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June 1993

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submitted to The Astrophysical Journal Letters, January 1993

FERMI ACCELERATION TO THE HIGHEST ENERGIES

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ABSTRACT

Ball, Melrose and Norman have raised a number of criticisms of a paper in which we suggest that stochastic Fermi acceleration, despite being currently out of favour, might offer a solution to the long-standing problem of the acceleration of cosmic rays to the highest observed energies. While the points made in criticism prompt the setting of the Fermi process in new perspective, they are found not to detract from the conclusion that small-scale (sub-parsec) regions of strong magnetic field ($\sim 10\mu T$) with random motion at speeds $\sim 10^{-3}$ c could, in principle, account for acceleration to $\sim 10^{20}eV$ within the constraints imposed by current knowledge of the random component of the galactic magnetic field.

1. INTRODUCTION

Ball et al (1992b, see also Ball et al 1992a) have raised a number of objections to the suggestion (Bryant et al 1992a) that Fermi acceleration, when recognized and treated as a true stochastic process, may be able to account for cosmic ray acceleration to the highest observed energies $\sim 10^{20}eV$. Their criticisms may be enumerated as (i) No new physical effects or new physical implications result from the statistical approach, (ii) Inclusion of the bias favouring energy-gaining collisions makes the statistical treatment equivalent to the standard treatment, (iii) Acceleration through ten orders of magnitude cannot be accomplished within the Hubble time, and (iv) The magnetic fields required to reach the highest energies are implausibly high.

We accept that all of these points are important, and respond to each individually below. It should be stressed from the outset, though, that the key feature of our earlier paper, not acknowledged in these criticisms, was the small scale size (sub parsec) found for the scattering centres. This factor is instrumental in establishing a suitably short mean free path and acceleration time, and in reducing the average random magnetic field to observed levels.

2. DISCUSSION

(i) New Physical Effects

We have become aware, as an indirect result of the criticism, that the statistical analysis employed in our recent study (Bryant et al 1992a) yielded a result identical to that obtained by Davis (1956), relatively soon after the Fermi (1949) theory was advanced. His finding that random walk in energy might offer a solution overcoming the slowness of the systematic progress with which Fermi acceleration was originally, and continues to be, identified, does not appear to have been firmly registered in the subject which currently tends to dismiss Fermi acceleration for the highest energy particles. The reason for its neglect may lie partly in the remark by Davis that pure random walk acceleration approaches the steady state at any energy in *about* the time required for a particle to reach this energy by the steady acceleration treated by Fermi. This important qualification in this statement was omitted by Ball et al in declaring that the characteristic time for the asymptotic power law spectrum to be set up at a given energy *is* the time required for systematic acceleration to that energy. In contradiction to Davis, Ball et al 1992b declare that acceleration is due to the systematic, not the diffusive, effect. Thus, while our consideration of distribution broadening, as opposed to systematic acceleration, is, we now realise, not new, it is an effect that appears to have been neglected. The implications are discussed below in relation to acceleration time.

The dismissal of the "high flyers" of the statistical analysis as "*just* the diffusive high-energy tail" seems to us to be inappropriate when it is precisely the high-energy tail of the distribution that is at issue. It is clearly the case that particles which by chance gain energy rapidly play a significant role in forming the high-energy tail. Their role will be particularly important if they also have long residence times. We fully accept that the residence time is crucial, and that our earlier estimate was of a maximum rather than an effective acceleration rate. We explore this point further in (iii) below.

(ii) Effect of Bias

In an exercise parallel to our own, but with the standard bias included, Ball et al (1992a) derive for random and systematic acceleration combined a negative spectral power-law exponent of

$$\gamma_{r+s} = \frac{1}{2}\left[1 + \sqrt{1 + \frac{8}{N_0\beta^2}}\right] \tag{1}$$

This is to be compared with our result with no bias, in accord with Davis's earlier finding, that

$$\gamma_r = 1 + \sqrt{\frac{2}{N_0\beta^2}} \tag{2}$$

The values of $N_0\beta^2$ to give the observed $\gamma \simeq 3$ are $\frac{1}{3}$ in (1), and $\frac{1}{2}$ in (2). Note that this represents a change by a factor of only 1.5. When carried forward in the earlier analysis, this factor increases the limiting energy E_{lim} by a factor of ${}^6\sqrt{1.5}$, and reduces the critical magnetic field strength by ${}^3\sqrt{1.5}$. The bias, therefore, enhances the process, as it must, but the enhancement is negligible.

(iii) Acceleration Time

Ball et al show how the cosmic-ray energy spectrum resulting from injection at a single energy would approach the asymptotic power law form as N/N_0 increases. This demonstrates that, for the asymptotic spectrum to be set up over 10 decades of energy, some particles must reside in the galaxy many times longer than the mean escape time. With this we, of course, concur. They then appeal to an argument based on the systematic acceleration of typical particles to show that, in a system of dimensions very different to the ones resulting from our analysis, the 92 nominal residence times required for this would take 10^{10} — $10^{12}yr$, of order of or greater than the Hubble time ($\approx 10^{10}yr$). This conclusions is based on an arbitrary $\approx 100pc$ separation of scattering centres. If, however, we evaluate $92N_0$ using the parsec and sub parsec mean free paths from our original table 1, we find that the times required range from $6 \times 10^6 yr$ for $\beta = 10^{-1}$ to $6 \times 10^9 yr$ for $\beta = 10^{-4}$. Acceleration through as many as 10 orders of magnitude is, therefore, not ruled out, even considering systematic acceleration alone.

Acceleration by random walk, although of the same order, eases the position still further. If we extend fig 1 of Ball et al to include higher values of N/N_0 we find that for the distribution to reach 80 percent of the asymptotic value (a criterion used by Davis(1956), consistent with a more recent assessment by Watson (1984)), acceleration through 10 decades of energy requires residence times corresponding to $N/N_0 \approx 25$. This is just the conclusion reached analytically by Davis. He found that the numbers of orders of magnitude x_r over which the asymptotic distribution is approached to within 20 percent by random walk in energy alone is given (using present terminology) by:

$$\left(log\frac{E}{E_0}\right)_r = \frac{1}{2.3}\sqrt{N_0\beta^2}\left[\sqrt{2}\frac{N}{N_0} - \sqrt{\frac{N}{N_0}}\right] \qquad , N \geq 4N_0 \qquad (3)$$

The corresponding result for systematic acceleration is:

$$\left(log\frac{E}{E_0}\right)_s = \frac{1}{2.3}N\beta^2 \qquad (4)$$

These are plotted in figure 1 together with the above-mentioned conclusion by Ball et al that:

$$\left(log\frac{E}{E_0}\right)_s = \frac{1}{4.6}N\beta^2 \qquad (5)$$

Although these are of the same order of magnitude, random walk is clearly significantly faster than systematic acceleration. It is also overwhelmingly dominant when $N/N_0 \gg 1$. It is readily show by numerical simulation that random walk would be equivalent to systematic acceleration only if the spectrum were required to rise to within 0.1 percent of the asymptotic value. It is interesting to note that the rate at which the spectrum is established increases very rapidly with time, in what might be described as an avalanche effect.

The times required to complete $N = 25N_0$ scatterings in the four original scenarios follow directly from the values of T_E in table 1 of Bryant et al 1992a. They are for $\beta =$

$10^{-1}, 10^{-2}, 10^{-3}$ and 10^{-4}, approximately $1.5 \times 10^6, 1.5 \times 10^7, 1.5 \times 10^8$ and $1.5 \times 10^9 yr$. Therefore, even the extreme requirement of producing $10^{20} eV$ particles from initial energies of as low as $10^{10} eV$, ie in the solar proton range, may be accomplished in less than the Hubble time. For higher initial energies, as might be anticipated for injection from supernovae, the time taken to reach the highest observed energies would, of course, be shorter. The well-known "knee" in the spectrum near $5 \times 10^{15} eV$ (eg Hillas 1984) may indeed indicate that the limit of the injection spectrum is as high as this, reducing the required range for the high energy process to approximately 5 orders of magnitude, which could be accomplished in approximately one third of the times given above.

(iv) Magnetic Fields

Ball et al mention in passing that the magnetic fields required for acceleration to the highest energies are implausibly high, an opinion also expressed by Chi et al (1992). We recognize that the field strengths B_{crit} which our analysis shows to be necessary for acceleration to proceed to the synchrotron-radiation limit, E_{lim}, are several orders of magnitude higher than the $\sim 0.5nT$ quoted for the random field of the galaxy derived from Faraday rotation by e.g. Rand and Kulkarni (1989). However, it is important to remember that the measurements apply to distance scales $\sim 50pc$, while the hypothetical scattering regions derived from the statistical analysis are on parsec and sub-parsec scales. The relation between the fields on these two different scales may be estimated by taking into account the filling factor and random orientation (Bryant et al 1992b). Such considerations yield

$$B_l \sim k \left(\frac{a}{d} \sqrt{\frac{d}{l}} \right)^3 B_a$$

where B_l is the apparent field over a distance scale l, B_a is the field on the perturbation scale a, and d is the mean spacing of scattering centres. $k = \frac{1}{\sqrt{3}}$ for $l \gg d$, and $k \rightarrow 1$ as $l \rightarrow d$. At the current resolution of $l \sim 50pc$, the values of B_{crit} $(6, 10, 30$, and $60\mu T)$, derived for the four original scenarios, would appear as approximately 9, 0.6, 0.04 and 0.003 nT, respectively. All, except the first, for $\beta = 10^{-1}$ lie within the values deduced from Faraday rotation. As we pointed out originally, magnetic fields below the critical values would reduce the limiting energy in the same proportion. A reduction by a factor of ten in deference to measurement would still permit protons energies to exceed $10^{19}eV$ in this scenario. Thus, while fine-scale (sub-parsec) regions of magnetic field enhanced enough to scatter cosmic rays of the highest energies represent a theoretical challenge, they do not appear to be ruled out by measurement. Consideration of heavy ions raises still further the possibilities for energies attainable within the measured magnetic fields. For iron nuclei the limiting energy is 24 times that for protons.

As a final point of discussion we should mention that the above analyses have all assumed for simplicity an energy-independent mean free path. An evaluation of possible effects of departure from this approximation are crucial to further development of the theory.

3. CONCLUSIONS

The issues raised by Ball et al in the form of criticisms do not invalidate the proposal. However they have prompted a clarification of the distinction between the systematic and random effects in Fermi acceleration. Contrary to current descriptions of the process, random walk in energy is shown to be overwhelmingly the dominant means of acceleration. The generally accepted limitations are seen to derive partly from preoccupation with systematic effects and with as-yet unnecessary constraints on galactic magnetic fields. Energy enhancements of ten orders of magnitude appear to be possible within the Hubble time, well exceeding the apparent requirement.

Until debilitating constraints are identified, therefore, we wish to re-iterate Davis's suggestion that his modified (random walk) version of the Fermi mechanism should be considered very seriously as a contender for the acceleration mechanism for very high energy cosmic rays.

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FIGURE CAPTIONS

1. Energization as a function of residence for random walk (thin line, from equation 3) and systematic acceleration (medium and thick lines, from (4) and (5) respectively). The upper scale shows the time taken when $\beta = 10^{-3}$ and the scattering mean free path is $10^{17}m$. The dominance of random walk is readily apparent.

2. Evolution of cosmic ray spectrum under random walk (thin line, from numerical simulation) and systematic acceleration (medium, and thick lines in accordance with (4) and (5), respectively). The dotted line is the asymptotic spectrum, with $\gamma = 3$. This again shows the dominance of random walk.

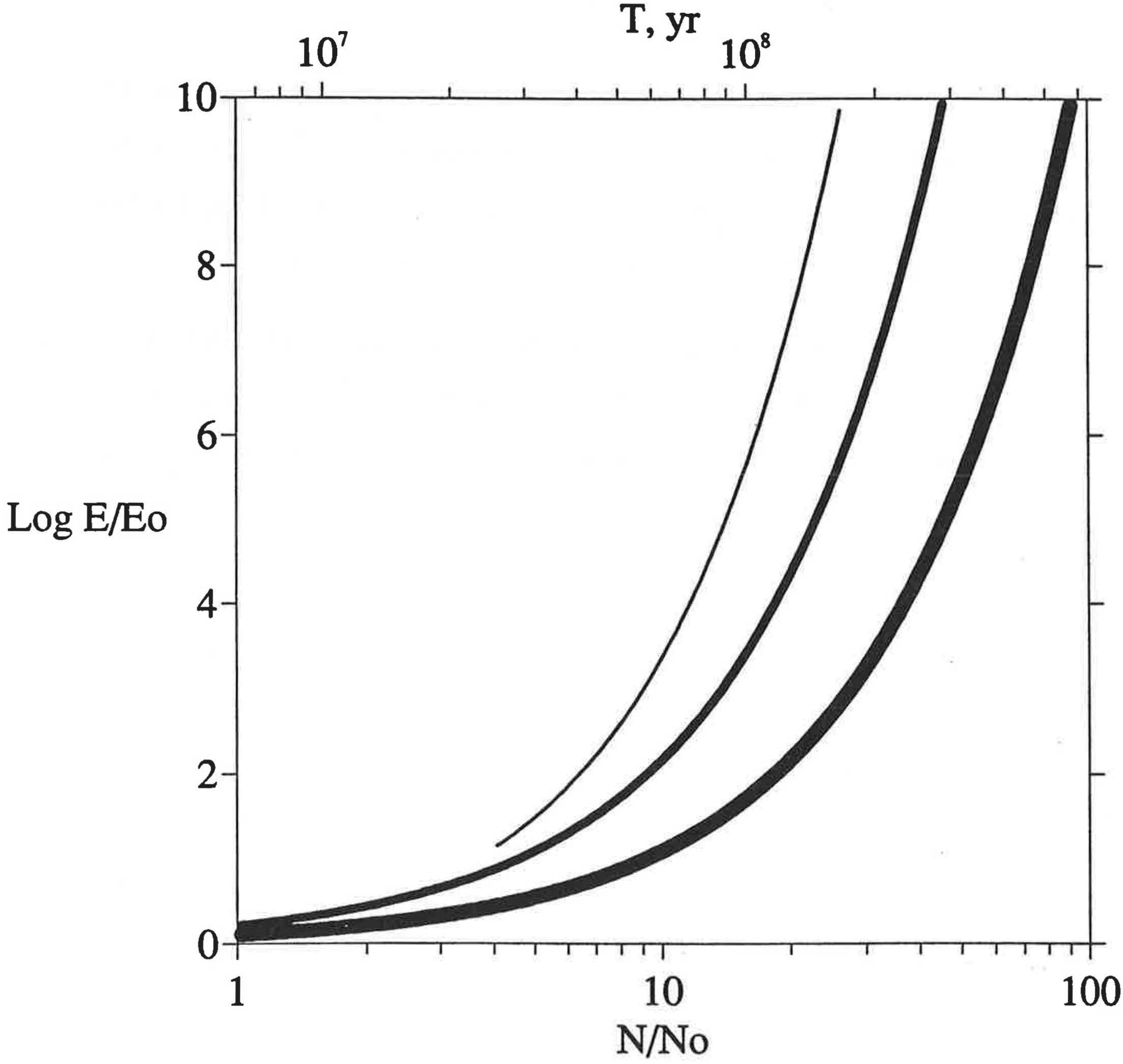

Figure 1

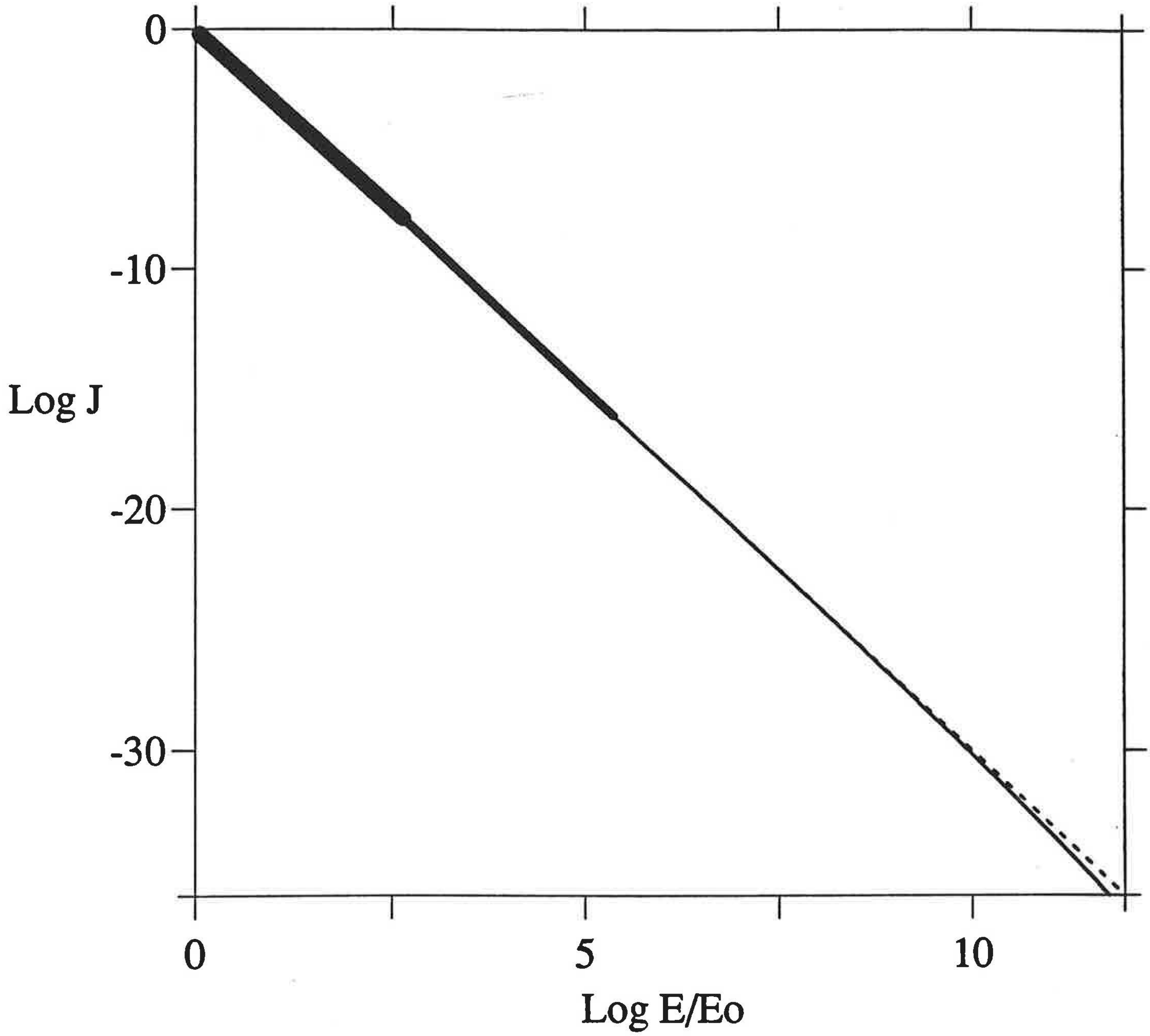

Figure 2