RAL-92-073

Science and Engineering Research Council

Rutherford Appleton Laboratory
Chilton DIDCOT Oxon OX11 0QX

RAL-92-073

On the Validity of Perturbative Evolution of Structure Functions from Low Q^2

J R Forshaw

December 1992

RAL 92-073

On the Validity of Perturbative Evolution of Structure Functions from Low Q^2

J.R.Forshaw

Particle Physics Department,
Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon,
England.

Abstract

The validity of using QCD perturbation theory to generate dynamically the parton distribution functions of hadrons, starting from a valencelike input at low Q^2, is discussed. In particular, we consider the prescription of Barone *et al* who evolve from $Q^2 = 0$, and that of Glück *et al* who start evolution from $Q^2 \simeq (2\Lambda_{QCD})^2$.

November 1992

1 Introduction

Recently, there has been significant interest in deriving parton (quark and gluon) distribution functions of hadrons by dynamically evolving from very low resolution scales [1,2]. The basic idea is to utilize the fact that, at low resolution, hadrons appear to be a collection of valence quarks. The details of the QCD dynamics allow one to generate the gluon and sea components which are known to be present at higher resolution scales. Such a program appears attractive since the input is reasonably well defined, and much of the work is entrusted to perturbative QCD (pQCD). Compare this with conventional approaches where one does not appeal to the valencelike structure of hadrons at low resolution and is therefore left with the task of constructing an input to the QCD evolution which must be extracted from the data, e.g. see refs.[3].

In this note, we wish to emphasise that great care must be taken when using pQCD evolution from low Q^2 low resolution scales, and that previous attempts are seriously flawed. In any perturbative calculation, one must be sure to sum all of the relevant diagrams, and which class of diagrams is relevant depends upon the kinematic regime under consideration. Often, it is not sufficient to work to leading order in the coupling, α_s, because there may well be large logarithmic factors present which seem to destroy the usefulness of α_s as an expansion parameter. The need to sum an infinite subset of the perturbative expansion is quite commonly encountered in pQCD calculations, in particular when calculating the dynamical evolution of the distribution functions. We first will briefly review the traditional calculation of the distribution functions, in particular for deeply inelastic scattering (DIS) where the spacelike virtuality of the photon ($-q^2 = Q^2$) provides the resolution scale.

In the parton model (where inter-parton correlations are negligible) the factorisation of the DIS cross section into a hard (perturbative) piece and a soft (non-perturbative) piece is straightforward – Bjorken scaling is predicted. As is well known, the violation of scaling is a consequence of QCD corrections to the basic parton model. The naive $\mathcal{O}(\alpha_s)$ corrections to the basic parton model come from the diagrams of fig.(1). However, a calculation of these diagrams reveals the presence of logarithms $\sim \ln(Q^2/\mu^2)$ (for fixed α_s), where the scale μ^2 is introduced to provide an infra-red cutoff. For large Q^2, the presence of terms $\mathcal{O}(\alpha_s \ln Q^2)$ seems to destroy the validity of a perturbative expansion. Fortunately, we are able to sum up the infinity of diagrams which possess a logarithm for each α_s. In an axial gauge, the contributors to this sum are the ladder diagrams, e.g. see fig.(2). We are able to relate the distribution functions at some scale Q^2 to their value at another scale Q_0^2. Our ignorance regarding the soft physics is contained in the input at Q_0^2. The choice of Q_0^2 must be sufficiently large to ensure the validity of the subsequent evolution procedure. In the language of the parton model, it is the Dokshitzer, Gribov, Lipatov, Altarelli and Parisi (DGLAP) evolution equations which perform this summation [4]. In terms of the light-cone operator product expansion (OPE), this summation is performed via the renormalisation group equation, which relates the Wilson coefficients at different values of Q^2 (and hence the moments of the structure functions) [5].

One might attempt to start the pQCD evolution from some low resolution scale: care must be taken. As one moves to lower scales, the presence of non-leading logarithmic terms will be felt more and more, as will higher-twist terms. Eventually, as $Q^2 \to \Lambda_{QCD}^2$, pQCD will breakdown as a meaningful expansion. In the language of the OPE, the light-cone expansion becomes less useful as Q^2 falls, since the dominant contribution is no longer on the light cone. In the next section, we concentrate on the parton model picture of pQCD evolution and discuss how one expects the DGLAP equations to fail at low Q^2. We discuss the modifications to DGLAP evolution advocated by Barone, Genovese, Nikolaev, Predazzi and Zakharov (BGNPZ), who claim to generate the parton content of hadrons by evolving from $Q^2 = 0$ [1]. We conclude that significantly more work is needed before one can claim to have even a reasonable phenomenological model of evolution from $Q^2 = 0$. We also comment on the procedure of Glück, Reya and Vogt (GRV), who evolve from $Q_0^2 \simeq 0.3$ GeV^2 [2].

2 QCD Evolution

Let us show how the summation of leading logs is performed. Consider the tree level process shown in fig.(1), where a quark from the parent hadron radiates a real gluon. As is well known, one encounters singularities in the cross section which must be regularised by taking into account the virtual corrections of fig.(1). The final result is renormalisation scheme dependent, it is leads to a modified quark distribution function given by:

$$
\begin{aligned}
q(x,Q^2) &= q(x) + \frac{2\alpha_s}{3\pi}\int_x^1 \frac{dy}{y} q(y) \left[\left(\frac{1+z^2}{1-z}\right)_+ \ln\frac{Q^2}{m_g^2} \right. \\
&+ (1+z^2)\left(\frac{\ln(1-z)}{1-z}\right)_+ - 2\frac{1+z^2}{1-z}\ln z \\
&- \left. \frac{3}{2(1-z)_+} + 4z + 1 - \left(\frac{2\pi^2}{3} + \frac{3}{4}\right)\delta(1-z) \right] . \qquad (1)
\end{aligned}
$$

The conventional 'plus prescription' is used to describe the effect of the virtual graphs and the non-logarithmic terms are determined in the massive gluon regularisation scheme. The quark masses are neglected.

As the gluon mass vanishes, we have a logarithmic divergence. This can be absorbed into a redefinition of the input, i.e. $q(y) \rightarrow q(y, \mu^2)$ where μ^2 is some factorisation scale. The perturbative expansion is only valid if Q^2 is sufficiently large, i.e. it is usual to insist that $Q^2 \gg \Lambda^2_{QCD}$. The presence of $\ln Q^2$ terms indicates that we should treat all terms which are $\mathcal{O}((\alpha_s \ln Q^2)^n)$ on an equal footing. They should be summed to ensure sensible results. Performing this summation, and neglecting all those terms which do not lie within the LL approximation leads to the DGLAP equations [4]:

$$\frac{\partial q_i(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}(P_{qq}(x/y)q_i(y,Q^2) + P_{qg}(x/y)g(y,Q^2)), \quad (2)$$

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}(\sum_{j=1}^{2n_f} P_{gq}(x/y)q_j(y,Q^2) + P_{gg}(x/y)g(y,Q^2)). \quad (3)$$

The splitting functions, P_{ij}, determine the probability for radiating a parton of type i from a parton of type j. For the process we considered, the LL form for P_{qq} is

$$P_{qq}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)_+ . \quad (4)$$

The strong ordering of transverse momenta is inherent in these equations, and is the approximation which results in selecting the $\ln Q^2$ terms which are essential for large Q^2, i.e.

$$k^2_{Ti} \gg k^2_{Tj} \quad (5)$$

is assumed. If one calculates the splitting functions to leading order (LO), then one is selecting all terms which have one logarithm for each α_s, this is the leading logarithmic (LL) approximation. A next-to-leading order (NLO) calculation of the splitting functions would result in the inclusion of the next-to-leading logarithmic (NLL) terms, i.e. those which are $\mathcal{O}(\alpha_s^n \ln^{n-1} Q^2)$. An example of a diagram which contributes to the quark structure function in the NLL approximation is shown in fig.(3).

It is clear that as Q^2 falls, the DGLAP equations run into serious difficulties. BGNPZ attempt to modify the evolution, so that it remains finite all the way down to $Q^2 = 0$. Let us outline their modifications. Note that we do not simply reproduce their prescription, rather we present it what we believe to be a more transparent way.

By appealing to the work of Gribov [6], they do not permit the coupling to become infinite as $Q^2 \to 0$. Rather, they introduce some low momentum scale which causes the coupling to freeze at low Q^2, i.e. they replace the leading order coupling with

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln((Q^2 + k_0^2)/\Lambda_{QCD}^2)}. \tag{6}$$

The scale k_0^2 is fixed by the requirement that it leads to the experimentally observed pion-nucleon total cross section, i.e. $k_0 \approx 0.44$ GeV. In this case, α_s/π remains small enough that perturbation theory may hopefully still apply.

The inclusion of quark masses is also necessary as $Q^2 \to 0$, as is the inclusion of a gluon mass (which serves the purpose of regularizing the gluon propagator, and confining the gluons). These are physical masses which determine the scale μ^2 in the $\ln(Q^2/\mu^2)$ factor. In this way, they avoid pushing the physics below $\sim \Lambda_{QCD}$ into the definition of the input.

To simplify things, it is assumed that one need only consider the radiation of gluons from quarks, i.e. the splitting functions P_{gg} and P_{qg} are neglected. This will be valid providing the gluon distribution function is sufficiently small, which will be the case for not-too-small x.

Since partons which are radiated with very low transverse momenta occupy a large transverse region of configuration space, it is possible that interference terms, like the one in fig.(4) may become important. To this end BGNPZ introduce a factor which is related to the two-quark form factor of the valencelike hadron. This factor is very powerful in regularizing the DGLAP kernel as $Q^2 \to 0$.

With the above modifications and simplifications in mind, the BGNPZ prescription corresponds to using the following evolution equations:

$$\frac{\partial q_i(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\tilde{P}_{qq}(x/y)\, q_i(y,Q^2) \tag{7}$$

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\sum_{j=1}^{n_f}\tilde{P}_{gq}(x/y)\, q_j(y,Q^2). \tag{8}$$

The freezing of α_s is understood to be operative and the modified splitting functions are:

$$\tilde{P}_{gq}(x) = V(x,Q^2)\frac{4Q^2}{3}\frac{\{[1+(1-x)^2]Q^2 + x^4 m_q^2\}}{x[Q^2+(1-x)m_g^2 + x^2 m_q^2]^2}, \tag{9}$$

$$\tilde{P}_{qq}(x) = \tilde{P}_{gq}(1-x). \tag{10}$$

The ggN vertex function is introduced to incorporate destructive interference terms, i.e. long wavelength partons probe the colour singlet hadron and hence decouple, it is given by

$$V(x,Q^2) = 1 - \exp\left(-\frac{R_{ch}^2}{2}\frac{Q^2 + x^2 m_q^2}{1-x}\right), \tag{11}$$

where R_{ch} is the charge radius of the nucleon ($\sim$ 0.8 fm).

Evolution is performed using the above equations starting from $Q^2 = 0$ assuming the nucleon to consist of three valence quarks only, i.e. their input valence quark distribution is determined by the three-quark light-cone wavefunction via

$$\begin{aligned} q_i(x) &= \int d^2\mathbf{k}_{t1} d^2\mathbf{k}_{t2} d^2\mathbf{k}_{t3}\, \delta(\textstyle\sum \mathbf{k}_{ti}) \\ &\times \int \frac{dx_2 dx_3}{x x_2 x_3}\delta(1-x-x_2-x_3)\left|\Psi(x_i,\mathbf{k}_{Ti}^2)\right|. \end{aligned} \tag{12}$$

They conclude that their results are relatively insensitive to the choice of wavefunction, making both Gaussian and dipole ansätze. Clearly the attraction of this approach is that the distribution functions appear to be totally calculable in pQCD. The inherent dependence upon the nucleon size is contained in the initial wavefunction, and is the only non-perturbative parameter needed.

Of course, for high enough Q^2, one must regain the traditional DGLAP equations. The P_{qg} and P_{gg} splitting functions are turned on at $Q^2 = 0.5$ GeV^2, where they expect the ggN vertex function to be close enough to unity and neglect of the quark and gluon masses to be justified.

In the original paper, the QCD evolution is not presented in a way that is quite so analogous to DGLAP evolution as the description above. Using the above description of the BGNPZ model, it becomes evident that a number of serious problems arise.

Inherent in the DGLAP approach, and the BGNPZ modification, is the assumption of strong ordering in transverse momenta. There is no justification in making this assumption if Q^2 is small, since the LL approximation is no longer a good one. The evolution kernel should depend upon the transverse momentum of the radiating parton, as well as on the radiated parton.

An example of an evolution equation which does not make the strong ordering assumption is the Balitsky, Fadin, Kuraev and Lipatov (BFKL) equation, which enables one to sum the diagrams relevant in the small x domain of QCD [7]. We emphasise that the construction of an evolution equation necessitates that one is able to: (1) classify the set of diagrams which need to be summed, and (2) derive those diagrams using basic building blocks (which determine the evolution kernel). The BFKL equation is designed to operate in the small-x region, and the presence of large logarithmic terms in $1/x$ (which can be classified) necessitates the construction of an evolution equation which can be expected to sum the dominant terms in the perturbative series. The BFKL equation has the structure:

$$\frac{\partial F(x,k^2)}{\partial \ln(1/x)} = \int dl^2 K(k^2,l^2) F(x,l^2), \tag{13}$$

where $F(x,k^2)$ must be integrated over k^2 to determine the gluon structure function.

Away from small x, we expect the appropriate set of evolution equations to be of the form:

$$F_i(x,k^2) = \int dl^2 dy\, K_{ij}(k^2,l^2,x,y)F_j(y,l^2). \tag{14}$$

Since there are no large logarithms to sum we have no idea which set of diagrams ought to be considered in deriving the kernel. The BGNPZ prescription amounts to summing a rather arbitrary subset of diagrams, i.e. at low Q^2 there is no reason to single out those diagrams which are within the LL approximation.

So, in the absence of any large logarithmic factors we are unable to single out any particular subset of the perturbation series and have no real hope of constructing a set of equations of the form determined in eqn.(14). To be consistent therefore, we ought to use α_s as the expansion parameter. The inclusion of the non-logarithmic terms (in eqn.(1) for example) is now imperative, for they are no longer negligible relative to the $\ln(Q^2/\mu^2)$ term. Let us make this more explicit. Ignoring the factor $V(x,Q^2)$ (and the running of α_s), the BGNPZ prescription gives, for the quark distribution function, logarithmic terms which are of the form

$$\ln\left(\frac{Q^2}{m_g^2} - 1\right)$$

and

$$\ln\left(\frac{Q^2}{m_q^2} - 1\right)$$

as the argument of the splitting function tends to zero and one respectively. This is a direct consequence of assuming the strong ordering of momenta, i.e. one integrates the quark virtuality over the range $0 < k_q^2 < Q^2$. The true limits lead to a different logarithmic variation of the structure function, as expressed in eqn.(1).

Thus for the BGNPZ prescription to make any sense one should abandon the strong ordering assumption and keep all terms in the splitting function calculations, using α_s as the expansion parameter. We no longer know how to derive the evolution kernel. It should be recognised that there exist large logarithms in $(1-x)$, which should be summed in order to ensure sensible behaviour as $x \to 1$.

Compounding the problems further, since α_s is so large we expect (so far uncalculated) NLO contributions to be significant. This point was realised in the slightly different case of LL and NLL evolution by GRV [2]. They emphasised the importance of considering NLL corrections when evolving from $\alpha_s \simeq 0.9$.

All our discussions so far have been confined to leading-twist processes. There are also higher-twist (HT) contributions (fig.(5)), which will depend upon the multi-parton distribution functions. There is no reason to neglect HT corrections at low Q^2, and it seems reasonable to expect that their inclusion would lead to an enhancement of the momentum carried by the u quarks relative to the d quarks within the proton, (i.e. uu pairs couple with spin-1, and ud pairs with spin-0 or spin-1, assuming a completely flavor symmetric quark distribution at some scale, then higher-twist corrections result in a lifting of the degeneracy of the spin-1 and spin-0 states within the proton. The higher level is the spin-1 state and it follows that the flavor symmetry is broken with u quarks carrying a larger fraction of the proton energy than one might naively expect [8]). Thus, even to first order in α_s, the inclusion of HT terms seems a necessary supplement to the BGNPZ approach.

We have so far emphasised the technical difficulties which one encounters when attempting to evolve from low Q^2 (especially $Q^2 \sim 0$). There is also a more fundamental difficulty, within the modified pQCD approach of BGNPZ, which is concerned with the absence of any dynamical scale serving delineate asymptotic freedom from confinement. As a clear example, consider the following discussion.

In the case of the photon structure function, it is reasonably well established by experiment that the photon (structure function) at low Q^2 resembles (that of) the ρ^0 (up to factors of α_{em}) [9]. This leads to the vector meson dominance hypothesis.

Physically, one can understand such an effect in terms of non-perturbative QCD. If the photon radiates a low-p_T $q\bar{q}$ pair then gluon emission is favoured by the largeness of the coupling ($\alpha_s(p_T^2)$) and the pair bind non-perturbatively to form a vector meson. In the BGNPZ model, it is perfectly reasonable to emit a gluon from a valence quark with a low p_T (i.e. compared with the p_T of the $q\bar{q}$ pair discussed in the context of the photon). However, it is assumed that no strong binding occurs subsequently between the gluon and valence quark, which would appear to be in contradiction with the existence of a vector meson contribution to the photon structure function.

The resolution of this paradox could be provided if one assumes that the non-perturbative physics is added, by hand, at the outset. It is unlikely that the BGNPZ modified perturbation theory, with non-perturbative physics added independently is equivalent to traditional QCD, where the onset of non-perturbative physics is signalled as the dynamical scale Q^2 tends to Λ_{QCD}. We point out that the work of Gribov is intended to account for confinement within a QCD-like framework – it is not simply manifest by freezing the coupling [6].

To conclude, let us say a few words on the approach of GRV [2]. Since they start evolution at $Q^2 \simeq 4\Lambda_{QCD}^2$, the LL approximation may well be useful. Indeed the dominance of the leading logarithmic terms is supported by the NLL calculation, which (although seen to be significant) results in a small correction to the LL result (for the structure function F_2). However, the fact that the data seem to indicate the onset of suppression due to the non-pertubative form factor

$$\left(\frac{Q^2}{Q^2+\nu^2}\right)^\lambda$$

for Q^2 as high as 1 GeV^2 is worrying, and may well signal the importance of HT effects below this Q^2. This should not be surprising, since a conservative choice for ν^2 would be 0.3 GeV^2 and the Regge intercept (λ) is 1/2 for valence quarks, giving a suppression factor of (at least) 0.9 at $Q^2 = 1$ GeV^2, falling to (at least) 0.7 at $Q^2 = 0.3$ GeV^2.

It may well be that the GRV approach is unreasonable for $Q^2 < 1$ GeV^2 and is only designed to produce a structure function which fits the data at $Q^2 \simeq 1$ GeV^2 (and hence beyond). If this is the case then one is left with one of two conclusions. Firstly, it may be that, through a judicious choice of (valencelike) input, one is able to fit the high-Q^2 data more-or-less by accident (if this is the case no benefit over more traditional structure function analyses can be claimed). Secondly, given the clear importance of the form factor suppression at low Q^2, one must conclude that the higher-twist terms are effectively de-coupled from the leading-logarithmic leading-twist terms, the origin of the de-coupling would then need to be explained.

Finally, although GRV claim to make serious small-x predictions we feel this to be wholly unjustified. The presence of large logs in $1/x$ cannot be ignored in a perturbative analysis and one must therefore use the BFKL equation (with appropriate shadowing corrections [10]). The small x regime of QCD is a subject of much controversy, and we await the data which will soon come from HERA to clarify the situation.

Acknowledgments

I should like to thank Frank Close and Dick Roberts for useful discussions.

References

[1] Barone *et al*, Torino preprints DFTT 14/92 (to appear in Z.Phys.C); DFTT 8/92 (to appear in Int.J.Mod.Phys.A).

[2] M.Glück, E.Reya, A.Vogt, Z.Phys. **C48**, 471 (1990); **C53**, 127 (1992).

[3] P.N.Harriman *et al*, Phys.Rev. **D42**, 798 (1990); J.F.Owens, Phys.Lett. **B266**, 126 (1991).

[4] L.N.Lipatov, Sov.J.Nucl.Phys. **20**, 94 (1975); V.N.Gribov and L.N.Lipatov, Sov.J.Nucl.Phys. **15**, 438 (1972); Y.Dokshitzer, Sov.Phys.JETP **46**, 641 (1977); G.Altarelli and G.Parisi, Nucl.Phys. **B126**, 298 (1977).

[5] H.Georgi and H.D.Politzer, Phys.Rev. **D9**, 416 (1974); D.J.Gross and F.Wilczek, Phys.Rev. **D9**, 980 (1974).

[6] V.N.Gribov, Lund preprint LU TP 91-7 (1991).

[7] E.A.Kuraev, L.N.Lipatov and V.S.Fadin, Sov.Phys.JETP **45**, 199 (1977); Ya.Ya.Balitsky and L.N.Lipatov, Sov.J.Nucl.Phys. **28**, 822 (1978).

[8] F.E.Close, 'An Introduction to Quarks and Partons', Academic Press, London (1979).

[9] Ch.Berger and W.Wagner, Phys.Rep. **146**, 1 (1986).

[10] L.V.Gribov, E.M.Levin and M.G.Ryskin, Phys.Rep. **100**, 1 (1983); J.Kwieciǹski, Z.Phys. **C29**, 147 (1985); A.H.Mueller and J.Qiu, Nucl.Phys. **B268**, 427 (1986).

FIGURE CAPTIONS

Figure 1 : Lowest order tree level amplitudes which contribute to the quark-to-quark splitting function, and the virtual graphs which regularise the $x \to 1$ singularities.

Figure 2 : A typical ladder graph, of the type that must be summed in the leading log approximation.

Figure 3 : A typical contribution which must be considered in the next-to-leading log approximation.

Figure 4 : Interference term between gluon distribution function amplitudes. The gluons originate from different quarks.

Figure 5 : Higher-twist contribution, the calculation of which necessitates an understanding of the diquark distribution function.

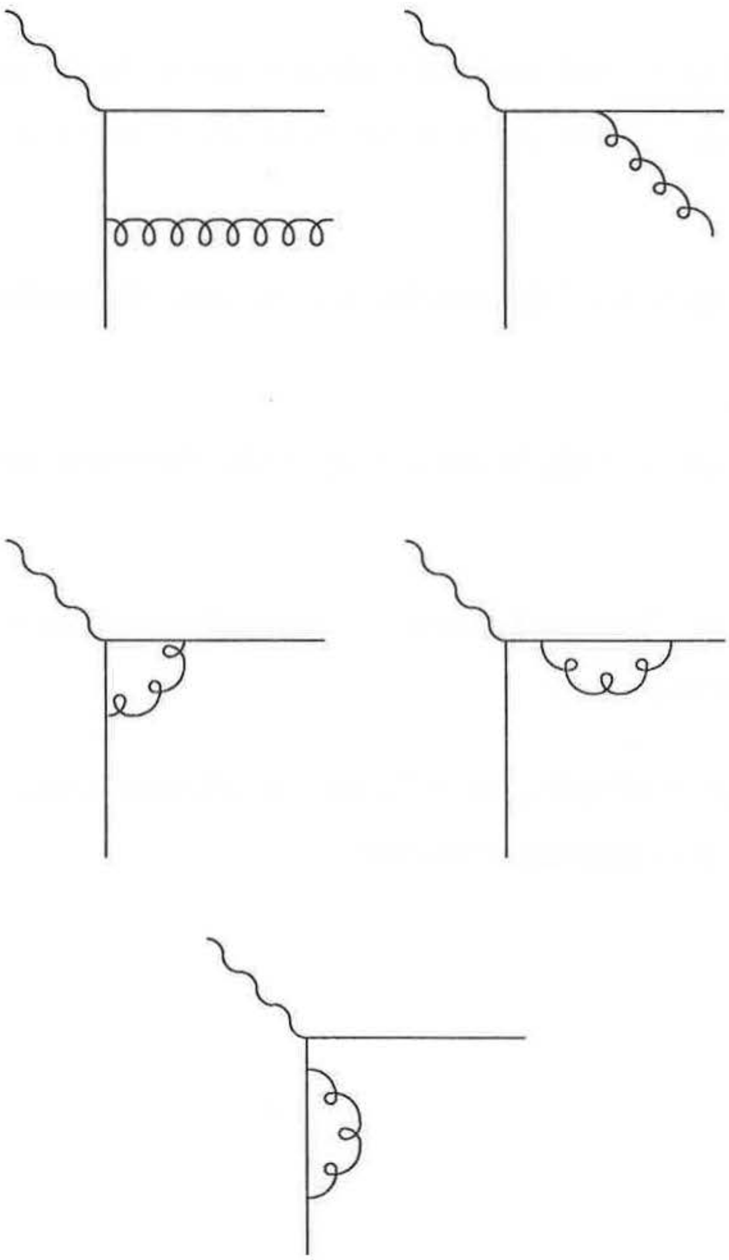

Figure 1

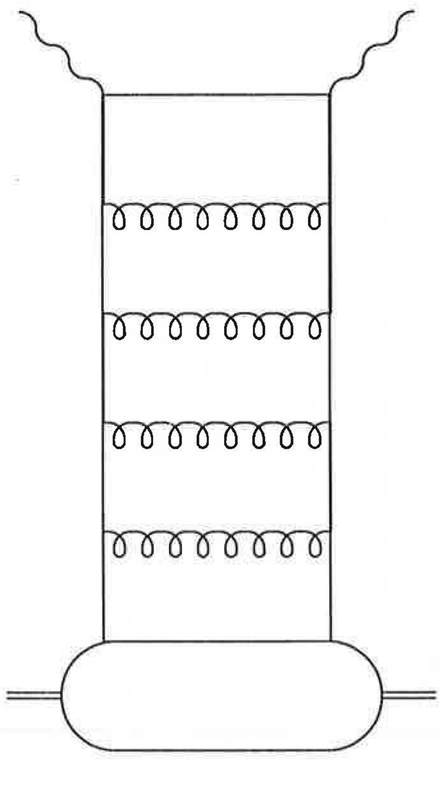

Figure 2

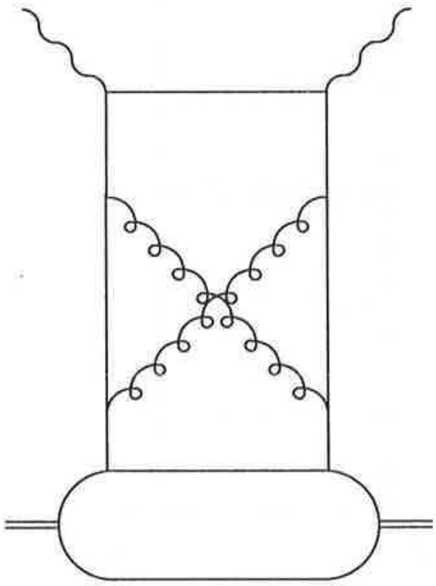

Figure 3

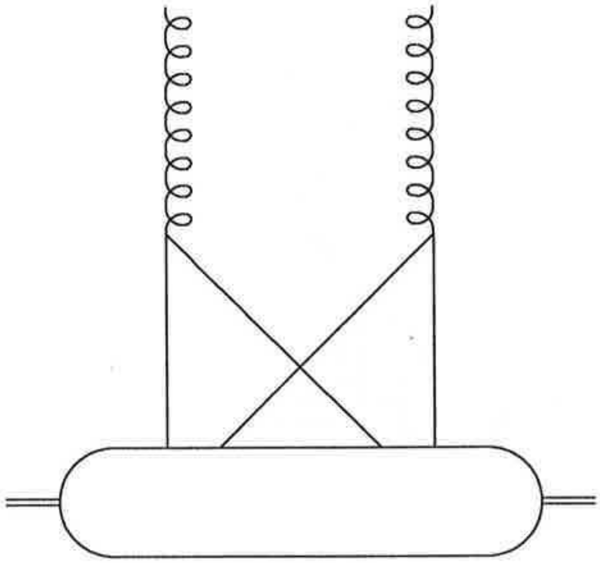

Figure 4

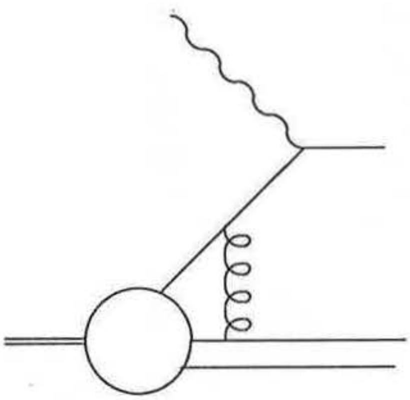

Figure 5