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Dynamic Response Factor Estimation: a Point Algebraic Method

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DYNAMIC RESPONSE FACTOR ESTIMATION: A POINT ALGEBRAIC METHOD

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ABSTRACT

Thermal response factors offer an accurate way to characterise the performance of building components. Although the existing techniques use time series methods they also require special environmental apparatus and boundary conditions in order to achieve consistent results. In this work a novel method for estimating the response of building components in unsteady boundary conditions is developed and then compared against the standard U-value method. The novel method uses a point algebraic technique to extract response factor values directly from the data. The response values being estimated from time series statistical central moments. The response factors represent the dynamic thermal transmission and the area under the response factor is the steady state U-value.

The U-value is determined and then compared to the theoretically predicted value. Linear vector and non-linear thermal systems are considered.

Keywords. Thermal response.

INTRODUCTION

Mitalas and Stephenson (Mitalas and Stephenson 1967 and 1968) pioneered the use of the response factor approach to the thermal characterisation of rooms. Their method is used in heating and cooling load calculations for building design (ASHRAE Handbook of Fundamentals 1989); however, it has proved difficult to extract consistent response factors from experimental data. In order to assess the impact of a particular construction on the building energy loads the relationship between the thermal processes and the thermophysical properties must be understood. Generally speaking the ability of current data analysis methods to characterise accurately the thermal properties of a building under actual meteorological conditions is severely limited. There is thus a need to develop and refine data analysis techniques which can quantify the thermophysical processes and their interactions. In this work a technique based on time series methods is developed that can estimate the response factors from experimental data. The new method can be used when general meteorological boundary conditions prevail. The data need only be collected for a duration equivalent to several time constants of the thermal process. The new method can be extended to nonlinear processes, such as are found at the solid fluid interface; whereas the current method only works when the process is linear.

RESPONSE FACTOR ESTIMATION

The heat flux flowing through a building component may be expressed in terms of the components response factors and the temperature gradient that is driving the flux. An ordered sequence of data in time is called a time series and the relationship between two time series may be characterised in terms of response factors. The heat flux may be expressed as a convolution between the observed temperature gradient and the thermal response factor, assuming that the thermophysical properties are linear, time invariant and that the heat flux f(t) and local temperature gradient $\nabla T(t)$ constitute a complete description of the process. Mathematically these conditions may be expressed as (Joseph and Preziosi 1989)

$$f(t) = \int_{t-\mu}^{t} h_{f\nabla T} (\tau)\nabla T(t-\tau)d\tau \tag{1}$$

which is an integral form of the equation where τ denotes lag, and where μ is the finite memory of the process; ie the Borel theorem of convolution (Luikov 1968). This is a scalar equation and assumes that the heat flux has only one variable, the temperature gradient. The response factor, $h_{f\nabla T}(\tau)$ is related to the steady state thermal conductivity k (Joseph and Preziosi 1989)

$$k = \int_{0}^{\mu} h_{f\nabla T}(\tau)d\tau \tag{2}$$

That is, the area under the response function between the local heat flux and temperature gradient is equal to the steady state thermal conductivity (Kusuda 1978) or more generally the area represents the steady state gain of the variables (Irving 1992).

Under certain circumstances the convolution equation may be simplified. If the wall is kept under steady state conditions then we obtain

$$f = -U \nabla T \tag{3}$$

where U is the U-value of the wall, which can be seen to be $U = \int h_{f\nabla T}(\tau)d\tau$. In steady state the temperature gradient may be replaced by a finite difference so that

$$f = U(T_2-T_1). \tag{4}$$

This leads to a definition (Pratt 1981) that the steady state thermal transmittance is the amount of heat which flows per unit area per unit time when a one degree steady state temperature difference is maintained across the structure. The U-value estimate offers a simple but effective measure against which to compare the integrated response factors.

The convolution equation may be Fourier transformed into the frequency domain with

$$f(\omega) = H_{f\nabla T}(\omega)\ \nabla T(\omega)$$

where $H_{f\nabla T}(\omega)$ is the frequency response between f and ∇T.

The convolution equation (1) allows the heat flux through a region of a building component to be estimated as a function of the local temperature gradient and response factors. The wall of a building experiences unsteady temperature and heat flux conditions on both sides simultaneously. This is a vector problem and the heat flux and temperatures on one side of the structure will be a function of the heat flux and temperatures on the other side of the wall. The most simple representation is in the form of a two port network equation which relates the heat flux and temperature fields as an input output convolution. If we denote $T_e(t)$ and $f_e(t)$ as the external surface temperature field and heat flux, and $T_i(t)$ and $f_i(t)$ as the internal surface temperature field and heat flux then these may be related by the vector equation

$$\begin{vmatrix} T_i(t) \\ f_i(t) \end{vmatrix} = \begin{vmatrix} h_{TeTi}(t) & h_{feTi}(t) \\ h_{Tefi}(t) & h_{fefi}(t) \end{vmatrix} \circledast \begin{vmatrix} T_e(t) \\ f_e(t) \end{vmatrix} \tag{5}$$

where one port is used as the input and the other port is used as the output of the system, and where $\circledast$ denotes the convolution operation. Equation (5) may be considered as a generalisation of the matrix form which is given in Carslaw and Jaeger (1946), which is only strictly valid for homogeneous materials under steady state periodic boundary conditions.

Equation (5) can be seen to be two linear simultaneous equations which have four unknown response function values. However, it is a straightforward matter to generate four simultaneous linear equations in terms of statistical central moments, which may then be solved for the four unknown response factor values.

If the input $\{x(t)\}$ and output $\{y(t)\}$ time series are stationary then the convolution equations may be expressed in terms of time delayed central moments. These equations for the two port case are

$$C_{x_1 y_1}(\tau_1) = \int h_{11}(\sigma_1)\, C_{x_1 x_1}(\tau_1,\sigma_1)d\sigma + \int h_{12}(\sigma_1)\, C_{x_1 x_2}(\tau_1,\sigma_1)d\sigma_1$$

$$C_{x_1 y_2}(\tau_1) = \int h_{21}(\sigma_1)\, C_{x_1 x_1}(\tau_1,\sigma_1)d\sigma_1 + \int h_{22}(\sigma_1)\, C_{x_2 x_2}(\tau_1,\sigma_1)d\sigma_1$$

$$C_{x_2 y_1}(\tau_1) = \int h_{11}(\sigma_1)\, C_{x_2 x_1}(\tau_1,\sigma_1)d\sigma_1 + \int h_{12}(\sigma_1)\, C_{x_1 x_2}(\tau_1,\sigma_1)d\sigma_1$$

$$C_{x_2 y_2}(\tau_1) = \int h_{21}(\sigma_1)\, C_{x_2 x_1}(\tau_1,\sigma_1)d\sigma_1 + \int h_{22}(\sigma_1)\, C_{x_2 x_2}(\tau_1,\sigma_1)d\sigma_1 \tag{6}$$

where $C_{xy}(\tau_1) = E[y(t)\ x(t-\tau_1)]$ is the correlation and
where $C_{xx}(\tau_1,\sigma_1) = E[x(t-\tau_1)\ x(t-\sigma_1)]$ is the auto correlation,
where E[ ] is the averaging operation and τ and σ denote delay.

Note that we are using an absolute time frame of reference for the moments rather than the usual retarded time frame of reference.

For a general linear system the vector correlation equations are

$$\begin{vmatrix} C_{x_1 y_j}(\tau_1) \\ \vdots \\ C_{x_i y_j}(\tau_1) \\ \vdots \\ C_{x_n y_j}(\tau_1) \end{vmatrix} = \begin{vmatrix} C_{x_1 x_1}(\tau_1,\sigma_1) & \cdots & C_{x_1 x_n}(\tau_1,\sigma_1) \\ \vdots & & \vdots \\ C_{x_i x_1}(\tau_1,\sigma_1) & \cdots & C_{x_i x_n}(\tau_1,\sigma_1) \\ \vdots & & \vdots \\ C_{x_n x_1}(\tau_1,\sigma_1) & \cdots & C_{x_n x_n}(\tau_1,\sigma_1) \end{vmatrix} \otimes \begin{vmatrix} h_{j1}(\sigma_1) \\ \vdots \\ h_{ji}(\sigma_1) \\ \vdots \\ h_{jn}(\sigma_1) \end{vmatrix} \tag{7}$$

where each individual term is a linear superposition of the form

$$C_{x_i y_j}(\tau_1) = \sum_k \int C_{x_i x_j}(\tau_1,\sigma_1)\, h_{jk}(\sigma_1)\, d\sigma_1 \tag{8}$$

In the frequency or Z domains this may be written as

$$\begin{vmatrix} \phi_{x_1 y_j}(\omega_1) \\ \phi_{x_2 y_j}(\omega_1) \\ \vdots \\ \phi_{x_i y_j}(\omega_1) \\ \vdots \\ \phi_{x_n y_j}(\omega_1) \end{vmatrix} = \begin{vmatrix} \phi_{x_1 x_1}(\omega_1) & \cdots & \phi_{x_1 x_j}(\omega_1) & \cdots & \phi_{x_1 x_n}(\omega_1) \\ \phi_{x_2 x_1}(\omega_1) & \cdots & \phi_{x_2 x_j}(\omega_1) & \cdots & \phi_{x_2 x_n}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \phi_{x_i x_1}(\omega_1) & \cdots & \phi_{x_i x_j}(\omega_1) & \cdots & \phi_{x_i x_n}(\omega_1) \\ \vdots & & \vdots & & \vdots \\ \phi_{x_n x_1}(\omega_1) & \cdots & \phi_{x_n x_j}(\omega_1) & \cdots & \phi_{x_n x_n}(\omega_1) \end{vmatrix} \begin{vmatrix} H_{j1}(\omega_1) \\ H_{j2}(\omega_1) \\ \vdots \\ H_{ji}(\omega_1) \\ \vdots \\ H_{jn}(\omega_1) \end{vmatrix} \qquad (9)$$

which may be solved for the $(i,j)^{th}$ response value at the frequence ω_1 using for example Cramer's rule.

NONLINEAR RESPONSE FACTORS: A POINT ALGEBRAIC APPROACH

The Volterra series is a nonlinear generalisation of the linear convolution equation (Volterra 1959) and is Volterra's integral representation of the Taylor's expansion. The Volterra Kernels are the same as nonlinear response factors and their values may be evaluated from central moment estimates.

First consider a linear time invariant system which is stimulated by a sequence of data $\{x(t)\}$, then the most general expression which simultaneously satisfies both the superposition and time invariant properties of the system (Seibert 1986) is the linear convolution equation, where

$$y(t) = \int_{-\infty}^{\tau} h_{xy}(\tau_1)\ x(t-\tau_1)d\tau_1$$

where $\{y(t)\}$ is the output sequence and $h_{xy}(\tau)$ is the first order Volterra kernel function or linear response factor. This may be generalised in the form of a Volterra series and may be written as (Volterra 1959)

$$y(t) = \sum_{n=1}^{N} \frac{1}{n!} \int_{-\infty}^{t} d\tau_1 \ldots \int_{-\infty}^{t} d\tau_n \, h_{yx^n} (\tau_1, \ldots, \tau_n) \prod_{i=1}^{n} x(t-\tau_i)$$

for continuous data or for the discrete data case as

$$y(t) = \sum_{n=1}^{N} \frac{1}{n!} \sum_{\tau_1=-\infty}^{t} \ldots \sum_{\tau_1=-\infty}^{t} h_{yx^n} (\tau_1, \ldots, \tau_n) \prod_{i=1}^{n} x(t-\tau_i) \qquad (10)$$

where we consider the thermal properties to be nonlinear to order N.

We thus have N unknown coefficients and only one equation so there is a need to generate N equations and then to solve them.

When the data sequences $\{x(t)\}$ and $\{y(t)\}$ are drawn from stochastic sequences then they and their interactions may be described in terms of statistical central moments.

If the different order nonlinear response factors are separated out and time series moments taken then we have the following set of equations (Dewson and Irving 1992).

$$E[y(t)\ x(t-\tau_1)] = \sum_{\sigma_1=-\infty}^{\tau_1} h_{yx}(\sigma_1)\ E[x(t-\tau_1)x(t-\sigma_1)] \tag{11}$$

$$\vdots \qquad\qquad \vdots$$

$$E[y(t) \prod_{i=1}^{n} x(t-\tau_i)] = \sum_{\sigma_1=-\infty}^{\tau_1} \cdots \sum_{\sigma_n=-\infty}^{\tau_n} h_{yx^n}(\sigma_1,\ldots,\sigma_n)\ E[\prod_{i=1}^{n} x(t-\tau_i) \prod_{j=1}^{n} x(t-\sigma_j)] \tag{12}$$

which is known as the isolated kernel approximation, where E[ ] is the averaging operation and τ and σ denote time delay relative to time t. The time series cross moments may be written as

$$C_{x^n y}(\tau_1,\ldots,\tau_n) = E[y(t) \prod_{i=1}^{n} x(t-\tau_i)]$$

and auto moments as

$$C_{x^n x^n}(\tau_1,\ldots,\tau_n,\sigma_1,\ldots,\sigma_n) = E[\prod_{i=1}^{n} x(t-\tau_i) \prod_{j=1}^{n} x(t-\sigma_j)].$$

Note that these equations are in the absolute time frame of reference and not the usual time retarded frame of reference.

Separating out each of the higher order terms from the expansion does not in itself facilitate the estimation of the Volterra kernels. However, there are experimental situations which arise where the estimation of the Volterra kernel values may be straightforward. One well known and extensively studied class of experiments is the so called Wiener white noise technique (Wiener 1942, Schetzen 1980), where the input data sequences $\{x(t)\}$ are drawn from Gaussian white noise distributions. Recently the Wiener identification technique has been extended for cases where the input data are stationary but not white noise (Dewson and Irving 1992). The response factor values may be extracted algebraically when the future independent form of the central moments is used. If a future independent form is assumed then equation (10) may be written as

$$C_{xy}(\tau_1) = \sum_{\sigma_1=0}^{\tau_1} h_{yx}(\sigma_1)\, C_{xx}(\tau_1,\sigma_1) \tag{13}$$

which is the time delay domain Borel theorem of convolution. This may be rearranged to yield the response factor value $h_{yx}(\tau_1)$ at a given time delay τ_1

$$h_{yx}(\tau_1) = \frac{C_{xy}(\tau_1) - \sum_{k_1=0}^{\tau_1-1} h_{yx}(k_1)\, C_{xx}(\tau_1,k_1)}{C_{xx}(0,0)} \tag{14}$$

which yields a unique value for $h_{yx}(\tau_1)$.

For a non-linear system the response factor values may be estimated using (Dewson and Irving 1992)

$$h_{x^n y}(\tau_1,\ldots,\tau_n) = \frac{C_{x^n y}(\tau_1,\ldots,\tau_n) - B_{x^n y}(\tau_1,\ldots,\tau_n)}{C_{x^n x^n}(0,\ldots,0)} \tag{15}$$

As an example consider the first non-linear response factor where

$$h_{xxy}(\tau_1,\tau_2) = \frac{C_{xxy}(\tau_1,\tau_2) - B_{xxy}(\tau_1,\tau_2)}{C_{xxxx}(0,0,0,0)} \tag{16}$$

and the value of $B_{xxy}(\tau_1,\tau_2)$ is estimated using

$$\begin{aligned} B_{xxy}(\tau_1,\tau_2) &= \sum_{\sigma_1=0}^{\tau_1-1}\sum_{\sigma_2=0}^{\tau_2-1} h_{xxy}(\sigma_1,\sigma_2)\, C_{xxxx}(\tau_1,\tau_2,\sigma_1,\sigma_2) \\ &+ \sum_{\sigma_1=0}^{\tau_1} h_{xxy}(\sigma_1,\tau_2)\, C_{xxxx}(\tau_1,\tau_2,\sigma_1,0) \\ &+ \sum_{\sigma_2=0}^{\tau_2} h_{xxy}(\tau_1,\sigma_2)\, C_{xxxx}(\tau_1,\tau_2,0,\sigma_2) \end{aligned} \tag{17}$$

Thus the method can, in principle, be extended to any order of nonlinearity and may be used with general boundary conditions. It does, however, require the isolated kernel approximation and the future independence assumptions to be valid and also that the moments be well defined (basically this means that the data should be almost stationary).

Example Application of the Method

Many experimental studies of heat transmission through walls have been performed in the past 50 years. Even so the measured U-values and response factors are not very accurate (typically 10%) and are not robust to changes in meteorological and environmental conditions. In the present work time series data was collected and analysed from a passive solar test cell that was exposed to the external meteorological conditions. The time series meteorological measurements collected were dry bulb temperature, wind speed, wind direction, global horizontal irradiance, diffuse horizontal irradiance and the nett irradiance between the test cell roof and the sky. Inside the test cell time series data were collected for dry bulb temperature and nett irradiance between the north facing wall and the south facing window. The passive solar test cell was not heated for this experiment and the ventilation rate was maintained at less than 0.05 air changes per hour.

In order to obtain the response factor and U-value of the north facing wall time series measurements were obtained from thermistors and TNO heat flux mats, each being embedded at a depth of approximately 5mm in the plywood which formed part of the wall's internal and external surface. The time series data were collected at one minute intervals for a duration of one month and the data was analysed on the CRAY XMP at the Rutherford and Appleton Laboratory.

Samples of the temperature and heat flux time series are shown in Figure 1 and Figure 2, and in Figure 3 is a schematic diagram of the experimental configuration used for the response factor measurements.

The sample covariance, $C_{xy}(\tau)$, and auto covariance $C_{xx}(\tau_1,\tau_2)$, are estimated from the time series temperature and heat flux values. The response factor values are then obtained using equation (13). The values obtained for the response factor values, $h_{feTi}(\tau)$, and the temperature gain $h_{TeTi}(\tau)$ are shown in Figures 4 and 5.

Integrating the response factor $h_{feTi}(\tau)$ yields an estimate of the U-value with

$$\int h_{feTi}(\tau)d\tau = 0.350 \pm 0.016 \text{ W m}^{-2} \text{ K}^{-1}$$

and this may be compared to the theoretical value of (Martin 1989)

$$U = 0.373 \text{ W m}^{-2} \text{ K}^{-1}$$

which is determined on the basis of the following resistance values

Layer	Thickness (mm)	Resistance (Km^2/W)
Plasterboard	12.7	0.085
Air gap	20.0	0.170
Glass fibre	100.0	2.326
Plywood skin	12.7	0.070
Plywood guard	5.0	0.028
Total		2.679

$$\text{U-value of construction} = \frac{1}{2.679} = 0.373 \text{ Wm}^{-2} \text{ K}^{-1}$$

The experimental and theoretical values agree well on the basis of a two tailed Student's t-test, which for these data has a value of

$$t_{9,7.5}\% = \frac{0.373 - 0.350}{0.016} = 1.6 \leq 2.26$$

which indicates that the two values are the same in a statistical sense.

Although the agreement is good in the present example further experiments and analysis should be undertaken to identify the accuracy, consistency and sensitivity of the method before firm recommendations for its use may be made.

CONCLUSIONS

The findings of this paper may be summarised as follows: a time series method has been developed that enables the thermal response factor values to be estimated from unsteady temperature and heat flux observations. This method was used to extract the response factors of a wall in a test cell. The area under the response factor is identified to be the U-value of the wall which was estimated and which compared favourably to the theoretical value. The linear vector form of the method is equivalent to the matrix method of Carslaw and Jaeger when the boundary conditions are steady state periodic. The scalar form was extended to the nonlinear case, so that known nonlinear process can now also be included in the analyses. The point algebraic method uses the isolated kernel approximation and the future independence assumption. The scalar and vector forms assume linearity and use the temperature field rather than the temperature gradient which drives the heat flux, as would be expected for the integral form of the diffusion equation. A programme of experimental and theoretical work is currently ongoing with the universities of Bristol and Newcastle to examine the validity of those approximations and assumptions. The findings from that work will be reported. In addition independent experimental work is needed to confirm the aptness of the method and to determine its inherent accuracy and range of appropriate use.

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REFERENCES

ASHRAE, 1989, ASHRAE Handbook - Fundamentals, Atlanta: American Society of Heating, Refrigeration and Air-Conditioning Engineers, Inc.

Carslaw H S and Jaeger J C, 1946, Conduction of Heat in Solids, Clarendon Press, Oxford.

Dewson T and Irving A D, 1992, Nonlinear Response Function Estimation: II Isolated Kernel Case, Submitted to J. Phys. A, See also Rutherford Appleton Laboratory Reports RAL-91-067 and RAL-91-068, 1991.

Irving A D, 1992, Stochastic Sensitivity Analysis, Applied Mathematical Modelling, 16, January, p 1091-1104.

Joseph D D and Preziosi L, 1989, Heat Waves, Reviews of Modern Physics, Vol 61, No 1, January, p 41-73

Kusuda T, 1978, NBSLD The Computer Program for Heating and Cooling Loads in Buildings, National Bureau of Standards Building Science Series 69, December, US Dept of Commerce.

Luikov A V, 1968, Analytical Heat Diffusion Theory, Academic Press, New York.

Martin C, 1989, Private Communication.

Martin C and Wilson M, Data Acquisition for the Rutherford Appleton Laboratory, Energy Monitoring Company Report, Unit 3, Cahyole Court, Newport Pagnell, Buckinghamshire.

Mitlas G P and Stephenson D G, 1968, Calculation of Heat Flows Through Walls and Roofs, ASHRAE TRANS., Vol 74, p 182-188.

Mitlas G P and Stephenson D G, 1967, Room Thermal Response Factors, ASHRAE TRANS., Vol 73.

Pratt A W, 1981, Heat Transmission in Buildings, John Wiley, Chichester.

Schetzen M, 1980, The Volterra and Wiener Theories of Non-linear Systems, John Wiley, New York.

Seibert W, 1986, Circuits, Signals and Systems, the MIT press, Cambridge, Massachusetts.

Stephenson D G and Mitlas G P, 1967, Cooling Load Calculations by the Thermal Response Factor Method, ASHRAE TRANS., Vol 73.

Volterra V, 1959, Theory of Functionals and of Integral and Integro-differential Equations, Dover, New York.

Wiener N, 1942, Response of a Non-linear Device to Noise, MIT Radiation Laboratory Report No 165.

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FIGURE CAPTIONS

Figure 1. Sample of the time series temperatures $T_e(t)$ and heat flux values $f_e(t)$ observed at the external surface of the test cell wall.

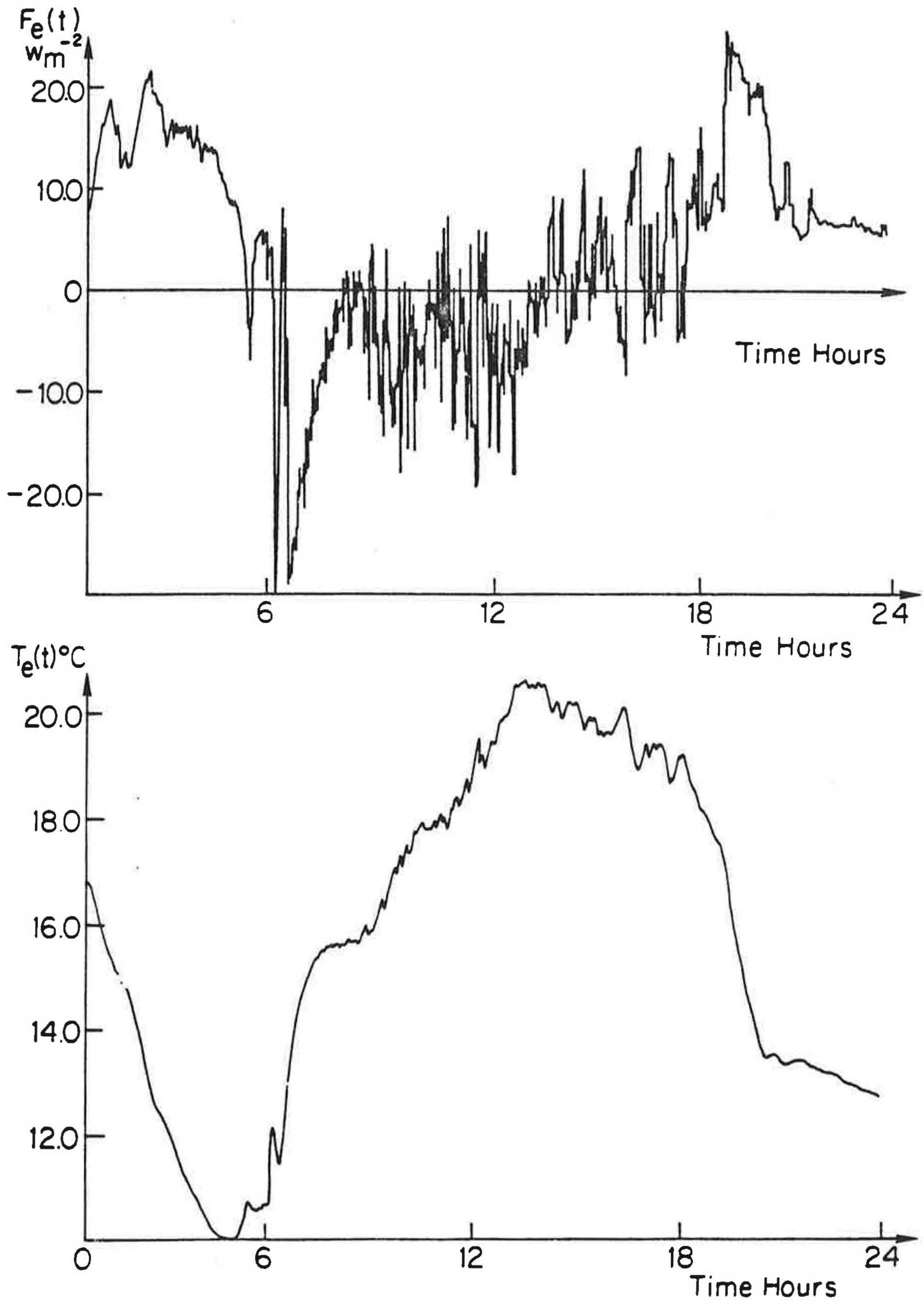
Figure 2. Sample of the time series temperatures $T_i(t)$ and heat flux values $f_i(t)$ observed at the internal surface of the test cell wall.

Figure 3. A schematic diagram of the experimental design used for the response factor measurement.

Figure 4. The estimated values of the response factor $h_{feTi}(\tau)$ using the point algebraic method.

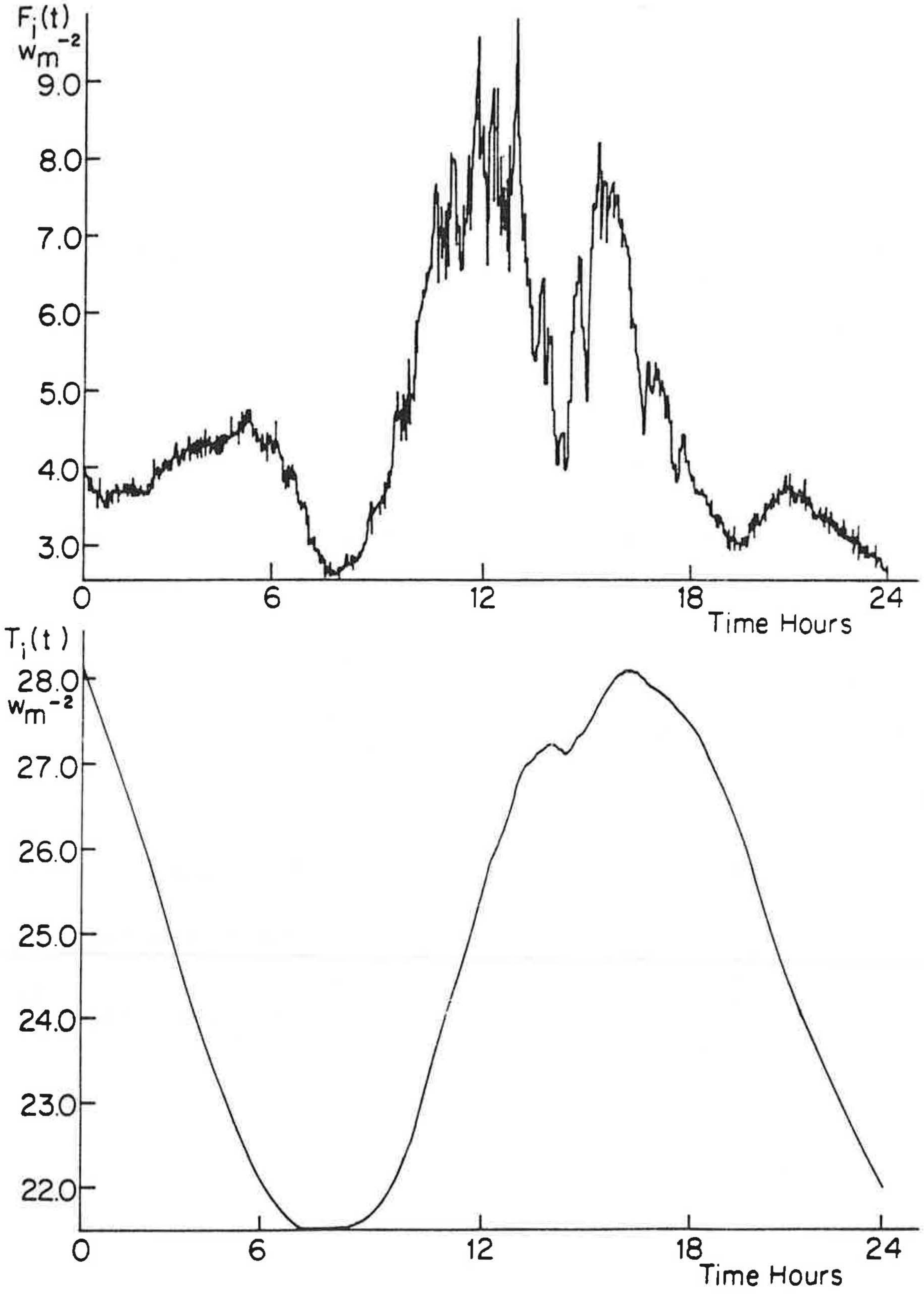
Figure 5. The estimated values of the response factor $h_{TeTi}(\tau)$ using the point algebraic method.

Figure 1.



$F_e(t)$
Wm^{-2}
20.0
10.0
0
-10.0
-20.0
Time Hours
6
12
18
24
Time Hours
$T_e(t)$°C
20.0
18.0
16.0
14.0
12.0
0
6
12
18
24
Time Hours

Figure 2.



$F_i(t)$
wm^{-2}
9.0
8.0
7.0
6.0
5.0
4.0
3.0
0
6
12
18
24
Time Hours
$T_i(t)$
28.0
wm^{-2}
27.0
26.0
25.0
24.0
23.0
22.0
0
6
12
18
24
Time Hours

Figure 3.

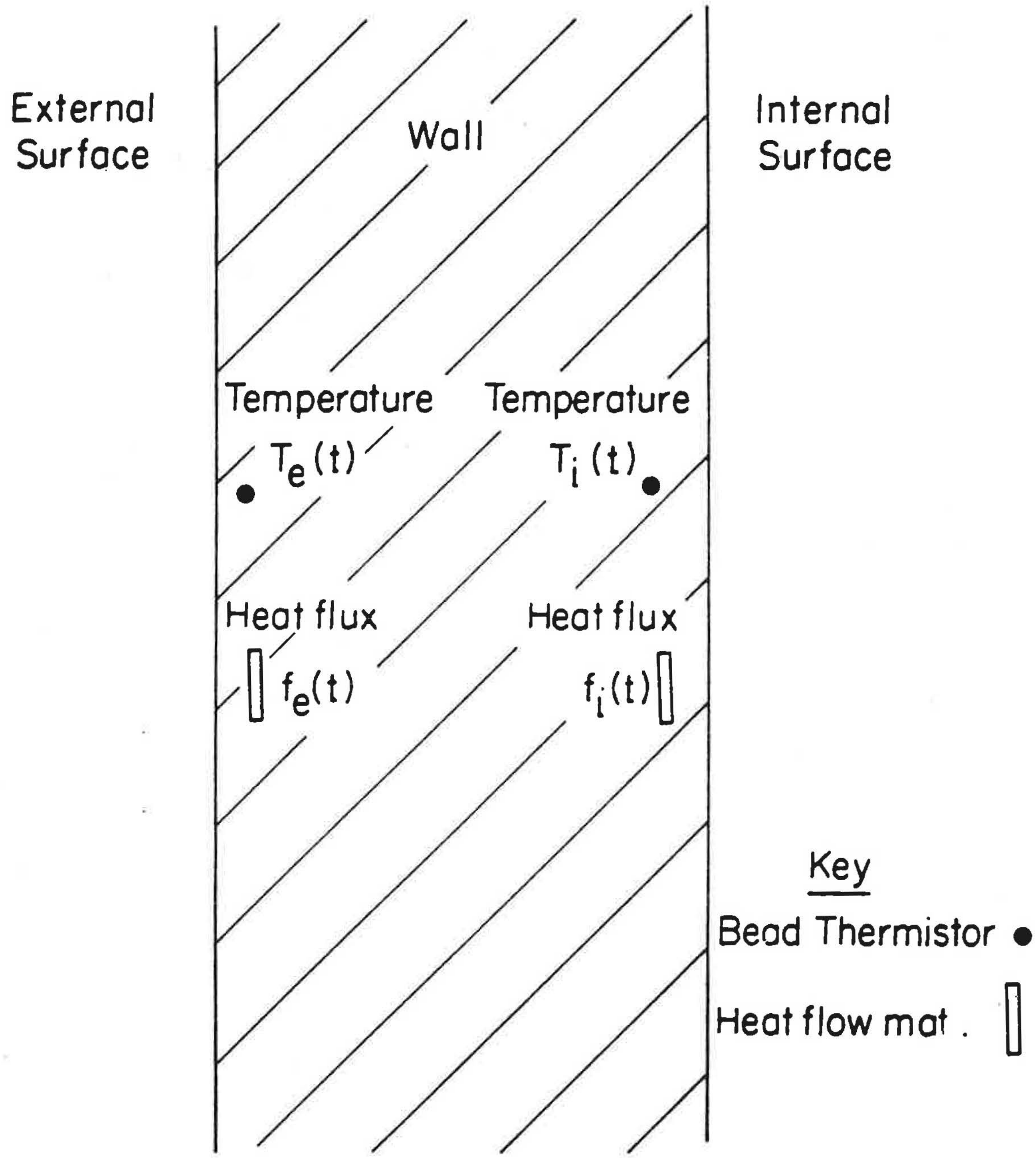

External Surface
Wall
Internal Surface
Temperature
$T_e(t)$
Temperature
$T_i(t)$
Heat flux
$f_e(t)$
Heat flux
$f_i(t)$
Key
Bead Thermistor
Heat flow mat .

Figure 4.

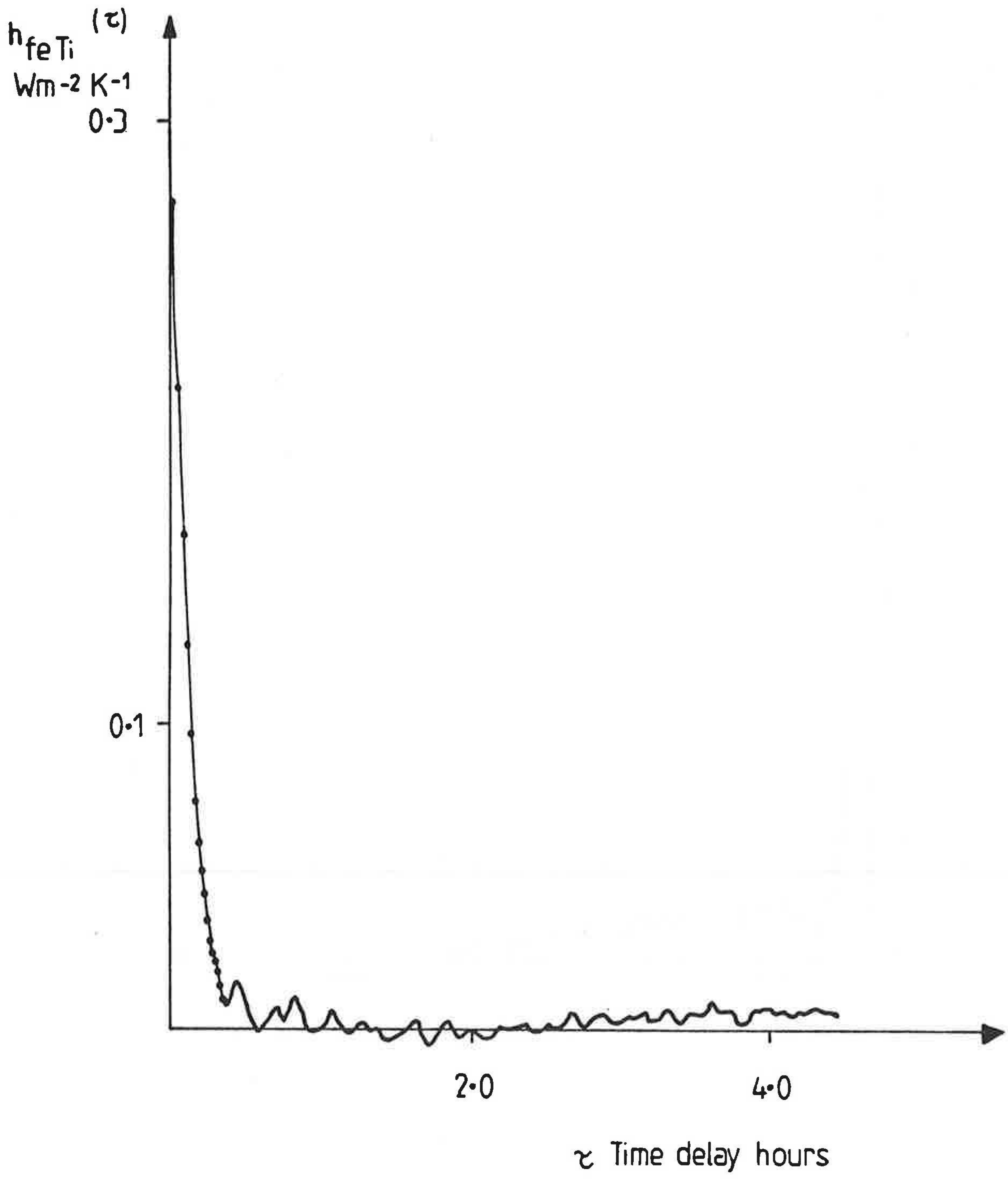


Figure 5.