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Computer modelling of single-stage Stirling-cycle cryocoolers

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1. INTRODUCTION

The Rutherford Appleton Laboratory (RAL) has been involved in the design and manufacture of single stage and two stage Stirling cycle coolers for space use for many years.[1]

The design of these machines can be optimised for different temperature regimes and heat loads by analysis of the thermodynamic cycle and losses involved. This can yield the optimal dimensions of the machine and indicate the effect of varying operating parameters such as displacer and piston strokes, filling pressure and frequency on performance.

Two computer models of Stirling cycle coolers are used at RAL to assist in cooler design and optimisation. The first of these analyses the cooler as a series of connected adiabatic and isothermal control volumes. This results in a calculation of the total refrigeration produced by a cooler of given geometry and displacements. The effect of the various loss mechanisms is then found. A second computer model is used to find the effect of incomplete heat transfer and finite heat capacity in the regenerator matrix.

2. THE THERMODYNAMIC CYCLE ANALYSIS

A representation of the cooler used in the thermodynamic cycle model is shown in Figure 1.

The order of the calculations is shown in Figure 2. The volume variations in the cooler are calculated from the specified geometry. An isothermal model internal to the program provides the start conditions for the adiabatic analysis.

The adiabatic model assumes that there is no heat transfer in the cylinders, all of the heat transfer is assumed to take place in the heat exchangers. From the start conditions provided by the isothermal analysis the program integrates a series of simultaneous linear differential equations for the mass of the gas in the cylinders and the pressure around the cycle. The cooling power and various other parameters are calculated before the process is repeated. When steady state conditions are established (usually five cycles are required) the mass flows in the cooler are calculated.

The mass flows are used in a separate program, described below, to calculate the regenerator loss. The pressure drop through the cooler is calculated and used to find reduction in pressure swing at the cold end which decreases the cooling power available.

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[1] T W Bradshaw, A H Orlowska, "Miniature closed cycle refrigerators", Proceedings of NATO Advisory Group for Aerospace Research and Development meeting on "Applications of superconductivity to Avionics", 7-9th May 1990, Bath, England, AGARD-CP-481.

Due to the high operating frequency an allowance for the phase shift in the pressure wave at the cold end, with respect to that in the compression space, is made based on the speed of sound in each element of the cooler.

The program is configured so that a range of displacer diameters and lengths can be covered in one computer run.

The assumptions and methodology are described in more detail in the following sections.

2.1 Isothermal cycle

The following assumptions are applied:-

a) No fluid flow dissipation ie:- the instantaneous pressure is the same throughout the system.

b) Steady state conditions are established.

c) All heat transfer occurs isothermally.

d) The mass of gas in the machine is a constant.

e) The working fluid is a perfect gas.

f) The kinetic energy of the gas is neglected.

The cooler is separated into five volumes with the following volumes, temperatures and masses of gas:

compression space Vc, Tex, Mc

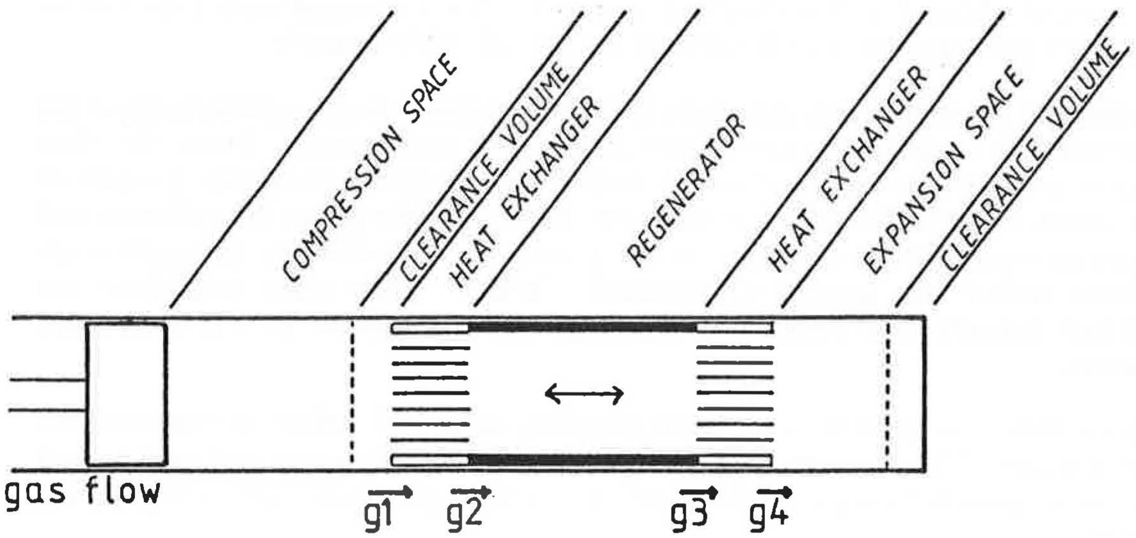

Figure 1. The series of control volumes used in the model

compression space heat exchanger Vcd, Tcx, Mcd
regenerator Vd, Td, Md
cold end heat exchanger Vxd, Tex, Mex
expansion space Ve, Te, Me

where

$$Td = \frac{(Tc - Te)}{\ln(Tc/Te)}$$

is the logarithmic mean of the temperature difference.

The volume variations for the split cycle are given by;

$$Vc(\theta) = Vclc + (Vc/2)(1 + \cos\theta) + (Ve/2)(1 - \cos(\theta + \alpha))$$

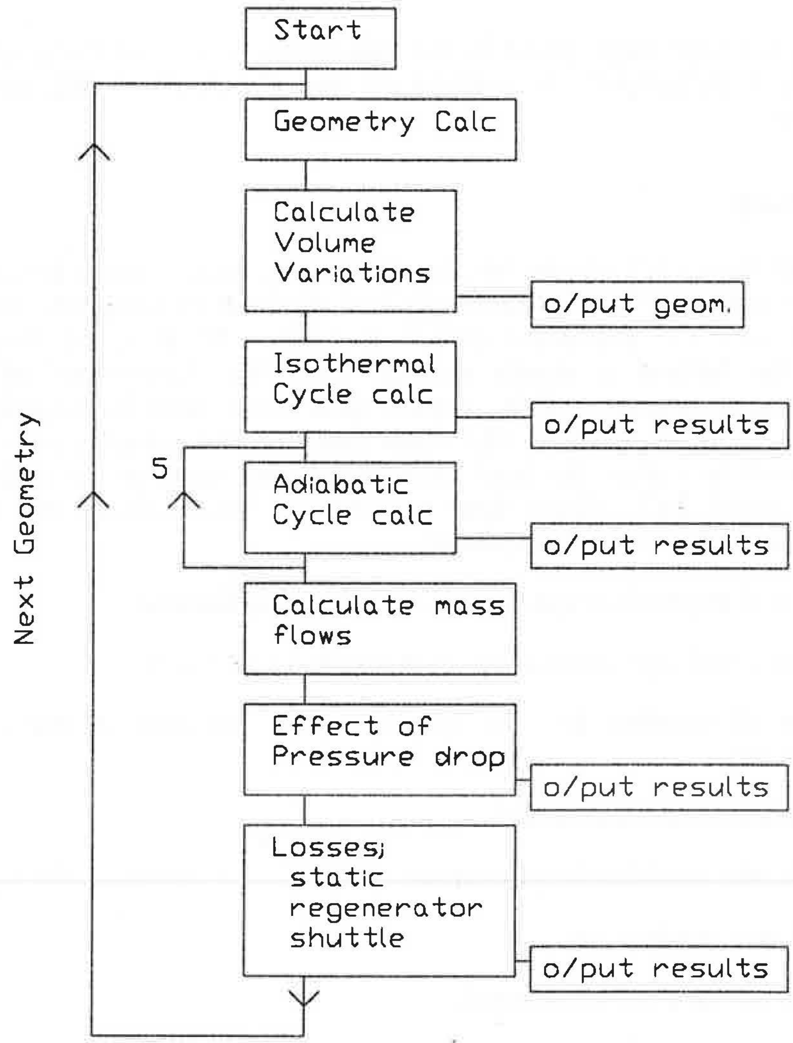

Figure 2. Flow diagram of the computer model

$$Ve(\theta) = Vcle + (Ve/2)(1 + \cos(\theta + \alpha))$$

Where θ is the "crank angle".

The temperature of the gas is specified by the wall temperature in each part of the cooler. The relation

$$PV = mRT$$

is used to calculate the pressure distribution.

To find the work done in the cycle and the heat extracted we need to calculate;

$$Q = \int P dVc(\theta)$$

$$Qe = \int P dVe(\theta)$$

$$W = Qc + Qe$$

By definition, all heat exchange takes place in the expansion or compression spaces so in this model the heat exchangers are redundant and the model yields perfect (100% Carnot) efficiency.

2.2 Ideal Adiabatic Model

In the isothermal model heat exchangers are, by definition, redundant whereas in an actual cooler most of the heat transfer takes place in the heat exchangers. At the frequency of operation of these miniature cryocoolers (35 - 55 Hz), the thermal penetration depth in the helium is much smaller than the dimensions of the compression and expansion spaces. This implies that little heat is transferred between the gas and the cylinder walls in the main body of the cylinder and that heat transfer is only possible within the heat exchangers and close to the cylinder walls. The adiabatic model only allows heat transfer to occur within the heat exchangers and makes the following assumptions;

a) The compression and expansion spaces are completely adiabatic.

b) The heat exchangers and regenerator are completely isothermal.

c) There is no fluid dissipation ie:- the instantaneous pressure is the same throughout the system.

d) Steady-state conditions are established.

e) The mass of gas in the machine is a constant.

f) The working fluid is a perfect gas.

g) The kinetic energy of the gas is neglected.

For the purpose of this study the connecting pipe from the compressor to the displacer is considered as part of the heat exchanger.

There are important differences between this model and the isothermal model. The temperature of the gas in the compression and expansion spaces is no longer a fixed quantity. Heat transfer takes place only in the heat exchangers and the regenerator.

A set of simultaneous linear differential equations for the change in pressure and mass of gas in the compression and expansion spaces are solved. They are integrated around the cycle with arbitrary starting conditions until a steady state is reached. A Runge-Kutta method is used with a one degree step interval to start the cycle from then on a fourth order Adams-Bashford method is used. From initial starting conditions the cycle converges within five iterations. The heat transferred in each of the heat exchangers and the regenerator can then be calculated. This provides a reliable test of convergence since the total heat transferred in the regenerator over one cycle tends towards zero as the solution converges.

The mass flows in the machine are calculated and give useful information about the gas dynamics in the machine.

2.3 Results from the Program

The program makes the following results available;

a) Cooling power

b) Input power

c) Static losses

d) Regenerator loss

e) Shuttle heat transfer

f) Mass flow

g) Volume variations

h) Temperature variations within the cylinders

i) Heat transferred in the regenerator and cylinders

These results can be output to files as necessary and plotting programs provide graphical output.

2.4 Adiabatic analysis including pressure drop

An order of magnitude calculation using the mass flows in the standard adiabatic analysis shows the pressure drop along the regenerator to be significant. This has the effect of reducing the pressure swing in the expansion space.

The pressure drop along the regenerator is calculated using the expressions given by Martini[2] which is a fit to the data presented by Kays and London.[3]

For an estimate of the maximum pressure drop that we can expect during the cycle the average of the peak mass flow is used. This is calculated from the adiabatic analysis of the cycle discussed earlier.

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[2] Martini W.R., "Stirling Engine Design Manual", NASA CR 135382

[3] Compact Heat Exchangers (2nd Ed.) W Kays and A L London, McGraw-Hill 1964.

The pressure drop along the cold end annular heat exchanger calculated using the friction factor obtained from data in Kays and London an annular duct assuming laminar flow. The transition between laminar and turbulent flow is not very well defined and is arbitrarily taken as the crossover point between the two expressions this occurs at a Reynolds number of approximately 2042.

The pressure drop along the connecting pipework is calculated using standard correlations.

These calculations allow the pressure distribution along the cooler to be calculated at any time interval. The PV integrals are then performed which give the revised cooling power.

2.5 Model Verification

The model has been verified in several ways;

a) The program code has been checked against models presented in the literature and is verified each time a change in the source code is made. To ease this operation the model can handle different volume variations including the split Stirling cycle and the classical configuration.

b) If the specific heat ratio is set to unity then the model becomes isothermal and the results compared to an analysis such as the Schmidt closed form analysis.

This model provides a wide range of information about the cooler geometry including the mass flow at various points, cooling power, input power, heat transferred in the heat exchangers, and pressure swing (see Figure 3 for typical results). This figure shows the mass flow at four positions in the cooler and the volumes in the compression and expansion spaces, designated g1...g4, as a function of 'crank angle'. They are the mass flow at the exit of the compression space, the exit of the first heat exchanger, the entrance to the cold end heat exchanger and the entrance to the expansion space respectively.

3. PARASITIC LOSSES

The losses in the cycle fall broadly into two groups, those that can be treated as pure heat loads on the cold end and need only to be subtracted from the gross cooling power, and those that introduce departures from the ideal adiabatic cycle.

The model assumes that the regenerator and pressure drop losses are perturbations on the ideal cycle and are calculated from the values of the mass flow derived from the program. The static, shuttle and regenerator losses are treated as parasitic heat loads and are subtracted directly from the gross cooling power.

The static heat loads on the cold end are found and a calculation of the shuttle heat transfer is made. The shuttle heat transfer is derived from the formula given by

Zimmerman[4] which has been found to give reasonable correlation with experimental results[5] .

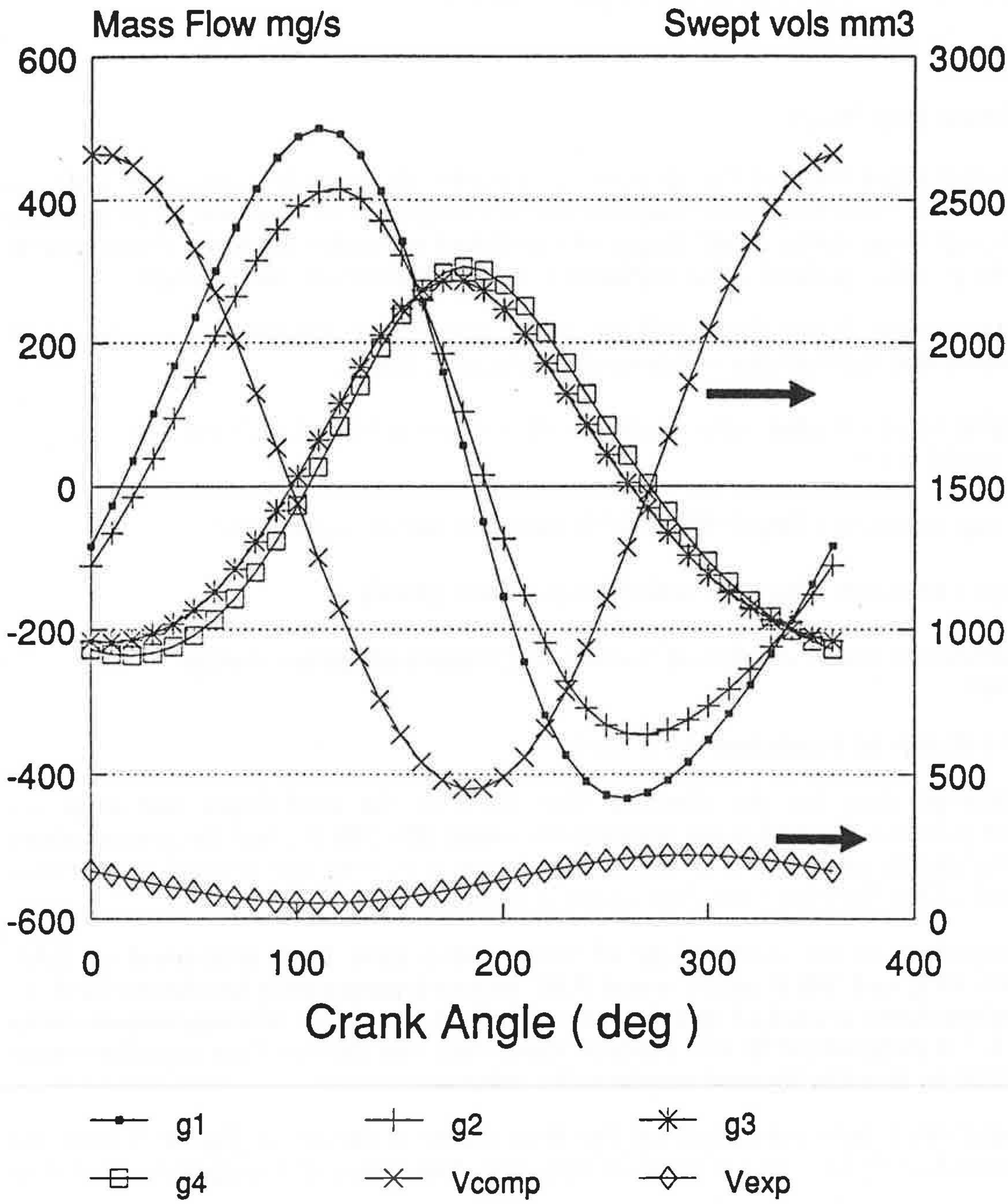

Figure 3. Mass flows and volume variations in the cooler

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[4] F.J Zimmerman and R.C. Longsworth,in "Advances in Cryogenic Engineering", Vol.16, Plenum Press, New York (1970), p342.

[5] A.H. Orlowska, Proc ICEC 11 (1986)

The various losses are subtracted from the gross cooling power to produce the available refrigeration.

The model has been applied to the BAe single stage cooler and the results compared to the measured performance. A correction to the temperature at which the modelling is performed was used in the model to allow for imperfect cold end heat transfer. This is described in detail later in the text.

3.1 Static heat losses

A detailed breakdown of the elements in a cooler displacer is required in order to calculate the static heat load. Measurements performed by Kotsubo et al[6] indicate conduction losses down a cold finger of a switched off cooler are lower than those in a working cooler probably due to changes in the temperature distribution.

The static losses in the cooler will be determined by the temperature gradient and the conduction through the components of the cold finger.

- an outer titanium alloy tube which has the copper cold end and contains the working gas

- a plastic displacer (Vespel SP3) which moves inside the outer tube

- an inner titanium alloy tube which supports the plastic

- a regenerator which consists of a stack of 250 mesh phosphor bronze gauzes

There will also be a radiation heat load.

Conductivity data for the titanium alloy used in the cold finger and displacer support tube was found for the temperature range 20 - 300 K.[7] but the contributions from the plastic and especially the regenerator matrix were not so easily calculated. A search of the literature for data on these elements was unsuccessful.

Measurements of the conductivity of Vespel SP-3 have been performed at RAL between 70 K and 300 K and a recent RAL research programme has determined the conduction down a stack of metal gauzes in helium at a range of temperatures down to 60 K.[8] A polynomial fit was made to these data and the resulting equations were integrated to give the thermal conductivity integrals.

The total static loss calculated for the BAe cooler is shown in Figure 4 with the measured data of Kotsubo et al for comparison. The warm end temperature is taken

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[6] V. Kotsubo, D.L. Johnson and R.G. Ross, Jr. "Cold-tip off-state conduction loss of miniature Stirling cycle cryocoolers" To be Published in Advances in Cryogenic Engineering Vol 36

[7] Purdue University, "TPRC Series - Thermophysical Properties of Matter" edited by Touloukian and Ho. Plenum.

[8] to be published

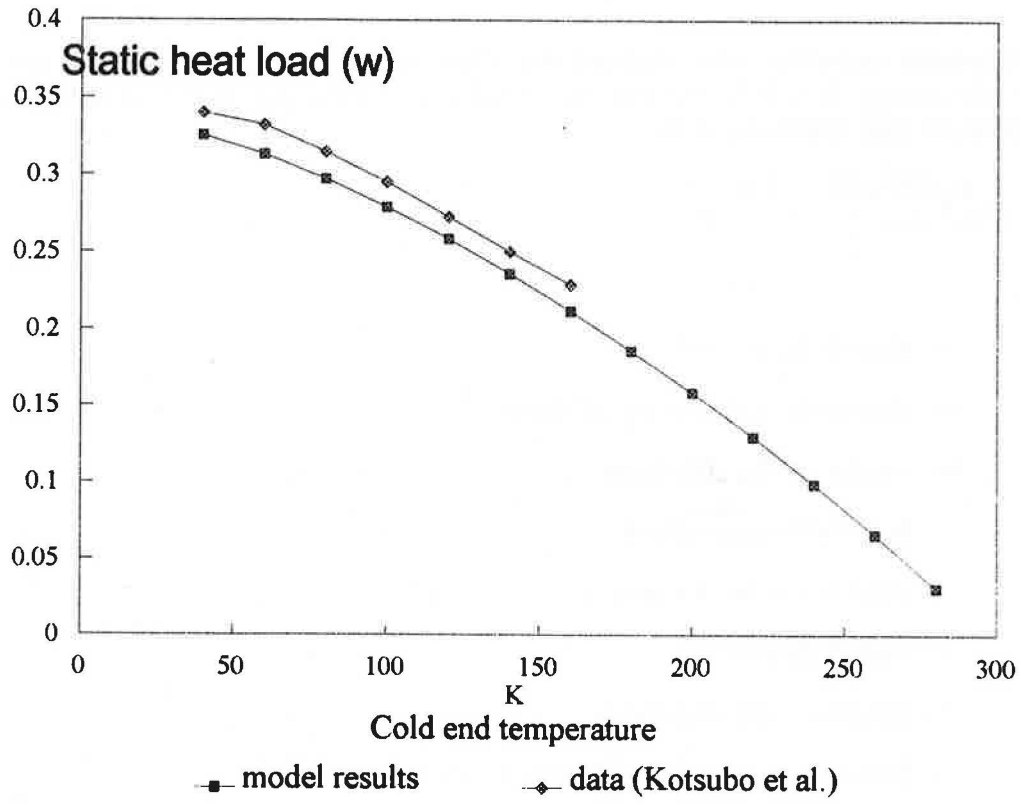

Figure 4. Conduction loss through BAe cold finger

as 295 K (the ambient temperature during the measurements of Kotsubo et al). The agreement between measured and calculated data is very good.

The accuracy of all of these calculations depends on the quality of the data used and the final figures should not be taken to be better than +/- 20%.

3.1.1 Regenerator Loss

The purpose of the regenerator is to act as a highly efficient heat exchanger and to admit gas to the cold end as near to the cold end temperature as possible. A convenient way of calculating the regenerator efficiency is to use Tiplers formula[9]. The mass flow rate used in these calculations was taken from the output of the adiabatic analysis program. This analysis was used within the adiabatic model to provide an estimate of the regenerator losses. The separate, more detailed, regenerator model is described below.

3.1.2 Shuttle Heat Transfer

There is a heat load on the cold end from energy transport due to the reciprocating motion of the displacer. In its mid-position there is no temperature difference between the displacer and the outer tube, however, when the displacer is at the top or the bottom of its stroke, there is a temperature difference and radial heat transfer takes place.

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[9] See for example Bretherton A, Granville, W H and Harness J B, Advances in Cryogenic Engineering, Vol 16, (1970), p333.

The following equation was derived by Zimmerman[10] and is for the condition where the energy flow is limited by the conduction of the gas in the annulus between the displacer and the outer tube.

$$Qs = C \frac{K_g D S^2 (Th - Tc)}{t} P$$

where

Qs	=	shuttle heat load
Kg	=	thermal conductivity of the gas
S	=	stroke of the displacer
Th	=	hot end temperature
Tc	=	cold end temperature
t	=	radial clearance
P	=	length of the displacer
C	=	factor due to type of displacer motion

4. REGENERATOR MODEL

The regenerator loss, dominant in cryocoolers at low temperatures, is one of the least amenable to theoretical calculation since it depends on knowledge of forced convection heat transfer between the working gas and the matrix in unsteady flow. Many computer programs have been written to model this loss, most making assumptions about the flow which are not applicable to conditions in small coolers operating at relatively high frequencies.

At RAL a computer program has been written to predict regenerator losses due to incomplete heat transfer and finite matrix heat capacity, using instantaneous mass flows and pressure data obtained from the adiabatic cooler model.

Regenerator loss measurements have been made in single stage coolers[11] and also in the coldest stage of two stage coolers. The losses predicted by this model are in good agreement with the experimental data.

4.1 Governing equations

Helium is used as the working fluid and is assumed to be a perfect gas with an equation of state given by PV=mRT and a constant specific heat but with

[10] Zimmerman, F.J. and Longsworth, R.C. Lafayette College PA and Air Products Inc., PA. Advances in Cryogenic Engineering vol. 16, 1970.

[11] Orlowska, A. H., and Davey, G. "Measurement of losses in a Stirling-cycle cooler" Cryogenics Vol 27 pp 645-651 (1987)

temperature dependent viscosity and conductivity. The matrix consists of gauze discs and has a temperature dependent specific heat.

The regenerator is divided into M cells (in the working model $M=50$ but it could take any value great enough to give reasonable accuracy but not so great as to require inordinate amounts of computer time) with an initial linear temperature gradient between the temperatures of the cold and hot ends of the regenerator. These two temperatures are assumed to be constant.

Initially the gas temperature in each cell is the same as the matrix temperature in that cell and the temperature of the gas at the boundary between two adjacent cells is taken as the average of the gas temperatures in those cells.

In each time interval the pressure changes by an amount calculated by the adiabatic cooler model and a quantity of gas enters the cell from the previous cell. This gas is at the temperature of the boundary between the cells. (For the first cell the mass input is taken from the adiabatic cooler model)

This results in a change in the mass of gas in the cell and in its temperature, and in the matrix temperature since a quantity of heat is transferred from the matrix to the gas (or vice versa).

The following equations are used to describe the mass flow of the gas and the heat flow between it and the matrix.

- Conservation of mass
- Equation of state for the gas ($PV=mRT$)
- Heat transfer between gas and matrix
- Conservation of energy for the matrix
- Conservation of energy for the gas (enthalpy flow, work done, heat transferred)

4.1.1 Solution of equations

The heat transfer coefficient can be found from a steady-state heat transfer correlation of the form

$Nu=f(Pr,Re)$

where Nu, Pr, and Re are the Nusselt, Prandtl, and Reynolds numbers respectively.

The heat transfer correlation is crucial to any regenerator model and can be one of the weakest points. Many studies of heat transfer in gauze disc matrices have been performed under various conditions, none of which accurately represent conditions found in a working miniature refrigerator. In this model a correlation is used which was found to give reasonable accuracy in measurements of regenerator losses in a single stage 80K cooler.

In this model a correlation is used which was proposed by Mikulin and Shevich[12] and found to give reasonable accuracy in measurements of regenerator losses in a single stage 80K cooler.[13]

Using this information the governing equations are solved and the mass of gas that enters the next cell is found. The program can then move on to repeat the calculations for the next cell of the regenerator. The program is cycled with the final gas and matrix temperatures from one cycle used as the initial values in the next.

5. THE EFFECT OF COLD END HEAT TRANSFER

It is known that when a heat load is applied to the cold end of a cooler the finite heat transfer leads to a temperature difference between the gas inside the cooler and the measured temperature on the cold tip. This ΔT will be a function of the heat load and may also be dependent on operating parameters such as displacer stroke, frequency, cold end temperature and filling pressure, since all will affect the gas dynamics inside the cold tip.

Modelling of the flow conditions and heat transfer is difficult and several attempts have been made to measure the ΔT both directly and indirectly.

5.1 Measurements of Cold end Heat Transfer

Direct measurements using differential thermocouples were made in the Department of Engineering Science at Oxford in 1985 and yielded a ΔT of 10 +/-2 K/W.

Measurements at RAL in 1983 of the temperature response to a step change in the heat load yielded a ΔT between 6 K/W and 10.2 K/W (for displacer strokes 3.5-3 mm). This was found to be a strong function of displacer stroke with a larger stroke giving the lower value.

Similar measurements at the Department of Atmospheric Physics gave ΔT s between 8.6 and 19.3 K/W (for displacer strokes 3-2.5 mm), again with lower temperature differences corresponding to higher displacer strokes.

Cold end geometry is important in calculation of this parameter and results of ΔT between 13.2 K/W and 23.9 K/W have been obtained with various cold end heat exchangers.

5.2 Effect on performance modelling

The large uncertainty in this data poses problems when this phenomenon is to be incorporated into models. For this reason it has been decided to assume a ΔT of 12 K/W which is in line with the data above.

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[12] Reported by Walker, G. in "Cryocoolers" Plenum Press (1983)

[13] A H Orlowska, G Davey, Cryogenics 1987

6. BAE COOLER MODELLING

The input data for this series of calculations was taken from BAe supplied drawings with a connecting pipe between the compressor and displacer of 300 mm. A compressor temperature of 299 K was used in early runs.

A typical data set is shown in Figure 6 on page 15.

6.1 Results

The results of the analysis are shown in Figure 5 on page 14. The figure shows the BAe experimental data. The horizontal error bars indicate the range of performance achieved with several different coolers of the same design at no applied heat load (base temperature), and with a heat load of 800 mwatts.

The results of the computer modelling are shown with and without the corrections to the model to allow for imperfect heat transfer in the cold end of the cooler. The vertical error bars are the results of the computer modelling with a 5% error on the amplitudes of the compressor and displacer motions.

The results of the computer modelling are in good agreement with the experimental data.

Following discussions with BAe the cooler models were run at an increased compressor temperature of 308 K which represents an increase of 15 deg. C above the laboratory temperature. The results can be seen in Figure 7 on page 16. There is an improvement in the fit to the experimental data.

7. REGENERATOR PRESSURE DROP CALCULATIONS

A simple model for the additional heat load imposed by the pressure drop through the regenerator has been added to the regenerator model. The correlation used for the pressure drop through the matrix has been verified against measurements performed at RAL. When applied to the BAe cooler the model predicted that the work required to overcome the pressure drop was 0.106 W. Previous measurements of this parameter[14] in a similar displacer indicated that the work done was around 0.09 W.

[14] A H Orlowska, 1985

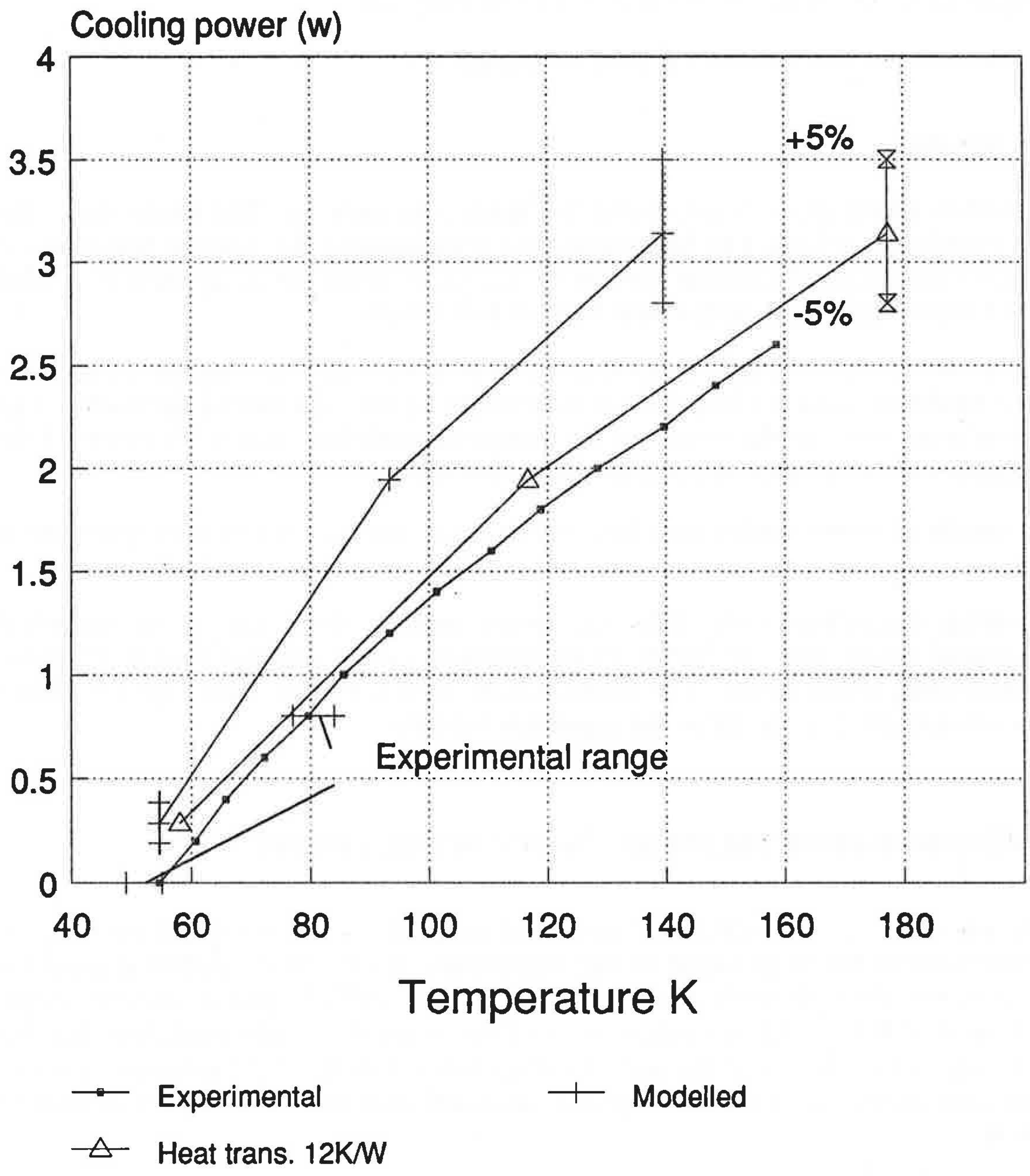

Figure 5. Computer modelling of BAe cooler performance.: The effect of allowing for cold end heat transfer in the model is shown together with 5% variation in compressor and displacer strokes.

```
15MAY91  10 45 40
 SPLIT CYCLE VOLUME VARIATIONS.
 Compressor stroke=    8.00[mm]
   TEX=   46.00      TCX=299.90        TD=135.43
    VC= 0.251E-05  VE= 0.204E-06   H= 0.175E-01
  PBAR= 0.122E+07 SPK=     1.667  CF= 0.144E+05    R=2078.0
  VCLC= 0.369E-06VCLE= 0.550E-07 VCD= 0.757E-06  VXD= 0.52160E-07

Regenerator and heat exchanger properties..
   Length=       0.048 flow area=   0.390E-04    por.=.73
wire dia.=    0.000040  Wire den=    8874.000   Visc2= 0.120E-04
    Acreg= 0.28496E-04        Ap= 0.50907E-01
Pipe dia.=    0.001500   length=       0.340   Viscl= 0.205E-04
    Clear=    0.000150     xlen=       0.005   Visc3= 0.633E-05
Carnot COP=      0.181     beta=      70.000    freq= 40.0
       VD= 0.13764E-05       A= 0.665E-11
  reg dia=    0.007050    Vreg=0.188545E-05     Vpr= 0.50907E-06
 reg mass=    0.004518
Comp str.=    0.008000Piston dia=    0.020000Max str.=    0.010000
Disp str.=    0.002600
-----------------------------------------------------
 1    1    RL=    48.3 DIA=    10.0mm  VE= 204.203E-09
   I    QCOM      QEXP       QW    % rel carn  QRT        PSW(Bar)
isothermal values....
    0 -0.45242  0.06939 -0.38302    100.00   0.000000     5.728
standard analysis....
    1 -0.62736  0.07431 -0.55305     74.16  -0.080571     6.699
    2 -0.59099  0.07622 -0.51476     81.73   0.038131     6.598
    3 -0.58520  0.07652 -0.50869     83.02  -0.005511     6.579
    4 -0.58433  0.07656 -0.50777     83.22  -0.004043     6.575
    5 -0.58421  0.07656 -0.50765     83.24  -0.001486     6.575
 Qc [w]=  -23.37 Qe=    3.06 Qw=  -20.31
with pressure drop....
    6 -0.58421  0.07666 -0.50756     83.36  -0.000623     6.575
 Qc [w]=  -23.37 Qe=    3.07 Qw=  -20.30
  Gbar= 0.21610E-03 PRDbar= 0.11444E+05
Temperature allowing for imperfect heat transfer= 58.77
Cooling power allowing for cold end phase error = 3.193    5deg
 TEX= 46.00
 QTO= 0.106  QPL= 0.072 QTI= 0.074
QUME= 0.149 QRAD= 0.008

1.4QR= 1.244
  QDP=-0.004
  QSH= 0.548
stat.= 0.408
total= 2.196 cooling power = 0.998
 Spring con=   25818.594 Moving mass=      0.409
```

Figure 6. A typical data set

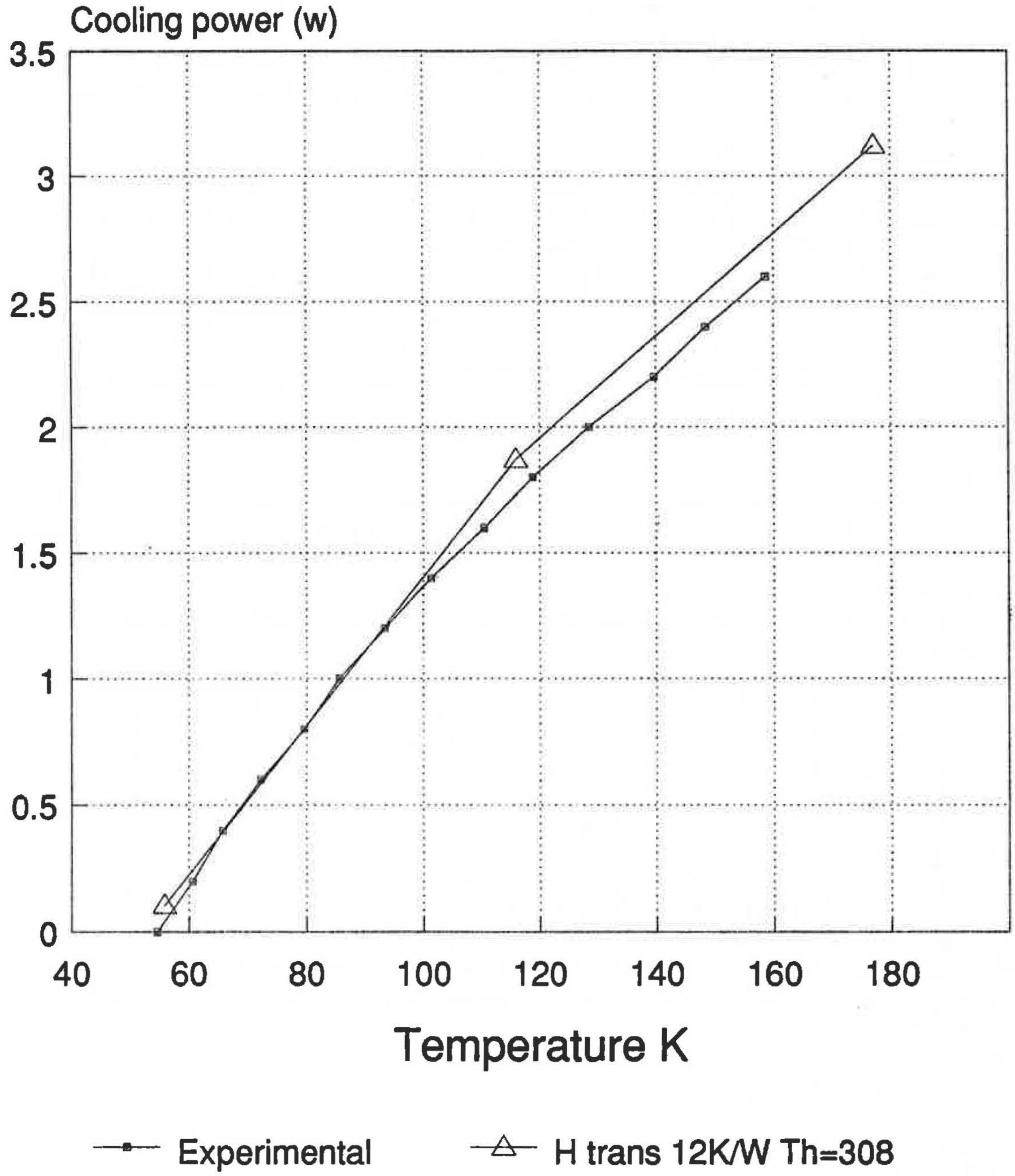

Figure 7. BAe cooler modelling at a temperature of 308 K