University of Wisconsin Madison

MAD/PH/755

RAL-93-024

April 1993

Phenomenological Implications of the m_t RGE Fixed Point for SUSY Higgs Boson Searches

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ABSTRACT

In minimal SUSY-GUT models with $M_{SUSY} \lesssim 1$ TeV, the renormalization group equations have a solution dominated by the infrared fixed point of the top Yukawa coupling. This fixed point predicts $m_t = (200\ \mathrm{GeV}) \sin\beta$; combined with the LEP results it excludes $m_t \lesssim 130$ GeV. For m_t in the range 130-160 GeV, we discuss the sensitivity of the m_t fixed point result to GUT threshold corrections and point out the implications for Higgs boson searches. The lightest scalar h has mass 60-85 GeV and will be detectable at LEPII. At SSC/LHC, each of the five scalars h, H, A, $H^{\pm}$ may be detectable, but not all of them together; in one parameter region none will be detectable.

For a large top quark mass $m_t > M_W$, the corresponding Yukawa coupling λ_t is expected to be large at the unification scale M_G in grand unified theories (GUTs). Then the renormalization group equations (RGEs) cause λ_t to evolve rapidly toward an infrared fixed point at low mass scales [1]. The prediction for m_t depends on the particle content below M_G. Recent success in achieving gauge coupling unification based on a low-energy supersymmetry (SUSY) [2,3], with minimal SUSY particle content at $M_{SUSY}\lesssim 1$ TeV and $\mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ evolution below M_G, has stimulated renewed interest in Yukawa coupling unification and fixed points [4–9]. In the present Letter, we discuss the origins and uniqueness of m_t-fixed-point solutions, for the case that $130 < m_t < 160$ GeV, and examine the implications for Higgs boson phenomenology.

From the one-loop SUSY standard model RGE

$$\frac{d\lambda_t}{dt} = \frac{\lambda_t}{16\pi^2}\left[-\sum c_i g_i^2 + 6\lambda_t^2 + \lambda_b^2\right] , \tag{1}$$

with $c_1 = 13/15$, $c_2 = 3$, $c_3 = 16/3$, the couplings evolve toward a fixed point close to where the quantity in square brackets in Eq. (1) vanishes. Here λ_t is related to the running mass by $m_t(m_t) = \lambda_t(m_t) v \sin\beta/\sqrt{2}$ where $\tan\beta = v_2/v_1$ is the ratio of the two scalar vevs, with $v_1^2 + v_2^2 = v^2 = (246\mathrm{GeV})^2$. If the λ_b contribution can be neglected and only the dominant g_3 coupling is retained, the approximate fixed-point prediction is

$$\lambda_t(m_t) = \frac{4}{3}\sqrt{2\pi\alpha_s(m_t)} \approx 1 . \tag{2}$$

giving

$$m_t(m_t) \approx \frac{v}{\sqrt{2}}\sin\beta . \tag{3}$$

More precise two-loop RGE evaluations [7] give

$$m_t(m_t) = (192\ \mathrm{GeV})\sin\beta , \tag{4}$$

in the regime where $\lambda_b << \lambda_t$, taking $\alpha_s(M_Z) = 0.118$. (The current experimental average is $\alpha_s(M_Z) = 0.118 \pm 0.007$ [10].) The pole mass is related to the running mass by [11]

$$m_t(\text{pole}) = m_t(m_t)\left[1 + \frac{4}{3\pi}\alpha_s(m_t)\right] \simeq (200\text{ GeV})\sin\beta\,. \tag{5}$$

All subsequent results will be expressed in terms of this pole mass. Thus in the SUSY-GUT fixed-point solution m_t is naturally large but dependent on the value of β. When evolving from the GUT scale to electroweak energies, the fixed point is approached more rapidly from above. If the top quark Yukawa coupling is below the fixed point at the GUT scale, the convergence to the fixed point is much more gradual, and in that case strong statements about the relationship between m_t and $\sin\beta$ cannot be made.

Many SUSY-GUT theories explain the observed m_b/m_τ ratio via a unification constraint $\lambda_b = \lambda_\tau$ at the GUT scale [12]. The one-loop evolution equation for $R_{b/\tau} \equiv \lambda_b/\lambda_\tau$ is

$$\frac{dR_{b/\tau}}{dt} = \frac{R_{b/\tau}}{16\pi^2}\left(-\sum d_i g_i^2 + \lambda_t^2 + 3\lambda_b^2 - 3\lambda_\tau^2\right)\,, \tag{6}$$

with $d_1 = -4/3$, $d_2 = 0$, $d_3 = 16/3$. If the bottom quark mass is sufficiently small, then a large top quark Yukawa coupling is required to counteract the evolution from the gauge couplings; neglecting λ_t would give a value of m_b too high. Taking the Gasser-Leutwyler [11] value $m_b(m_b) = 4.25 \pm 0.10$ GeV gives rise to the fixed point solution. Larger values of the strong coupling constant require a larger top quark Yukawa coupling and hence yield solutions that are more strongly of the fixed point character. The uncertainties on $\alpha_s(M_Z)$, $m_b(m_b)$, and the scale of the supersymmetric threshold M_{SUSY} introduce a correction in the coefficient in Eq. (5) of up to 10%. GUT scale threshold corrections will be discussed below.

There are also fixed-point solutions at $\lambda_b \approx 1$; this avenue leads to very large $\tan\beta \approx 60$. In Fig.1 we show SUSY-GUT solution regions in the $(m_t,\ \tan\beta)$ plane, with the boundary condition $\lambda_b(M_G) = \lambda_\tau(M_G)$, but without yet imposing any $m_b(m_b)$ constraint; the upper and lower bands are the λ_b and λ_t fixed-point regions, respectively. In each case, Yukawa couplings λ_i that are > 1 at the GUT scale evolve to values in tight agreement with the corresponding fixed-point value at the electroweak scale. There is a region where these fixed points overlap, and it was noted in Ref. [8] that the m_b/m_τ ratio can be obtained from λ_t, λ_b, λ_τ fixed points without necessarily imposing the unification constraint $\lambda_b = \lambda_\tau$. We adopt the perturbative criterion [7] that the two-loop contribution to the evolution of the Yukawa couplings must not exceed one quarter of the one-loop contribution; this gives perturbative bounds $\lambda_t < 3.3$ and $\lambda_b < 3.1$, eliminating the region $\tan\beta > 65$.

Threshold corrections at the GUT scale can introduce model-dependent modifications into the unification criterion $\lambda_b = \lambda_\tau$, in the same way that mass splittings in the GUT scale spectra introduce corrections to gauge coupling unification. These corrections are expected to be larger when $\lambda_t(M_G)$ is large. In the following we impose the low energy boundary condition on $m_b(m_b)$ and consider relaxations of the $\lambda_b(M_G) = \lambda_\tau(M_G)$ unification. In Figure 2(a) we show the effects of threshold corrections for $\alpha_3(M_Z) = 0.118$ taking $m_b(m_b) = 4.25$ GeV. Corrections up to 20% are displayed for $\lambda_b(M_G) < \lambda_\tau(M_G)$. Large threshold corrections are not possible for $\lambda_b(M_G) > \lambda_\tau(M_G)$ since in this case the top quark coupling is pushed up against the Landau pole. In any case, one sees from Fig. 2(a) that for $\alpha_3(M_Z) = 0.118$, threshold corrections even as large as 10% do not destroy the fixed point solution. Large threshold corrections have a more severe impact on the fixed point solutions for significantly smaller values of $\alpha_s(M_Z)$. In Figure 2(b) the two-loop evolution of the top quark Yukawa coupling is plotted for different threshold corrections along with the value obtained from setting $d\lambda_t/dt = 0$ in Eq. (1), which gives an approximation good to about 10% to the fixed point.

Once m_t is known, the fixed-point relationship of Eq. (5) will uniquely determine $\sin\beta$. Global analyses of all electroweak data give a mass $m_t = 134^{+19+15}_{-24-20}$ GeV [13]. The present Tevatron lower bounds are $m_t > 103$ GeV from the D0 collaboration and $m_t > 108$ GeV from the CDF collaboration [14], giving $\sin\beta > 0.54$ through Eq. (5). However, there exist possible top candidate events, one from D0 and two from CDF, that have no obvious alternative theoretical interpretations. For the D0 event, a maximum-likelihood analysis is consistent with a mass in the range $130 < m_t < 160$ GeV (10% - 90% interval), and the production rates in both experiments are consistent with a similar range for m_t [14]. We shall also see below that the fixed point relation Eq. (5) plus LEP Higgs searches exclude $m_t \lesssim 130$ GeV. Given this admittedly circumstantial evidence, we are motivated to consider the implications if m_t is indeed in the range $130-160$ GeV, and hence from Eq. (5) that

$$0.85 < \tan\beta < 1.35 \, . \tag{7}$$

In general, the SUSY RGE analysis gives two solutions for $\tan\beta$ at given m_t [3,7] as shown in Fig.2 (except the value of $\tan\beta$ is not determined for m_t approaching 200 GeV). However, for $m_t < 160$ GeV, the large $\tan\beta$ solution is disfavored by perturbative criteria, as shown in Fig.2, and by models [4,5,7] that give $|V_{cb}| = \sqrt{\lambda_c/\lambda_t}$. We therefore select the λ_t fixed point solution. A range of small $\tan\beta$ ($1 \lesssim \tan\beta \lesssim 5$) is also found in the standard SU(5) supergravity solutions of Ref. [15,16], with the upper bound obtained from proton decay limits; however other authors [17] find a weaker requirement $\tan\beta \lesssim 85$ from considerations of proton decay bounds.

The Higgs spectrum in minimal SUSY models [18] consists of two CP-even scalars h and H ($m_h < m_H$), a CP-odd state A and two charged scalars $H^{\pm}$; at tree level they are fully described by two parameters m_A and $\tan\beta$. Important one-loop corrections depend on m_t and various SUSY parameters (mainly the top-squark mass $m_{\tilde{t}}$), giving mass corrections of order $\Delta m_h^2 \sim G_F m_t^4 ln(m_{\tilde{t}}/m_t)$. One usually selects typical values of m_t and $m_{\tilde{t}}$ and then analyzes how the Higgs masses, couplings and detectable signals vary across the $(m_A, \tan\beta)$ plane. Existing LEPI data [19] already exclude some areas of this plane. Extensive analyses [20–23] of future possibilities at LEPII and SSC/LHC agree that almost the whole parameter space can be explored, but that an inaccessible region remains (with boundary depending on m_t and $m_{\tilde{t}}$) where none of the Higgs scalars would be discoverable at either LEPII or SSC/LHC. On the other hand, possible future e^+e^- colliders above the LEPII range could complete the coverage, guaranteeing the discovery of at least one Higgs scalar [24]. It is therefore interesting and important to know about any further theoretical constraints that could reduce the expected range of parameters and make it easier to confirm or exclude the minimal SUSY Higgs scenario. In the present Letter, we point out that the fixed-point SUSY-GUT solution imposes a severe constraint through Eq. (7), and we spell out the consequences for Higgs searches at LEPI, LEPII and SSC/LHC and future higher energy

e^+e^- colliders.

(i) LEPI searches. In the narrow range of Eq.(7), the $e^+e^- \to Z^*h$ channel gives the dominant signals at LEPI. The cross section differs from that of the corresponding Standard Model (SM) process by a factor $\sin^2(\beta-\alpha) \gtrsim 0.7$, where α is the h-H mixing angle, and the detection efficiency for h decays is approximately the same as that for H_{SM} (if we neglect the possibility of $h \to \tilde{Z}_1\tilde{Z}_1$ decays to an invisible lightest neutralino). Hence, with appropriate one-loop radiative corrections, the combined LEP bound $m(H_{SM}) > 61.0$ GeV [25] can be translated into a bound in the $(m_A, \tan\beta)$ plane, or alternatively a bound in the $(m_h, \tan\beta)$ plane, making use of Eq. (5). These bounds are shown in Fig.3; they imply that $m_t \gtrsim 130$ GeV and $m_h \lesssim 85$ GeV. The left-hand limit on m_h comes from LEP data; the right-hand m_h limit is the intrinsic upper limit at given $\tan\beta$. As higher statistics accumulate, the reach of LEPI will increase; then either h will be discovered or the left-hand limit will move to the right.

The bounds in Fig.3 depend somewhat on the one-loop radiative correction parameters. We have determined m_t via Eq. (5) and have chosen $m_{\tilde{t}} = 1$ TeV for illustration (with other SUSY parameters set as in Ref. [23] and playing little role). Lowering $m_{\tilde{t}}$ makes the bounds more stringent; the bound in Fig.3(a) moves right, the left-hand bound in Fib.3(b) becomes more vertical, and the right-hand bound in Fig.3(b) moves left. Had we chosen $m_{\tilde{t}} = 0.3$ TeV instead, the LEPI bounds would have essentially excluded the entire range $\tan\beta < 1.35$ and $m_t < 160$ GeV of our fixed-point solutions.

(ii) LEPII searches. If $m_t \lesssim 160$ GeV, Figure 3(b) shows that 60 GeV$\lesssim m_h \lesssim$85 GeV. Then h will be discoverable at LEPII [26]; furthermore, the experimentally difficult situation $m_h \simeq M_Z$ is unlikely to occur. In the allowed kinematic range of $(m_A, \tan\beta)$, we find that $m_{H^\pm} \gtrsim 105$ GeV; hence $H^\pm$ will not be discoverable at LEPII. In principle A could be discovered via $e^+e^- \to Ah$, but the cross section is too small through almost all the allowed range, except for a small corner around $\tan\beta \simeq 1.3$, $m_A \simeq 75$ GeV, $m_h \simeq 60$ GeV. H production is forbidden by kinematics.

(iii) SSC/LHC searches. In the allowed region of Fig.3(b), previous analyses have shown that several Higgs signals will be viable [20–23]. Figure 4 shows the signal regions at the SSC from Ref. [23] for $h \to \gamma\gamma$, $H \to 4\ell$, $A \to \gamma\gamma$ and $H^\pm \to \tau\nu$; a signal from $A \to Zh \to \ell\ell\tau\tau$ will also be detectable [27] in a region somewhat smaller than that for $A \to \gamma\gamma$. There appear to be good prospects for detecting at least one more Higgs scalar H or A or $H^\pm$ in addition to the h detectable at LEPII (the LEPII reach for h is approximated by the contour $m_h = 90$ GeV). Note however that the $h \to \gamma\gamma$ signal is not expected to be viable for $m_h \lesssim 80$ GeV, because of steeply rising backgrounds [28]; the lower boundary of this signal region is essentially the $m_h = 80$ GeV contour. Note also that the $H \to 4l$ and $A \to \gamma\gamma$ (and $A \to Zh$) signals are cut off for $m_H \simeq m_A \gtrsim 2m_t$ by competition from $H, A \to t\bar{t}$ decays. Hence there is a parameter region ($m_A \gtrsim 2m_t, m_h \lesssim 80$ GeV, $\tan\beta \lesssim 1.2$) where no Higgs signal will be detectable at SSC/LHC and Higgs discovery (h alone) relies on LEPII.

(iv) Higher-energy e^+e^- linear collider searches. The m_t-fixed-point solutions above have small $\cos^2(\beta - \alpha) < 0.3$, 0.05 for $m_t < 160$, 145 GeV, respectively, tending toward zero as m_A increases; this factor suppresses the $e^+e^- \to Ah$, ZH and virtual WW, $ZZ \to H$ production channels. But the channels $e^+e^- \to Zh$, AH have factors $\sin^2(\beta - \alpha)$ and are unsuppressed, while $e^+e^- \to H^+H^-$ has no such factors; copious h production is therefore guaranteed, with H, A, $H^\pm$ too if they are not too heavy.

ACKNOWLEDGMENTS

This research was supported in part by the University of Wisconsin Research Committee with funds granted by the Wisconsin Alumni Research Foundation, in part by the U.S. Department of Energy under contract no. DE-AC02-76ER00881, and in part by the Texas National Laboratory Research Commission under grant no. RGFY9273. PO was supported in part by an NSF graduate fellowship.

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Figure Captions

Fig.1: Contours of constant Yukawa couplings $\lambda_i^G = \lambda_i(M_G)$ at the GUT scale in the $(m_t^{\rm pole}, \tan\beta)$ plane, obtained from solutions to the RGE with $\lambda_t^G = \lambda_b^G$ unification imposed. The GUT scale Yukawa coupling contours are close together for large λ^G. The fixed points describe the values of the Yukawa couplings at the electroweak scale for $\lambda_t^G \gtrsim 1$ and $\lambda_b^G \gtrsim 1$.

Fig.2: RGE results for $\alpha_s(M_Z) = 0.118$ with the boundary condition $m_b(m_b) = 4.25$ GeV. (a) GUT threshold corrections to Yukawa coupling unification. The solutions strongly exhibit a fixed point nature, for threshold corrections $\lesssim 10\%$. Taking a larger supersymmetric threshold M_{SUSY} or increasing $\alpha_s(M_Z)$ moves the curves to the right, so that the fixed point condition becomes stronger. (b) Evolution of the top quark Yukawa coupling for $\tan\beta = 1$. The dashed line indicates $\frac{d\lambda_t}{dt} = 0$ which gives an approximation to the electroweak scale value of m_t with accuracy of order 10%.

Fig.3: Fixed-point solution regions allowed by the LEPI data: (a) in the $(m_A, \tan\beta)$ plane, (b) in the $(m_h, \tan\beta)$ plane. The top quark masses are m_t(pole), correlated to $\tan\beta$ by Eq. (5).

Fig.4: SSC/LHC signal detectability regions, compared with the LEPI-allowed region of fixed-point solutions from Fig.3(a) and the probable reach of LEPII. The top quark masses are m_t(pole).