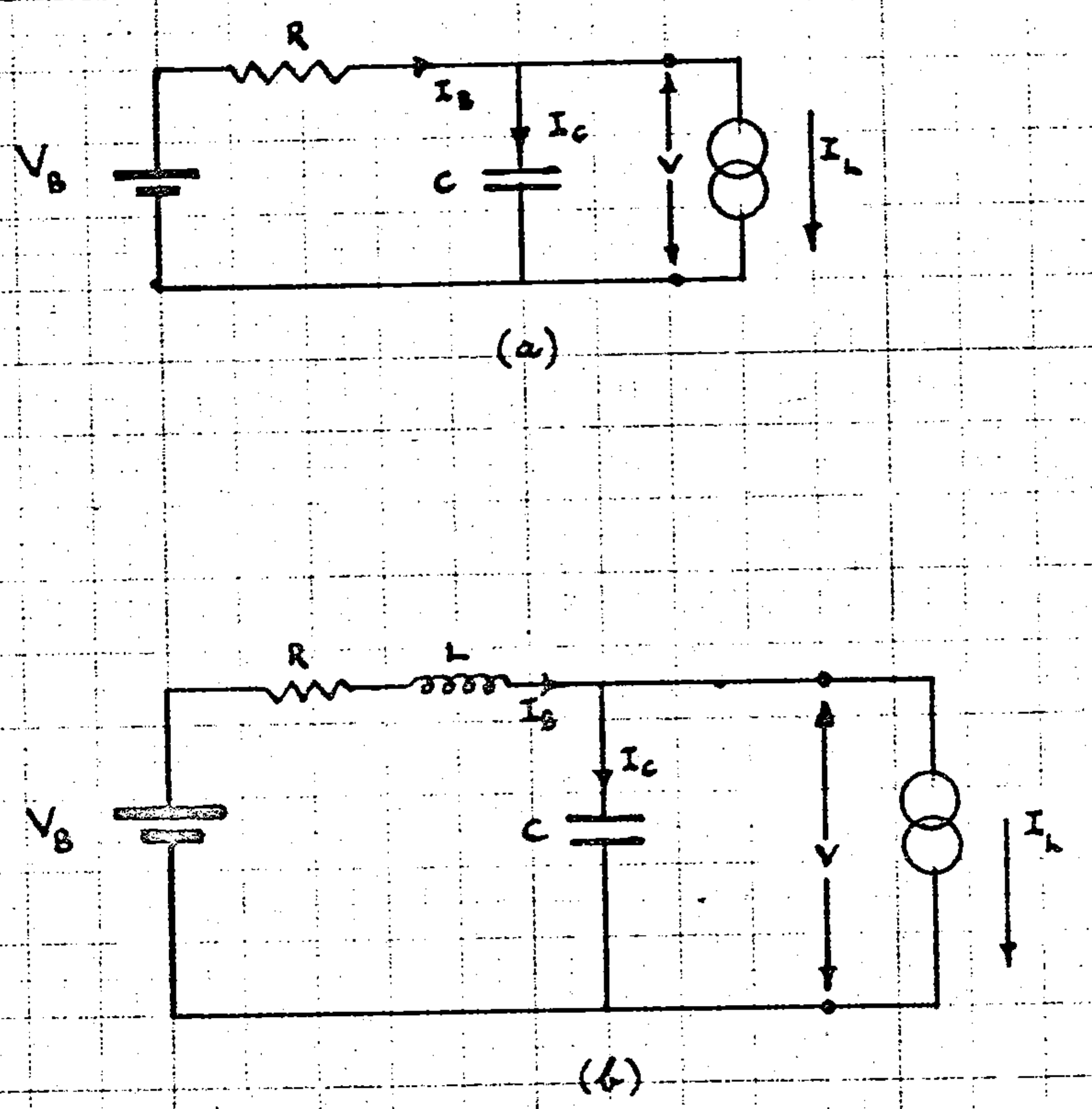


FIG 3.



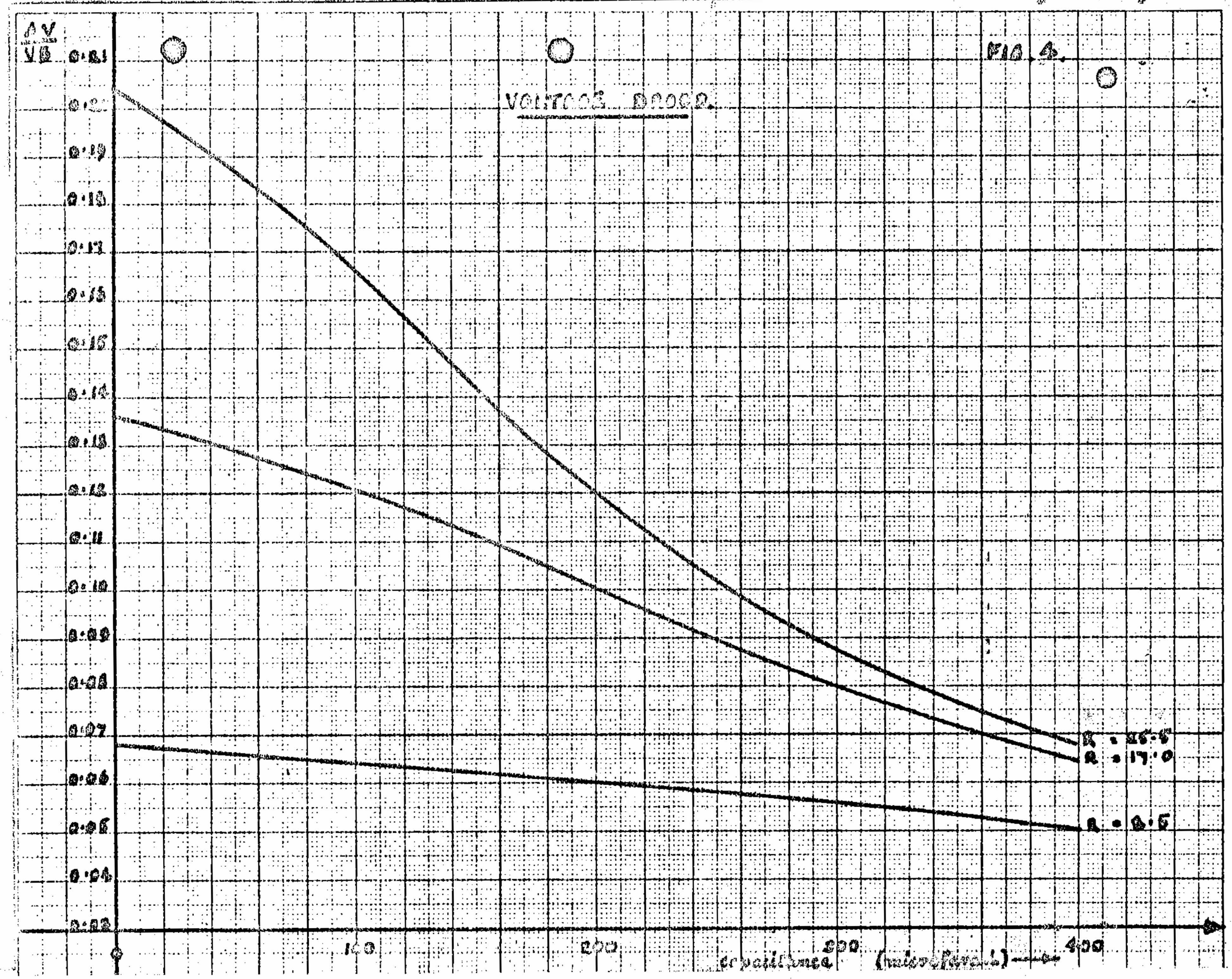
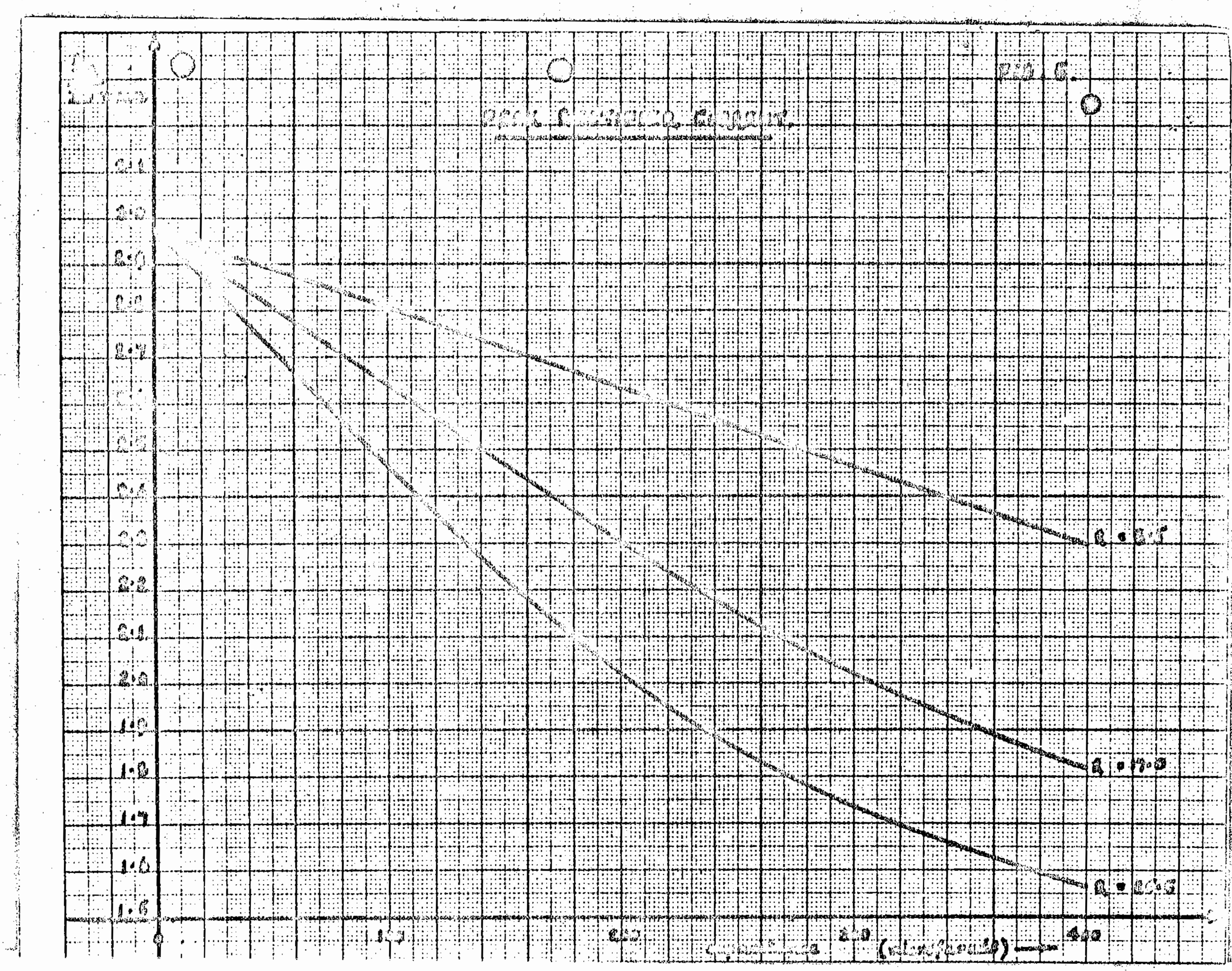

VOLTAGE DROOP.
ΔV / VB
0·21
0·20
0·19
0·18
0·17
0·16
0·15
0·14
0·13
0·12
0·11
0·10
0·09
0·08
0·07
0·06
0·05
0·04
100
200
300
400
capacitance (microfarads)
R = 17·0
R = 8·5

FIG. 4.



1·8
1·7
1·6
400

FIG. 6.

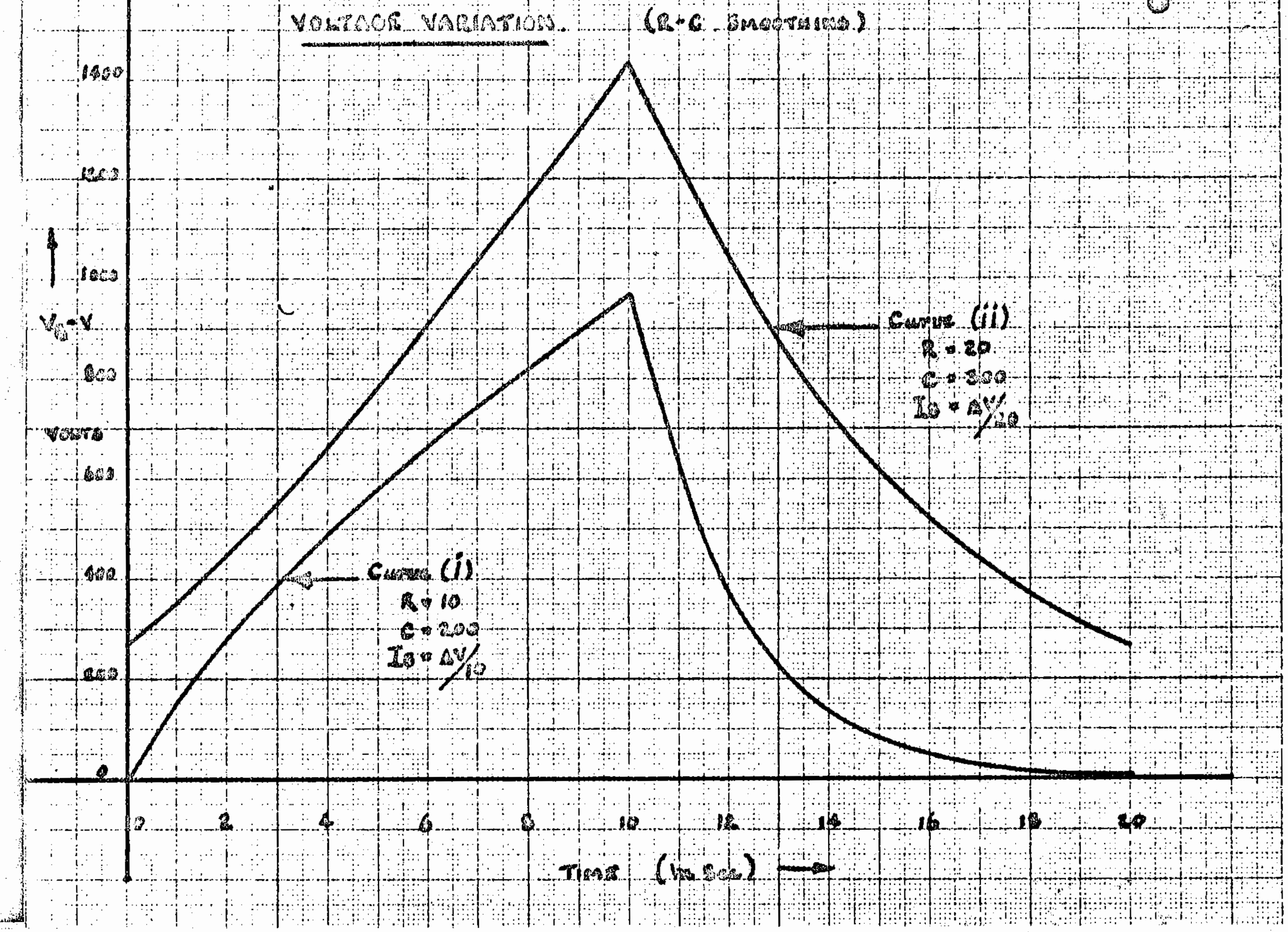
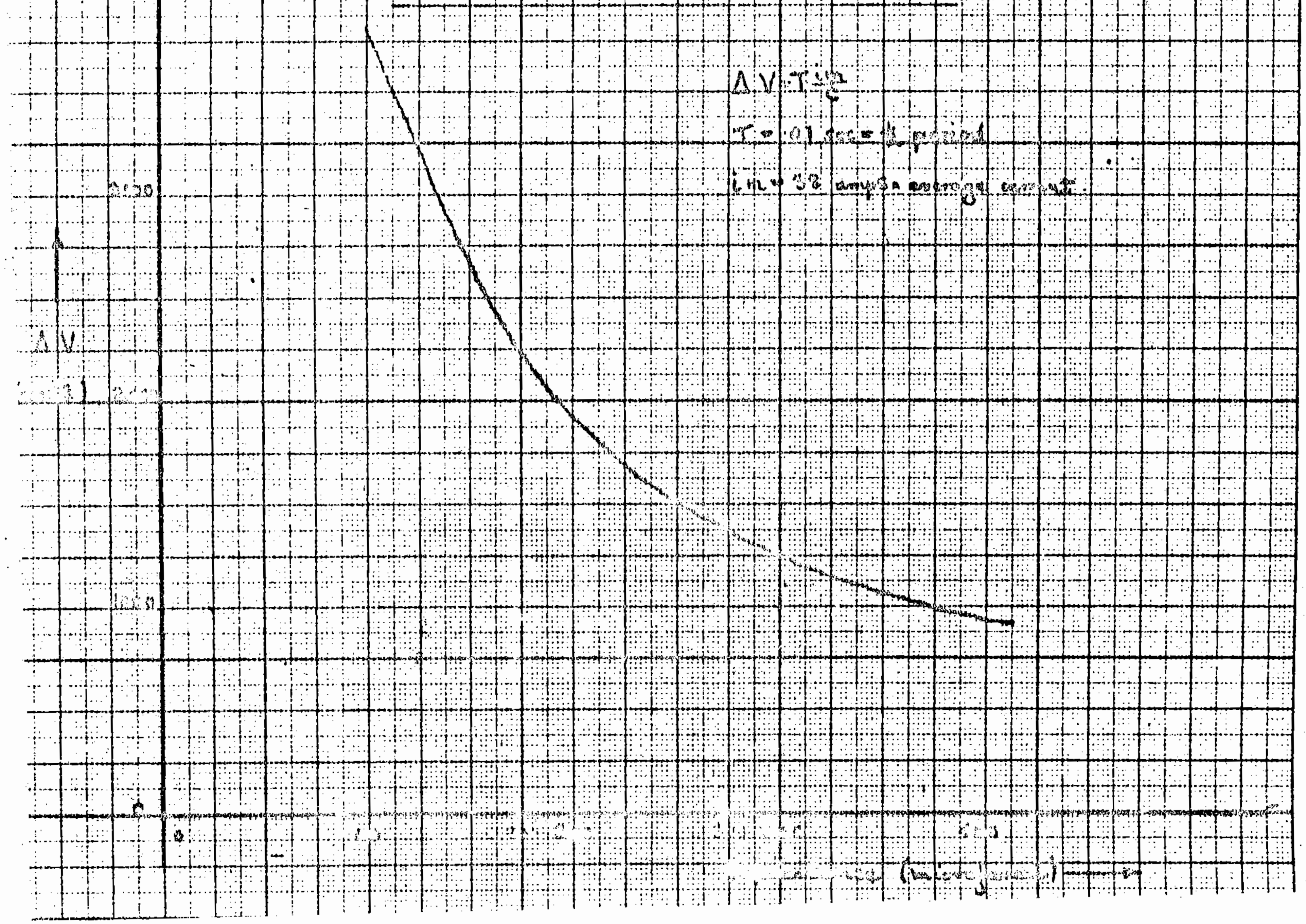
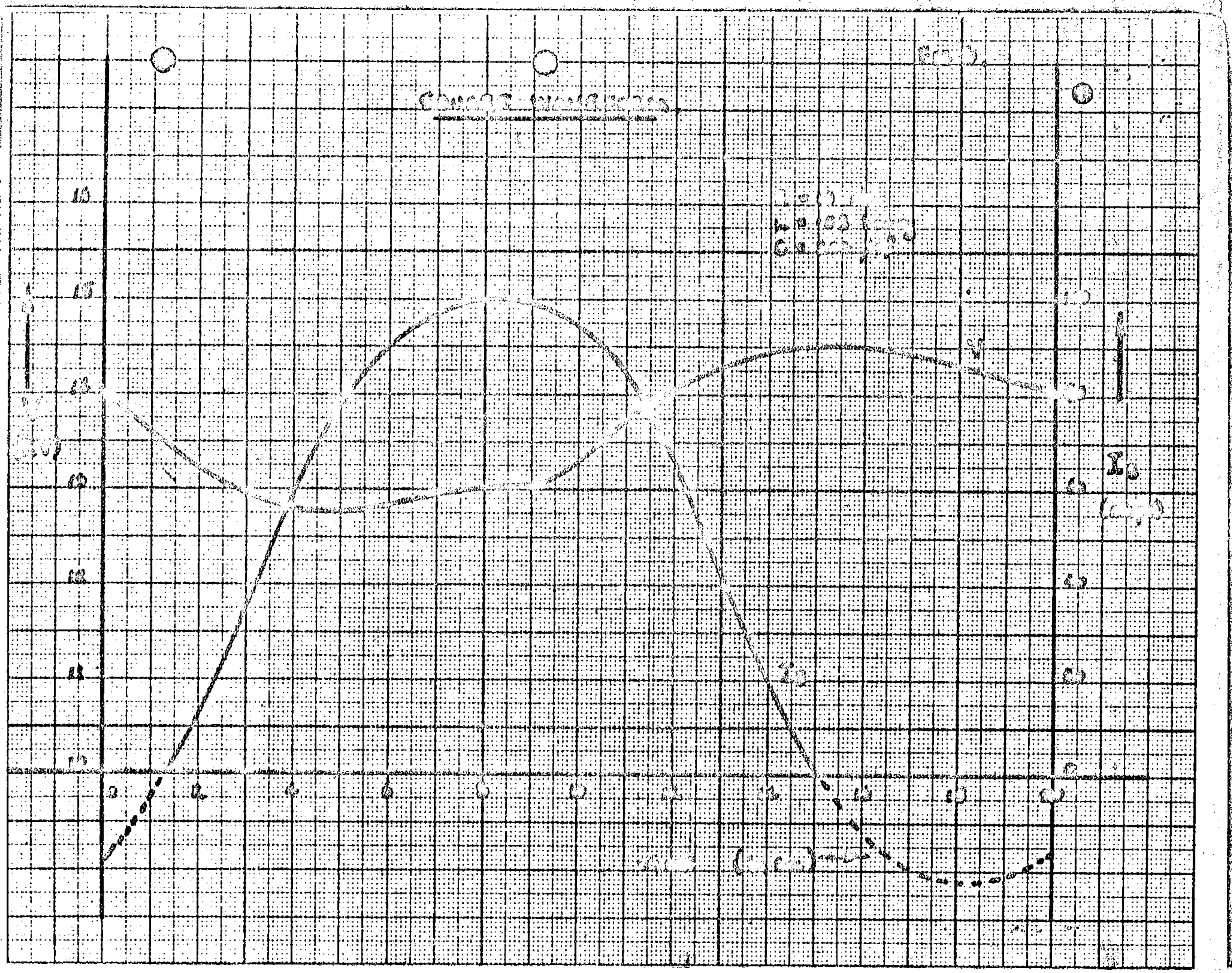

VOLTAGE VARIATION. (R-C SMOOTHING)
1400
1200
1000
800
600
400
200
0
VOLTS
$V_G - V$
0
2
4
6
8
10
12
14
16
18
20
TIME (in sec.)
Curve (i)
R = 10
C = 200
$I_B = \Delta V/10$
Curve (ii)
R = 20
C = 300
$I_B = \Delta V/20$

FIG 7

VOLTAGE VARIATION FOR INFINITE INDUCTANCE





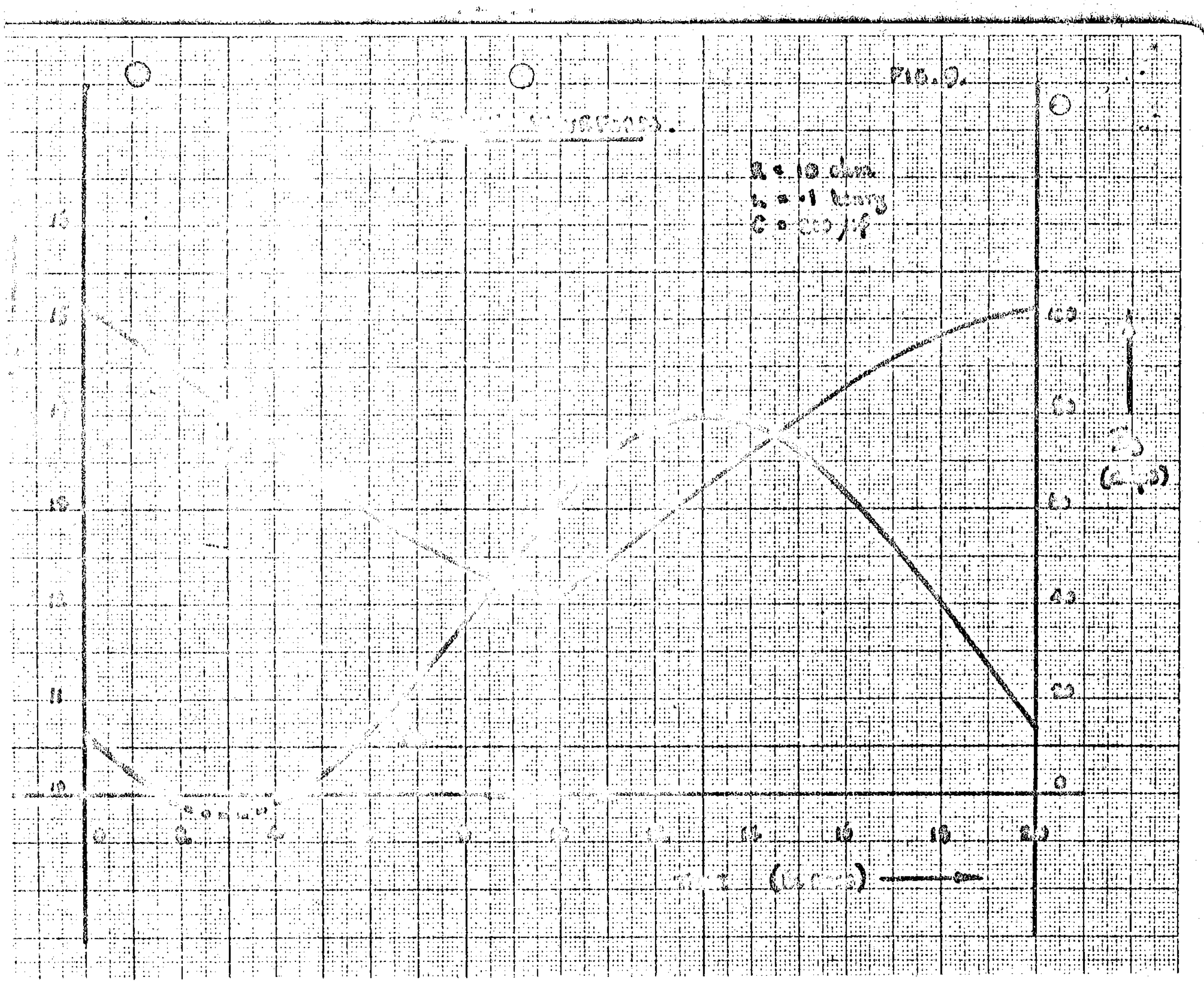
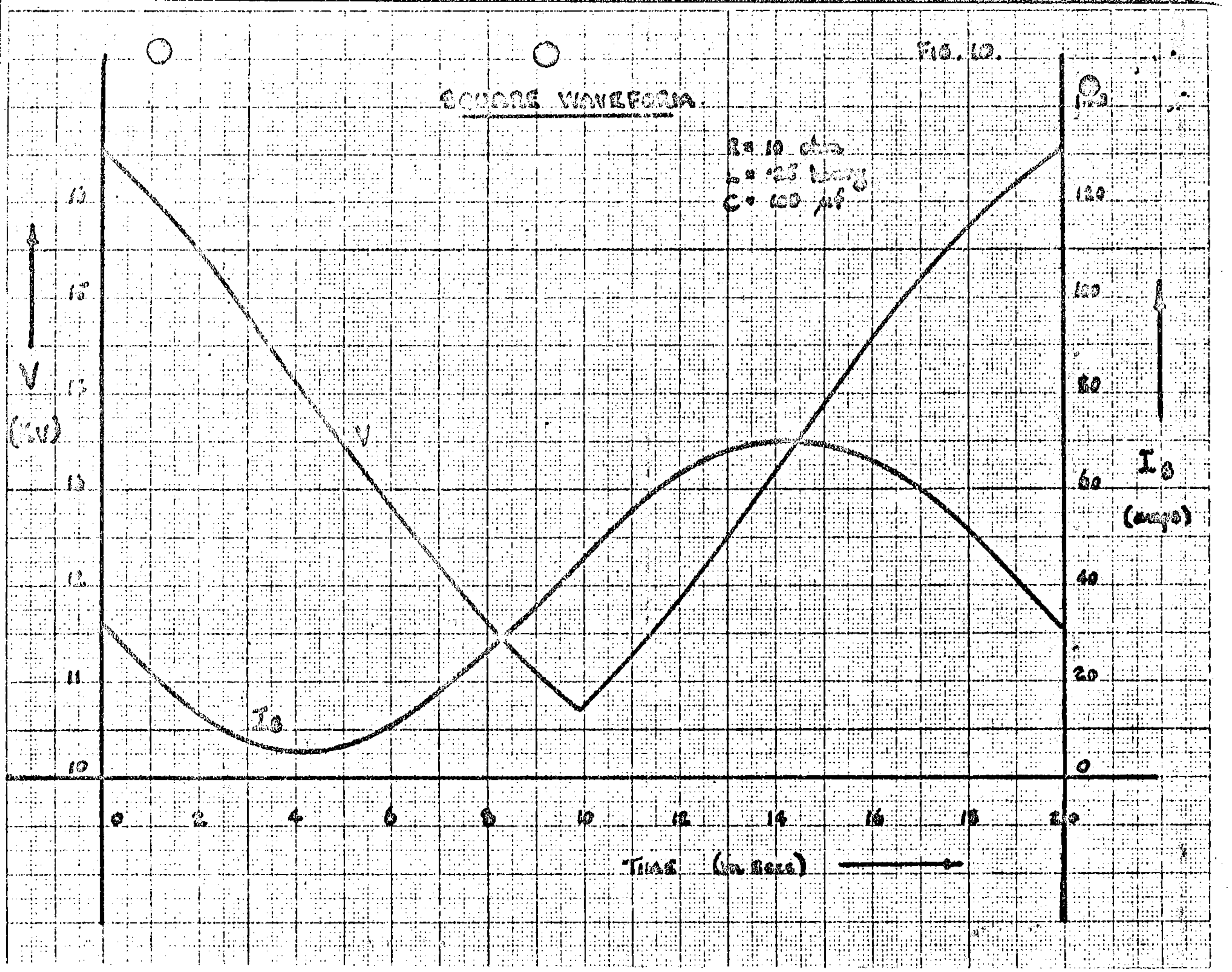


FIG. 9.


SQUARE WAVEFORM
R = 10 ohm
L = ·25 Henry
C = 100 μf
V
(kV)
10
11
12
13
14
15
16
I_a
(amps)
0
20
40
60
80
100
120
140
0
2
4
6
8
10
12
14
16
18
20
TIME (m secs)
V
I_a
Fig. 10.

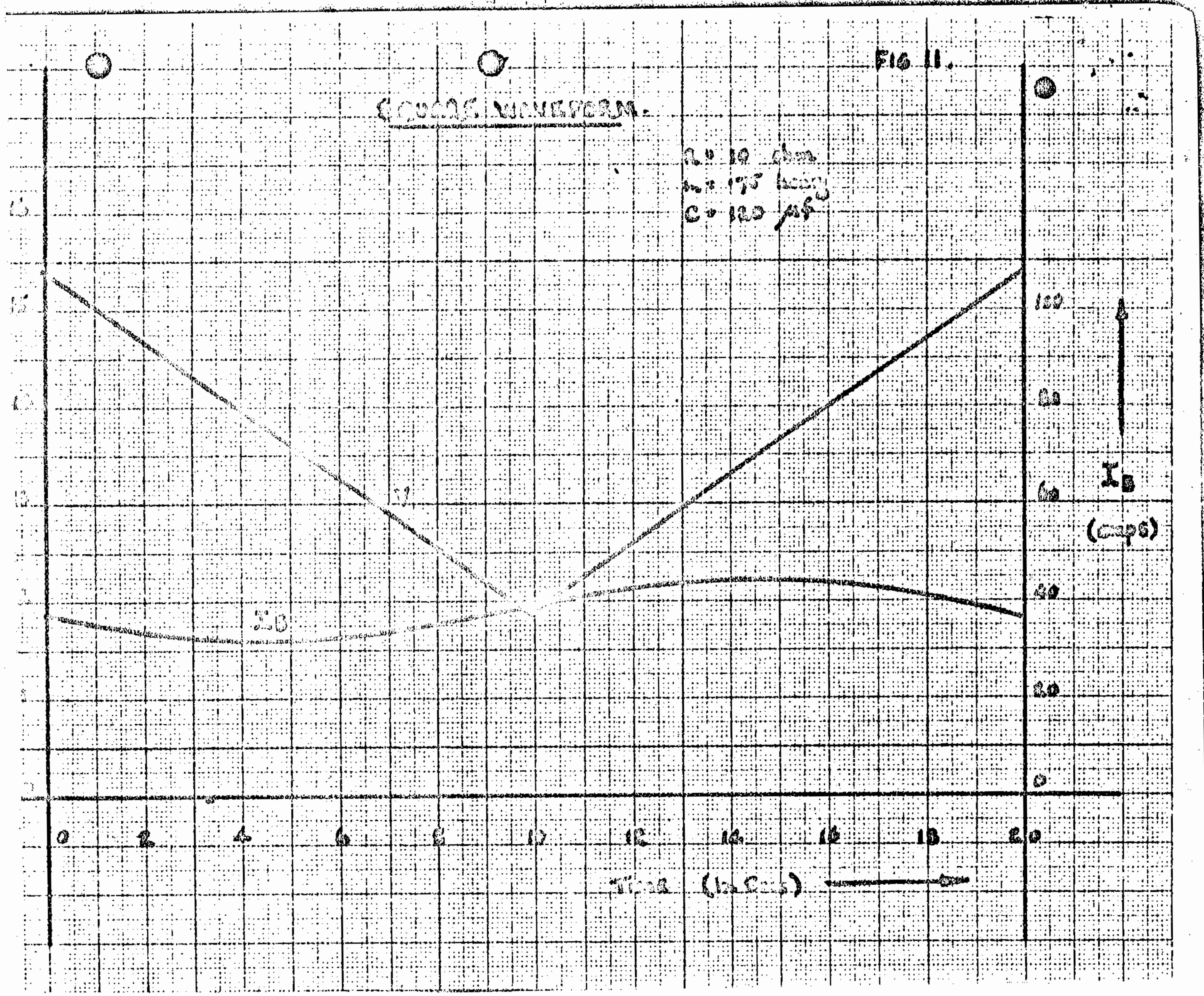


FIG 11.
C = 120 μf
V
I B
0
2
4
6
8
10
12
14
16
18
20
0
20
40
60
100
I_B
(amps)