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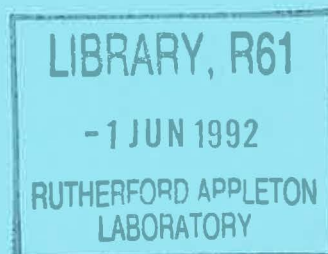
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## Reexamining the Fermion Mass Relations in the Supersymmetric $SU(4) \otimes O(4)$ GUT Model

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May 1992

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# Reexamining the Fermion Mass Relations in the Supersymmetric $SU(4) \otimes O(4)$ GUT Model

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## Abstract

We study in detail the charged fermion masses in the context of the minimal GUT-version of the supersymmetric  $SU(4) \otimes O(4)$  model. We show how the presence of vacuum expectation values (VEVs) of the order of the GUT mass scale for the right-handed (RH) sneutrinos may modify the “naive” equality of the charged-lepton and the down-quark masses at  $M_G = 10^{16}$  GeV. In particular, we indicate how one can successfully modify the problematic low-energy relation between  $m_s$  and  $m_\mu$ , while preserving the good prediction  $m_b/m_\tau = r \simeq 2.7$  ( $r$  being the renormalization parameter). The fact that the VEVs of the RH sneutrinos are predicted to be of order  $\sim M_G$ , may also have interesting consequences in the mechanism of symmetry breaking of the model.

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One of the outstanding problems of particle physics is certainly related to the understanding of the structure of fermion masses. In particular, since the discovery of Grand Unified Theories (GUTs), it was noticed that many of these models lead to particular relations among the fermion masses which can be tested experimentally. In general, this may result in stringent constraints on the models, in addition to those obtained, for example, from the limit on the proton lifetime and the measurement of the Weinberg angle at low energy. Certainly, the most famous mass relations are those which were derived in the (minimal)  $SU(5)$  GUT model [1], namely the equality of the down-type quark and the charged lepton masses at the GUT mass scale ( $M_G \simeq 10^{16}$  GeV):  $m_{d_i}(M_G) = m_{e_i}(M_G)$ , where  $i = 1, 2, 3$  labels the various generations. Introducing the renormalization parameter,  $r$ , which takes into account the different running of the mass of the quarks with respect to the charged leptons down from  $M_G$  to some scale  $\mu$ , and which may be evaluated by the following expression (which is actually appropriate for the SUSY  $SU(5)$  model):

$$r(\mu) = \left( \frac{\alpha_3(\mu)}{\alpha(M_G)} \right)^{\frac{4}{9-2n_g}} \left( \frac{\alpha_1(\mu)}{\alpha(M_G)} \right)^{\frac{1}{2n_g}}, \quad (1)$$

these mass relations predict<sup>1</sup>:

$$m_d = r m_e, \quad m_s = r m_\mu, \quad m_b = r m_\tau. \quad (2)$$

In eq.(1)  $n_g$  is the number of fermion generations,  $\alpha_3(\mu)$  is the QCD running coupling constant and  $\alpha_1(\mu) = [5\alpha_{em}(\mu)/3 \cos^2 \theta_W(\mu)]$ . For  $\mu=1$  GeV and with  $n_g=3$ , eq.(1) gives<sup>2</sup>:  $r \simeq 2.7$ , in good agreement with the observed mass ratio  $m_b/m_\tau$  (at  $\mu=1$  GeV). This success of the third relation in eq.(2) has in fact been one of the stronger successes of the idea of GUTs. Unfortunately, this agreement does not hold for the first two families, since the actual masses give  $m_d/m_e \simeq 20$  and  $m_s/m_\mu \simeq 1.6$ , in both cases different than the value of  $r$  ( $\simeq 2.7$ ) obtained from eq.(1). If this disagreement may be understood for the first family saying that the down quark mass is dominated by non-perturbative QCD,  $m_d$  being much lighter than  $\Lambda_{QCD} \sim 100$  MeV, this argument cannot be of any help for the second family. In fact, apart from the fact that we expect only a marginal contribution

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<sup>1</sup> Throughout this paper, all masses must be understood as the running masses evaluated at the scale  $\mu = 1$  GeV.

<sup>2</sup> In the non-supersymmetric case  $r$  turns out to be only slightly different:  $r \simeq 2.9$ .

on the  $s$ -quark mass from non-perturbative QCD (since  $m_s \sim \Lambda_{QCD}$ ), its effects would in any case imply an even smaller mass originated by the electro-weak symmetry breaking, making therefore the inconsistency of the second relation in eq.(2) even more serious. The optimum would be to find a solution which modifies these mass relations only for the first two families, while keeping the good  $m_b = r m_\tau$  prediction. In minimal  $SU(5)$ , for example, two suggestions were pointed out: (i) the enlargement of the Higgs sector by the addition of a **45** (of  $SU(5)$ ) dimensional representation, and (ii) the introduction of suitable non-renormalizable terms, which were thought to result as the effect at low energies of some unspecified interaction (*e.g.*, supergravity, etc.). In the first case, the mass matrices at the GUT scale were modified as follows:  $M_e = M_e(\mathbf{5}) + M_e(\mathbf{45}) = M_d(\mathbf{5}) - 3M_d(\mathbf{45})$ , and  $M_d = M_d(\mathbf{5}) + M_d(\mathbf{45})$ , removing therefore the naive relation  $M_e = M_d$  obtained in the absence of the **45** Higgs multiplet. Unfortunately, in this way one loses any correlation between  $M_e$  and  $M_d$ , and therefore not only destroys the unwanted mass relations involving the first two families, but also the successful relation between  $m_b$  and  $m_\tau$ . The same thing usually occurs also in the more complicated models. In  $SO(10)$ , for example, with the presence of all the Higgs multiplets belonging to the product  $\mathbf{16} \otimes \mathbf{16} = \mathbf{10}_S \oplus \mathbf{120}_A \oplus \mathbf{126}_S$  (the indices “S” and “A” mean, respectively, symmetric and antisymmetric Yukawa couplings in generation space), one does not get any correlation between the fermion masses in the different sectors [1]. In this case the **126** Higgs representation plays the same role as the **45** in the  $SU(5)$  model, ruining the naive relations (2) obtainable if only the **10** were present.

More recently, the study of the GUT models derivable as effective field theories from the string has indicated two promising models, which have the nice feature of not requiring the presence of the adjoint or the other large self-conjugate Higgs representations (which cannot be present in string inspired theories with Kac-Moody level  $K = 1$ ) for obtaining the correct symmetry breaking of the gauge group down to the Standard Model (SM) and for producing the well-known doublet-triplet mass splitting which is essential for avoiding a fast proton decay induced by the exchange of light colour-triplet Higgs scalars. These models, which are based on the gauge groups “flipped”  $SU(5) \otimes U(1)$  [2] and  $SU(4) \otimes O(4)$  [3], have been extensively studied in the literature, both in their simple GUT versions [2-5] as well as in their “string” type of version [2,6,7]. In the flipped  $SU(5) \otimes U(1)$  model, the word “flipped” refers to the exchange in the fermionic representations of the up-type with the

down-type fields ( $u^{(c)} \leftrightarrow d^{(c)}$ ,  $\nu^{(c)} \leftrightarrow e^{(c)}$ ), with respect to the “standard”  $SU(5)$  model. It can be shown that, contrary to this last model, the flipped version [2] does not lead to any relation among the charged fermion masses. On the other hand, the  $SU(4) \otimes O(4)$  group [3], which is isomorphic to the left-right symmetric group  $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ , naively, reproduces the results of the minimal  $SU(5)$ , leading to the same mass relations (2). In such a model, one must therefore face the problem of modifying the relations which are not in agreement with the experimental information, without affecting the successful one which relates  $m_b$  to  $m_\tau$ . In this paper we suggest a solution based on the possible presence of large vacuum-expectation-values (VEVs) for the scalar partners of the right-handed (RH) neutrinos. In a particular example, we shall see how it is possible to modify only the relation involving the second generation fermions, with the strange quark mass given as a function of  $\langle \nu_\mu^c \rangle$ . Of course, we cannot solve at the same time the problem associated with the relation  $m_d = r m_e$ , but, as discussed above, this may just be a consequence of the importance of non-perturbative QCD effects for the down quark mass. Our main result is that by fitting the s-quark mass we are able to predict, under quite general assumptions concerning the free parameters of the model, that  $\langle \nu_\mu^c \rangle \sim M_G$  within one order of magnitude. This means that it is possible that, at least in part, the gauge group  $SU(4) \otimes O(4)$  is broken down to the SM not only by the Higgs supermultiplets, but also by the scalar neutral component of the RH matter superfields.

In the next paragraph we shall briefly review the main characteristic features of the model and fix our notations. Then, we shall construct and analyse in detail the mass matrix for the down-type quarks, treating the “naive” mass matrix which yields the mass relations (2) as the *unperturbed* matrix, and considering the effect of  $\langle \nu^c \rangle \neq 0$  as a correction. This will give some insight on the type of modification of the mass matrices which can be obtained. At last, we shall show a particular example which allows the evaluation of the mass eigenvalues, and therefore, through the fitting of  $m_s$ , gives a prediction for  $\langle \nu_\mu^c \rangle$ .

The superfield content of the  $SU(4) \otimes O(4)$  model, in its minimal GUT version, is given in the Table, which has been taken from ref.[5], to which we also refer for the notation. The fifteen standard fermions plus the RH neutrino  $\nu^c$  of each generation fit in the reducible sum  $\mathbf{F}(4, 2, 1) \oplus \bar{\mathbf{F}}(\bar{4}, 1, 2)$  of  $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ , identifiable with the standard fermionic **16** representation of  $SO(10)$ . The gauge group is spontaneously



broken down to the Standard Model at the GUT mass scale through the VEVs of the scalar components of the Higgs supermultiplets, belonging to two incomplete (spinorial) representations of  $SO(10)$ :  $\mathbf{H}(4, 1, 2)$  and  $\bar{\mathbf{H}}(\bar{4}, 1, 2)$ ; we denote these VEVs by  $\langle \bar{\nu}_H^c \rangle$  and  $\langle \nu_H^c \rangle$ , respectively. Then, the subsequent electro-weak symmetry breaking down to  $U(1)_{em}$  is produced by a bidoublet Higgs field  $\mathbf{h}(1, 2, 2)$ , whose two VEVs,  $\langle h^0 \rangle \equiv v_u$  and  $\langle \bar{h}^0 \rangle \equiv v_d$ , constrained by the condition  $\sqrt{v_u^2 + v_d^2} = v \simeq 246$  GeV, produce mass terms in the “up” and the “down”-type fermion sectors, respectively. The model also contains a  $\mathbf{D}(6, 1, 1)$  multiplet, which forms with  $\mathbf{h}(1, 2, 2)$  a 10-dimensional representation in the  $SO(10)$  embedding of the model, and whose role is essentially to generate the desired doublet-triplet mass-splitting producing massive states ( $\sim M_G$ ) with the coloured triplets  $d_H^c$  and  $\bar{d}_H^c$  of  $\bar{\mathbf{H}}$  and  $\mathbf{H}$ . Finally, there are  $n_g + 1$  gauge singlet superfields  $\phi_m$  of which one (say,  $\phi_0$ ) has the scalar component developing a VEV at the electroweak scale (we shall take, for simplicity,  $\langle \phi_0 \rangle \equiv X = v$  GeV). These singlets, besides producing a suitable seesaw mechanism for suppressing the neutrino masses [4,5], are essential for generating a correct Higgs mixing [2,3] and so prevent the appearance of an unacceptable electroweak axion. The model is then completely specified by the most general superpotential satisfying the discrete  $Z_2$  symmetry  $\bar{\mathbf{H}} \rightarrow -\bar{\mathbf{H}}$ , which is essential for forbidding a heavy tree-level Majorana mass for the standard left-handed (LH) neutrinos. As in ref.[5], we write such superpotential as:

$$\begin{aligned} \mathcal{W} = & \lambda_1^{ij} F_i \bar{F}_j h + \lambda_2^{im} \bar{F}_i H \phi_m + \lambda_3 H H D + \lambda_4 \bar{H} \bar{H} D + \lambda_5^m h h \phi_m \\ & + \lambda_6^{mnq} \phi_m \phi_n \phi_q + \lambda_7^{ij} F_i F_j D + \lambda_8^{ij} \bar{F}_i \bar{F}_j D + \lambda_9^n D D \phi_n, \end{aligned} \quad (3)$$

where in general all the nine Yukawa-type coupling constants  $\lambda_i$  are independent. In the case of the  $SO(10)$  embedding of the model, however, one gets [5] the following constraints:  $\lambda_1 = \lambda_7 = \lambda_8$  and  $\lambda_5 = \lambda_9$ ; in what follows, in addition to these conditions, we shall also choose  $\lambda_3 = \lambda_4$ . After spontaneous symmetry breaking the only surviving (*i.e.*, *uneaten*, or which do not acquire large masses via the supersymmetric Higgs mechanism) particles in  $\mathbf{H}$  and  $\bar{\mathbf{H}}$  are, in addition to a single combination of  $\nu_H^c$  and  $\bar{\nu}_H^c$ , the d-type fields  $\bar{d}_H^c$  and  $d_H^c$ , which, mixing with the ordinary down-type quarks (as well as with the colour triplets belonging to  $\mathbf{D}(6, 1, 1)$ ), will play an important role in our discussion on the d-type mass matrix. At first, let us discuss the *standard* charged fermion<sup>3</sup> mass terms arising from the

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<sup>3</sup> The neutral fermion sector has been studied in detail in refs.[4,5].

superpotential  $\mathcal{W}$ , that is without assuming any VEV for the RH sneutrinos. Later, we shall give the new terms, due to  $\langle \nu^c \rangle \neq 0$ .

The first term in eq.(3) gives rise to the standard masses for the ordinary fermions (plus Dirac masses for the neutrinos), which lead to the naive mass relations discussed in the introduction:

$$M_u = r' M_\nu^D = \lambda_1 v_u, \quad (4a)$$

$$M_d = r M_e = \lambda_1 v_d, \quad (4b)$$

where  $v_u = \langle h^0 \rangle$ ,  $v_d = \langle \bar{h}^0 \rangle$ , and  $r, r' (\simeq 3)$  are the renormalization parameters appropriate for comparing the quark and the lepton masses at a low mass scale,  $\mu = 1$  GeV. We shall take  $v_u/v_d \equiv \tan \beta = m_t/m_b \simeq 30$ , corresponding to  $m_t(\mu=1 \text{ GeV}) = 150$  GeV and to a physical mass  $m_t(phys) \simeq 100$  GeV. The  $\lambda_2$ -term produces a mass term equal to  $\lambda_{2,i} \langle \phi_0 \rangle \equiv X_i$ , ( $i = 1, 2, 3$  labels the different generations), mixing the RH component of the ordinary down-type quarks  $d_{Ri}$  with  $\bar{d}_H^c$ . Furthermore, the third and the fourth terms of  $\mathcal{W}$  lead to identical masses for the mixed states  $D_3 d_H^c$  and  $\bar{D}_3 \bar{d}_H^c$ , given by  $G \equiv \lambda_3 \langle \bar{\nu}_H^c \rangle = \lambda_3 M_G$ . The equality of these mass terms holds in virtue of our condition  $\lambda_3 = \lambda_4$  and of the assumption  $\langle \nu_H^c \rangle = \langle \bar{\nu}_H^c \rangle \equiv M_G$ . Notice, however, that in contrast to the flipped  $SU(5) \otimes U(1)$  model, in the present case the  $D$  and  $F$  flatness does not necessarily imply the equality of the VEVs of  $\mathbf{H}$  and  $\bar{\mathbf{H}}$  [3]. Finally, the  $\lambda_9$ -term generates the following diagonal mass term of order of the weak scale:  $\bar{D}_3 D_3 \lambda_9 \langle \phi_0 \rangle \equiv X_9 \bar{D}_3 D_3$ . All these contributions lead then to the standard results for the charged fermion masses, which, up to negligible corrections of order  $\mathcal{O}(v/M_G) \sim 10^{-14}$ , give the naive mass relations in eq.(2). However, here we wish to point out that in the presence of possibly large VEVs for the scalar RH neutrinos,  $\langle \nu_i^c \rangle$ , ( $i = 1, 2, 3$ ), the resulting mass matrix for the d-type colour-triplet particles allows us to construct an “effective” mass matrix for the ordinary down quarks, which yields mass relations modified with respect to those in eq.(2). In fact, the  $\lambda_8$ -term produces mass terms like the following:

$$\sum_{j=1}^3 \lambda_8^{ij} \langle \nu_j^c \rangle (\bar{d}_{Ri} D_3) \equiv S_i (\bar{d}_{Ri} D_3), \quad (5)$$



which mix the right-handed states  $d_{Ri}$  with  $D_3$ . In what follows we shall use  $\lambda_8 = \lambda_1$  ( $=rM_e/v_d$ ), a condition obtainable in the embedding of the model in  $SO(10)$ . Of course, we do not consider also the consequences of possible non-vanishing VEVs of the left-handed (LH) sneutrinos, since, the  $\langle \nu_i \rangle$ 's being at most of the order of the electro-weak scale, they could only have negligible effects on the standard fermion masses, suppressed by  $(\langle \nu \rangle / M_G) \leq 10^{-14}$ . On the other hand,  $\langle \nu^c \rangle$ , being the VEV of a doublet under  $SU(2)_R$ , but a singlet under  $SU(2)_L$ , can be as large as  $M_G$ .

The mass terms listed in the previous paragraph allow us to write the following tree-level mass matrix<sup>4</sup> for the  $d$ -type (colour-triplet) fermions:

$$\mathcal{M}_d = \begin{matrix} & d_{Li} & \bar{d}_H^c & D_3 \\ \bar{d}_{Ri} & \begin{pmatrix} M_d & X & S \\ 0 & 0 & G \\ 0 & G & X_9 \end{pmatrix} \end{matrix} \equiv \begin{pmatrix} M_d & A \\ 0 & \Gamma \end{pmatrix}, \quad (6)$$

where the block sub-matrices are such that  $M_d = rM_e$  is  $3 \times 3$ ,  $X$  and  $S$  are  $3 \times 1$  column vectors,  $A \equiv (X; S)$  is  $3 \times 2$ , and  $\Gamma$  is  $2 \times 2$ . Notice that this mass matrix is non-symmetric, as a consequence of the discrete symmetry  $\bar{\mathbf{H}} \rightarrow -\bar{\mathbf{H}}$  imposed on the superpotential  $\mathcal{W}$ , and because only the scalar partners of the RH neutrinos can develop a large VEV. As usual, in order to evaluate the mass eigenvalues of a non-hermitian mass matrix, one must consider<sup>5</sup> the matrix  $\mathcal{M}^T \mathcal{M}$ , whose eigenvalues are just the positive masses squared. In our case, we get:

$$\mathcal{M}_d^T \mathcal{M}_d = \begin{pmatrix} M_d^T M_d & M_d^T X & M_d^T S \\ X^T M_d & G^2 + X^T X & GX_9 + X^T S \\ S^T M_d & GX_9 + S^T X & G^2 + S^T S + X_9^2 \end{pmatrix} = \begin{pmatrix} M_d^T M_d & M_d^T A \\ A^T M_d & A^T A + \Gamma^2 \end{pmatrix}, \quad (7)$$

which yields the following “effective”  $3 \times 3$  mass matrix for the three ordinary down-type quarks:

$$\mathcal{M}_{d(eff)}^2 \simeq M_d^T M_d - M_d^T A (A^T A + \Gamma^2)^{-1} A^T M_d. \quad (8)$$

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<sup>4</sup> The SUSY protection of the gauge hierarchy ensures the absence of large ( $\geq M_W$ ) mass contributions generated radiatively [8].

<sup>5</sup> For simplicity, we assume all elements of our matrices to be real.

The inverse matrix in this equation may be approximately evaluated by noticing that  $A^T A + \Gamma^2$  is diagonal up to very small correction terms of orders  $\mathcal{O}(v/M_G)$  and  $\mathcal{O}(\langle \nu^c \rangle v/M_G^2)$ . Therefore,

$$(A^T A + \Gamma^2)^{-1} \simeq G^{-2} \begin{pmatrix} 1 & 0 \\ 0 & z \end{pmatrix},$$

where  $z \equiv [1 + (|S|/G)^2]^{-1}$  and  $|S| \equiv (\sum_{i=1}^3 S_i^2)^{1/2}$ . Then, eq.(8) gives:

$$\mathcal{M}_{d(eff)}^2 \simeq M_d^T \left\{ I - \frac{1}{G^2} [(XX^T) + z(SS^T)] \right\} M_d, \quad (9)$$

where  $(XX^T)$  and  $(SS^T)$  represent the matrices whose  $(i, j)$  elements are just the products  $X_i X_j$  and  $S_i S_j$ , respectively, and  $I$  is the  $2 \times 2$  identity matrix. Since the terms proportional to  $(XX^T)$  are suppressed by  $\sim (v/M_G)^2$ , they may be neglected and our final effective mass matrix can be written in a very simple and convenient way:

$$\mathcal{M}_{d(eff)}^2 \simeq r M_e^T \left\{ I - \frac{1}{|S|^2 + G^2} (SS^T) \right\} r M_e, \quad (10)$$

where we have used  $M_d = r M_e$  from eq.(4b). Because of the form of eq.(10), it is natural to consider the “naive” result  $\mathcal{M}_{d(eff)}^2 = M_d^T M_d = r^2 (M_e^T M_e)$  as the *unperturbed* mass matrix squared, and the second term, proportional to  $(S_i S_j)$ , as the *perturbation*. At this point it is helpful to use a vector basis where the charged lepton mass matrix,  $M_e$  (for simplicity, assumed to be real and symmetric), is diagonal; *e.g.*, we may write:

$$E^T (r M_e) E = r \text{diag}(m_e, m_\mu, m_\tau) \equiv r \hat{M}_e,$$

where  $E$  is an orthogonal matrix. In this new basis the column vector  $S$  is correspondingly transformed into:

$$S' = E^T S = E^T \lambda_1 V = (r/v_d) \hat{M}_e E^T V \equiv \left( \frac{r}{v_d} \right) \hat{M}_e V',$$

where, besides the condition  $\lambda_8 = \lambda_1$ , we have used the obvious fact that the  $3 \times 3$  matrix  $\lambda_1$  is also diagonalized by the orthogonal matrix  $E$ :  $E^T \lambda_1 E = r \hat{M}_e / v_d$ ;  $V$  and  $V'$  represent the column vectors of the VEVs of the RH sneutrinos in the initial and in the new basis,

respectively. If we neglect any possible (s)neutrino mixing in the RH sector<sup>6</sup>, we may say that  $V' \equiv E^T V$  contains just the VEVs  $\langle \nu_l^c \rangle$ , ( $l = e, \mu, \tau$ ). In this case,  $S'$  is given by:

$$S' = \left( \frac{r}{v_d} \right) \begin{pmatrix} m_e \langle \nu_e^c \rangle \\ m_\mu \langle \nu_\mu^c \rangle \\ m_\tau \langle \nu_\tau^c \rangle \end{pmatrix}, \quad (11)$$

and our final effective mass matrix may be written as:

$$\mathcal{M}_{d(eff)}^2 \simeq r \hat{M}_e \left\{ I - \frac{1}{|S|^2 + G^2} (S' S'^T) \right\} r \hat{M}_e. \quad (12)$$

This formula, which gives the leading term of the correction to the naive result of eqs.(2) and (4b) as a function of the three VEVs  $\langle \nu_l^c \rangle$ , is our first result.

At this point we wish to impose the condition that the successful mass relation between  $m_b$  and  $m_\tau$  is not modified. Generally speaking, this could mean that all the correction terms along the third row (or column) in the second term of eq.(12) must be smaller than  $(r m_\tau)^2$ . However, it may be more instructive to give here a specific example, which, while preserving exactly the relation  $m_b = r m_\tau$ , allows the fitting of the  $s$ -quark mass, and consequently gives a prediction for  $\langle \nu^c \rangle$ . Since we do not want to affect the third-generation fermion masses, and since we do not pretend to solve the puzzle concerning the masses of the first generation, the structure of the “correction” mass matrix in eq.(12) ( $\propto S' S'^T$ ) suggests that the simplest choice might be to assume only  $\langle \nu_\mu^c \rangle \neq 0$  (so that only  $S'_2 \neq 0$ ), in which case  $\mathcal{M}_{d(eff)}^2$  is diagonal and gives directly the following mass eigenvalues:

$$m_d = r m_e, \quad (13a)$$

$$m_s = \sqrt{z} r m_\mu, \quad (13b)$$

$$m_b = r m_\tau. \quad (13c)$$

This shows, in fact, that with respect to the naive results of eq.(2), only the second mass relation is changed, and allows us to evaluate  $\langle \nu_\mu^c \rangle$  by using the observed value for  $m_s$ :

$$\frac{\langle \nu_\mu^c \rangle}{M_G} \simeq \frac{\lambda_3 v}{m_s} \left( \frac{m_b}{m_t} \right) \sqrt{1 - \left( \frac{m_s}{r m_\mu} \right)^2}, \quad (14)$$

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<sup>6</sup> In any case, our results do not depend in an important way on this assumption.

where we have used:  $|S| = S_2 = (r m_\mu/v_d) \langle \nu_\mu^c \rangle$ ,  $G = \lambda_3 M_G$ , and  $v_d \simeq v(m_b/m_t)$ . Notice that, as it might be expected,  $\langle \nu_\mu^c \rangle$  would vanish if the naive mass relation  $m_s = r m_\mu$  had been successful. In particular, choosing  $\lambda_3 = 0.1$ , and with  $m_t/m_b \simeq 30$  (at  $\mu = 1$  GeV), eq.(14) gives:

$$\frac{\langle \nu_\mu^c \rangle}{M_G} \sim \mathcal{O}(1). \quad (15)$$

This result is very interesting; it means that, in order to modify correctly the second generation fermion masses (and fitting therefore  $m_s$ ),  $\langle \nu_\mu^c \rangle$  must be of the order of  $M_G = \langle \nu_H^c \rangle$ . In this case the  $SU(4) \otimes O(4)$  gauge group would not be broken down to the Standard Model only by the Higgs fields  $\mathbf{H}$  and  $\bar{\mathbf{H}}$ , but also by the VEV of the scalar component of the RH matter supermultiplet  $\langle \bar{\mathbf{F}}_{(2)}(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \rangle = \langle \nu_{(\mu)}^c \rangle \sim \mathcal{O}(M_G)$ . The possible consequences in the neutrino sector of allowing these large VEVs for the scalar component of the right-handed (RH) neutrinos, will be analysed elsewhere [9].

### Table Caption

The superfield content of the model. The various supermultiplets are labelled by their transformation properties under  $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ , ( $i$  is the generation index).

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**Table**

$SU(4) \otimes SU(2)_L \otimes SU(2)_R$
$\mathbf{F}_i = (4, 2, 1) = \begin{pmatrix} u \\ d \end{pmatrix}_i \oplus \begin{pmatrix} \nu \\ e \end{pmatrix}_i$
$\bar{\mathbf{F}}_i = (\bar{4}, 1, 2) = \begin{pmatrix} \bar{u}^c \\ \bar{d}^c \end{pmatrix}_i \oplus \begin{pmatrix} \nu^c \\ e^c \end{pmatrix}_i$
$\mathbf{H} = (4, 1, 2) = \begin{pmatrix} \bar{u}_H^c \\ \bar{d}_H^c \end{pmatrix}_i \oplus \begin{pmatrix} \bar{\nu}_H^c \\ \bar{e}_H^c \end{pmatrix}_i$
$\bar{\mathbf{H}} = (\bar{4}, 1, 2) = \begin{pmatrix} u_H^c \\ d_H^c \end{pmatrix}_i \oplus \begin{pmatrix} \nu_H^c \\ e_H^c \end{pmatrix}_i$
$\mathbf{h} = (1, 2, 2) = \begin{pmatrix} h^0 & \bar{h}^+ \\ h^- & \bar{h}^0 \end{pmatrix}$
$\mathbf{D} = (6, 1, 1) = D_3 \oplus \bar{D}_3$
$\Phi_m = (1, 1, 1), \quad m = 1, \dots, n_g + 1$





