Science and Engineering Research Council

Rutherford Appleton Laboratory

Chilton DIDCOT Oxon OX11 0QX

RAL-93-019

Practical Gain Formulae for Proportional Counters

J E Bateman

March 1993

PRACTICAL GAIN FORMULAE FOR PROPORTIONAL COUNTERS

J E Bateman

Rutherford Appleton Laboratory, Chilton, Didcot, OX11 0QX, UK

In studying the sensitivity of the gain of flow gas counters to ambient conditions it is found that a simple model of the avalanche process can be adapted to give an excellent description of the dependence of the gas gain on pressure (P) and temperature (T) around ambient. Three types of detector are studied: the wire counter, the pin detector and the parallel gap detector. It is found that a simple linear servo equation using the variable P/T can be used to stabilise counters by adjusting the applied anode potential.

1. INTRODUCTION

At Rutherford Appleton Laboratory we have developed and operated large numbers of gas counters chiefly for use in Particle Physics and are well accustomed to take account of the gain variations induced by ambient conditions (notably the barometric pressure). In recent years we have developed applications which demand a level of stability beyond the requirements usually imposed on us. In particular, our system for the assay of beta activity in immunological studies [1] is required to have a secular efficiency stability of a few percent independent of ambient conditions. Beta energy spectra are of such a form that gain variation in the detector translates efficiently into sensitivity variation so placing a stringent gain control requirement on the detector, particularly in the face of ambient pressure swings of from 950 to 1050 millibars. (The counters operate in flow mode and so follow barometric pressure.)

Since the addition of a pressure control system would approximately double the cost of the detector system, attention turned to the option of using an electronic control system to servo the EHT and thus compensate for the pressure-induced gain shifts. At the same time a systematic study of the efficiency (i.e. the gas gain) of the system was carried out while monitoring the ambient thermodynamic variables - pressure (P) and temperature (T). The expected inverse dependence of the gas gain on pressure was observed along with a direct dependence on temperature. Turning to the extensive literature on proportional counter gain [2-11] I was surprised to discover no formulation which makes explicit any temperature dependence. In order to derive useful parameterisations of the gas gain as a function of P and T, I instituted an experimental survey of the gain of a cylindical proportional counter, a pin detector (spherical anode) and a parallel gap counter over the dynamic range of the variables experienced in my laboratory ($940 < P < 1030$mb and $18 < T < 24$C). These results were analysed in terms of the simplest possible model and stabilisation algorithms developed which enabled the gains of the various devices to be controlled to within an error of a few percent.

2. THE GAIN MODEL

In a single wire proportional counter the gas gain is given by the expression:

$$\ln G = -\int_{r_0}^{a} \alpha \, dr \qquad (1)$$

where G is the gas gain, a the radius of the anode wire, α the first Townsend coefficient and r_0 the radius at which avalanching commences.

If we postulate for the time being only a single interaction of the drifting electrons with the counter gas (viz. ionisation) then we can write:

$$\alpha = \frac{1}{\lambda} \exp \frac{-W}{E\lambda} \qquad (2)$$

where W is the threshold energy for ionisation in eV, λ the electron's mean free path for the process and E is the local value of the electric field. If σ is the molecular cross-section for this process we also have:

$$\lambda = \frac{1}{N\sigma}$$

where $$N = \frac{N_a}{R}\frac{P}{T} \qquad \text{(the molecular density)}$$

(N_a is Avogadro's number and R is the gas constant).

Thus λ and therefore α are functions of the variable P/T (q).

The problem is that there are several inelastic scattering processes involved in the avalanche with cross-sections which vary with electron energy, and the experimentally observed α does not behave as (2) predicts when P is varied. The commonest approach to this problem is to parameterise in terms of the variables $s = \alpha/P$ and $S = E/P$ and find an empirical fit or approximation to relate these two quantities and this has been done in various ways with various levels of success (see [10] for a comparative analysis). However, the temperature dependence is made implicit and so the results are not useful in the case of flow counters. (It will be noted that in the case of a sealed (rigid) counter the gain is independent of P and T since, by definition, N cannot change.)

Given our requirements of finding a gain expression valid only around NTP it seemed useful to proceed on the basis of expressions (1) and (2) and see if the non-ideal behaviour of α could be incorporated in a simple way which explicitly depended on P and T. As a further simplification r_0 was set to ∞. This removed an undefined constant from the gain equation while still providing an excellent fit. Integrating (1) with (2) substituted and using $E = V/(r\ln(b/a))$ gives:

$$\ln G = V/A \exp(-B/V) \qquad (3)$$

where $A = W \ln(b/a)$ and $B = W \ln(b/a)\ a\ Na/R\ \sigma\ q$
V is the anode potential and a and b the anode and cathode radii. The other parameters are defined above.

As figure 1 shows this formula provides an excellent fit to the gain over a range of 10^2 to 10^4, the practical gain range of proportional counters. We see that (3) predicts a linear dependence of B on q. We shall see below that experiment confirms this expectation and the physical complexities of the avalanche processes show up simply as an offset on the B versus q curve.

Applying the same reasoning to the case of the pin detector with its spherical electrode where $E = V/(r^2(1/a - 1/b))$ we get:

$$\ln G = \sqrt{\pi}/2 \sqrt{(V/A)}\ \mathrm{erfc}\sqrt{(B/V)}$$

where A and B are functions of a,b, W, λ.

This equation is very unfriendly for purposes of analysis and I found that equation (3) fitted the pin detector gain curves just as well so permitting the same analysis to be performed for both detectors (with, of course, different fitting parameters).

The integration of (1) in the case of the parallel gap counter is simplified by the fact that $E = V/d$ where d is the width of the amplifying gap. We have:

$$\ln G = A \exp(-AB/V) \qquad (4)$$

where $A = d/\lambda$ and $B = W$

In this case we expect A to be proportional to q.

3. EXPERIMENTAL RESULTS

The experimental measurements were carried out on three detectors:

1. A standard cylindrical wire counter with a gold-plated tungsten anode wire of 20µm diameter inside a brass tube of 20mm diameter.

2. A single pin detector as described in [1] with an anode consisting of a 2mm diameter sphere mounted on a coned shaft. The cathode structures are approximately 50mm from the anode.

3. A parallel gap counter consisting of a 10mm deep conversion space separated from a 1mm wide avalanche gap by a stainless steel mesh. The drift field was kept low (typically 30V across the 10mm gap) to minimise charge loss on the mesh which was kept at earth while positive EHT was applied to the back electrode.

A flow of 100cc/min of argon + 7.5%CH_4 was used in all cases.

Measurements were carried out over the range of ambient conditions obtaining in my laboratory which during the period of the experiment was $940 < P < 1030$mb and $18 < T < 24$C giving a range of approximately $3.25 < q < 3.5$ (mb/K).

As figure 1 shows the fit of equation (3) for the gain of the wire counter is extremely good over the gain range of $70 < G < 17000$. As P and T varied the constants A and B were determined by a standard fitting procedure from the measured gain versus EHT curve for each q value. A and B were both found to vary with q. However, it was found that no statistically significant change occurred in the fitting error if A was fixed at the average derived from the first six q values so that all the q dependence could be confined to the B parameter. After a few weeks a sufficient span of q values had been accumulated to allow a plot of B versus q. As figure 2 shows this gives a very reasonable straight line fit: B = 219q + 307 (V).

The procedure was repeated with the pin detector. As noted above, the wire formula (3) was applied to the pin detector gain curves and found to fit as well as the formula derived from the spherical field description. Figure 3 shows the B versus q plot for this detector. Again we have a good straight line fit: B = 1915q + 663 (V).

As usual the parallel gap counter was the most difficult device to obtain reproducible results from, due to its high sensitivity to gas purity as well as to q. However, eventually a consistent set was obtained. In this case the B parameter (equation 4) was averaged and a single parameter fit performed on the A parameter which was expected to dominate the q dependence. As figure 4 shows we again observe a straight line fit for A: A = 916q-91.4 (V).

4. GAIN STABILISATION

As a result of the above measurements we now have a complete description of the gain of a device in terms of q and V. Thus for the wire counter we have:

$$\ln G = V/74.0 \exp(-(222q+297)/V) \qquad (5)$$

If we now select a desired operating point for the counter such as: Po=990mb, To=20C, Vo=1330V (Go=3565.4) then the relation between V and q defined by the equation

$$\ln(3565.4) = Vs/74.0 \exp(-(222q+297)/Vs) \qquad (6)$$

yields the anode potential (Vs) which keeps the counter gain set on Go at any ambient condition defined by q. Solving this equation numerically yields data that can be accurately fitted by the straight line Vs = 124q + 910. Thus on any particular occasion when the counter is in use one obtains the ambient pressure and temperature, calculates q and thence the set value for the EHT from this relation. Figure 5 shows the measured gas gain of the wire counter over a reasonably wide range of q under (manual) servo control. A standard deviation of 0.8% is observed; without correction a gain change of 25% would occur over this range of q.

Similarly we have for the pin detector:

$$\ln G = V/60.0 \exp(-(1915q+663)/V)$$

Choosing a set point of Vo=3700V (Go=7855) with Po and To as above and solving the equivalent equation to (6) above we get Vs

= 655q + 1485. Applying this servo control to the usual range of ambient variation ($3.3 < q < 3.48$) the gain is held constant to a standard deviation of 1.27%. Over this range of q the gain, if uncorrected, would vary by a factor of 2.3:1.

Solving the equation for a range of set-points reveals the useful information that the coefficients of q in the equation for Vs are themselves linear functions of the set-point voltage Vo. Thus the servo relation can be calculated for any chosen gain using the formula:

$$Vs = (0.116Vo+226)q + 0.586Vo - 720$$

For the parallel gap detector we obtain the gain formula:

$$\ln G = (91.6q-91.4) \exp(-23.864(91.6q-91.4)/V)$$

Choosing a set point of Vo=1520V (Go=1218) with Po and To as above and again solving the equivalent of equation (6) for the parallel gap case we obtain the servo equation for the set-point voltage: Vs = 453q - 11. Measurements show that this relation permits the gain to be stabilised to within a standard deviation of 4.15%. (The uncorrected gain variation over $3.3 < q < 3.5$ is 4.1:1).

5. DISCUSSION

The above results show that a simple modification to the simplest model for gas counter gains gives algorithms which can describe the behaviour of the gains of the main types of flow gas counter accurately in the range of ambient conditions normally encountered in practical operation. The results show that (as the physics predicts) the key parameter for determining environmental effects is P/T (ambient pressure/ambient absolute temperature). It is relatively easy to adapt these algorithms to produce servo control formulae which permit the gain of the counter to be stabilised by appropriate adjustment of the counter voltage. The fact that these formulae are simple linear expressions in q (P/T) make the option of automatic electronic control of the gain very attractive and much cheaper to implement than stabilising the pressure and temperature of the detector.

The (perhaps) surprising fact that the solutions to equations such as (6) are so accurately linear is simply due to the limited range of q over which we must work. Differentiating (6) with respect to q shows that dV/dq is indeed approximately constant under these conditions. Given the knowledge that Vs is a linear function of q, the servo equation for any detector can obviously be measured directly as ambient conditions vary and the required function quickly obtained.

The results also show that the dependence of the gain on the ambient conditions is much more serious in the case of the pin detector and the parallel gap counter. In fact, differentiating the gain with respect to q for each of the three cases and substituting the measured parameters and typical running conditions gives the following results for 1/G(dG/dq): wire counter -1.4, pin detector -4.6 and parallel gap counter -8.4. The stabilisation accuracy of the servo process declines in parallel with these parameters. The parallel gap counter is so sensitive to ambient conditions that a significant gain shift is induced by doubling the counter flow rate and thus slightly increasing the back pressure from the oulet tube. The different sensitivities exhibited by the various counters probably results from the differing number of mean free paths involved in the respective gain processes. In the case of the 20µm diameter wire the gain occurs almost entirely in about 20 mean free paths, in the pin detector there is probably of order 100 and in the parallel gap device the fit to equation (4) indicates some 220 mean free paths. Clearly, the best flow detector design for minimising the effects of ambient fluctuations is a wire counter with as small a diameter anode as possible. The narrow anode strip widths of the new gas microstrip detectors and the very compact high field region near the anode should keep the avalanche very short and make the gain very stable against ambient fluctuations.

For each detector I have fitted the gain curve to q and V with three parameters which clearly depend on both the detector dimensions and the filling gas. Separating out these dependencies would entail a further sequence of measurements which would eventually produce constants for the gas. The methods described above are simply intended to provide a convenient method for stabilising flow counters against ambient conditions in an ad hoc manner and not as a method for investigating the physics of the avalanche process. However, additional measurements with varying counter parameters should generate general formulae predictive of the gas gain of any cylindrical wire counter in conditions around the ambient range.

REFERENCES

[1] J E Bateman, A Joyce, S C Knight and P Bedford, Nucl Instr and Meth A310 (1991) 354-358

[2] M E Rose and S A Korff, Phys Rev 59 (1941) 850

[3] W Diethorn, NYO-6628 (1956)

[4] A Williams and R I Sara, Int J Appl Rad Isotopes 13 (1962) 229

[5] A Zastawny, J Sci Instr 43 (1966) 179

[6] M W Charles, J Phys E 5 (1972) 95

[7] W Bambynek, Nucl Instr and Meth 112 (1973) 103

[8] R S Wolff, Nucl Instr and Meth 115 (1974) 461

[9] J Planinic, Nucl Instr and Meth 136 (1976) 165

[10] H Miyahara, M Watanabe and T Watanabe, Nucl Instr and Meth A241 (1985) 186-190

[11] Y Uozumi, T Sakae, M Matoba, H Ijiri,, N Koori, Nucl Instr and Meth A324 (1993) 558-564

FIGURE CAPTIONS

Figure 1 A fit of the gain formula proposed for a cylindrical wire counter over a wide range of gain (70 - 17000).

Figure 2 A plot of the fitted values of the parameter B in the cylindrical wire counter formula as ambient conditions change (q = P/T)

Figure 3 A plot of the fitted values of the parameter B in the gain formula applied to pin detector as ambient conditions change.

Figure 4 A plot of the fitted vales of the parameter A in the gain formula for the parallel gap counter as ambient conditions change.

Figure 5 A plot of the gas gain of the cylindrical wire counter as ambient conditions change while the anode potential is adjusted in accordance with the "servo" formula derived from the measurements shown in figure 2.

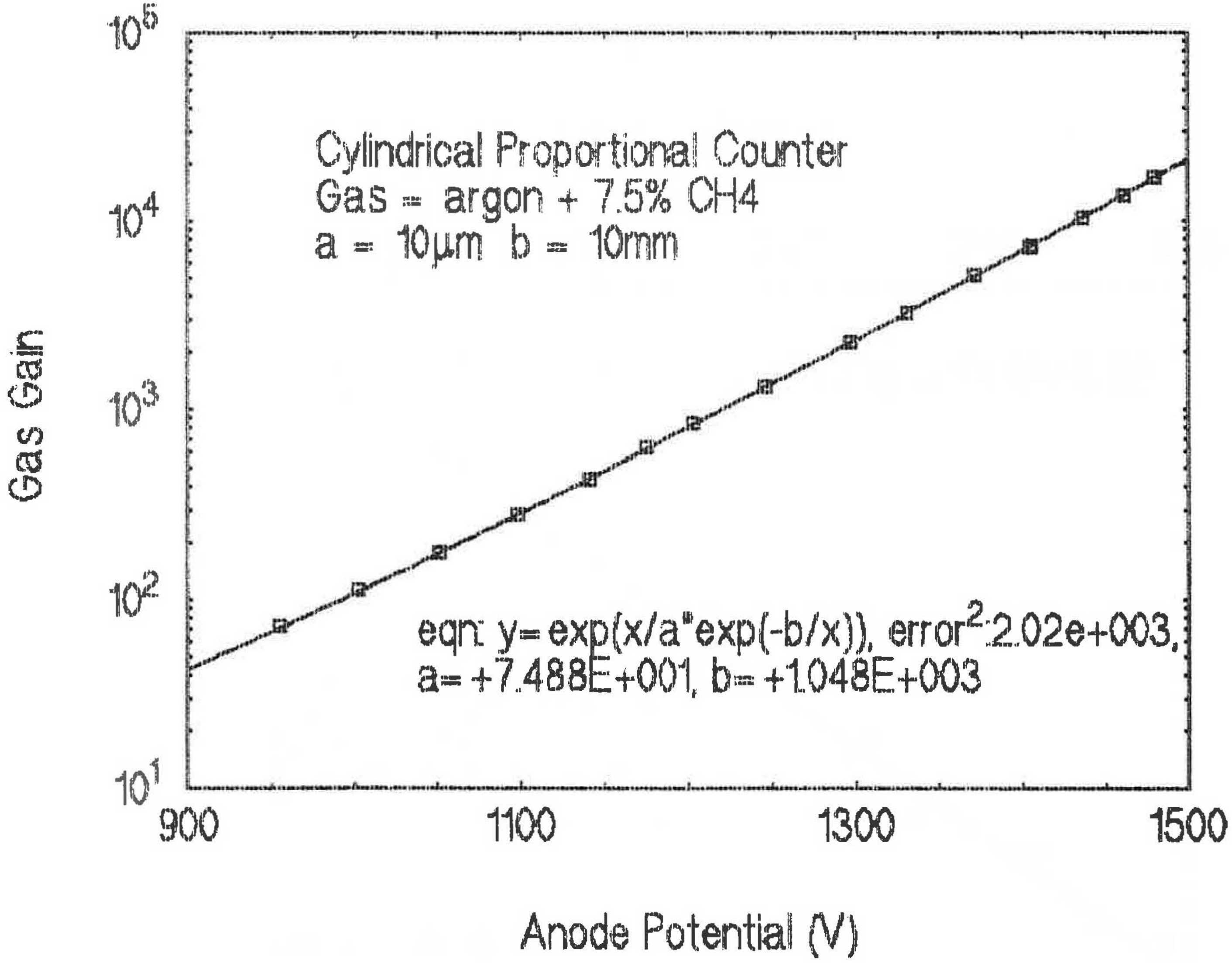
Cylindrical Proportional Counter
Gas = argon + 7.5% CH4
a = 10μm b = 10mm
eqn: y=exp(x/a*exp(-b/x)), error²:2.02e+003,
a= +7.488E+001, b= +1.048E+003
Gas Gain
Anode Potential (V)
900
1100
1300
1500

Figure 1

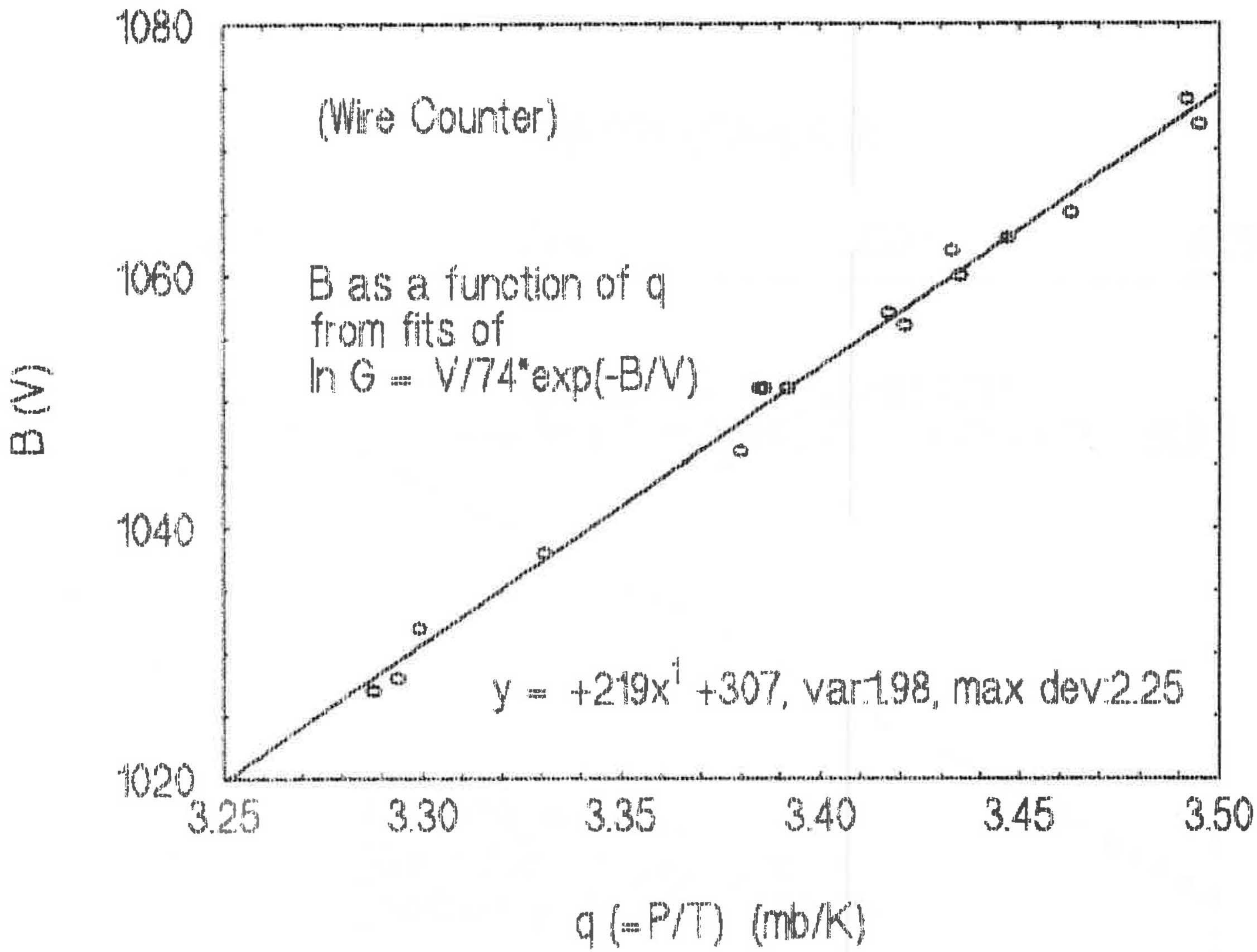
1080
1060
1040
1020
(Wire Counter)
B as a function of q
from fits of
ln G = V/74*exp(-B/V)
y = +219x^1 +307, var:1.98, max dev:2.25
B (V)
3.25
3.30
3.35
3.40
3.45
3.50
q (=P/T) (mb/K)

Figure 2

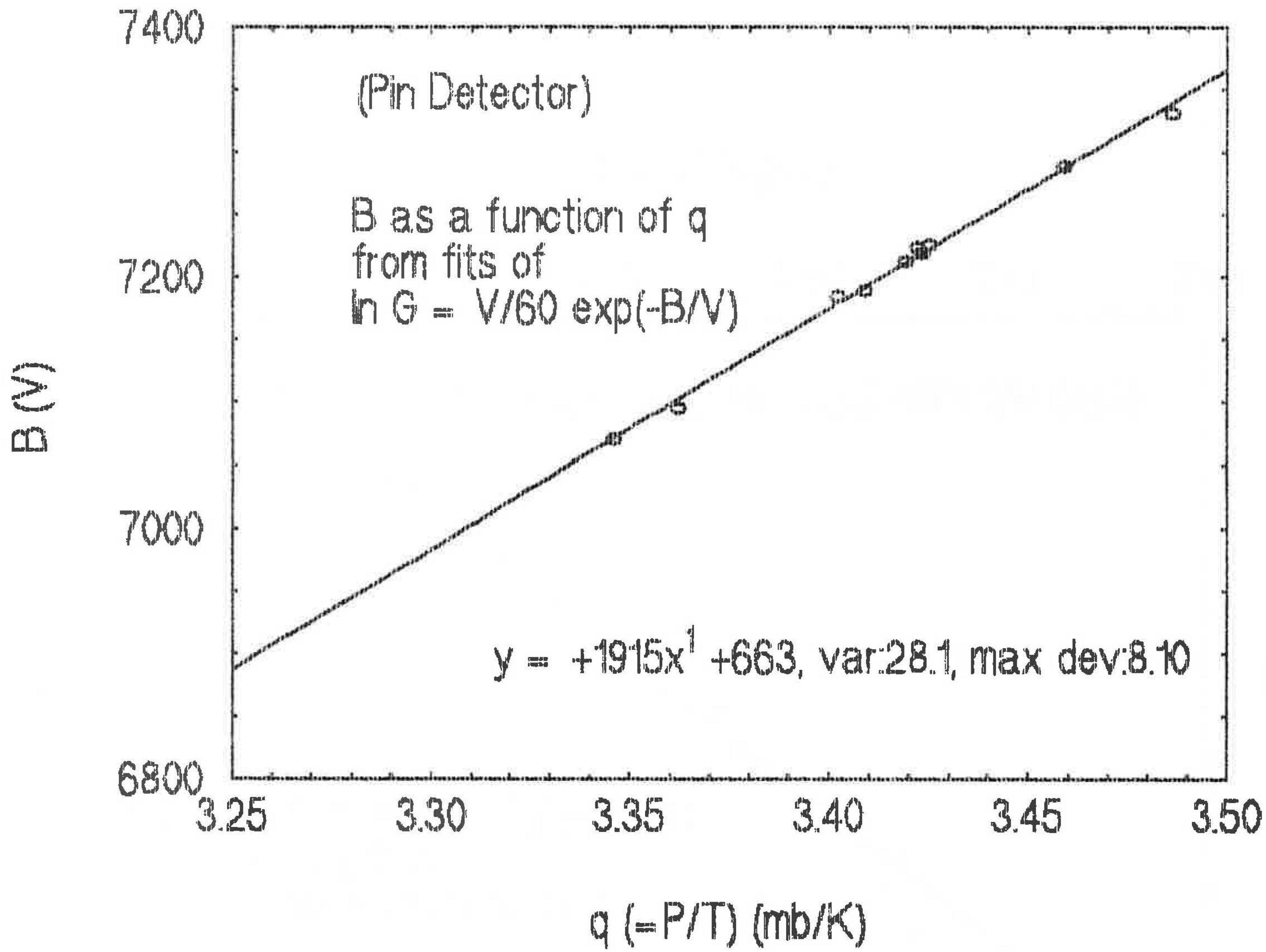
(Pin Detector)
B as a function of q
from fits of
ln G = V/60 exp(-B/V)
y = +1915x^1 +663, var:28.1, max dev:8.10
7400
7200
7000
6800
3.25
3.30
3.35
3.40
3.45
3.50
B (V)
q (=P/T) (mb/K)

Figure 3

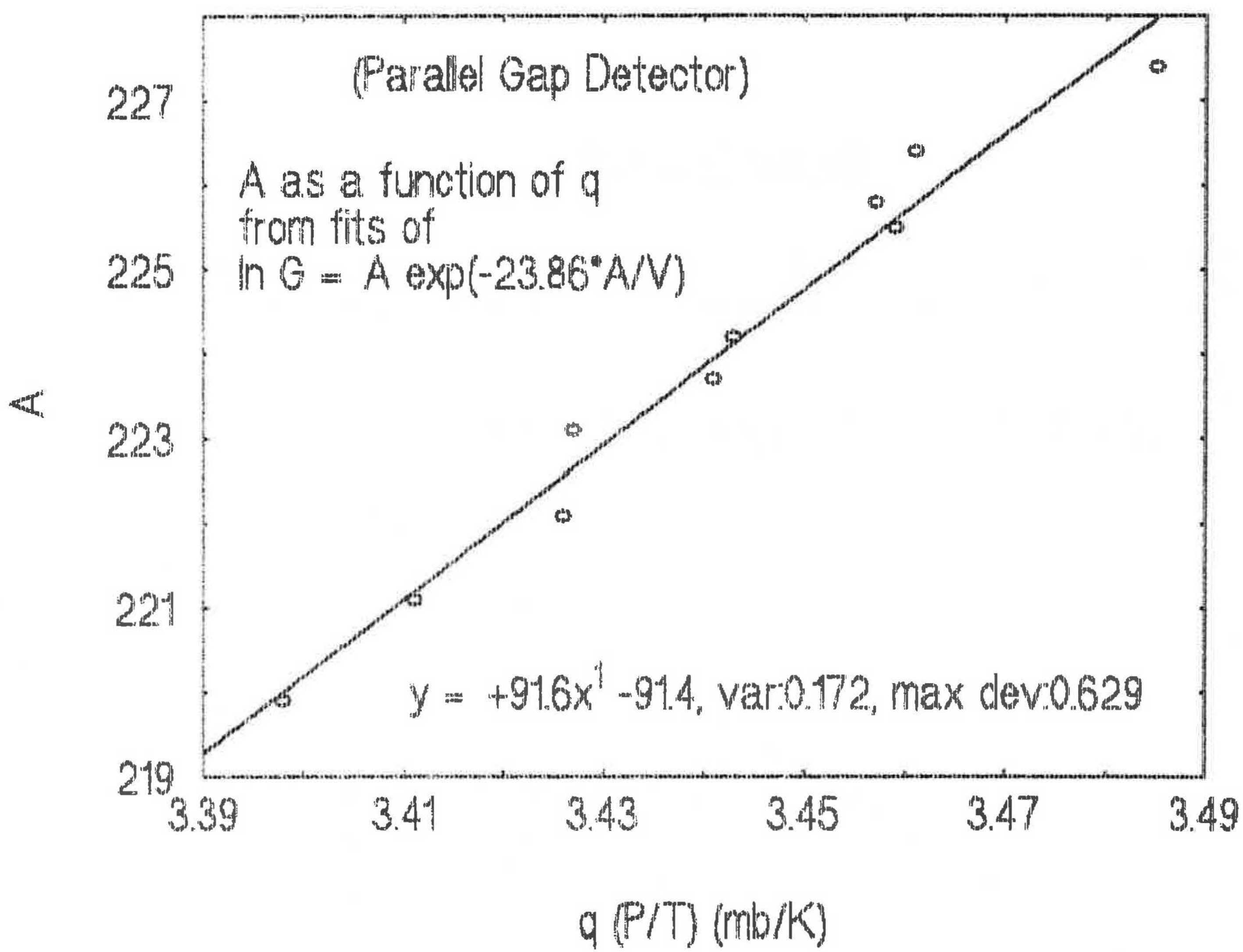
(Parallel Gap Detector)
A as a function of q
from fits of
ln G = A exp(-23.86*A/V)
y = +91.6x^1 -91.4, var:0.172, max dev:0.629
227
225
223
221
219
A
3.39
3.41
3.43
3.45
3.47
3.49
q (P/T) (mb/K)

Figure 4

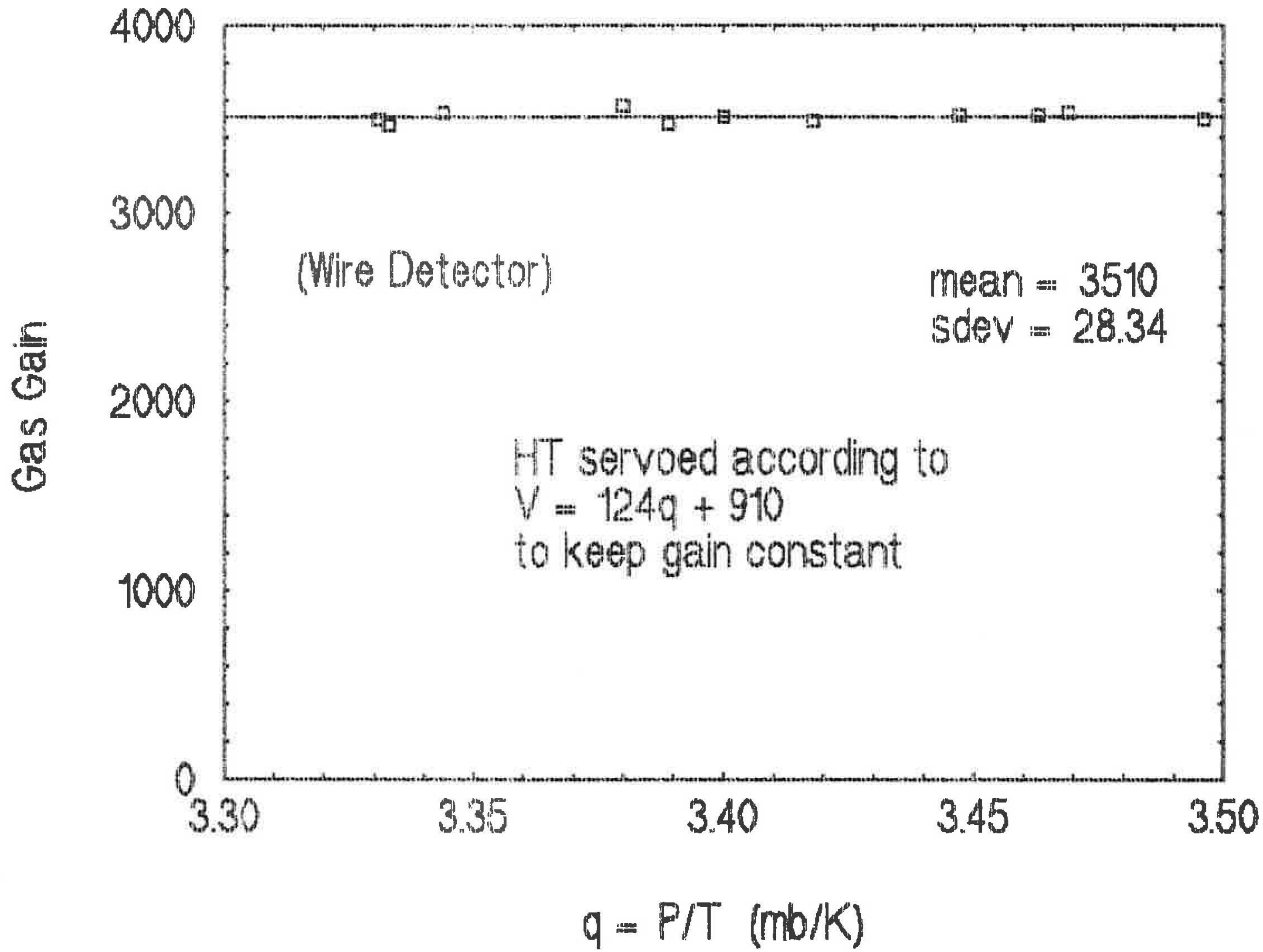
4000
3000
2000
1000
0
Gas Gain
(Wire Detector)
mean = 3510
sdev = 28.34
HT servoed according to
V = 124q + 910
to keep gain constant
3.30
3.35
3.40
3.45
3.50
q = P/T (mb/K)

Figure 5