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Finite Amplitude Light Bullets

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Abstract

The generalized Schrödinger equation with the Kerr and saturation nonlinearities is used to investigate the filamentation instability of arbitrarily large amplitude optical pulses in a nonlinear medium. The existence of finite amplitude optical clumps (optical solitons or light bullets) is demonstrated. Conditions under which the filamentation instability and light bullets arise, are presented.

In the past, a number of authors[1-3] have investigated the modulation and filamentation instabilities as well as self-focusing of optical pulses in a homogeneous nonlinear media. In order to study the nonlinear propagation of optical pulses, the cubic Schrödinger equation[4] has been often used. The cubic Schrödinger equation contains the Kerr nonlinearity which arises from the modification of the refractive index due to the optical pulse intensity. When the pulse dispersion is exactly balanced by the Kerr nonlinearity, we have the possibility of the optical solitons[4] (light bullets[5]) or a periodic sequency of optical clumps. It is thought that the latter are the final state of the modulational/filamentation instability. Optical solitons have been observed in laboratory experiments[6] and they might serve the purpose of transmitting rapid information in optical fibres.

However, in addition to the Kerr nonlinearity, there exists the so-called saturation nonlinearity in a nonlinear medium. Thus, Akhmediev et.al.[7] incorporated the combined effect of these two nonlinearities in the numerical study of non-stationary longitudinal modulational instability of an optical pulse.

In the Letter, we present an analytical investigation of the flamentation instability and light bullets involving finite amplitude optical pulses in a nonlinear medium with the Kerr and saturation nonlinearities. Our results should strengthen the understanding of the nonlinear optical pulse propagation, which plays a vital role in telecommunications, infrared spectroscopy, medicine and many other areas of physics.

The interaction of a finite amplitude optical pulse with a nonlinear medium can give rise to a slowly varying optical field envelope, which is governed by the generalized nonlinear Schrödinger equation[7,8]

$$i\partial_z\varphi + \left(\nabla_\perp^2 + \partial_\tau^2\right)\psi - \psi + f\left(|\,\psi\,|^2\right)\psi = 0 \tag{1}$$

where the function ψ, which basically represents the electric field of the optical pulse, and all other variables are appropriately normalized to include all material parameters. Furthermore, z in the normalized longitudinal coordinate, $\nabla_\perp^2 = \partial_x^2 + \partial_y^2 \gg \partial_z^2$, τ is the retarded time, and[7]

$$f\left(|\psi|^2\right) = \frac{|\psi|^2}{1+\beta|\psi|^2} \equiv \frac{I}{1+\beta I}, \tag{2}$$

where β is the saturation parameter, which varies in the interval $0 < \beta < 1$. For $\beta = 0$, $F(|\psi|)^2 = |\psi|^2$ and (1) becomes the cubic Schrödinger equation, which describes the propagation of an optical pulse in a nonlinear medium with the Kerr nonlinearity. The βI-term in (2) is the contribution of the saturation nonlinearity. Note that (1) is valid for an optical pulse with anomalous dispersion.

In the following, we present an investigation of the filamentation instability and the formation of light bullets involving finite amplitude optical pulse. To investigate the filamentation instability of a constant amplitude (ψ_o) optical pump wave, we let

$$\psi = (\psi_o + \psi_1)\, exp\,(i\delta z - i\Omega\tau)\ , \tag{3}$$

and assume $\psi_1 \ll \psi_o$, where ψ_1 is the complex amplitude of the perturbation. Here δ and Ω are the nonlinear wave number and frequency shifts, respectively. Inserting (3) in (1), we obtain

$$1 + \delta + \Omega^2 = I_o/\left(1 + \beta I_o\right) \equiv I_{KS} \tag{4}$$

and

$$i\partial_z\psi_1 + \nabla_\perp^2\psi_1 + I_{KS}\left(1 + \beta I_{KS}\right)\left(\psi_1 + \psi_1^*\right) = 0, \tag{5}$$

where the asterisk denotes the complete conjugate.

Supposing that $\psi_1 = (\psi_r + i\psi_i)\, exp\left(G\, z + i\vec{k}.\vec{r}_\perp\right)$, where ψ_r and ψ_i are real and imaginary parts of ψ_1, respectively, G is the spatial amplificiation rate along the z-axis, and $\vec{k}$ is the wave vector of the modulation in a direction perpendicular to the optical pulse propagation, we can readily derive the nonlinear dispersion relation of the filamentation instability from (5). The result is

$$G^2 = k^2\left[2I_{KS}\left(1 + \beta I_{KS}\right) - k^2\right]\ . \tag{6}$$

The spatial amplification of the optical pulse along the direction of the pulse propagation is possible provided that

$$2I_{KS}\left(1 + \beta I_{KS}\right) > k^2 \tag{7}$$

The minimum length over which the amplification occurs involve $k = I_{KS}^{1/2}(1+\beta I_{KS})^{1/2} \equiv k_m$. Thus, the minimum amplification length is $1/k_m$.

Next, we focus on the formation of finite amplitude optical bullets. Thus, we let $\psi = \Psi exp(i\lambda z - i\gamma t)$, where Ψ is the complex amplitude of the modulated wave envelope and λ and γ are the nonlinear wave number and frequency shifts, and assume steady propagation of the bullet along the z-direction. For one-dimensional problem, (5) takes the form

$$\partial_x^2\Psi - \Delta\Psi + \mid \Psi \mid^2 \Psi / \left(1 + \beta \mid \Psi \mid^2\right) = 0 \, , \tag{8}$$

where $\Delta = 1 + \lambda + \gamma^2$

Multiplying (8) by $\partial_x\Psi$, and integrating once we obtain the energy integral

$$\left(\partial_x\Psi\right)^2 + V\left(\Psi\right) = 0, \tag{9}$$

where we have assumed $\Psi, \partial_x\Psi \rightarrow 0$ at $\mid x \mid \rightarrow \infty$, and have defined

$$V(\Psi) = \left(\frac{1}{\beta} - \Delta\right)\Psi^2 - \frac{1}{\beta^2}\log\left(1 + \beta\Psi^2\right). \tag{10}$$

For localized bullets to exist, $V(\Psi) < 0$ for $0 \leq \Psi < \Psi_m$, where Ψ_m is the maximum amplitude of the bullet. Thus, $\Delta > 0$ is required. Furthermore, the condition $V\left(\Psi = \Psi_m\right) = 0$ and $\left(\partial_x\Psi\right)_{\Psi=\Psi_m} = 0$ give

$$1 + \beta\Psi_m^2 - exp\left[\left(\beta - \Delta\beta^2\right)\Psi_m^2\right] = 0 \, , \tag{11}$$

which determines the maximum amplitude Ψ_m. Furthermore, the bullets are formed if $\left(\partial V/\partial\Psi\right)_{\Psi=\Psi_m} > 0$, yielding

$$\beta\Psi_m^2 / \left(1 + \beta\Psi_m^2\right) > \Delta\beta \, . \tag{12}$$

Equations (11) and (12) are the necessary conditions for the formation of finite amplitude optical bullets in a nonlinear medium. For the small amplitude bullets, we assume $\beta\Psi^2 \ll 1$ and obtain $V\left(\Psi\right) = -\Delta\Psi^2 + \frac{1}{2}\Psi^4$. Thus, (9) admits the well-known secant hyperbolic profile for the pulse electric field. Clearly, the saturation nonlinearity (the βI-term) does not play any role in the small amplitude limit.

Finally, it is instructive to mention that although the condition (12) should also hold for cylindrically symmetric optical bullets, the profile of the latter is determined from

$$\frac{1}{r}\frac{d}{dr}(r\frac{d\Psi}{dr}) - \Delta\Psi + \mid \Psi \mid^2 \Psi / \left(1 + \beta \mid \Psi \mid^2\right) = 0 \,. \tag{13}$$

Equation (13) can be formulated in terms of the variational principle

$$\delta \int dr L = 0 \,, \tag{14}$$

where the Lagrangian is given by

$$L = -\frac{1}{2r}(\frac{d\Psi}{dr})^2 - \frac{1}{2} r \Delta \Psi^2 + \frac{1}{2} r \int_0^{|\Psi|^2} f(x) dx \,. \tag{15}$$

Following Anderson[9] we can use Gaussian trial functions and a Ritz optimization procedure in order to obtain approximate solutions for bullet width, bullet amplitude, and nonlinear frequency chirp. On the other hand, we note that pulse-like solutions will exist corresponding to the bound-state solutions of (13) provided that $\Delta > 0$. We have not been able to obtain an analytical solution to Eq. (13) and, therefore, one should resort to numerical integration subject to the boundary conditions $\Psi(r = 0) = 0 = \Psi(r = \infty)$. This investigation as well as the variational problem for cylindrical light bullets are under consideration and the results shall be presented in a detailed version elsewhere.

To summarize, we have analytically investigated the filamentation instability and flat light bullets involving finite amplitude optical pulses in a nonlinear medium with the Kerr and saturation nonlinearities. It is found that the saturation nonlinearity plays a decisive role when the amplitude of the pulse is arbitrarily large. The present investigation supplements our current understanding of the nonlinear optical pulse propagation in nonlinear media, such as the optical fibres.

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