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Abstract

It is shown that an antisymmetric rank-two tensor gauge potential of the type first found in string and supersymmetry theories occurs also in ordinary Yang-Mills theory when formulated in loop space, where it appears as a Lagrange multiplier for a zero curvature constraint necessary and sufficient for removing the inherent redundancy of loop variables. It is then further shown that the tensor potential acts there as the parallel 'phase' transport for monopoles.

Recently there has been renewed interest in antisymmetric tensor gauge potentials [1, 2, 3, 4, 5, 6, 7], especially in the 'nonabelian' version suggested by Freedman and Townsend in 1981 [8]. Although the present emphasis is on the quantization of the theory, the physical or geometrical significance of these potentials remain somewhat obscure in that it is unclear whether they function as connections for parallel 'phase' transport as ordinary vector gauge potentials do, and if they do so, what the 'phase' is that they transport. In this paper, we wish to point out first that, though originally discovered in string and supersymmetric theories, these tensor potentials occur also in ordinary Yang-Mills theory when formulated in loop space, where they appear as Lagrange multipliers for the constraint needed to remove the intrinsic redundancy of loop variables, and second, that in this context they function as the parallel transport of the 'phases' of monopoles. It is hoped that these observations will help with the general understanding of the physical significance of tensor gauge potentials.

Yang-Mills theory can be fully described by means of the loop variables:

$$F_\mu[\xi|s] = \frac{i}{g}\Phi[\xi]^{-1}\frac{\delta}{\delta\xi^\mu(s)}\Phi[\xi], \tag{1}$$

introduced by Polyakov [9], where $\Phi[\xi]$ is the phase factor:

$$\Phi[\xi] = P_s \exp ig \int_0^{2\pi} A_\mu(\xi(s))\dot{\xi}^\mu(s)ds. \tag{2}$$

In (2), a dot denotes differentiation with respect to the loop parameter s, P_s denotes ordering in s, from right to left for increasing s in our convention, and ξ represents a parametrized loop passing through a fixed reference point P_0, namely:

$$\xi = \{\xi(s), s = 0 \to 2\pi, \xi(0) = \xi(2\pi) = P_0\}. \tag{3}$$

In terms of ordinary space-time variables, $F_\mu[\xi|s]$ can be expressed as:

$$F_\mu[\xi|s] = \Phi_\xi^{-1}(s,0)F_{\mu\nu}(\xi(s))\dot{\xi}^\nu(s)\Phi_\xi(s,0), \tag{4}$$

where:

$$\Phi_\xi(s_2,s_1) = P_s \exp ig \int_{s_1}^{s_2} A_\mu(\xi(s))\dot{\xi}^\mu(s)ds \tag{5}$$

is the parallel transport from $\xi(s_1)$ to $\xi(s_2)$ along the loop ξ.

From (4), it follows that $F_\mu[\xi|s]$ can depend on the loop coordinate $\xi(s')$ only for $s' \leq s$, i.e.:

$$\frac{\delta}{\delta\xi^\nu(s')}F_\mu[\xi|s] = 0, \quad s' > s, \tag{6}$$

which is a consequence of our ordering convention P_s in (2), and that it has only components transverse to the loop, namely that:

$$F_\mu[\xi|s]\dot{\xi}^\mu(s) = 0, \tag{7}$$

which is equivalent to the fact that $\Phi[\xi]$ is by definition independent of how the loop ξ is parametrized. These two properties (6) and (7) which in effect just reduce the range of the arguments and the number of components of $F_\mu[\xi|s]$ will henceforth be regarded as understood and absorbed into the notation.

The variables $F_\mu[\xi|s]$, like the phase factors $\Phi[\xi]$, are gauge invariant except for an x-independent rotation at the reference point P_0 which is easily handled. But, like $\Phi[\xi]$ also, they form a highly redundant set and have to be severely constrained. By the latter assertion, we mean that in order effectively to employ $F_\mu[\xi|s]$ as variables to describe Yang-Mills theory, which we know is already adequately described by $A_\mu(x)$, we have to ensure that the loop variables can indeed be expressed in terms of some vector potential $A_\mu(x)$ in the manner of (1) and (2). However, this may not be true for just any given set of $F_\mu[\xi|s]$.

The conditions that $F_\mu[\xi|s]$ have to satisfy for (1) and (2) to hold is best stated in terms of the quantity [9]:

$$G_{\mu\nu}[\xi|s] = \frac{\delta}{\delta\xi^\nu(s)}F_\mu[\xi|s] - \frac{\delta}{\delta\xi^\mu(s)}F_\nu[\xi|s] + ig[F_\mu[\xi|s], F_\nu[\xi|s]]. \tag{8}$$

From its definition (1), we note that $F_\mu[\xi|s]$ can be interpreted as a connection in loop space for parallel transport of the phase $\Phi[\xi]$. In this sense then, $G_{\mu\nu}[\xi|s]$ is the corresponding curvature. Now it can readily be seen that so long as $F_\mu[\xi|s]$ is expressible in terms of an $A_\mu(x)$ through (2) and (1), then it satisfies the following condition:

$$G_{\mu\nu}[\xi|s] = 0, \tag{9}$$

meaning that, as a connection, it is pure gauge, giving thus zero curvature. Conversely, it can also be shown, though less readily and perhaps for this reason less widely recognized, that provided $F_\mu[\xi|s]$ satisfies (9), (6) and (7) being understood, then there will exist an $A_\mu(x)$ in terms of which $F_\mu[\xi|s]$ can be expressed through (1) and (2) as required [10]. In other words, the constraint (9) removes exactly the redundancy of the loop variables $F_\mu[\xi|s]$ and makes a description of the theory in terms of them equivalent to the original description in terms of the gauge potential $A_\mu(x)$.

From (4), it follows that the Yang-Mills action:

$$\mathcal{A}_F^0 = -\frac{1}{16\pi}\int d^4x \,\mathrm{Tr}\{F_{\mu\nu}(x)F^{\mu\nu}(x)\} \tag{10}$$

can be rewritten in terms of loop variables as:

$$\mathcal{A}_F^0 = -\frac{1}{4\pi\bar{N}} \int \delta\xi \int_0^{2\pi} ds \, \mathrm{Tr}\{F_\mu[\xi|s]F^\mu[\xi|s]\}\dot{\xi}^{-2}(s), \tag{11}$$

where $\bar{N}$ is a normalization factor:

$$\bar{N} = \int_0^{2\pi} ds \int \prod_{s' \neq s} d^4\xi(s'). \tag{12}$$

The variables $F_\mu[\xi|s]$, however, are not independent variables, but, as stated above, have to satisfy the constraint (9) for all ξ and s. Hence, introducing the Lagrange multipliers $L_{\mu\nu}[\xi|s]$ antisymmetric in μ, ν, and incorporating the constraint into the action, we have:

$$\mathcal{A}_F = \mathcal{A}_F^0 + \int \delta\xi ds \, \mathrm{Tr}\{L^{\mu\nu}[\xi|s]G_{\mu\nu}[\xi|s]\}. \tag{13}$$

For example, to find the field equations, we extremize (13) with respect to $F_\mu[\xi|s]$, obtaining:

$$(4\pi\bar{N}\dot{\xi}^2(s))^{-1}F_\mu[\xi|s] = -\mathcal{D}^\nu(s)L_{\mu\nu}[\xi|s], \tag{14}$$

where

$$\mathcal{D}_\nu(s) = \frac{\delta}{\delta\xi^\nu(s)} - ig[F_\nu[\xi|s], \quad] \tag{15}$$

is a kind of covariant derivative in loop space with $F_\mu[\xi|s]$ as connection. From (14), one deduces that:

$$(2\pi\bar{N}\dot{\xi}^2(s))^{-1}\frac{\delta}{\delta\xi_\mu(s)}F_\mu[\xi|s] = -[\mathcal{D}^\mu(s), \mathcal{D}^\nu(s)]L_{\mu\nu}[\xi|s], \tag{16}$$

where the right-hand side is just $ig[G^{\mu\nu}[\xi|s], L_{\mu\nu}[\xi|s]]$, and hence from the constraint (9) one obtains:

$$\frac{\delta}{\delta\xi_\mu(s)}F_\mu[\xi|s] = 0, \tag{17}$$

which, as pointed out by Polyakov [9], is the loop space statement of the Yang-Mills equation.

One sees that apart from some trivial changes in notation, (13) is exactly the loop space version of the first order Freedman-Townsend action [8] for a tensor gauge field with $L_{\mu\nu}[\xi|s]$ as the potential. Indeed, under the transformation:

$$\Delta F_\mu[\xi|s] = 0, \tag{18}$$

and:

$$\Delta L_{\mu\nu}[\xi|s] = \epsilon_{\mu\nu\rho\sigma}\mathcal{D}^\rho(s)\Lambda^\sigma[\xi|s], \tag{19}$$

the action (13) is invariant for arbitrary functions $\Lambda_\mu[\xi|s]$ by virtue of the Bianchi identity of $G_{\mu\nu}[\xi|s]$. We notice then that the dual of $L_{\mu\nu}[\xi|s]$ defined by:

$$^*L_{\mu\nu}[\xi|s] = -(1/2)\epsilon_{\mu\nu\rho\sigma}L^{\rho\sigma}[\xi|s] \tag{20}$$

will transform as:

$$\Delta^*L_{\mu\nu}[\xi|s] = \mathcal{D}_\nu(s)\Lambda_\mu[\xi|s] - \mathcal{D}_\mu(s)\Lambda_\nu[\xi|s], \tag{21}$$

exactly as the tensor gauge potential in Freedman and Townsend.

The question now is: what does this transformation represent? It is not the original Yang-Mills gauge transformation which has already been removed by going over into the gauge-invariant (apart from a trivial x-independent transformation at the reference point P_0) loop variables $F_\mu[\xi|s]$. What we shall show in fact is that it represents a local change in 'phase' of 'colour' monopoles in the Yang-Mills fields, in much the same way that the original Yang-Mills gauge transformation represents a local change in 'phase' of 'colour' sources. Further, it is for this 'phase' of 'colour' monopoles that the tensor potential $L_{\mu\nu}[\xi|s]$ is acting as parallel transport.

We should first make clear that by 'colour' monopoles of Yang-Mills fields, we do not mean here the 't Hooft-Polyakov soliton solutions of 1974 [11, 12] which are abelian monopoles embedded in a nonabelian Yang-Mills-Higgs field. We mean rather the generalization of the Dirac point monopole [13] to nonabelian theories as first suggested by Lubkin [14], Wu-Yang [15] and Coleman [16]. These latter are characterized by nontrivial nonabelian bundles over S^2, whereas the former are characterized by nontrivial abelian subbundles of a trivial nonabelian bundle as in the original 't Hooft-Polyakov papers. In this language then, the charge of a 'colour' monopole in a Yang-Mills theory with gauge group G takes values in the fundamental group $\pi_1(G)$. In particular, for the simplest pure Yang-Mills theory with gauge algebra $su(2)$ and gauge group $G = SO(3)$, the monopole charge ζ can take only values in Z_2 and can thus be labelled just by a sign $\pm$ [17].

The monopole charge so defined which is enclosed inside any given surface Σ is given by the holonomy over this surface [10]:

$$\Theta_\Sigma = \zeta_\Sigma, \tag{22}$$

where the surface Σ passing through the loop-space reference point $P_0 = \{\xi_0^\mu\}$ is considered as a closed loop in loop space. This holonomy Θ_Σ can be written explicitly as:

$$\Theta_\Sigma = P_t \exp ig \int_0^{2\pi} dt \int_0^{2\pi} ds F_\mu[\xi_t|s] \frac{\partial \xi_t^\mu(s)}{\partial t} \tag{23}$$

for any parametrization $\{\xi_t^\mu(s)\}$ of Σ:

$$\Sigma = \{\xi_t(s); s = 0 \to 2\pi, t = 0 \to 2\pi, \xi_t(0) = \xi_t(2\pi) = \xi_0(s) = \xi_{2\pi}(s) = \xi_0\}. \quad (24)$$

For example, in the pure $su(2)$ Yang-Mills theory with gauge group $SO(3)$, the quantity in (23) for any Σ will be an element of the group $SU(2)$ taking the values $\pm I$ in its centre $Z_2 = SU(2)/SO(3)$. The value $-I$ for Θ_Σ will then signify that there is a 'colour' monopole enclosed inside the surface Σ.

The existence of a monopole charge at some space-time point x means that the loop space curvature $G_{\mu\nu}[\xi|s]$ of (8) will fail to vanish at $\xi(s) = x$. In other words, a monopole charge may be interpreted as a source of loop space curvature [10, 18]. This is not in contradiction with the statement above in (9) since the presence of a monopole necessitates patching of the gauge potential so that at the position of the monopole $A_\mu(x)$ is not defined, violating thus the conditions for (9) to hold. Indeed, since geometrically the curvature is just a differential version of the holonomy, it follows from (22) that at the monopole position, $G_{\mu\nu}[\xi|s]$ must take some value $4\pi\tilde{g}\kappa$ in the gauge Lie algebra where κ satisfies:

$$\exp i\pi\kappa = \zeta \quad (25)$$

for a monopole of charge ζ. Thus, in particular, for a monople of charge $-$ in the $SO(3)$ theory, κ will take a value $n\sigma$ for n odd and $\sigma = \alpha_i\tau^i$, where α is a unit vector and τ^i are the Pauli matrices. Notice that in ascribing the algebra element κ to the monopole, one has assigned to it a 'phase' or orientation in internal symmetry space which it did not originally possess.

Suppose now that there is a monopole moving along a world-line $Y(\tau)$, then (9) will be replaced by:

$$G_{\mu\nu}[\xi|s] = 4\pi\tilde{g}\kappa[\xi|s]\epsilon_{\mu\nu\rho\sigma}\dot{\xi}^\rho(s)\int d\tau \frac{dY^\sigma(\tau)}{d\tau}\delta^4(Y(\tau) - \xi(s)). \quad (26)$$

It can be shown that provided this is satisfied, the existence of the gauge potential $A_\mu(x)$ is still guaranteed at all points in space-time [10], except of course on the monopole world-line where we cannot expect the potential to be defined in any case. Hence, the condition (26) will again remove the redundancy from the loop variables as (9) did before, meaning now that for all loops not passing through $Y(\tau)$, it is assured that we may still write (2) and (1) as before.

Replacing (9) by (26) as constraint in (13), however, means that there is coupling now between the field as represented by $F_\mu[\xi|s]$ and the monopole coordinate $Y(\tau)$ or that there is an induced interaction between the field and the monopole.

Indeed, this seems a natural way to define the interaction and it has been shown that in this way equations of motion for the field-monopole system can be derived which for the abelian theory reduce exactly to the Maxwell and Lorentz equations for the magnetic charge [18, 19].

Our primary interest here, however, is not monopole dynamics but the meaning of the antisymmetric tensor potential $L_{\mu\nu}[\xi|s]$ as connection. To see this, we shall need to extend the discussion to a quantum monopole described by a wave function so that we may follow the variation of the wave function's 'phase' under parallel transport as we did for the electron wave function in the Bohm-Aharonov experiment [20].

We notice that the right-hand side of (26) is basically just the monopole current. Indeed, in the special case of an abelian theory, the equation reduces by virtue of (4) simply to:

$$\partial_\nu {}^*F^{\mu\nu}(x) = -4\pi\tilde{e}\int d\tau \frac{dY^\mu(\tau)}{d\tau}\delta^4(x - Y(\tau)), \tag{27}$$

in which, with $\tilde{e}$ being the magnetic charge, it is exactly the magnetic current which appears on the right-hand side. In replacing a classical monopole by a quantum monopole described by a (say Dirac) wave function, the classical current on the right-hand side of (27) is replaced by the quantum current, so that instead of (27) we have:

$$\partial_\nu {}^*F^{\mu\nu}(x) = -4\pi\tilde{e}\bar{\psi}(x)\gamma^\mu\psi(x). \tag{28}$$

Equivalently, in loop space notation, we have:

$$G_{\mu\nu}[\xi|s] = 4\pi\tilde{e}\epsilon_{\mu\nu\rho\sigma}\dot{\xi}^\rho(s)\bar{\psi}(\xi(s))\gamma^\sigma\psi(\xi(s)). \tag{29}$$

For the general Yang-Mills case, the corresponding constraint reads as [19]:

$$G_{\mu\nu}[\xi|s] = 4\pi\tilde{g}\epsilon_{\mu\nu\rho\sigma}\dot{\xi}^\rho(s)\Omega_\xi^{-1}(s,0)\{[\bar{\psi}(\xi(s))\gamma^\sigma\tau^i\psi(\xi(s))]\tau_i\}\Omega_\xi(s,0), \tag{30}$$

where the 'current' inside the curly brackets is again an element of the gauge Lie algebra with a 'phase' or orientation in internal symmetry space. This 'phase', however, is measured in the local monopole frame at $\xi(s)$, hence the rotation matrices $\Omega_\xi(s,0)$ to transform back to the reference frame at ξ_0 in which $G_{\mu\nu}[\xi|s]$ on the left-hand side is measured.

Suppose now we write an action for the field-monopole system. We shall then write the free action as usual as:

$$\mathcal{A}^0 = \mathcal{A}_F^0 + \int d^4x\, \bar{\psi}(x)(i\partial_\mu\gamma^\mu - m)\psi(x), \tag{31}$$

where we may again express $\mathcal{A}^0_F$ in terms of the loop variables $F_\mu[\xi|s]$ as in (11). The loop variables have again to be constrained, but now by (30) instead of (9). Incorporating then the constraint into the action by means of Lagrange multipliers $L_{\mu\nu}[\xi|s]$ as before, we have:

$$\mathcal{A} = \mathcal{A}^0 + \int \delta\xi ds\, \mathrm{Tr}\{L^{\mu\nu}[\xi|s](G_{\mu\nu}[\xi|s] + 4\pi J_{\mu\nu}[\xi|s])\}, \tag{32}$$

where $J_{\mu\nu}[\xi|s]$ denotes $-1/4\pi$ times the right-hand side of (30).

We ask now what happens if we perform the transformations (18) and (19) on $\mathcal{A}$. We have seen already that the field part of the action remains invariant. Next, let us examine the last term on the right of (32), namely:

$$4\pi \int \delta\xi ds\, \mathrm{Tr}\{L^{\mu\nu}[\xi|s] J_{\mu\nu}[\xi|s]\} = \tilde{g} \int d^4x \bar{\psi}(x) \tilde{A}_\mu(x) \gamma^\mu \psi(x), \tag{33}$$

where, from the definition of $J_{\mu\nu}[\xi|s]$ as the right-hand side of (30):

$$\tilde{A}_\mu(x) = -8\pi \int \delta\xi ds \Omega_\xi(s,0) {}^*L_{\mu\nu}[\xi|s] \Omega_\xi^{-1}(s,0) \dot{\xi}^\nu(s) \delta^4(x - \xi(s)). \tag{34}$$

The transformation (19) will give an increment to $\tilde{A}_\mu(x)$ of the form:

$$\Delta_L \tilde{A}_\mu(x) = -8\pi \int \delta\xi ds \Omega_\xi(s,0) \mathcal{D}_\mu(s) \Lambda_\nu[\xi|s] \Omega_\xi^{-1}(s,0) \dot{\xi}^\nu(s) \delta^4(x - \xi(s)), \tag{35}$$

where we have used the fact that loop quantities have no derivatives longitudinal to the loop because of reparametrization invariance. Next, using the Bianchi identity satisfied by $G_{\mu\nu}[\xi|s]$, we can deduce from (26) and the conservation of the monopole current that [19]:

$$\frac{\delta}{\delta\xi^\mu(s)} \Omega_\xi(s,0) = -ig \Omega_\xi[\xi|s] F_\mu[\xi|s], \tag{36}$$

so that:

$$\frac{\delta}{\delta\xi^\mu(s)} \{\Omega_\xi(s,0) \Lambda_\nu[\xi|s] \Omega_\xi^{-1}(s,0)\} = \Omega_\xi(s,0) \{\mathcal{D}_\mu(s) \Lambda_\nu[\xi|s]\} \Omega_\xi^{-1}(s,0). \tag{37}$$

Substituting this into (35), one obtains after some manipulation:

$$\Delta_L \tilde{A}_\mu(x) = \partial_\mu \tilde{\Lambda}(x), \tag{38}$$

where:

$$\tilde{\Lambda}(x) = -8\pi \int \delta\xi ds \Omega_\xi(s,0) \Lambda_\nu[\xi|s] \Omega_\xi^{-1}(s,0) \dot{\xi}^\nu(s) \delta^4(x - \xi(s)). \tag{39}$$

Such an increment in $\tilde{A}_\mu(x)$ will induce a corresponding change in the action $\mathcal{A}$ of (32), which will however be cancelled if we perform at the same time the following transformation on the monopole wave function $\psi(x)$:

$$\psi(x) \longrightarrow (1 + i\tilde{g}\tilde{\Lambda}(x))\psi(x), \tag{40}$$

meaning a rotation of the local 'phase' of the monopole, since this will induce a similar rotation in the frame-transformation factors $\Omega_\xi(s,0)$, thus:

$$\Omega_\xi(s,0) \longrightarrow [1 + i\tilde{g}\tilde{\Lambda}(\xi(s))]\Omega_\xi(s,0), \tag{41}$$

which gives the total transformation of $\tilde{A}_\mu(x)$ the standard transformation for a nonabelian potential:

$$\Delta\tilde{A}_\mu(x) = \partial_\mu\tilde{\Lambda}(x) + i\tilde{g}[\tilde{\Lambda}(x), \tilde{A}_\mu(x)]. \tag{42}$$

The increment from the term (33) in the action (32) will therefore cancel in a familiar manner with the variation in the free action term (31) from the transformation (40) of the wave function $\psi(x)$.

One sees thus that the transformation (19) of the tensor potential $L_{\mu\nu}[\xi|s]$ does indeed correspond to a 'phase' rotation of the monopole wave function, and that the action for the field-monopole system is invariant under this transformation. We stress again that this is not the original Yang-Mills gauge invariance which has already been absorbed into the formulation by adopting the 'gauge-invariant' loop quantities $F_\mu[\xi|s]$ as variables. The total gauge symmetry is now therefore $SU(N) \times SU(N)$ where the second $SU(N)$ carries (because of the ϵ symbol occuring in, for example, (30)) a parity opposite to that of the first, original Yang-Mills $SU(N)$.

What is the origin of this new additional gauge symmetry? We recall that the monopole charge was originally defined as an element in the centre of the group $SU(N)$; in particular for $SU(2)$, it is labelled only by a sign $-$. It does not therefore have at first a 'phase'. But when its dynamics was formulated through the imposition of the constraint (26) or (30) as was done above, it was obliged to make a choice of 'phase', since the constraints imposed are equations in the algebra. This choice of 'phase' is 'local', depending not only on the loop ξ, but also on the 'end-point' labelled by the parameter s. However, the actual physics cannot depend on this choice of 'phase' since the monopole charge itself has none. Hence the dynamics must be invariant under an arbitrary rotation in 'phase' at every ξ and s, which is indeed the freedom enjoyed by the gauge parameter $\Lambda_\mu[\xi|s]$ in the symmetry transformation (19) found above.

Since one is allowed to rotate the monopole 'phase' arbitrarily for each ξ and s, one needs a parallel transport or 'connection' to specify what is meant by the same 'phase' at different values of ξ and s, namely to play the role here of the gauge potential $A_\mu(x)$ in the original Yang-Mills symmetry. This is provided by the tensor potential $L_{\mu\nu}[\xi|s]$, which depends on ξ and s as expected. The reason why it should carry two indices μ, ν instead of just one index μ as in the ordinary gauge potential $A_\mu(x)$ is that the monopole charge as defined above is specified by a closed 2-dimensional surface enclosing a 3-volume, and a 3-volume element in 4-dimensional space has a direction. For this reason, the measured monopole charge depends not only on the position but also on a direction in space-time. The parallel transport has therefore to specify 'phase' variations not only for neighbouring positions but also for 'neighbouring' directions, hence the extra index.

Although our observations here on the tensor potential are made specifically only within the framework of the loop space formulation of Yang-Mills fields, it is hoped that they will be useful also for understanding the geometrical meaning of tensor potentials in general.

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