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A Comment on the Quark Mixing in the Supersymmetric $SU(4)_{\otimes}O(4)$ GUT Model

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A Comment on the Quark Mixing in the Supersymmetric SU(4)⊗O(4) GUT Model

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Abstract

The $SU(4) \otimes O(4)$ and the "flipped" $SU(5) \otimes U(1)$ models seem to be the only possible GUT's derivable from string theories with Kac-Moody level K=1. Naively, the $SU(4) \otimes O(4)$ model, at least in its minimal GUT version, is characterized by the lack of any mixing in the quark sector. In this "Comment" we show that, although some mixing may be generated as a consequence of large vacuum-expectation-values for the scalar partners of the right-handed neutrinos, it turns out to be too small by several orders of magnitude, in net contrast with our experimental information concerning the Cabibbo mixing. Our result, which therefore rules out the minimal $SU(4) \otimes O(4)$ GUT model, also applies to "flipped" $SU(5) \otimes U(1)$ in the case of the embedding in $SO(10)$.

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The understanding of the observed pattern of quark mixing is one of the problems that the Standard Model (SM), as well as most of the present Grand Unified Theories (GUT's), cannot solve satisfactorily . A particularly appealing solution would be the "calculation" of the elements of the mixing matrix (the so-called Cabibbo-Kobayashi-Maskawa matrix, parametrized by three angles and one phase in the case of three generations, or a single angle in the simple case of only two families) in terms of the observed quark masses, with results consistent with our present experimental information. This procedure has often been used as a method for testing the consistency of specific GUT models.

In this *comment* we shall study the quark mixing in the context of a supersymmetric GUT based on $SU(4) \otimes O(4)$. Our result will be that, in spite of its several nice features, the model is unable to account for the observed large quark mixing between the first two generations represented by the Cabibbo angle.

Recently, there has been a considerable effort in the study of the GUT models derivable as effective field theories from the superstring. In particular, two promising models have been singled out, having the nice feature of not requiring the presence of the adjoint or any other large self-conjugate Higgs representation (not allowed in models based on string theories with Kac-Moody level K=1), usually necessary for obtaining the correct symmetry breaking down to the Standard Model and for producing the so-called doublet-triplet mass splitting, essential for avoiding an unplausible fast proton decay mediated by the exchange of light color-triplet scalars. Both these models, which are based on the gauge groups "flipped" $SU(5) \otimes U(1)$ [1] and $SU(4) \otimes O(4)$ [2] (which is isomorphic to the left-right symmetric $SU(4) \otimes SU(2)_L \otimes SU(2)_R$), have been studied in detail in their simple GUT versions [1-6] as well as in their "string"-type of version [1,7,8]. In spite of many similarities, the two models lead to different relations among the masses of the various charged fermion sectors. In fact, while in the *flipped* model the quark and the charged-lepton masses are uncorrelated[1], arising from independent terms of the superpotential, in the case of $SU(4) \otimes O(4)$ one recovers, naively, the same mass relations between the down-type quarks and the charged leptons as in the standard (minimal) $SU(5)$ model, namely $m_{d_i} = m_{e_i}$ (i

[1] In the case of the $SO(10)$ embedding of the flipped model, however, one obtains certain constraints among the Yukawa couplings, which yield the same mass relations as in the $SU(4) \otimes O(4)$ model [4].

labelling the different generations), where the masses are the running masses at the GUT mass scale, $M_G = 10^{16}$ GeV. On the other hand, both models predict the equality (at M_G) of the up-quark mass matrix with the Dirac neutrino mass matrix, implying therefore the need of a seesaw-type of mechanism for the suppression of the neutrino masses. The detailed study of the possible seesaw scenarios which may be implemented in the two models, both in the supersymmetric and in the non-supersymmetric case, has been given in Refs.[2-4,6].

The mass relations between the d-type quarks and the charged leptons which can be derived in the $SU(4) \otimes O(4)$ model and which we have mentioned above, are consistent with the actual masses only for the third generation, corresponding to the famous relation $m_b \simeq 3m_\tau$, where the factor of 3 arises from the different mass renormalization in the two fermion sectors, down from M_G to a low-energy mass scale (say, $\mu = 1$ GeV). The success of this formula has been, indeed, one of the stronger indications in favour of the GUT idea. On the other hand, the masses of the fermions of the first two generations do not satisfy at all the above relations. If this disagreement may be understood for the first generation, in view of possible large effects of non-perturbative QCD on the light d-quark mass, this explanation cannot be used for the second generation, since $m_s \geq \Lambda_{QCD}$. A way for preserving the successful relation between m_b and m_τ, while improving the one involving m_s and m_μ, is the one we have recently suggested in Ref.[5], by assuming non-vanishing vacuum-expectation-values (VEV's) for the scalar partners of the right-handed (RH) neutrinos. In particular, we have found that in order to fit m_s and m_μ to their actual values, $< \nu_\mu^c >$ must be set at the GUT mass scale, M_G. Interestingly, this means that the gauge group $SU(4) \otimes O(4)$ is broken down to the SM not only by the Higgs supermultiplets $\mathbf{H(4,1,2)}$ and $\mathbf{\bar{H}(\bar{4},1,2)}$, but also by the RH matter supermultiplets $\mathbf{\bar{F}_i(\bar{4},1,2)}$, the three quantum numbers specifying the transformation properties under $SU(4) \otimes SU(2)_L \otimes SU(2)_R$. The consequences of this result on the possible structure of the neutrino mass spectra have been studied in detail in Ref.[6], where it was shown that sizeable (*i.e.*, non-negligibly small) masses could be obtained without introducing non-renormalizable terms [4], or considering the non-supersymmetric model [3,4] where radiative mechanisms for generating large Majorana masses for the RH neutrinos are possible à la Witten.

In this paper we shall investigate, in the context of the supersymmetric $SU(4) \otimes O(4)$

minimal GUT model, the effects that large VEV's for some of the RH sneutrinos may have on the quark mixing . In fact, since naively the model is characterized by a trivial CKM matrix equal to the identity, the hope is that the modification of the effective d-type mass matrix induced by such VEV's may generate the correct quark mixing, in agreement with the experimental information.

The superfield content of the model has been given in the Tables of Refs.[4,5], to which we refer for our notations. The matter superfields $\mathbf{F_i}(\mathbf{4},\mathbf{2},\mathbf{1})$ and $\bar{\mathbf{F}}_\mathbf{i}(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$ (i=1,...,n_g, where n_g is the number of generations) form the 16-dimensional spinorial representation of $SO(10)$. The Higgs superfields responsible for the symmetry breaking at the GUT mass scale, $\mathbf{H}(\mathbf{4},\mathbf{1},\mathbf{2})$ and $\bar{\mathbf{H}}(\bar{\mathbf{4}},\mathbf{1},\mathbf{2})$, whose VEV's $< \bar{\nu}^c_H >$ and $< \nu^c_H >$ ($\simeq M_G$) are for simplicity assumed to be equal[2], belong to two incomplete 16-dimensional representations of $SO(10)$. The electroweak Higgs supermultiplet $\mathbf{h}(\mathbf{1},\mathbf{2},\mathbf{2})$, whose two VEV's v_u and v_d are constrained by the condition $(v_u^2+v_d^2)^{1/2} \equiv v = 246$ GeV, forms with a $\mathbf{D}(\mathbf{6},\mathbf{1},\mathbf{1})$ a 10-dimensional representation of $SO(10)$. Furthermore, the model includes a certain number (=n_g+1) of singlet superfields $\phi_\mathbf{m}(\mathbf{1},\mathbf{1},\mathbf{1})$, of which only one has the scalar component developing a non-vanishing VEV of the order of the electroweak scale (for simplicity we have set $< \phi_0 > \equiv X = v$). These singlets are essential for generating a correct electroweak Higgs mixing and for producing a suitable seesaw suppression mechanism for the neutrino masses [2-4,6]. The model is then completely specified by the superpotential, taken to be the most general one satisfying the discrete symmetry $\bar{\mathbf{H}} \to -\bar{\mathbf{H}}$, which must be imposed in order to prevent heavy tree-level Majorana masses for the ordinary LH neutrinos. Following the Refs.[2-6], we shall write the superpotential in the following form:

$$\begin{aligned} \mathcal{W} =& \lambda_1^{ij} F_i \bar{F}_j h + \lambda_2^{im} \bar{F}_i H \phi_m + \lambda_3 H H D + \lambda_4 \bar{H} \bar{H} D + \lambda_5^m h h \phi_m \\ &+ \lambda_6^{mnq} \phi_m \phi_n \phi_q + \lambda_7^{ij} F_i F_j D + \lambda_8^{ij} \bar{F}_i \bar{F}_j D + \lambda_9^n D D \phi_n \, . \end{aligned} \tag{1}$$

In general, all Yukawa-type coupling constants λ_i are independent, but in the interesting case of the $SO(10)$ embedding of the model, one has the constraints: $\lambda_1 = \lambda_7 = \lambda_8$ and $\lambda_5 = \lambda_9$; here, in addition to these conditions, we shall also choose $\lambda_3 = \lambda_4$. After the spontaneous breaking of the gauge symmetry down to the SM, the only *"uneaten"* (or which do not acquire a large mass through the super-Higgs mechanism) fields belonging to

[2] In fact, their equality is implied by the minimization of the F and the D terms only in the flipped $SU(5) \otimes U(1)$ model.

$\mathbf{H}$ and $\bar{\mathbf{H}}$ are, in addition to a single combination of $\bar{\nu}^c_H$ and ν^c_H (see the Table in Ref.[5]), the $\bar{d}^c_H$ and the d^c_H, which mix with the ordinary down-type quarks and the color-triplets (D_3 and $\bar{D}_3$) belonging to $\mathbf{D(6,1,1)}$. At the subsequent electroweak symmetry breaking, with $\mathbf{h(1,2,2)}$ developing the VEV's v_u and v_d, the first term of the superpotential gives masses to all the ordinary fermions, resulting in the following relations (at the GUT mass scale) among the corresponding mass matrices:

$$M_u = \lambda_1 v_u = M_\nu^D \,, \qquad M_d = \lambda_1 v_d = M_e \,, \tag{2}$$

where we have suppressed the generation indices and where M_ν^D is the Dirac mass matrix for the neutrinos. However, eqs.(2) show that both the up- and the down-quark mass matrices are proportional to the same Yukawa coupling matrix λ_1, and are therefore proportional to each other. As a consequence, they are diagonalized by the same unitary transformation, implying the vanishing of any quark mixing and resulting in a trivial CKM matrix equal to the identity. Of course, this is in evident contrast with our experimental information, in particular with the quite large mixing observed between the first two families, as given by the Cabibbo angle, $\sin\theta_c \simeq 0.22$. In general, this result is not much affected by the presence of mixing terms between the d-quarks and the heavy fields D_3 and $\bar{d}^c_H$, essentially because the latter decouple effectively at low-energy. Nevertheless, important changes may occur in presence of non-vanishing (and large) VEV's for the RH scalar neutrinos, as discussed in Ref.[5], where their effects were used to modify the naive equality (at M_G) of the down-type quark and the charged lepton masses. Motivated by this fact, here we shall check if, in a simple two-generation model, the presence of some large $<\nu^c>$ may reproduce the observed Cabibbo mixing. Unfortunately, it will turn out that, even in the case where both $<\nu^c_e>$ and $<\nu^c_\mu>$ are of order M_G, the Cabibbo angle can never be larger than $\sim 10^{-7}$, too small by six orders of magnitude! This means that the present model, in its minimal GUT version [2-6] as well as in the various "string" inspired versions [7,8] which reduce to the former at low-energy, are ruled out by the present analysis.

Our starting point is the mass matrix for the d-type ($-1/3$ charged) color-triplet fermions which has been given in Ref.[5], and which follows direcly from the superpotential in eq.(1):

$$\mathcal{M}_d = \begin{array}{c} \\ \bar{d}_{Ri} \\ d^c_H \\ \bar{D}_3 \end{array} \begin{array}{c} \begin{array}{ccc} d_{Li} & \bar{d}^c_H & D_3 \end{array} \\ \begin{pmatrix} M_d & X & S \\ 0 & 0 & G \\ 0 & G & X_9 \end{pmatrix} \end{array} . \tag{3}$$

In this equation $M_d = \lambda_1 \, v_d$ is the *"naive"* mass matrix ($n_g \times n_g$) for the down-type quarks, $X_9 = \lambda_9 < \phi_0 >= \lambda_9 \, v$, $G \equiv \lambda_3 < \bar{\nu}^c_H >= \lambda_3 \, M_G$, X is a column-vector whose i-th element is given by $X_i = \lambda_{2,i} < \phi_0 > \simeq \lambda_{2,i} \, v$. The only term which is due to the presence of non-vanishing VEV's for the RH sneutrinos is the column-vector S whose i-th element is given by:

$$S_i = \sum_{j=1}^{n_g} \lambda_8^{ij} < \nu_j^c > , \tag{4}$$

where, under the assumption of the $SO(10)$ embedding of the model, $\lambda_8 = \lambda_1 \equiv M_u/v_u$, M_u being the up-quark mass matrix. The full mass matrix (3) is non-symmetric as a consequence of the discrete symmetry $\bar{\mathbf{H}} \to -\bar{\mathbf{H}}$ imposed on the superpotential and because only the scalar partners of the RH neutrinos can develop large VEV's (eventually as large as M_G); the VEV's of the LH sneutrinos being at most of the order of the electroweak mass scale, may give effects on the standard fermion masses which are suppressed by $< \nu > /M_G \leq 10^{-14}$. Since $\mathcal{M}_d$ is therefore non-hermitian[3], in order to obtain the corresponding (real and positive) eigen-masses, we must consider the matrix $\mathcal{M}_d^T \mathcal{M}_d$ (given in eq.(7) of Ref.[5]), whose eigenvalues are in fact the positive masses squared. Using the hierarchical structure of the elements of this matrix and neglecting the terms containing X_i ($\propto v$) with respect to those proportional to S_i ($\propto < \nu^c >$), we may then evaluate the *"effective"* mass matrix squared for the n_g lightest d-type colour-triplet fermions (see eq.(10) of Ref.[5]), which here we write in terms of M_u:

$$\mathcal{M}^2_{d(eff)} \simeq \cot^2\beta \; M_u^T \, \{ I - \frac{1}{|S|^2 + G^2} (SS^T) \} \, M_u \, , \tag{5}$$

where $|S| = (\sum_{i=1}^{n_g} |S_i|^2)^{1/2}$ and $\cot\beta \equiv v_d/v_u$. Now, using $\lambda_8 = \lambda_1 = M_u/v_u$, we can express the column-vector S as $S = M_u V/v_u$, where V is the column-vector of the VEV's of the n_g RH sneutrinos. This allows to write eq.(5) in the following form:

[3] Here, as in Refs.[4-6], we assume for simplicity all the mass matrices to be real.

$$\mathcal{M}^2_{d(eff)} \simeq \cot^2\beta \, (M_u^T M_u) \; \{I - \frac{1}{(|S|^2 + G^2)\, v_u^2} (V V^T) \; (M_u^T M_u)\} \, . \tag{6}$$

Since, as we have said, in the present paper we are mainly interested in "calculating" the Cabibbo angle, that is the mixing between the first two families, we shall take n_g=2. This simplification does not restrict in any important way the relevance of our results. In the case we are considering, therefore, V includes only the VEV's of the electron and the muon RH sneutrinos, and can be written as:

$$V = \begin{pmatrix} < \nu_e^c > \\ < \nu_\mu^c > \end{pmatrix} \equiv \begin{pmatrix} \epsilon \\ \mu \end{pmatrix} .$$

Then, working in the basis where M_u is diagonal:

$$M_u = \begin{pmatrix} m_u & 0 \\ 0 & m_c \end{pmatrix} , \tag{7}$$

from eq.(6) we obtain:

$$\mathcal{M}^2_{d(eff)} \simeq \cot^2\beta \begin{pmatrix} m_u^2 [1 - (\epsilon\, m_u)^2\, \Omega] & -\Omega\, \epsilon\, \mu\, (m_u m_c)^2 \\ -\Omega\, \epsilon\, \mu\, (m_u m_c)^2 & m_c^2 [1 - (\mu\, m_c)^2\, \Omega] \end{pmatrix} , \tag{8}$$

where we have set:

$$\frac{1}{\Omega} = (G^2 + |S|^2)\, v_u^2 = (\lambda_3\, M_G\, v_u)^2 + (m_u\, \epsilon)^2 + (m_c\, \mu)^2 \, . \tag{9}$$

Now, we notice that for a typical value of the Yukawa-type coupling, $\lambda_3 \sim 0.1$, even if both the VEV's ($\epsilon \equiv < \nu_e^c >$ and $\mu \equiv < \nu_\mu^c >$) take their largest possible value, $\sim M_G$, the second terms in the diagonal elements of the matrix in eq.(8) may be neglected; therefore, we can express our effective mass matrix squared for the two lightest d-type fermions (essentially, the d and the s quarks) as:

$$\mathcal{M}^2_{d(eff)} \simeq \cot^2\beta \begin{pmatrix} m_u^2 & -\Omega\, \epsilon\, \mu\, (m_u\, m_c)^2 \\ -\Omega\, \epsilon\, \mu\, (m_u\, m_c)^2 & m_c^2 \end{pmatrix} , \tag{10}$$

where the off-diagonal elements may be approximated by the following formula:

$$\Delta \equiv \Omega\, \epsilon\, \mu\, (m_u\, m_c)^2 \simeq \left(\frac{\epsilon\, \mu}{M_G^2}\right) \left(\frac{m_u\, m_c}{\lambda_3\, v}\right)^2 . \tag{11}$$

Since the mass matrix (10) has been obtained in the basis where the up-quark mass matrix is diagonal, its diagonalization gives directly the Cabibbo mixing angle:

$$\sin\theta_c \simeq \frac{\Delta}{m_c^2 - m_u^2} \simeq \left(\frac{\epsilon\,\mu}{M_G^2}\right)\left(\frac{m_u}{\lambda_3\,v}\right)^2 \simeq 4\cdot 10^{-8}\left(\frac{\epsilon\,\mu}{M_G^2}\right). \tag{12}$$

Therefore, since the VEV's ϵ and μ cannot be larger than the GUT mass scale, this result shows that the Cabibbo angle can never be larger than $\sim 10^{-7}$, against the experimental evidence. Apart from the smallness of the mixing obtainable in the context of the present model, eq.(12) shows that in order to "generate" a non-zero mixing, it is necessary that both $<\nu_e^c>$ and $<\nu_\mu^c>$ are non-vanishing.

In addition to the difficulty of reproducing the observed Cabibbo mixing, we notice that also the eigenvalues of $\mathcal{M}^2_{d(eff)}$ fail to reproduce the actual quark masses; in fact, we find:

$$m_1 \simeq \cot\beta\; m_u \leq m_u/30\,, \qquad m_2 \simeq \cot\beta\; m_c \leq m_c/30 \simeq 50\text{MeV}\,,$$

where we have used $\cot\beta \simeq m_b/m_t \leq 1/30$, corresponding to a *physical* top-quark mass of about 90 GeV. That m_1 is completely different than the actual value of m_d may not wonder too much, in view of the probable dominance of non-perturbative QCD contributions on the mass of the light quarks of the first generation ($m \ll \Lambda_{QCD}$). On the other hand, the result $m_2 \leq 50$ MeV, in net contrast with the experimental indication $m_s \geq 150$ MeV, appears more problematic, since in this case we expect much smaller non-perturbative effects ($m_s \geq \Lambda_{QCD}$). As a last comment we wish to point out that the results obtained in the present paper do not apply to the "flipped" $SU(5)\otimes U(1)$ model, except when it is embedded in $SO(10)$; in such a case, in fact, the *flipped* and the $SU(4)\otimes O(4)$ models turn out to be very similar, as shown in Ref.[4].

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