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Solar Neutrino Oscillations

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SOLAR NEUTRINO OSCILLATIONS *

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1. Introduction.

Standard Solar Models (SSM) predict the ν_e flux of Fig.1 with some uncertainties[1, 2]. Measurements by capture in ^{37}Cl [3], $\nu - e$ scattering[4] and capture in ^{71}Ga [5, 6], with differing E_ν thresholds, find three different deficits:

Detection	*Threshold*	*Observation/SSM*[1]	*Observation/SSM*[2]
$\nu - e$	$7.5 MeV$	$0.51 \pm .07 \pm .07$	$0.66 \pm .09 \pm .16$
^{37}Cl	$0.81 MeV$	$0.29 \pm .03 \pm .04$	$0.36 \pm .04 \pm .08$
^{71}Ga	$0.24 MeV$	$0.62 \pm .10 \pm .03$	$0.67 \pm .11 \pm .04$

where the first error is experimental, the second is from the SSM. These numbers suggest a differential suppression, with the top and bottom of the accessible range less suppressed than the middle. They pose the Solar Neutrino Problem 1994.

Re-tuning the solar model gives no easy solution[7]. A lower central temperature would suppress ^8B production and the $\nu - e$ rate, but to explain ^{37}Cl rates the ^7Be line must then be obliterated - a bit unlikely given that ^8B is made from ^7Be.

Neutrino oscillations offer several possible explanations, that I briefly compare.

2. Long Wavelength Vacuum Oscillations (LWVO).

Suppose the weak eigenstate ν_e, emitted by β-decays in the Sun, is actually a superposition of two mass eigenstates: $\nu_e = \nu_1 cos\theta - \nu_2 sin\theta$ with $\nu_\mu = \nu_1 sin\theta + \nu_2 cos\theta$. The mass eigenstates propagate independently with time t, each picking up a different phase factor $exp(-im_i^2 t/2E)$, so that after a distance $L = ct$ the projection back onto ν_e becomes $A(\nu_e \to \nu_e) = [\cos^2\theta exp(-im_1^2 L/2E) + \sin^2\theta exp(-im_2^2 L/2E)]$. The probability that this evolved state can interact like ν_e is then

$$P(\nu_e \to \nu_e) = |A|^2 = 1 - sin^2(2\theta) sin^2(\delta m^2 L/4E).$$

where $\delta m^2 = m_2^2 - m_1^2$. Figure 2 illustrates this oscillatory probability. For $E/\delta m^2 >> 1$ there is negligible effect; for values $\sim 0.1 - 1$ there are resolvable oscillations; for values $<< 0.1$ the oscillations are averaged in practice, either by source/detector size or by energy resolution. Averaged 2-neutrino oscillations suppress by at most 1/2, but n-neutrino mixing can give 1/n; however this suppression is flat and therefore unsuited to the 1994 solar problem. On the other hand, resolved oscillations provide

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a strongly varying suppression. Try overlaying Figs.1 and 2 (they have the same horizontal log scale). If we tune δm^2 such that the first minimum falls around a few MeV, the Kamiokande $\nu - e$ rate will be somewhat suppressed; if we fine-tune to put the $860 KeV$ ^7Be line in a minimum, the ^{37}Cl rate will be somewhat more suppressed; meanwhile the ^{71}Ga rate suffers less, since its dominant pp neutrinos encounter only average suppression. We clearly have the makings of one or more solutions here, with these "just-so" oscillations[8]. Figure 3 shows typical recent fits [9] in the $(sin^2 2\theta, \delta m^2)$ parameter plane; the disconnected regions put the ^7Be line in different minima of P. Note that for $\nu_e - \nu_\mu$ mixing, $\nu_\mu - e$ scattering contributes a bit to the Kamiokande signal and helps to explain why it is less suppressed than ^{37}Cl; $\nu_e - \nu_x$ sterile flavour mixing lacks this help and is harder to fit.

Two special features arise from LWVO resolved oscillation patterns [8].
i) There is an oscillatory modulation on the shape of the high- energy ^8B spectrum contribution. The shape (though not the magnitude) of this SSM component is model-independent; the modulation would be detectable at SNO[10] and Super-Kamiokande. See Figs. 8,9 at the end.
ii) The ^7Be line with fixed E_ν has oscillatory strength, because the Earth-Sun distance L has small seasonal variations. This line strength could be measured directly by Borexino[11]; the effect is diluted in ^{37}Cl and ^{71}Ga signals.
(Present $\nu - e$, Cl and Ga data constrain (i) and (ii) rather weakly).

But mixing ν_e with ν_μ or ν_τ affects ν_e spectra from supernovae; SN1987A data may disfavour large $sin^2(2\theta) > 0.7 - 0.9$ [12], including solutions like Fig.3a.

3. Oscillations in matter.

Coherent forward scattering in matter generates a refractive index and affects propagation[13]. Z-exchange processes are the same for ν_e, ν_μ, ν_τ, generating a common phase that can be ignored, but W-exchange contributes only to $\nu_e - e$ scattering and significantly changes the propagation equation:

$$4iE\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\alpha\end{pmatrix} = \begin{pmatrix} m_1^2 + m_2^2 - \delta m^2 cos2\theta + 4\sqrt{2}G_F\rho_e E & \delta m^2 sin2\theta \\ \delta m^2 sin2\theta & m_1^2 + m_2^2 + \delta m^2 cos2\theta \end{pmatrix}\begin{pmatrix}\nu_e\\ \nu_\alpha\end{pmatrix}$$

where θ is the vacuum mixing angle, ρ_e is the electron number density and $\nu_\alpha = \nu_\mu$ or ν_τ (for sterile ν_x see later). Diagonalizing the propagation matrix above, we find that the mixing angle in matter θ_m depends on $\rho_e E$:

$$tan2\theta_m = \frac{tan2\theta}{1 - (2\sqrt{2}G_F\rho_e E)/(\delta m^2 cos2\theta)}.$$

If $\delta m^2 > 0$ the mixing is enhanced; it becomes maximal ($\theta_m = \pi/4$) where the denominator vanishes - sometimes called a resonance. As neutrinos travel out from the solar core, the mixing angle θ_m and the matter-propagation eigenstates ν_{1m}, ν_{2m} change continuously.

This gives a possibility for efficient $\nu_e \to \nu_\mu(\nu_\tau)$ conversion via adiabatic level crossing (the MSW effect[14]). Suppose that $\rho_e E$ is far above the resonance value at the point of ν_e creation in the solar core (i.e. $\theta_m \sim \pi/2$); then $\nu_e \simeq \nu_{2m}$ here. If subsequent propagation is adiabatic, the local eigenstate components are essentially preserved: $\nu_{jm} \to \nu_{jm}(j=1,2)$. Emerging from the Sun, the dominant ν_{2m} component becomes the vacuum mass eigenstate $\nu_2 = -\nu_e sin\theta + \nu_\mu cos\theta \simeq \nu_\mu$ if the vacuum mixing angle θ is small; thus initial ν_e ends up as mostly ν_μ. Fig.4 shows how the two eigenvalues m_j^2 of the propagation matrix behave versus ρ_e; solid lines give the case of no mixing, $\theta = 0$; dashed lines show how the eigenvalues cross over when mixing is present. If ρ_e changes slowly enough for the mixing to act (adiabatically), the physical state follows the full eigenstates (dashed lines); but if ρ changes too suddenly, the physical state follows the unmixed eigenstates (solid lines).

There are 2 conditions for adiabatic level crossing.
(i) Central density is above the resonance value:

$$E(MeV)/\delta m^2(eV^2) > cos2\theta/[2\sqrt{2}G_F\rho_e(max)] \simeq 10^5,$$

(ii) Density changes slowly enough near the resonance for adiabaticity: from the Landau-Zener approximation we obtain[15]

$$E(MeV)/\delta m^2(eV^2) << sin^2 2\theta/[\rho_e^{-1} d\rho_e/dR] cos2\theta \simeq 3 \times 10^8 sin^2 2\theta.$$

The detailed consequences require big calculations, but we need no computer to see the main features. The MSW effect gives a bathtub-shaped suppression factor; see Fig.5. The steep left-hand end is determined by the resonance-crossing condition (i); the sloping right-hand end covers the range where adiabaticity breaks down, determined by condition (ii). We can choose almost any bathtub we please, versus energy E_ν, by selecting $\delta m^2 cos2\theta$ to get the left-hand end and $tan^2 2\theta$ to get the length (and also the depth) of the bathtub. However, condition (ii) excludes MSW effects in the LWVO region. Notice also that efficient $\nu_e \to \nu_\mu$ conversion does not require big vacuum mixing θ; on the contrary, the best conversion is with small θ.

The MSW bathtub offers an immediate explanation of the apparent differential suppression of the ν_e spectrum: let the sloping end lie across the $\nu - e$ scattering range $E_\nu > 7.5MeV$ (moderate $\nu - e$ suppression); let the flat bottom lie across the rest of the ^{37}Cl capture range $E_\nu > 0.9MeV$ (more ^{37}Cl suppression); let the steep end fall near $E_\nu \sim 0.2MeV$ at the top of the pp spectrum contribution (less ^{71}Ga suppression). This simple prescription leads to the best MSW solution: $\delta m^2 \sim 10^{-5} eV^2$ with $sin^2 2\theta \sim 10^{-2}$, Fig.6a shows a typical recent fit[16]. There is also a large-θ region, not really a good solution but a local χ^2 minimum, where the bathtub is much shallower and wider.

Matter effects can also arise in the Earth. They are not MSW (no chance of adiabatic level crossing), just amplified vacuum oscillations through which ν_μ can convert

back to ν_e when the sun is below the horizon, giving day/night and summer/winter asymmetries in counting rates. There are 2 conditions for big Earth effects.
(i) Near-resonant amplification in the Earth ($\tan 2\theta_m$ large):

$$\delta m^2(eV^2)cos2\theta/E(MeV) \sim 2\sqrt{2}G_F\rho_e \simeq 3\times 10^{-7},$$

assuming rock density $\sim 4gm/cm^3$.
(ii) Matter oscillation wavelength ($\lambda = 4\pi E/\delta m_m^2$) less than Earth diameter ($10^7 m$). At resonance the matter-eigenvalue difference is $\delta m_m^2 = \delta m^2 sin2\theta$, giving

$$\delta m^2(eV^2)sin2\theta/E(MeV) > 2.5\times 10^{-7}.$$

Both these conditions can be approached or satisfied in a small region of $(\delta m^2, sin^2 2\theta)$ for given E, but θ cannot be very small. For the Kamiokande $\nu - e$ range, $E \sim 10$ MeV, a region near the MSW large-θ solution is sensitive to Earth effects; the absence of a day/night asymmetry[4] excludes this region (labelled "excluded 90% C.L." in Fig.6). Future experiments will enlarge this region of sensitivity.

Similar things can happen for ν_e mixing with sterile ν_x, but now Z-exchange no longer drops out; coherent $\nu - e$ and $\nu - p$ Z-exchanges cancel and the net effect is to replace ρ_e above by $(\rho_e - \frac{1}{2}\rho_n)$ where ρ_n is the neutron number density[17]. The critical parameters change a bit and the large-θ solution vanishes (Fig.6b).

Three- or four-flavour neutrino mixing offers more complicated possibilities, with more free parameters, that we do not need yet and shall not discuss today.

4. Exotic neutral current effects.

If there are new neutral-current interactions, such as $\nu_e d \to \nu_e d, \nu_\tau d$ flavour-conserving or flavour-flipping scattering via R-parity-violating b-squark exchanges[18], new terms will appear in the matter-propagation matrix. In the most general case with diagonal and off-diagonal contributions from scattering on e, p, n distributions in matter, this matrix can be put in the form

$$\begin{pmatrix} m_1^2 + m_2^2 - \delta m^2 cos2\theta + 4\sqrt{2}G_F\rho_e E & \delta m^2 sin2\theta + \epsilon 4\sqrt{2}G_F\rho_e E \\ \delta m^2 sin2\theta + \epsilon 4\sqrt{2}G_F\rho_e E & m_1^2 + m_2^2 + \delta m^2 cos2\theta + \epsilon' 4\sqrt{2}G_F\rho_e E \end{pmatrix}$$

in the approximation $\rho_n \simeq \rho_e$, with just two constant parameters ϵ and ϵ' describing the new physics in units of the standard matter effect. If $\epsilon \neq 0$, we have mixing and oscillations even in the absence of vacuum mixing ($\delta m^2 sin2\theta = 0$)[13].

These new terms modify the previous MSW solutions. Fig.7 compares $\nu_e - \nu_\tau$ solutions in the cases $\epsilon = 0$, $\epsilon = 0.04$, $\epsilon = -0.04$ (with $\epsilon' = 0$)[19]. Adding this small exotic mixing scarcely affects the large-θ solution but distorts or even splits the small-θ solution.

5. Outlook.

The different two-flavour-mixing scenarios can be distinguished (or rejected) by future measurements of the ^8B spectrum modulation (SNO, Super-Kamiokande, see Figs.8,9), the ^7Be 0.86 MeV line contribution (Borexino), and possible day/night effects (SNO, Super-Kamiokande, ICARUS):

$Measurement$	$LWVO$	$MSWsmall-\theta$	$MSWlarge-\theta$
$^8B\ modulation$	yes	yes	$none$
$^7Be\ line$	$seasonal$	$small$	$medium$
$day/night\ effects$	$none$	$small$	$medium$

Furthermore, the charged-current/neutral-current event ratio [SNO, Borex, ICARUS] will distinguish whether the neutrino flavour mixed with ν_e is active (ν_μ, ν_τ) or sterile (ν_x). The problem will become much more clearly defined.

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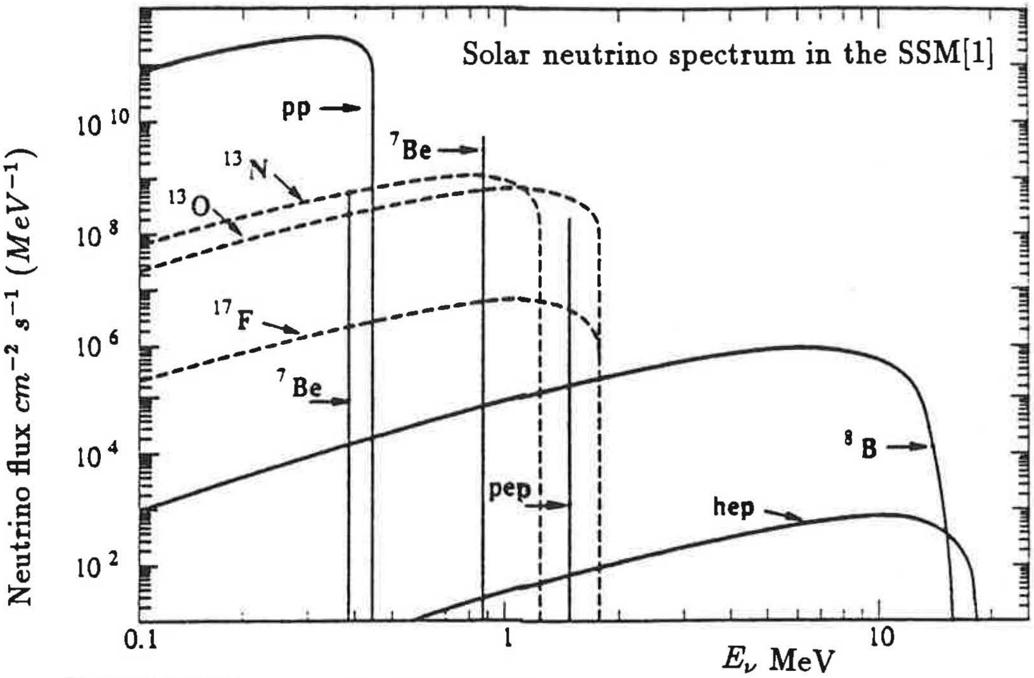
Fig.1.



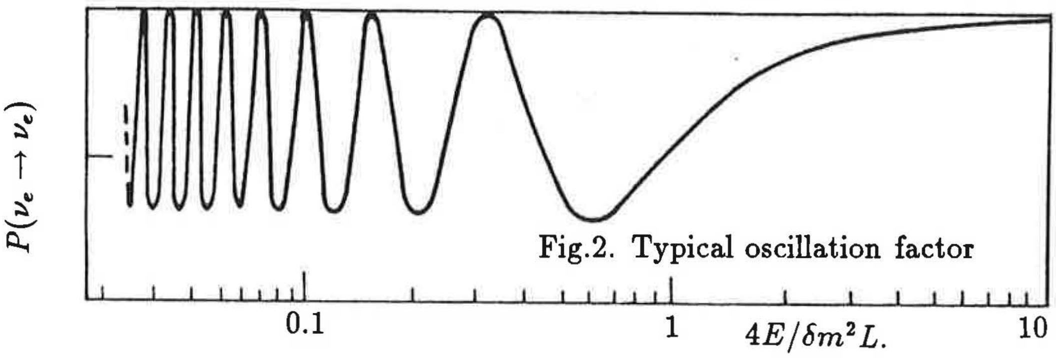


Fig.2. Typical oscillation factor

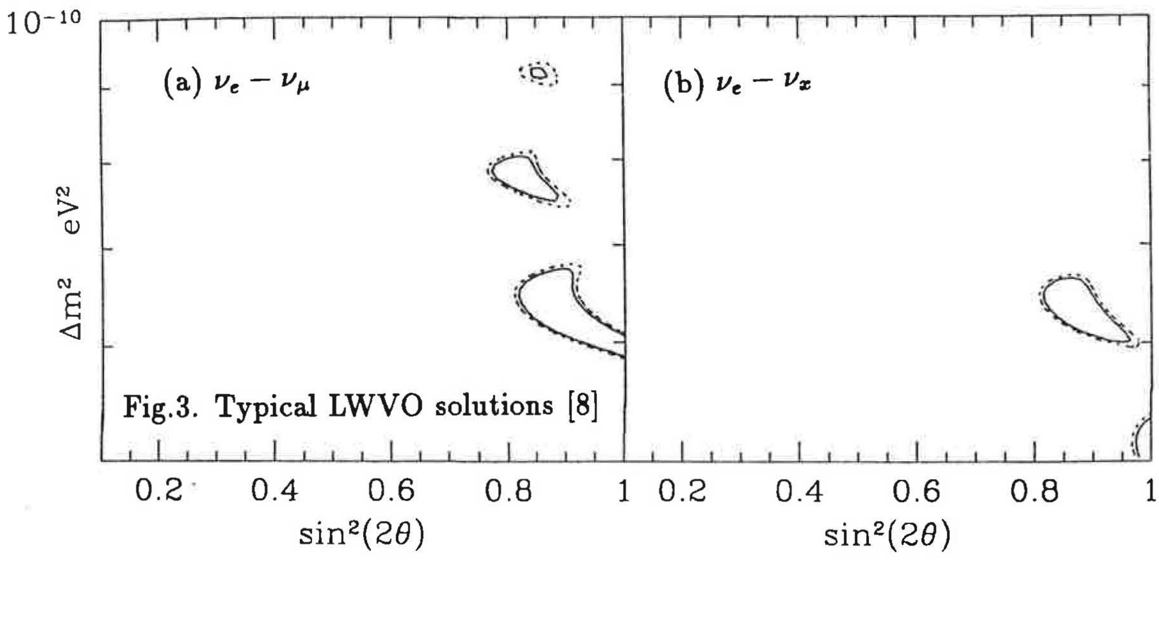


Fig.3. Typical LWVO solutions [8]

Fig.4. Eigenvalues of propagation matrix versus density ρ_e

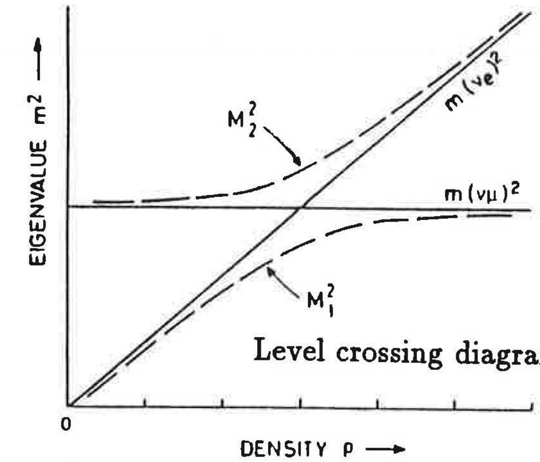
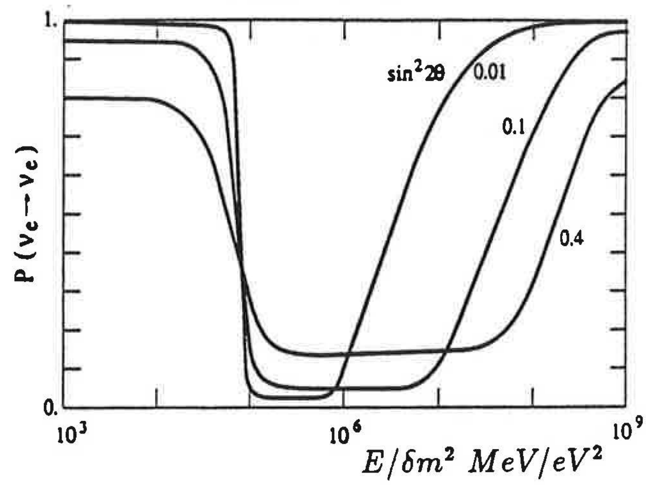


Fig.5. The MSW bathtub: suppression factor P versus $E/\delta m^2$



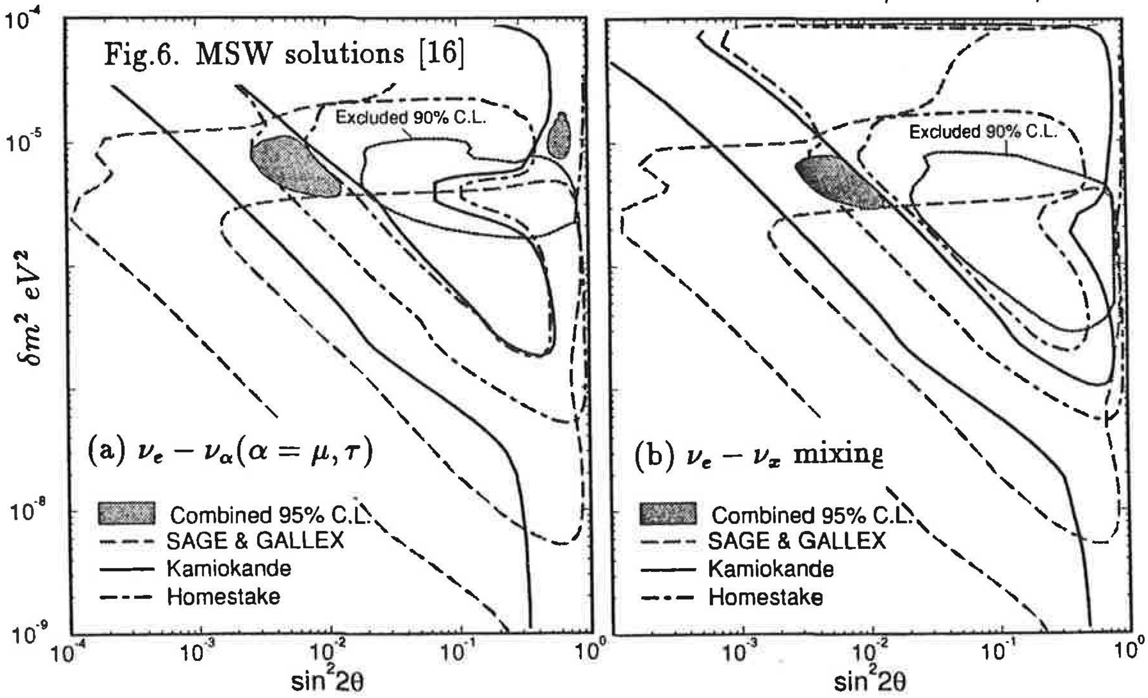


Fig.6. MSW solutions [16]

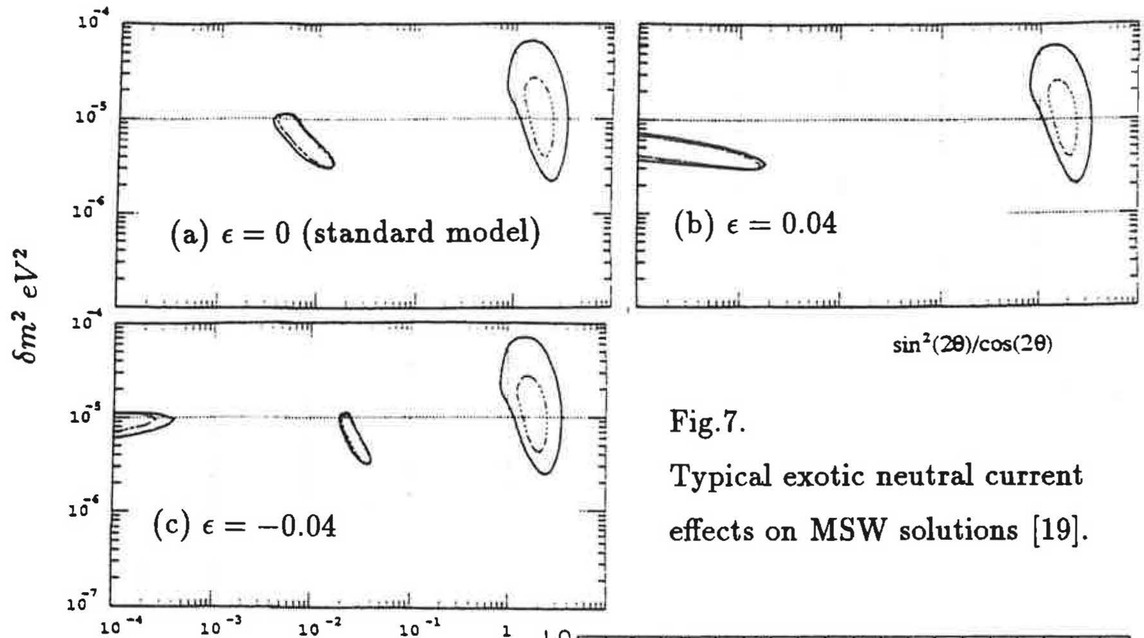


Fig.7.
Typical exotic neutral current effects on MSW solutions [19].

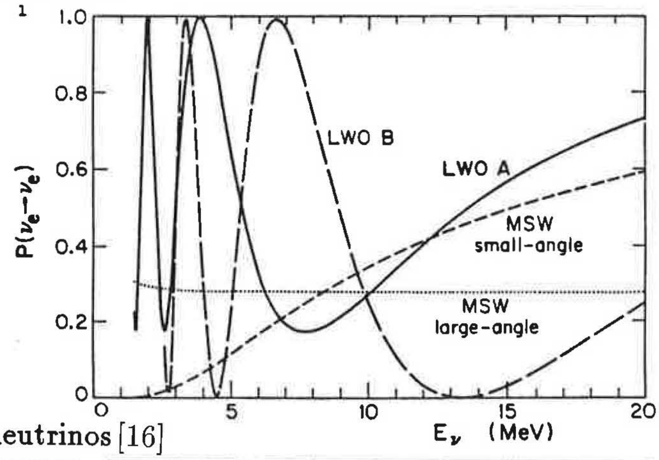


Fig.8. Modulation factors for ${}^8B$ neutrino spectrum
LWO A has $\delta m^2 = 0.6 \times 10^{-5} eV^2$
LWO B has $\delta m^2 = 1.1 \times 10^{-5} eV^2$

Fig.9. Electron spectra due to ${}^8B$ neutrinos [16]