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The Nucleon Spin Crisis Bible

F E Close

June 1993

Science and Engineering Research Council

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F E Close

Rutherford Appleton Laboratory,
Chilton, Didcot, Oxon, OX11 0QX, England.

Abstract

When the new data on polarised lepton nucleon scattering are compared at the same value of Q^2 and with a common set of assumptions, a consistent picture of the spin content of the nucleon begins to emerge. Higher order effects in $O(\alpha_s)$, higher twist effects, modern data on unpolarised structure functions and an updated value for F/D are all important in analysing the data. The detailed x dependences of the polarisation asymmetry in the valence quark region are shown to confirm 20 year old predictions of the quark model and I argue that these are an important ingredient in decoding the nucleon spin puzzle.

Invited talk at the 6th ICTP Workshop, Trieste
May 3-7, 1993

Deep inelastic scattering of polarised leptons off polarised nucleons reveals the internal spin structure of the target. In 1988 the EMC results for such an experiment generated considerable excitement as their data seemed to imply that very little of a proton's spin is due to its quark constituents [1,2]. This generated considerable activity both by theorists and experimentalists. Theorists were inspired to look more deeply into the foundations of polarised inelastic formalism; the role of the anomaly, polarised gluons and polarised strangeness within the proton have all been intensively examined. On the experimental side, a round of second generation experiments was planned and, just weeks ago, we learned of the first data on neutron polarisation. It is for this reason that the question of the nucleon's spin has received the lead attention at this conference.

The new experimental data have been described in detail already. SMC uses a frozen polarised deuteron target [3], SLAC experiment E142 uses a polarised ^3He gas target [4]. To the extent that ^3He is dominantly in an S-state such that its two protons couple to net spin zero (by the Pauli principle), the ^3He polarisation directly leads to neutron polarisation. There will certainly be debate about the effect of nuclear binding on the interpretation of data and extraction of the nucleon structure functions (indeed this has already begun [5]). In this talk I shall assume that the experiments have satisfactorily taken this into account.

The material is organised as follows.

I shall begin by recalling the Genesis of the excitement, namely the proton spin measurement by EMC. We shall see that both the original measured value for the integral

$$I^p \equiv \int dx g_1^p(x)$$

of the polarised structure function has been modified as has the theoretical prediction for its magnitude. The outcome is that the spin "crisis" has been much reduced and is now only a 2σ effect (for the proton at least).

Most attention has focused on the magnitude of the integral which sums over valence and sea collectively and may even be affected by gluon polarisation. In "Exodus" I shall leave the integral and concentrate instead on the x -dependence of the polarisation asymmetry

$$A(x, Q^2) = g_1(x, Q^2)/F_1(x, Q^2)$$

This is the quantity which is most directly measured and for which predictions were made within the framework of a naive valence quark model more than 20 years ago. We shall see that the data on both proton and, now, neutron are in

remarkable agreement with prediction throughout the valence quark region ($x \gtrsim 0.1$). The message, for me at least, is that the valence quarks are polarised as “expected”; hence if the total integral I^p violates the theoretical expectation, this implies that the sea (\bar{q} and/or gluons) has a non-trivial role.

In “Numbers” I review the theoretical expectations for the magnitudes of the integrals. These depend on the F/D parameter for the octet of beta-decays and I present an updated value for this quantity and, in turn, for the sum rules.

I shall then return to the question of the integrals and in “Deuteronomy” shall compare the new information from SMC (deuteron) and E142 (3He , in effect neutron). Combining these results with the original EMC proton data reinforces the message that care is needed in interpretation of these data and that there is less of a “crisis” than advertised hitherto.

In “Revelations” I show that when all data sets are analysed at the same Q^2 (11 GeV^2) and with the modern unpolarised $F_1(x)$, a consistent picture emerges. The result is that the net quark spin is about 50% of that measured at low Q^2 , and in accord with expectation from QCD evolution. It implies that the sea is polarised and I conclude by discussing a possible way of probing a polarised sea by means of inclusive K^- production at HERA (HERMES collaboration).

1 Genesis

Most interest has centred on the integral

$$I^{p(n)} \equiv \int dx g_1^{p(n)}(x)$$

of the polarised structure function. There is an important sum rule due to Bjorken [6] which follows rigorously from QCD. Its expected value varies slightly with momentum transfer Q^2 .

$$I^p - I^n = 0.210 \left(1 - \frac{\alpha(Q^2)}{\pi}\right) Q^2 = 11 \text{ GeV}^2 \xrightarrow{\quad} 0.192$$

where I have specialised to $Q^2 = 11 \text{ GeV}^2$ as a suitable reference value. (Ref [7] has evolved all data to a common value in order to make a unified comparison).

Polarisation measurements from both protons and neutrons are needed to test this fundamental sum rule and, until recently, only proton data were available. Gilman [8] had shown how to write sum rules for p, n separately in terms of an (unknown) SU(3) singlet contribution. This was developed by Ellis and Jaffe [9]

who made the assumption that the strange sea within the proton and neutron is unpolarised, and predicted $I^{p,n}$. Their sum rule was expressed as a function of the F/D parameter that summarises nucleon and hyperon beta-decays.

The sum rule for the proton was written

$$I^p = \frac{1}{12} \left(\frac{g_A}{g_V} \right) \left[1 + \frac{5}{3} \frac{3F - D}{F + D} \right]$$

and if $F/D = 0.63 \pm 0.02$ (as assumed in ref 1, 2) the theoretical value becomes 0.20 (reduced to 0.191 after $O(\alpha_s)$ QCD corrections are included). The original measurement of I^p by EMC quoted

$$I^p(\text{EMC, ref 1}) = 0.116 \pm 0.012 \pm 0.026$$

and was compared with

$$I^p(\text{TH}) = 0.191 \pm 0.002$$

This shortfall of 40% in the integral translated into the astonishing conclusion that the net quark spin

$$\Delta q \simeq 0$$

Thus was the so-called proton “spin crisis” born.

It is important to note that **both experimental and theoretical values have changed relative to the original numbers** quoted above. The modern values are

$$I^p(\text{EMC, ref 2}) = 0.126 \pm 0.011 \pm 0.014$$

while the theoretical expectation is more appropriately

$$I^p(\text{TH}) = 0.175 \pm 0.007$$

leading to a much reduced significance for the discrepancy.

I would first like to bring up to date the experimental situation concerning I^p and the status of criticisms that Roberts and I made at the time [10]. We made three particular criticisms; namely the sensitivity to

- i) F/D
- ii) extrapolation as $x \rightarrow 0$
- iii) unpolarised $F_1(x)$ used to obtain $g_1(x)$ from $A(x)$.

Point (i): I shall review the status of F/D in section (3).

Point (ii): we considered both a hypothetical $[x \ln^2(x)]^{-1}$ and x^α form. The former, it is now generally agreed, is not well motivated and is withdrawn; the

latter (Regge) power fit for $x \lesssim 0.1$ has been used widely and contributes typically $I(0;0.01) \simeq 0.002$. In view of our deepening understanding of the $x \rightarrow 0$ behaviour of $F_1(x)$ this may merit re-evaluation in future but I shall continue to adopt this conventional form here.

Point (iii) The choice of unpolarised structure functions in constructing $g_1(x)$ is a significant concern and there have been considerable developments here since the original EMC data appeared. To help estimate these uncertainties it is useful to recall the reasons why the quoted I^p has changed.

The contribution in the measured region ($0.01 < x < 0.7$) was originally computed using $F(x)$ of EMC. This gave

$$I(0.01;0.7) = 0.113 \pm 0.012$$

(at $Q^2 = 10.7 \text{ GeV}^2$). The contributions in the unmeasured regions were estimated to be

$$I(0;0.01) \simeq 0.002 \quad ; \quad I(0.7,1) \simeq 0.001$$

and hence the original reported value [1]

$$I^p = 0.116 \pm 0.012 \pm 0.026$$

In their subsequent paper EMC made a detailed comparison of how the integral varied when other $F_1(x)$ structure function data were used, notably those of BCDMS; this gives [2]

$$I^{BCDMS}(0.01;0.7) = 0.127 \pm 0.014$$

Using an average of I^{EMC} and I^{BCDMS} led to the total integral [2]

$$I^p(EMC + BCDMS) = 0.123 \pm 0.013 \pm 0.019$$

Independent of the above SLAC had measured their own I^p . The combination of the EMC(BCDMS) and SLAC raised I^p slightly and reduced the errors, giving the modern value [2]

$$I^p = 0.126 \pm 0.010 \pm 0.015$$

There are two developments that imply that reevaluation of this value is warranted. First, we now have improved understanding of the relative normalisations of the various $F_1(x)$ (this is reviewed in ref 11). In ref [10] we used BCDMS data, hence our $I^p \simeq 0.14$. The NMC data at $x < 0.1$ (which link smoothly onto the $x < 0.01$ data from HERA) suggest that the $F_1(0.01 < x < 0.1)$ may be

rather bigger than thought hitherto. This in turn would cause $g_1(x)$ to increase in this region with corresponding increase in the extrapolated $I(0,0.01)$. Thus a new computation of $g_1(x)$ from the measured $A(x)$ is called for. A preliminary investigation [7] suggests $I^p \simeq 0.138$.

I shall critically examine the theoretical prediction for the integrals later. First I shall concentrate on the directly measured asymmetry by means of a pedagogical example.

2 X-odus

Most analyses of the “spin crisis” have concentrated on the **integral** of $g_1(x)$. Here I shall concentrate on the detailed dependence of the **asymmetry** in the valence region. The x **dependence** will provide important clues as to the source of the “spin crisis”.

2.1 A simple pedagogic example and its generalisation

I will begin with the constituent quark model in order to give a specific example of more general ideas that will be developed later. Although this model has no a priori justification in the case of deep inelastic polarised structure functions, nonetheless it will be seen to give a remarkably good average (in x_{bj}) description of some of the data. Thus it may provide a suitable reference from which intuition may be developed and more legitimate models built.

Given that all pairs of coloured quarks attract into $\bar{3}$, which are antisymmetric under interchange, it follows from Pauli that the pairs must be symmetric under interchange of all other quantum numbers. Let us suppose that they are symmetric in momentum space (this is the case in the constituent picture where the quarks are in overall S -wave). This implies that identical flavours (such as uu in the proton or dd in the neutron) couple symmetrically in spin (thus to $S = 1$ in the nucleon rest frame).

Coupling the $S = 1$ with the $S = 1/2$ of the remaining valence quark to form overall $J=1/2$ (this is the rest frame constituent quark description) yields

$$p^\uparrow = \sqrt{\frac{2}{3}}(u^\uparrow u^\uparrow)d^\downarrow + \sqrt{\frac{1}{3}}(u^\uparrow u^\downarrow)d^\uparrow \quad (1)$$

where arrows refer to the \hat{z} component of spin. This implies that the spin weighted

numbers of flavours are

$$u^\uparrow = 5/3 \quad u^\downarrow = 1/3 \quad d^\uparrow = 1/3 \quad d^\downarrow = 2/3. \quad (2)$$

and so

$$\begin{aligned} u &\equiv u^\uparrow + u^\downarrow \equiv \int dx u(x) = 2 \\ d &\equiv d^\uparrow + d^\downarrow \equiv \int dx d(x) = 1 \\ \Delta u &\equiv u^\uparrow - u^\downarrow \equiv \int dx \Delta u(x) = 4/3 \\ \Delta d &\equiv d^\uparrow - d^\downarrow \equiv \int dx \Delta d(x) = -1/3 \end{aligned} \quad (3)$$

This model clearly has all of the proton's spin carried by its valence quarks

$$\Delta q \equiv \Delta u + \Delta d = 1 \quad (4)$$

and immediately demonstrates the “bad” SU(6) result for g_A/g_V since

$$\frac{g_A}{g_V} \equiv \Delta u - \Delta d = \frac{5}{3} \quad (5)$$

Closer examination of the derivation of this result suggests that it should read in general (see e.g. ref 13)

$$\frac{g_A}{g_V} = \frac{5}{3} \Delta q \longrightarrow \Delta q \simeq 0.75 \quad (6)$$

To the extent that nucleon magnetic moments probe the spin magnetic moments of the constituents and assuming that the latter are proportional to the electrical charges of the quarks, then the ratio μ_p/μ_n is in proportion to $\Delta u/\Delta d$

$$\frac{\mu_p}{\mu_n} = \frac{\frac{2}{3}\Delta u - \frac{1}{3}\Delta d}{\frac{2}{3}\Delta d - \frac{1}{3}\Delta u} = \frac{2(\frac{\Delta u}{\Delta d}) - 1}{2 - (\frac{\Delta u}{\Delta d})} \quad (7)$$

(where $\Delta d^n \equiv \Delta u^p \equiv \Delta u$ and $\Delta u^n \equiv \Delta d^p \equiv \Delta d$ in the above). We see that the empirical result follows successfully

$$\frac{\mu_p}{\mu_n} = -\frac{3}{2} \leftrightarrow \frac{\Delta u}{\Delta d} = -4. \quad (8)$$

Now consider the $\langle e_i^2 \rangle$ weighted Δq_i . This is intimately related to what is probed in spin dependent inelastic scattering of leptons on polarised nucleons in the $x \gtrsim 0.1$ region dominated by valence quarks [13].

This “asymmetry”

$$A = \Sigma e_i^2 \Delta q_i / \Sigma e_i^2 q_i. \quad (9)$$

For a neutron

$$A^n \simeq \frac{1}{9} \Delta u + \frac{4}{9} \Delta d \rightarrow 0 \quad (10)$$

which follows because $\Delta u = -4\Delta d$. For a proton

$$A^p = \frac{4}{9}\Delta u + \frac{1}{9}\Delta d \equiv \frac{1}{6}(\Delta u - \Delta d) + \frac{5}{18}(\Delta u + \Delta d) \rightarrow \frac{5}{9} \quad (11)$$

For future reference note that

$$A^n(\Sigma e_i^2 q_i)^n = \frac{5}{18}\Delta q - \frac{1}{6}\frac{g_A}{g_V} \quad (12)$$

$$A^p(\Sigma e_i^2 q_i)^p = \frac{5}{18}\Delta q + \frac{1}{6}\frac{g_A}{g_V} \quad (13)$$

The above picture would apply to the deep inelastic polarisation in a primitive picture where the nucleon is made of three quarks each of which, symmetrically, carries fraction $x = 1/3$ of the parent's momentum. Thus the structure function $\sim \delta(x - 1/3)$. In this toy model, the above results may be written

$$\begin{aligned} 2F_1^n(x) &= \frac{2}{3}\delta(x - 1/3) \quad ; \quad A^n(x) = 0 \\ 2F_1^p(x) &= \delta(x - 1/3) \quad ; \quad A^p(x) = \frac{1}{3}\frac{g_A}{g_V}\delta(x - 1/3) \end{aligned} \quad (14)$$

where F_1 is the unpolarised transverse structure function of the target (note that $2xF_1(x) \equiv F_2(x) = \Sigma e^2 xq(x)$).

It is well known that the ratio $\frac{F_1^n}{F_1^p}(x = 1/3) \simeq \frac{2}{3}$ in the real world; thus it was suggested over twenty years ago, in advance of any data for polarised asymmetries, that the $A^{n,p}$ predictions may also be realised when $x \simeq 1/3$. Thus it is quite remarkable that the $A^p(x = 1/3)$, measured in 1977 and 1987, and the new reports of A^n (from either polarised ^3He or deuteron) are in complete agreement with these predictions, (fig 1).

In 1972 Gilman suggested that eqn 14 be generalised in an integrated form for non- diffractive (valence quark) contributions, thus

$$\int dx (F_1^p(x) - F_1^n(x)) = 1/6 \quad (15)$$

$$\int dx (g_1^p(x) - g_1^n(x)) = 1/6 \frac{g_A}{g_V} \quad (16)$$

where $g_1(x) \equiv F_1(x)A(x)$. Thus one sees the origin of the Gottfried (eqn 15) and Bjorken (eqn 16) sum rules in this toy model [13]. But it also, through eqns (12, 13), leads to sum rules for the proton and neutron individually

$$\int dx g_1^{(n)}(x) = \pm \frac{1}{12}\frac{g_A}{g_V} + \frac{5}{36}\Delta q \quad (17)$$

These correspond to the Ellis-Jaffe sum rules in the particular case where $F/D = 2/3$. For general F/D one may write

$$\frac{g_A}{g_V} \equiv F + D ; \quad (18)$$

$$\Delta q \equiv 3F - D \quad (19)$$

and so eq (17) generalises to the Ellis-Jaffe form

$$\int dx g_1(x) = \frac{1}{12} \frac{g_A}{g_V} [\pm 1 + \frac{5}{3} \frac{3F - D}{F + D}] \quad (20)$$

The above picture at $\delta(x - 1/3)$ was “generalised” to an integrated form which is the Ellis-Jaffe sum rule. If the latter is violated by data, perhaps we can use intuition from the above example to see where the violation originates.

In particular we saw how the sum rule is derived from the measured asymmetry, $A(x)$, albeit somewhat remotely. The $A(x \simeq 1/3)$ predicted in the simple model is remarkably good. Let us return to that model and see what **predictions** it made for $A(x \gtrsim 0.1)$ **several years** prior to any data on polarised deep inelastic scattering.

Guided by the unpolarised data, where $\frac{F_1^n}{F_1^p}(x \rightarrow 1) < \frac{2}{3} \rightarrow \frac{1}{4}$ I suggested that $A^{p,n}(x \rightarrow 1) \rightarrow 1$. The foundations for this are described in some detail in chapter 13 of ref (13) and I shall not repeat them here. The latter prediction was put on more solid ground following the advent of QCD and the development of counting rules [14]. The latter imply [15] that for a system of N constituents

$$F_{a/A}(x) \sim (1 - x)^{2N-1+2|\Delta h|}$$

where Δh is the difference in helicity

$$\Delta h = |h_a - h_A|$$

and hence that $A(x \rightarrow 1) \rightarrow 1$. Prior to these helicity dependent counting rules I had advocated [12]

$$A^n(x \rightarrow 1) = A^p(x \rightarrow 1) = \xi$$

where ξ = fraction of hadron spin carried by quarks. Assuming this to be 75% generated the curves in ref (12). Specifically if $F_1^n/F_1^p(x) \equiv R(x)$ then

$$A^p(x) = \frac{19 - 16R(x)}{15} \xi$$

$$A^n(x) = \frac{2 - 3R(x)}{5R} \xi$$

These curves are compared with the subsequent proton and neutron data in fig 2. The agreement in the valence region is quite remarkable and suggests that **the polarisation response of valence quarks at large Q^2 is rather similar to that anticipated from our low energy experience.** Why this should be so is perhaps an interesting question in its own right, but it does suggest that any problems in the integrated structure function do not originate here. We must focus either on the sea polarisation (of either gluons or antiquarks) and/or on the strength of the unpolarised $F_1(x)$ by which the $A(x)$ is multiplied in order to reach $g_1(x)$.

In order to help determine what is the message of the sum rules, we must first establish what the sum rules are. The literature has not always been consistent on this.

3 Numbers

In order to define the sum rules, first write

$$I^p = I_3 + I_8 + I_0$$

$$I^n = -I_3 + I_8 + I_0$$

where [16]

$$\begin{aligned} I_3 &= \frac{1}{12} \left(1 - \frac{\alpha_s}{\pi}\right) a_3 \\ I_8 &= \frac{1}{36} \left(1 - \frac{\alpha_s}{\pi}\right) a_8 \\ I_0 &= \frac{1}{9} \left[1 - \left(1 - \frac{2N_f}{\beta_0}\right) \frac{\alpha_s}{\pi}\right] a_0 \end{aligned}$$

In terms of quark polarisation $\Delta q \equiv \int dx (q^\uparrow(x) - q^\downarrow(x))$, or the F and D parameters,

$$\begin{aligned} a_3 &= \Delta u - \Delta d && \equiv F + D \\ a_8 &= \Delta u + \Delta d - 2\Delta s && \equiv 3F - D \\ a_0 &= \Delta u + \Delta d + \Delta s \end{aligned}$$

so that a_0 corresponds to the net quark polarisation. Then since $F + D \equiv \frac{g_A}{g_V}$ we have the Bjorken sum rule

$$I^p - I^n = \frac{1}{6} \left(\frac{g_A}{g_V}\right) \left(1 - \frac{\alpha_s}{\pi}\right) \rightarrow \begin{cases} 0.210 & \alpha \rightarrow 0 \\ 0.192 & \alpha \simeq 0.25 \end{cases}$$

For the separate nucleons, and specialising to $N_f = 3$

$$I_n^p = \frac{1}{12} \left(\frac{g_A}{g_V} \right) \left\{ \left[\pm 1 + \frac{5}{3} \left(\frac{3F - D}{F + D} \right) \right] - \frac{\alpha_s}{\pi} \left[\pm 1 + \frac{7}{9} \left(\frac{3F - D}{F + D} \right) \right] \right\} + \frac{\Delta s}{3} \left(1 - \frac{\alpha_s}{3\pi} \right)$$

which reduces to the Ellis-Jaffe sum rule when $\alpha_s \rightarrow 0$ and if $\Delta s = 0$. Note that $(\frac{g_A}{g_V})$ at the front and $(F+D)$ in denominator are identical quantities: this has not always been maintained in analyses.

For the case of 3 flavours we may rewrite them in the form

$$\begin{aligned} I^p &= \frac{1}{6} \left(F + \frac{1}{3} D \right) \left(1 - \frac{\alpha}{\pi} \right) + \frac{1}{9} \Delta q \left(1 - \frac{\alpha}{3\pi} \right) \\ I^n &= \frac{1}{9} D \left(1 - \frac{\alpha}{\pi} \right) + \frac{1}{9} \Delta q \left(1 - \frac{\alpha}{3\pi} \right) \\ \frac{1}{2} I^{p+n} &= \frac{1}{12} \left(F - \frac{1}{3} D \right) \left(1 - \frac{\alpha}{\pi} \right) + \frac{1}{9} \Delta q \left(1 - \frac{\alpha}{3\pi} \right) \end{aligned}$$

Note that $I^{p,n}$ do **not** share a common factor of $(1 - \frac{\alpha}{\pi})$. (Contrast the expressions at eq 3 in ref 4 and ref (17) from which it originates. The analysis in ref 4 is thereby compromised). Furthermore E142 use $F = 0.47 \pm 0.04$ $D = 0.81 \pm 0.03$ from Jaffe-Manohar analysis [17], whose sum does not centre on the currently accepted value of $(g_A/g_V)_{np}$: this introduces further incongruity. As these F, D values [17] are determined from 1986 data, which are now significantly modified, a modern fit is called for.

I list the current world averages from the 1992 edition of the Particle Data Group [18], together with their F, D parameterisation and the $SU(6)$ value (arbitrarily) renormalised such that only 25% of the spin is “lost” to relativistic (lower components of spinors, see ref 12 and 13).

g_A/g_V	F, D	experiment[18]	$\frac{3}{4}(SU_6)$
np	$F + D$	1.2573 ± 0.0028	$5/4$
Λp	$F + \frac{1}{3}D$	0.718 ± 0.015	$3/4$
$\Xi \Lambda$	$F - \frac{1}{3}D$	0.25 ± 0.05	$1/4$
Σn	$F - D$	-0.340 ± 0.017	$-1/4$

The best fit, with $\chi^2 = 1.55$ for one degree of freedom (F/D with $F+D$ constrained to equal 1.257) is [7]

$$\left. \begin{aligned} F &= 0.459 \pm 0.008 \\ D &= 0.798 \mp 0.008 \end{aligned} \right\} F/D = 0.575 \pm 0.016$$

This is 1σ larger than in Close-Roberts 1988 paper [10] principally due to improved Λp and Σn data. I shall use these values in what follows; however there are two

caveats. First there is a systematic error, not included, which arises from the phase space or form factor corrections in the $\Delta S = 1$ examples [19]. The second is potentially more serious.

The quoted figures assume that in the hadronic axial current

$$A_\mu = g_A \gamma_\mu \gamma_5 - g_2 \frac{i \sigma_{\mu\nu} q^\nu \gamma_5}{m_i + m_j}$$

one has $g_2 = 0$. While this is assured in the limit where $m_i = m_j$ (such as $n \rightarrow p$) it is not necessarily so for $\Delta S = 1$. Indeed, in quark models one expects that $g_2 = 0(\frac{m_i - m_j}{m_{ij}})$ with $m_{ij} \equiv \frac{1}{2}(m_i + m_j)$. [20]

The Hsueh et al. analysis of Σn made a fit allowing for $g_2 \neq 0$ and found [21]

$$g_2 = -0.56 \pm 0.37$$

$$g_A = 0.20 \pm 0.08$$

This raises a tantalising possibility that the (g_A/g_V) throughout the octet are given by the naive quark model, all values being renormalised by 25% (such that the net spin is 0.75 rather than 1). This is illustrated in the fourth column of the above table. Such an eventuality would correspond to the realisation of the effective quark model result

$$F = 1/2, \quad D = 3/4 \quad ; \quad F/D = 2/3$$

We note that either of these solutions gives essentially the same predictions for the non-singlet contributions to $I^{p,n}$

$$\frac{1}{6}(F + \frac{1}{3}D) = 0.131 \text{ versus } 0.125$$

$$\frac{1}{9}D = 0.09 \text{ versus } 0.08$$

So the inferred Δq from I^p or I^n changes only marginally; the sensitivity is Δs [22] since

$$\Delta s \simeq -(F - \frac{D}{3}) + \frac{1}{3}\Delta q$$

and

$$F - \frac{1}{3}D = 0.19 \text{ versus } 0.25$$

It is the assumption that $\Delta s = 0$ in the Ellis-Jaffe sum rule and the sensitivity to the combination $F - \frac{1}{3}D$ that causes the sensitivity of the $I^{p,n}$ (THY) to F,D.

Using the best fit F, D in the standard $g_2=0$ approach, the predictions are

$$\begin{aligned}
I^p &= (0.121 \pm 0.001)(1 - \frac{\alpha}{\pi}) + \frac{1}{9}\Delta q(1 - \frac{\alpha}{3\pi}) \\
I^n &= (-0.089 \pm 0.001)(1 - \frac{\alpha}{\pi}) + \frac{1}{9}\Delta q(1 - \frac{\alpha}{3\pi}) \\
\frac{1}{2}I^{p+n} &= (0.016 \pm 0.001)(1 - \frac{\alpha}{\pi}) + \frac{1}{9}\Delta q(1 - \frac{\alpha}{3\pi})
\end{aligned}
\tag{21}$$

We are now in position to confront these with the new data.

4 Deuteronomy

SMC have measured the deuteron (essentially the sum of proton and neutron). Roberts and I [10] had pointed out that this has direct interest in that the I^{p+n} is less sensitive to errors than I^p or I^n separately if one wishes to extract the net quark spin.

The sum rule for the proton immediately highlights a sensitivity in the I^p data. To the extent that $I^p(EMC) \simeq 0.12$ then $\Delta q \rightarrow 0$ if $\alpha_s \rightarrow 0$, which is the much heralded spin crisis. However, the $\frac{1}{9}$ in front of Δq causes a rapid turn on of the inferred Δq when any small changes are included in I^p or α_s . For example

$$\Delta q \simeq \frac{9\alpha_s}{\pi}0.12 + 9(I^p - 0.12)(1 + \frac{\alpha}{3\pi})$$

The first term alone generates 10% spin; systematic and statistical errors in I^p each induce large uncertainties in Δq . This is what underlies the result that the deviation is only a 2σ effect. Moreover, higher order corrections in $O(\alpha_s)$ at finite Q^2 may also induce corresponding ninefold increases in the corrections to Δq . Misestimates of the contribution to I^p as $x \rightarrow 0$ will also affect the inferred value of Δq ; we shall see later that deuteron data may enable this particular problem to be circumvented.

I^p and I^n , are expressed in general as a linear sum of g_A/g_V and Δq . Thus when extracting Δq from I^p (or I^n) one must first remove the “unwanted” contribution (g_A/g_V). This is what, in part, causes small percentage errors in I^p to be larger percentagewise in $\Delta q \simeq 9(I^p - (g_A/g_V))$ and, furthermore, the factor of 9 makes things worse still.

It is this sensitivity that was the main message in ref [10], being more general than the particular example used as illustration there. In particular, the fluctuation in the three data points at $x \rightarrow 0$ in the EMC proton data could arguably

take care of (some of) the 2.5σ effect if future data shows that the asymmetry values given by these data points are at the upper end of their magnitudes. (Note that the NMC unpolarised structure function [23] tends to suggest that the $g_1(x)$ is underestimated here [7].)

We can form combinations of $I^{p,n}$ that emphasise the Δq or eliminate it. Thus two natural combinations are

$$I^p - I^n = 2I_3$$

$$I^p + I^n = I_8 + I_0$$

The former has eliminated Δq (and is the Bjorken sum rule combination). In ref 10 we noted that the orthogonal combination (essentially given by I^d) emphasises Δq .

First recall the individual nucleon integrals

$$I_n^p \simeq \begin{pmatrix} 0.12 \\ -0.09 \end{pmatrix} \left(\frac{g_A}{g_V} \right) \left(1 - \frac{\alpha_s}{\pi} \right) + \frac{\Delta q}{9} \left(1 - \frac{\alpha}{3\pi} \right)$$

The coefficient of $\left(\frac{g_A}{g_V} \right)$ saturated the I^p . But now form $I^N = \frac{1}{2}(I^p + I^n)$ which to good approximation may be measured directly with a deuteron target,

$$I^N \simeq (0.016) \left(\frac{g_A}{g_V} \right) \left(1 - \frac{\alpha_s}{\pi} \right) + \frac{\Delta q}{9} \left(1 - \frac{\alpha}{3\pi} \right)$$

We see that the “unwanted” $\frac{g_A}{g_V}$ term has been suppressed so that Δq has a chance of showing up. At first sight one may say that there is no free lunch and that $I^N \simeq 0$ so that one still has problems. This is indeed true for the total integral; however one may place an **upper bound** on Δq (and hence if this is small one has established the spin crisis).

The reason lies in the prediction (now confirmed) for the behaviour of $A(x)$, and hence $g_1(x)$, in the valence region, $x \gtrsim 0.1$. It is known that $A^p(x) > 0$, and was predicted [12, 24] that $A^n(x) > 0$ certainly for $x \gtrsim 0.3$. The $A^n(x \rightarrow 0) \lesssim 0$ and so theoretical prejudice (and maybe even data) suggest that

$$I^d(x < x_c) < 0 \quad ; \quad I^d(1 > x > x_c) > 0$$

for some small x_c . If we assume that $I^d(x \rightarrow 0)$ does not oscillate, one can immediately measure an upper bound to Δq .

$$\int_{x_c}^1 g_1^d(x) dx \geq \int_0^1 g_1^d(x) dx \simeq 0.016 \left(\frac{g_A}{g_V} \right) \left(1 - \frac{\alpha_s}{\pi} \right) + \frac{1}{9} \Delta q \left(1 - \frac{\alpha}{3\pi} \right)$$

We will now confront this with data and consider the implications of the new experiments. First we shall compare the experiments while ignoring the fact that they span different ranges of Q^2 . This is done merely for first orientation and to highlight the importance of careful treatment of Q^2 : in the next section we shall see how a comparison of the experiments at a **common** Q^2 value leads to rather different conclusions.

The SMC measurement on the deuteron gives [3]

$$I^n + I^p = 0.049 \pm 0.044 \pm 0.032$$

which on combining with I^p of EMC yields

$$I^n = -0.08 \pm 0.04 \pm 0.04.$$

In turn this would give

$$I^p - I^n = 0.206 \pm 0.06$$

which is compatible with the Bjorken sum rule (though with large uncertainties).

The E142 data are at rather small Q^2 and give for the neutron “directly”

$$I^n = -0.022 \pm 0.011$$

If this were the only datum available we would compare with the Ellis-Jaffe sum rule and conclude that all is well with the world. (Though, as we shall see in section 5, the small value of Q^2 require this to be re-evaluated.) If now one combines this result with the I^p from EMC one obtains

$$(I^p - I^n)(EMC \text{ and } E142) = 0.148 \pm 0.022$$

which is about two standard deviations below the fundamental Bjorken sum rule (this 2σ reflecting the 2σ shortfall of I^p from EMC relative to Ellis-Jaffe).

As a final way of combining the three experiments pairwise we can take I^n from E142 and I^{p+n} from SMC. This would imply

$$I^p = 0.071 \pm 0.055$$

and hence

$$I^p - I^n = 0.093 \pm 0.08$$

We can now take combinations of all experiments and compare their quoted integrals pairwise with the $I^{p,n}$ and I^{p-n} . This gives

	I^p	I^n	I^d	I^{p-n}
Theory	.173 (.015)	−.019 (.011)	.15 (.02)	.192
<i>EMC + SMC</i>	.13 (.02)	−.08 (.06)		.21 (.06)
<i>E142 + EMC</i>		−.02 (.01)	.10 (.03)	.15 (.02)
<i>SMC + E142</i>	.07 (.06)		.05 (.05)	.09 (.08)

Note that the “theory” value corresponds to $Q^2 = 11 \text{ GeV}^2$ whereas the experiments span a range of Q^2 . Superficially only the first of these appears to survive “self consistency” checks: proton and neutron each violate the E-J sum rule in a common manner that preserves the fundamental Bjorken sum rule. The second possibility violates Bjorken due to some anomaly that is manifested in proton data alone. The third combination also appears to have an equivalent “inconsistency” to the above and one may be tempted therefore to “blame” the SLAC neutron data; however, the neutron data satisfy the E-J sum rule and would appear “innocent” if that were all that we knew.

However this comparison is misleading because we have cavalierly compared and combined data from experiments at different Q^2 and whose $g_1(x)$ have been constructed from the $A(x)$ by invoking different data on $F_1(x)$. In ref 7 we have attempted to compare the experiments under common assumptions. This has an interesting consequence which I now summarise.

Revelations

The sensitivity to the unpolarised structure function, and Q^2 dependence, is highlighted by the following example [7]. Using the NMC data for $F_1^p(x)$ the EMC proton integral and inferred Δq become

$$I^p = 0.134 \pm 0.012 \quad ; \quad \Delta^p q = 22.9 \pm 12.0\%$$

For the E142 neutron, evolution of the data to a common value $Q^2 = 11 \text{ GeV}^2$ yields (assuming that the asymmetry is Q^2 independent)

$$I^n = -0.031 \pm 0.007 \quad ; \quad \Delta q = 49.8 \pm 6.7\%$$

The Bjorken sum rule is to be compared with

$$I^{p-n} = 0.165 \pm 0.014$$

There is a discrepancy between the I^d of SMC and that constructed from the $p+n$ of EMC and E142. When one examines the $g_1(x)$ directly one notices, in particular, that the $x \gtrsim 0.4$ data of SMC have large errors and give a lack of strength in this region relative to the $p+n$ combination. We suggest [7] that the $p+n$ “constructed data” be used as the guide here because

- a) they are consistent with the SMC and have smaller errors
- b) the $A^d(x \rightarrow 1)$ is predicted to be large (unity ?) as $x \rightarrow 1$ and this also favours the high end of the SMC error bars, consistent with the EMC + E142 combination.

Taking this as the most probable solution in the $x \gtrsim 0.4$ valence quark region, the constructed I^d and resulting Δq are

$$I^d = 0.037 \pm 0.018 \quad ; \quad \Delta q = 21.6 \pm 17.8$$

In fig (3) I summarise the various integrals and Δq from these “improved” data. It is particularly notable that **all combinations now give excellent agreement with the fundamental Bjorken sum rule**. (Note that at small Q^2 higher order corrections must be included such that [29] for three flavours

$$I^{p-n} \simeq \frac{1}{6} \frac{g_A}{g_V} \left(1 - \frac{\alpha}{\pi} - 3.6 \left(\frac{\alpha}{\pi} \right)^2 - 20 \left(\frac{\alpha}{\pi} \right)^3 \right)$$

If $\frac{\alpha}{\pi} = 0.1$ this reduces I^{p-n} to some 0.177. Similar effects may be expected for $I^{p,n}$ separately. Higher twist can also be important, especially in the case of the neutron [30,31]; $\delta I^n \simeq 0.04/Q^2$, $\delta I^p \simeq -0.004/Q^2$. Ref [30] has shown that these can make a significant effect on the analysis, especially as the neutron E142 data have low $Q^2 \simeq 2 \text{ GeV}^2$ and conclude that

$$\Delta q = 22 \pm 10\%$$

This is in accord with our results abstracted from d and p experiments above ($22 \pm 18\%$ and $23 \pm 12\%$ respectively) as is to be expected since these tended to have $\langle Q^2 \rangle$ large and hence be less susceptible to $1/Q^2$ corrections. However the value inferred for Δq depends rather sensitively on the assumed values of the higher twist coefficients. The above value is valid to the extent that one accepts the QCD sum rule input; if however one chooses to fit the A/Q^2 terms such that a common value of Δq obtains for each of the three experiments then [7] finds $\Delta q = 38 \pm 48\%$. Clearly a deeper understanding of the magnitude of $1/Q^2$ corrections

for p and n targets is required if Δq is to be extracted from low Q^2 data. Note again that

$$\Delta q = 41 \pm 6\%$$

is a minimum χ^2 fit if the neutron ($Q^2 = 2 \text{ GeV}^2$) data are included and evolved without any $1/Q^2$ higher twist contribution.

The conclusion seems to be that the nucleon spin structure as revealed in low energy data is **modified** when evolved to larger Q^2 but not totally destroyed. Higher twist and $O(\frac{\alpha}{\pi})^3$ corrections to the Ellis- Jaffe sum rules are probably needed before reliable conclusions on Δq can be drawn. The relative spin strength of the u and d valence quarks is preserved (as evidenced by the successful prediction of $A(x)$ in the valence region) but some of their overall strength is evolved into the (polarised) sea including glue and strangeness.

If the sea is polarised then we need experiments to reveal this directly. Processes that are dominated by gluons (such as $Q\bar{Q}$ production) may be tested for target polarisation dependence in order to measure $\Delta G(x)$ (see refs 25). The presence of polarised flavour in the sea [26] may be revealed by inclusive lepto-production of hadrons, in particular K^- .

From unpolarised deep-inelastic scattering the probability is estimated to be greater than 70% that the fastest forward-moving charged hadron with $z > 0.5$ contains a quark of the same flavour as the scattered quark [27].

In principle the production of $K^-(s\bar{u})$ at large $z(= E_K/E_\gamma)$ in $\ell_p \rightarrow \ell' K^- \dots$ is most clear in this regard. The lepton interacts with a q (or \bar{q}) which subsequently produces the detected hadron. As $z \rightarrow 1$ the most probable occurrence is that the hadron contains the struck $q(\bar{q})$ in its valence Fock state [13, 28]; thus a fast K^- is a signal for an s or \bar{u} having been struck by the current. As s, \bar{u} occur in the proton's sea, a target polarisation dependence of fast K^- production would indicate that the s or \bar{u} components of the proton sea are polarised; indeed the K^- inclusive asymmetry as $z \rightarrow 1$ is a direct measure of the amount and sign of the sea polarisation.

This simple example is too idealised. The limit $z \rightarrow 1$ is not accessible in practice and production of K^- at $z < 1$ is contaminated by the highly polarised valence quarks. However, in ref [26] we show that even for $z \leq 0.5$ there is a distinct dependence on sea polarisation. Measurement of the K^- dependence on beam-target polarisation promises to reveal polarisation in the flavoured sea. This experiment is planned to take place at HERA (the HERMES Collaboration) within a few years. This could clarify the way that a proton's spin is distributed

among its valence quarks and the flavoured $q\bar{q}$ sea.

Acknowledgements

I am indebted to E Gabathuler, RG Roberts, S Rock, GG Ross and T Sloan for discussions about the recent experiments.

References

- [1] EMC Collaboration, Phys. Lett. **B206**, 364 (1988).
- [2] EMC Collaboration, Nucl. Phys. **B328**, 1 (1989).
- [3] SMC Collaboration, B. Adera et al. Phys. Lett. **B302**, 553 (1993).
- [4] E142, P. Anthony et al. SLAC-PUB T/E-6101 (1993), Phys. Rev. Letters (1993).
- [5] C. Ciofidegli Atti et al. Perugia report DFUPG 75/93.
- [6] J.D. Bjorken, Phys. Rev. **148**, 1476 (1966); **D1**, 1376 (1970).
- [7] F.E. Close and R.G. Roberts RAL-93-040.
- [8] F.J. Gilman, Proc. of Scottish Universities Summer School in Physics (1973).
- [9] J. Ellis and R.L. Jaffe, Phys. Rev. **D9**, 1444 (1974).
- [10] F.E. Close and R.G. Roberts, Phys. Rev. Lett. **60**, 1471 (1988).
- [11] F.E. Close, Proc. of 25th International Conf. on HEP, Singapore, 1990; p 213;
A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Rev. **D47**, 867 (1993);
RAL-92-078 (Phys. Letters in press).
- [12] F.E. Close, Nucl. Phys. **B80**, 269, (1974);
F.E. Close, Proc. 19th International Conf. on HEP, Tokyo (1978) p 209;
see also F.E. Close, F.J. Gilman and I. Karliner, Phys. Rev. **D6**, 2533 (1972);
and F.E. Close, Proc. of Rencontre de Moriond, (1973).
- [13] F.E. Close, An Introduction to Quarks and Partons, Academic Press (1978).
- [14] S.J. Brodsky and G. Farrar, Phys. Rev. Lett. **31**, 1153 (1973).

- [15] A. Vainshtein and V. Zacharov, Phys. Letters **72B** 368 (1978).
- [16] J. Kodeira et al, Phys. Rev. **D20**, 627 (1979); J. Kodeira, Nucl. Phys. **B165**, 129 (1980).
- [17] R.L. Jaffe and Y. Manohar, Nucl. Phys. **B337**, 509 (1990).
- [18] Particle Data Group, Phys. Rev. **D45**, S1 (1992)
- [19] P. Ratcliffe, Phys. Lett. **242B**, 271 (1990).
- [20] J. Donoghue et al. Phys. Rev. **D35**, 934 (1987);
Z. Dziembowski and J. Franklin, Temple University report TUHE-89-11 (unpublished).
- [21] S. Hsueh et al, Phys. Rev. **D38**, 2056, (1988).
- [22] F.E. Close, Phys. Rev. Lett. **64**, 361 (1990).
- [23] NMC Collaboration, Nucl. Phys. **B371**, 3 (1992).
- [24] F.E. Close, Phys. Letters **43B**, 422 (1973).
- [25] See e.g. J. Guillet, Z. Phys. **C39**, 75 (1988);
Z. Kunszt, Phys. Lett. **B218**, 243 (1989).
- [26] F.E. Close and R.G. Milner, Phys. Rev. **D44**, 3691 (1991).
- [27] J.P. Albanese et al, Phys. Lett. **144B**, 302 (1984).
- [28] R.P. Feynman, Photon Hadron Interactions (Benjamin, 1972).
- [29] S. Gorishny and S. Larin, Phys. Lett. **B172**, 109 (1986);
S. Larin and J. Vermaseren, Phys. Lett. **B259**, 345 (1991).
- [30] J. Ellis and M. Karliner, CERN-TH-6898 (1993).
- [31] I.I. Balitsky et al, Phys. Lett. **242B**, 245 (1990).

Figure Captions

Figure 1. $A^{p,n}(x = 1/3)$ compared with prediction.

Figure 2. Predictions for $A(x)$ if $\zeta = 1$ (solid) or $\zeta = 3/4$ (dashed) for p and n , compared with data.

Figure 3. Diagrammatic representation of results and predictions.







