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Abstract

We apply Heavy Quark Effective Theory to the production of 0^- and 1^- $Q\bar{q}$ states in e^+e^- annihilation. We show that HQET implies that the electric quadrupole amplitudes vanish and we propose tests for this theory. We also show how HQET can be applied to distinguish the $^3D_1$ and $^3S_1$ $Q\bar{Q}$ states.

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Heavy Quark Effective Theory (HQET)[1] has been extensively investigated for the cases where a heavy quark undergoes a (flavour changing) current induced transition (such as $B \to D^* l\nu$). This has been most widely applied where $y = v \cdot v' \approx 1$ with v, v' the four-velocities of the initial and final hadrons (heavy quarks). There is another physical region where heavy quark interactions with (electromagnetic) currents are important, namely heavy flavour pair production in electron-positron annihilation (such as at a τ-Charm or B-factory). Some old results[2,3] have recently been reformulated within the context of HQET for such processes. It is the purpose of this note to advertise how e^+e^- annihilation may be analysed in order to test the HQET. Furthermore we shall show how the production of bottom, charm and even *strange* particles may be of interest and how these ideas may be exploited to determine the structure of the ψ, or Υ, resonance states.

In a quark model analysis where the pseudoscalar and vector $Q\bar{q}$ are assumed to be the $^1S_0$ and $^3S_1$ members of an $SU(2)_{spin}$ supermultiplet, it is known that the production of such states in e^+e^- annihilation involves only three a priori arbitrary form factors, in contrast to the most general case where five independent form factors are needed. Denoting these as the F_E (Electric), F_M (Magnetic) and F_Q (Quadrupole), then the relative production cross sections are[3]

$$\begin{aligned} \sigma(e^+e^- \to PP) : \sigma(e^+e^- \to PV + VP) : \sigma(e^+e^- \to VV) \ &= \\ 1 : 4\frac{s}{4M^2}(\frac{F_M}{F_E})^2 : 3 + 4\frac{s}{4M^2}(\frac{F_M}{F_E})^2 + \frac{8}{9}(\frac{s}{M^2})^2(\frac{F_Q}{F_E})^2 \end{aligned} \tag{1}$$

where $V \equiv B^*, D^*$, and $P \equiv B, D$, are $Q\bar{q}$ mesons made of heavy-light quarks. In eq(1), (as well as in the rest of this paper), it has been assumed that the e^+e^- energy is sufficiently above threshold that the V and P mesons can be considered to be degenerate. In this approximation the heavy quark mass M is taken to be equal to that of the heavy meson.

In HQET these channels are described by a single form factor. In the language of ref [3] this means that $F_E = F_M$ and $F_Q = 0$. The significance of these constraints was noted soon after the discovery of charm[2,3] but the HQET has recently put them on a sounder footing [4,5].

Setting $F_Q = 0$ in the general formula, eq(1), leads to a sum rule for the production differential cross sections at any angle ϑ to the initial e^+e^- axis,

$$3\frac{d\sigma}{d\theta}(e^+e^- \to PP) + \frac{d\sigma}{d\theta}(e^+e^- \to PV + VP) = \frac{d\sigma}{d\theta}(e^+e^- \to VV) \tag{2}$$

In HQET one expects that

$$V = {}^3S_1 + O(\frac{1}{M^2})^3D_1 \tag{3}$$

This implies that as the heavy quark mass $M \to \infty$, the vector meson $V \to {}^3S_1$ which in turn means that $F_Q \to 0$. So eq(2) may be interpreted as a test of HQET at leading order, and in particular as a test of the $^3S_1$ nature of the $Q\bar{q}$ vector.

The individual contributions of the various final states to the sum rule eq(2) depend upon the dynamic coupling of the *initial* e^+e^- to the heavy quarks (e.g. whether in the continuum or on an S or D-wave $Q\bar{Q}$ resonance). So by varying the beam energy we can expect different ratios of the individual contributions to eq(2).

In the continuum the direct coupling of the photon to the $Q\bar{Q}$ pair involves a γ_μ vertex together with perturbative QCD corrections which induce also a $\sigma_{\mu\nu}q^\nu$ form factor. In this case the ratios of the total cross sections are[4,5]

$$\sigma(e^+e^- \to PP : PV + VP : VV) = 1 + h : 4\frac{s}{4M^2} : 3(1+h) + 4\frac{s}{4M^2} \tag{4}$$

where

$$h \equiv -\frac{2\alpha_s}{3\pi}\sqrt{1-\frac{4M^2}{s}}log\{\frac{s}{2M^2} - 1 + \frac{s}{2M^2}\sqrt{1-\frac{4M^2}{s}}\} \tag{5}$$

describes the first order QCD corrections[5]. Note that eq(4) gives a particular realisation of eq(2), which is a consequence of the fact that the off shell photon has no electric quadrupole coupling to the VV final state.

On an S-wave (or D-wave) resonance we explicitly neglect D-wave (or S-wave) contributions respectively. Thus although we cannot predict the absolute magnitudes of the form factors, their relative strengths follow simply from angular momentum considerations alone and are independent of perturbative QCD corrections at the heavy quark production vertex. For an S-wave ($^3S_1(Q\bar{Q})$) bound state we find

$$< P(v_1)\bar{P}(v_2)|^3S_1, \epsilon >= M\frac{(1+2v_1^0)}{3}\xi(v_1 \cdot v_2)(v_1 - v_2)_\mu \cdot \epsilon^\mu \tag{6}$$

$$< V(v_1,\epsilon_1)\bar{P}(v_2)|^3S_1, \epsilon >= iM\frac{(1+2v_1^0)}{3v_1^0}\xi(v_1 \cdot v_2)\epsilon_{\mu\nu\lambda\sigma}\epsilon^\mu\epsilon_1^{*\nu}v_1^\lambda v_2^\sigma \tag{7}$$

$$\begin{aligned} < V(v_1,\epsilon_1)\bar{V}(v_2,\epsilon_2)|^3S_1, \epsilon >= M\xi(v_1 \cdot v_2)\frac{(1+2v_1^0)}{3(1+v_1^0)} \times \\ \{(1+v_1^0)(\epsilon_1^* \cdot \epsilon_2^*)(v_1-v_2)_\mu - (1+1/v_1^0)[(\epsilon_2^* \cdot v_1)\epsilon_{1\mu}^* - (\epsilon_1^* \cdot v_2)\epsilon_{2\mu}^*] \\ -\frac{1}{2v_1^0}(\epsilon_1^* \cdot v_2)(\epsilon_2^* \cdot v_1)(v_1-v_2)_\mu\}\epsilon^\mu \end{aligned} \tag{8}$$

where ϵ is the polarisation vector of the decaying state, $v_1^0 = \frac{\sqrt{s}}{2M}$ and v_1, v_2 are the four-velocities of the mesons in the final state. This leads to the following realization of eq(2) for the integrated cross sections

$$\sigma(e^+e^- \to PP : PV + VP : VV) = 1 : 4 : 7 \tag{9}$$

whereas for a D-wave ($^3D_1(Q\bar{Q})$) bound state one finds

$$< P(v_1)\bar{P}(v_2)|^3D_1, \epsilon >= -M\frac{2(v_1^0-1)}{3}\xi(v_1 \cdot v_2)(v_1 - v_2)_\mu \cdot \epsilon^\mu \tag{10}$$

$$< V(v_1,\epsilon_1)\bar{P}(v_2)|^3D_1,\epsilon >= iM\frac{(v_1^0-1)}{3v_1^0}\xi(v_1\cdot v_2)\epsilon_{\mu\nu\lambda\sigma}\epsilon^{\mu}\epsilon_1^{*\nu}v_1^{\lambda}v_2^{\sigma} \tag{11}$$

$$< V(v_1,\epsilon_1)\bar{V}(v_2,\epsilon_2)|^3D_1,\epsilon >= -M\xi(v_1\cdot v_2)\times$$
$$\{\frac{2(v_1^0-1)}{3}(\epsilon_1^*\cdot\epsilon_2^*)(v_1-v_2)_{\mu}+\frac{(v_1^0-1)}{3v_1^0}[(\epsilon_2^*\cdot v_1)\epsilon_{1\mu}^*-(\epsilon_1^*\cdot v_2)\epsilon_{2\mu}^*]$$
$$-\frac{1+2v_1^0}{6v_1^0(1+v_1^0)}(\epsilon_1^*\cdot v_2)(\epsilon_2^*\cdot v_1)(v_1-v_2)_{\mu}\}\epsilon^{\mu} \tag{12}$$

which leads to

$$\sigma(e^+e^-\to PP:PV+VP:VV)=1:1:4 \tag{13}$$

(see also[3,6])

Note that the continuum result differs from the S-wave only in the perturbative QCD corrections which vanish at threshold and are small in the kinematic region of interest. On the other hand the D-wave result given in eq(13) is very different from both eq(4) and eq(9) for the continuum and S-wave cases respectively. This is related to the fact that the D-wave contribution also vanishes at threshold like $\vec{v}^2$, which can be seen by inspection of eqs(10-12). The D-wave result in eq(13) naturally gives the dominant contribution on a D-wave resonance, provided that this resonance lies sufficiently above threshold to justify the neglect of the mass difference between the P and V states. Thus to the extent that eq(2) is realised in the data, we can use eq(13) to identify the ${}^3D_1\psi$ and Υ-like states. At this point it is worthwhile to emphasise that within the framework of HQET we can determine the internal structure of the $Q\bar{Q}$ resonance by studying only the branching ratios into various channels without need for detailed angular distributions.

These results suggest the following strategy.

Possibility 1

Eq(2) is violated or, in the continuum, eq(4) is violated. In this case HQET at leading order in M_Q is not a good approximation.

One particular source of such violation could be the presence of $F_Q \neq 0$. This can be tested by analysing the polarisation of the final state vector mesons[3,7]. According to our treatment, in this case the vector meson wavefunction should in general not be simply given by ${}^3S_1$; this would undermine some of the analysis of semileptonic decays of heavy flavours such as $B\to D^*l\nu$ (which, in HQET, implicitly assumes no ${}^3D_1$ component in the D^*). It may even be interesting to study $e^+e^-\to K^*\bar{K}^*$, e.g. at $DA\Phi NE$, where it would be natural to expect a non-vanishing F_Q, (with corresponding implications for HQET applications to $B,D\to K^*l\nu$).Independent of the specific interest in HQET, this is a unique way of measuring the quadrupole moments of vector mesons.

Possibility 2

Eq(2) and (in the continuum) eq(4) are satisfied, and $F_Q=0$.

In this case HQET is correct to a high level of accuracy and the vector state $V(Q\bar{q}) = {}^3S_1$.Such a direct measure of the vector meson's wavefunction would constrain models of substructure; in particular it would eliminate the possible presence of a significant mass-independent $\sigma \cdot \sigma$ interaction between the quarks as would arise from an elementary γ_5 pseudoscalar interaction. In such a case it will also be interesting to study the predictions of this theory not only for bottom and charmed states but also for production of strange mesons in order to help establish the extent to which the strange quark may be considered as heavy. If $e^+e^- \to K^*\bar{K}^*$ shows that $F_Q \neq 0$, then some applications of leading order HQET to $B(D) \to K^* l\nu$ or $b \to s\gamma$ may need reexamination.

If HQET is established in the continuum, then measurement of the production ratios on $Q\bar{Q}$ resonance may, courtesy of eqs (9, 13), be able to determine the ${}^3S_1, {}^3D_1$ content of the $Q\bar{Q}$ states.

The essential physics that enabled this analysis is that the initial state has well defined total angular momentum and parity quantum numbers, which in this case happen to be 1^-, with the fermions in either S or D orbital angular momentum. Such ideas could also be applied to final states consisting of spin-$\frac{1}{2}, \frac{3}{2}$ baryons in e^+e^- annihilation and to $\gamma\gamma$ physics where the $\vec{L}$ of the various χ states could be determined. For example, when $J^{PC} = 2^{++}$ the χ can be either ${}^3P_2$ or ${}^3F_2$. Ref.[8] has shown how the helicity dependence of $\gamma\gamma$ production is sensitive to this. These are particular examples of a more general possibility, namely that the $\vec{L}$ and $\vec{S}$ substructure of a heavy state of total $\vec{J}$ may be analysed by just the branching ratios to various channels.

To recapitulate: In $e^+e^- \to PP, PV, VV$ we have identified ways of determining how good HQET is to leading order in M_Q. If HQET turns out to be valid, such processes can then be used to study the substructure of heavy states.

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