

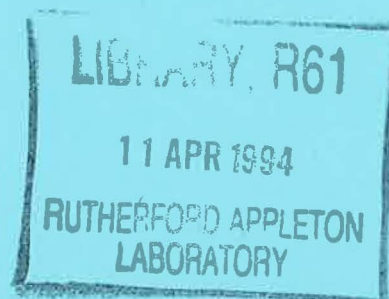
RAL 94028  
COPY 1 R61 RR  
ACCN: 9402  
RAL-94-028

Science and Engineering Research Council

**Rutherford Appleton Laboratory**

Chilton DIDCOT Oxon OX11 0QX

RAL-94-028



# **Direct Photoproduction of Jets in the $k_T$ -Factorisation Prescription**

**J R Forshaw and R G Roberts** *HEP.*

March 1994

**Science and Engineering Research Council**

"The Science and Engineering Research Council does not accept any responsibility for loss or damage arising from the use of information contained in any of its reports or in any communication about its tests or investigations"

## Direct photoproduction of jets in the $k_T$ -factorisation prescription

J.R. Forshaw and R.G. Roberts  
Rutherford Appleton Laboratory,  
Chilton, Didcot OX11 0QX, England.

### Abstract

We study the cross section for dijet production via direct photons, as seen at the HERA collider. Rather than assuming the standard factorisation formula for the cross section (where the incoming partons are assumed to be on-shell), we use the  $k_T$ -factorisation formula. This formalism ought to be more appropriate for jet production in regions where the cross section is sensitive to the small- $x$  parton distribution functions. We study the rapidity distribution and the azimuthal angular distribution of the jets and compare with the traditional approach. Our calculations reveal a breakdown of the traditional approach as one moves into the regime of small  $x$ .



In a previous paper [1], we illustrated the importance of studying the cross section for dijet production via direct photons at the HERA collider in order to gain information regarding the gluon content of the proton at small- $x$ . Since that paper was published, the ZEUS collaboration has shown that it is possible to separate their events into ‘direct photon’ events and ‘resolved photon’ events using a cut on the fraction of the photon energy which enters the hard scatter [2]. This encourages us to study further this important process and so, in this paper, we wish to return to the theoretical evaluation of the dijet cross section in the context of the ‘ $k_T$ -factorisation’ prescription, which ought to be used for small  $x \sim p_T^2/s$  ( $p_T$  is a typical jet transverse momentum and  $s$  is the  $ep$  centre-of-mass energy).

At large  $x \sim p_T^2/s$ , we know that the  $\ln p_T$  terms, which are present in the perturbative expansion of the cross section, can become significant and need to be summed – leading to the violation of the scaling predictions of the parton model. The summation of the leading logarithms (which arise as a result of multi-parton branching before the hard scatter) is performed using the DGLAP (Dokshitzer, Gribov, Lipatov, Altarelli, Parisi) equations [3]. The infra-red singularities (reflecting the presence of long distance physics) which arise in the calculation of these QCD corrections can be separated from the short distance contributions and thus we are led to cross sections which factorise (schematically) in the following way [4]:

$$\sigma \sim F(\dots, \mu^2) \otimes \hat{\sigma}(\dots, \mu^2, Q^2) \quad (1)$$

The function  $F$  contains the (unknown) long-distance physics whilst only short distance physics is contained in the ‘hard’ subprocess cross section  $\hat{\sigma}$ . It is usual to choose the factorisation scale ( $\mu^2$ ), to equal the hard scale ( $Q^2$ ), in order to subsume the leading logarithms within the function  $F$ . Central to this factorisation formula is the assumption that the ‘hard’ cross section can be computed assuming that the incoming partons have negligible transverse momenta (and virtualities) compared with the scale of the hard process. We shall call this the ‘ $Q^2$ -factorisation’ approach.

For small enough  $x$  we expect that the  $\ln x$  terms, which are present in the perturbative expansion, will become dominant. We refer to the summation of these leading logarithms in  $1/x$  as the BFKL (Balitsky, Fadin, Kuraev, Lipatov) summation [5]. If the logarithms in  $x$  are indeed dominant, then we are no longer allowed to use the formula of eqn.(1). We must use the  $k_T$ -factorisation formula of refs.[6, 7, 8], which states (again schematically) that

$$\sigma \sim F(\dots, k_{Ti}^2, \mu^2) \otimes \hat{\sigma}(\dots, k_{Ti}^2, \mu^2, Q^2) \quad (2)$$

The scales,  $k_{Ti}$ , are the transverse momenta carried by the partons entering the hard scatter.

In the specific case of direct photoproduction of dijets the  $Q^2$ -factorisation formula can be written:

$$\frac{d^3\sigma^{dir}}{dy_1 dy_2 dp_T^2} = z f_{\gamma/e}(z) x g(x, Q^2) \frac{d\hat{\sigma}(\gamma g \rightarrow q\bar{q})}{d\hat{t}} \quad (3)$$

The final state parton rapidities are denoted  $y_1$  and  $y_2$ , and  $p_T$  denotes their transverse momentum in the  $\gamma p$  centre-of-mass frame. The azimuthal angular distribution is trivial since the jets are produced back-to-back. The photon flux is determined by the function,  $f_{\gamma/e}(z)$ . The long distance physics is contained in the gluon structure function,  $x g(x, Q^2)$ , and the short distance physics is contained in the cross section:

$$\frac{d\hat{\sigma}(\gamma g \rightarrow q\bar{q})}{d\hat{t}} = \frac{2\pi\alpha\alpha_s}{\hat{s}^2} \left( \frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}} \right) \quad (4)$$

For simplicity we show only the gluonic contribution to the cross section (it is this contribution which dominates for jets produced in the electron direction, i.e. small  $x$ ). The subprocess Mandelstam variables are denoted  $\hat{s}, \hat{t}$  and  $\hat{u}$  and satisfy the usual kinematic relations.

The corresponding formula in the language of  $k_T$ -factorisation can be written thus [8]:

$$\frac{d^2\sigma^{dir}}{dy_1 dy_2} = \int \frac{d^2 p_T}{\pi} \frac{d^2 k_T}{\pi} d^2 p'_T \delta(p_T + p'_T - k_T) \mathcal{J}(\nu, p_T^2, k_T^2, \theta) z f_{\gamma/e}(z) \Phi(x, k_T^2) \frac{d\hat{\sigma}(\gamma g^* \rightarrow q\bar{q})}{d\hat{t}} \quad (5)$$

The final state partons have transverse momentum vectors  $p_T$  and  $p'_T$  which are no longer equal and opposite for non-vanishing gluon momentum,  $k_T$ . We can use the delta function to perform the  $p'_T$  integrals. One of the two remaining angular integrals can also be performed (since the integrand only depends upon  $\theta$ , the angle between  $k_T$  and  $p_T$ ). We can therefore write:

$$\frac{d^2\sigma^{dir}}{dy_1 dy_2} = \int dp_T^2 dk_T^2 \frac{d\theta}{2\pi} \mathcal{J}(\nu, p_T^2, k_T^2, \theta) z f_{\gamma/e}(z) \Phi(x, k_T^2) \frac{d\hat{\sigma}(\gamma g^* \rightarrow q\bar{q})}{d\hat{t}} \quad (6)$$

For the 'hard scatter' cross section we use [9]:

$$\frac{d\hat{\sigma}(\gamma g^* \rightarrow q\bar{q})}{d\hat{t}} = \frac{2\pi\alpha\alpha_s}{(\hat{s} + k_T^2)^2} \left( \frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}} - \frac{2k_T^2\hat{s}}{\hat{u}\hat{t}} + \frac{12k_T^2\hat{s}}{(\hat{u} + \hat{t})^2} \right) \quad (7)$$

$\mathcal{J}$  is the Jacobean and  $\nu = \exp(y_1 - y_2)$ . Some important kinematic relations follow.

$$p_T'^2 = p_T^2 + k_T^2 - 2k_T p_T \cos \theta \quad (8)$$

The momentum fractions  $x$  and  $z$  can then be written

$$\begin{aligned} x &= \frac{1}{2E_p} (p_T e^{-y_1} + p'_T e^{-y_2}) \\ z &= \frac{1}{2E_e} (p_T e^{y_1} + p'_T e^{y_2}) \end{aligned} \quad (9)$$

and

$$-\hat{t} = p_T^2 + p_T p'_T / \nu \quad (10)$$

Also,  $\hat{s} = 4xzE_e E_p$  and  $\hat{s} + \hat{t} + \hat{u} + k_T^2 = 0$ . We can write the Jacobean explicitly as:

$$xz\mathcal{J}(\nu, p_T^2, k_T^2, \theta) = \frac{p_T p'_T}{4\nu E_e E_p} \left[ \nu^2 + \nu \left( \frac{p'_T}{p_T} + \frac{\Delta^2}{p_T p'_T} \right) + \frac{\Delta^2}{p_T^2} \right] \quad (11)$$

where  $\Delta^2 = (p_T^2 + p'^2_T - k_T^2)/2$ .

To regain eq.(3) from eq.(6), one assumes that the integrand can be approximated by its value at  $k_T^2 = 0$  (the  $\theta$  integral is then trivial) and that the maximum value of  $k_T^2 = Q^2$ . This leads to the relation:

$$xg(x, Q^2) = \int_0^{Q^2} dk_T^2 \Phi(x, k_T^2) \quad (12)$$

We can re-arrange eq.(6) in order to examine the azimuthal distribution of the final state partons:

$$\frac{d^3\sigma^{dir}}{dy_1 dy_2 d\phi/2\pi} = \int dp_T^2 dk_T^2 \mathcal{J}(\nu, p_T^2, k_T^2, \theta) \frac{p'_T \cos \phi}{k_T \cos \theta - p_T \sin^2 \phi} z f_{\gamma/e}(z) \Phi(x, k_T^2) \frac{d\hat{\sigma}(\gamma g^* \rightarrow q\bar{q})}{d\hat{t}} \quad (13)$$

The angle between  $p_T$  and  $p'_T$  is  $\phi$  and

$$\cos \theta = \frac{p_T}{k_T} \sin^2 \phi \pm \left( 1 - \frac{p_T^2}{k_T^2} \sin^2 \phi \right)^{1/2} \cos \phi \quad (14)$$

In the limit of  $k_T^2 \rightarrow 0$ , the integrand becomes a delta function at the only allowed point in phase space, i.e. where  $\phi = \pi$ .

At this point we should emphasise that if one intends to use the standard DGLAP evolution equations to determine the  $Q^2$ -evolution of  $xg$  then it is formally inconsistent to construct  $\Phi$  by taking the derivative and then using the  $k_T$ -factorisation formula above. In the DGLAP formalism, the QCD corrections to the cross section contain mass singularities, which must be removed by factorisation. This should be contrasted with the BFKL formalism where there are no mass singularities. However, we acknowledge that the absence of such singularities does not guarantee sensible results and if our cross section depends critically on inherently infra-red physics we may not be able to trust the results.

These issues are dealt with nicely in the paper of Collins and Ellis [7]. They introduce a factorisation scale to isolate the infra-red physics and, working in moment space, i.e.

$$\tilde{\Phi}(j, k_T^2; \mu^2) = \int_0^1 dx x^{j-1} \Phi(x, k_T^2; \mu^2) \quad (15)$$

they find that

$$\bar{\Phi}(j, k_T^2; \mu^2) = \gamma_c(j) \frac{1}{k_T^2} \left( \frac{k_T^2}{\mu^2} \right)^{\gamma_c(j)} x \tilde{g}(j, \mu^2) \quad (16)$$

where  $\gamma_c(j)$  is the BFKL anomalous dimension function which takes on the value of (exactly)  $1/2$  when  $j = 1 + 12\alpha_s \ln 2/\pi$ . The BFKL corrections which multiply the gluon distribution are calculated by solving the BFKL equation with an infra-red cut-off of  $\mu$ . To obtain the final cross section,  $\Phi$  must be convoluted with the ‘impact factor’ – in our case this leads to the expression:

$$\begin{aligned} \frac{d^2 \sigma^{dir}}{dy_1 dy_2} &= \int dp_T^2 z f_{\gamma/e}(z) x g(x, \mu^2) \frac{d\hat{\sigma}(\gamma g \rightarrow q\bar{q})}{d\hat{t}} \\ &+ \int dp_T^2 \int_{\mu^2} dk_T^2 \frac{d\theta}{2\pi} \mathcal{J}(\nu, p_T^2, k_T^2, \theta) z f_{\gamma/e}(z) \Phi(x, k_T^2; \mu^2) \frac{d\hat{\sigma}(\gamma g^* \rightarrow q\bar{q})}{d\hat{t}} \end{aligned} \quad (17)$$

In this letter, we do not wish to focus on the specific solution for the function,  $\Phi$ . Rather, we want to investigate the general features of this new factorisation prescription – and to ask whether it might be expected to produce significant deviations from the more standard formalism.

To this end, let us use the following BFKL motivated form for the gluon distribution function (we ignore quarks throughout this paper – they are not included in the BFKL formalism since they do not lead to the dominant small  $x$  behaviour).

$$\Phi(x, k^2) = \mathcal{N} \frac{x^{-\lambda}}{k^2} (k^2)^{1/2} \exp \left( -\frac{\ln(k^2/k_0^2)}{2\lambda'' \ln(x_0/x + 1)} \right) \quad (18)$$

where

$$\begin{aligned} \lambda &= \frac{3\alpha_s}{\pi} 4 \ln 2 \\ \lambda'' &= \frac{3\alpha_s}{\pi} 28\zeta(3) \end{aligned} \quad (19)$$

The parameters,  $x_0$  and  $k_0^2$ , are treated as essentially free – and we show results for different choices. Let us briefly reflect upon our prejudices regarding their likely values. The scale,  $k_0^2$ , is determined by the size of the hadron, and we therefore expect rather small values,  $\sim \Lambda_{QCD}^2$ . In the limit of  $x \rightarrow 0$ , eq.(18) reduces to that which is predicted by the BFKL formalism. However, in order to ensure sensible behaviour at intermediate values of  $x$ , we modify the asymptotic logarithm, i.e.  $\ln 1/x \rightarrow \ln(x_0/x + 1)$ . This modification ensures a narrow distribution in  $k^2$  at large  $x$  (consistent with the  $Q^2$ -factorisation assumption that the partons are on-shell). One can think of  $x_0$  as the parameter which delineates the onset of BFKL behaviour from the more

traditional large  $x$  behaviour, which suggests  $x_0 \sim 0.01$ . The normalisation,  $\mathcal{N}$ , is a constant and unimportant for the purposes of this paper.

The following figures show the effect of  $k_T$ -factorisation on the cross section for direct photoproduction of dijets at HERA, i.e. 820 GeV protons colliding with 30 GeV electrons. The cross sections are  $ep$  cross sections, and the photon flux was approximated using the same Weiszacker-Williams formula as in ref.[1]. We took  $\lambda = 0.5$  and  $\lambda'' = 6.1$ , in eq.(19) but allowed a running coupling (with  $\Lambda_{QCD} = 200$  MeV and  $Q^2 = p_T^2/4$ ) in the hard scattering cross section.

In figs.(1), we show the variation of the cross section with the jet rapidity (the jets are fixed to have equal rapidity, i.e.  $y_1 = y_2 = y$ ) for four different combinations of the parameters  $k_0$  and  $x_0$ . In all cases, we find that the  $k_T$ -factorisation prescription (eq.(17)) produces a cross section which is significantly smaller than that which is obtained using the  $Q^2$ -factorisation prescription (eq.(3)) for those jets which are produced in the electron direction, i.e. small  $x$ . Not surprisingly, the effect increases as  $k_0$  and  $x_0$  are increased – since we move the peak of the  $k^2$ -distribution to larger energies as  $k_0$  increases and broaden the width of the  $k^2$ -distribution as  $x_0$  increases.

The transition from the  $k_T$ - to  $Q^2$ -factorisation prescriptions is illustrated in figs.(2). As the factorisation scale,  $\mu^2$ , is increased the cross section tends toward the standard,  $Q^2$ -factorisation, prescription result (shown as the horizontal dotted line on each plot). Exact agreement is never quite obtained since the upper limit on the  $k_T$ -factorisation integral is calculated using the exact kinematics (rather than assuming it to be equal to  $Q^2 = p_T^2/4$ , which is the hard scale taken in the  $Q^2$ -factorisation calculation). In fig.(2a), we show the  $\mu^2$  dependence at  $y = \pm 1$  using the BFKL motivated form for  $\Phi$  and typical values of  $k_0$  and  $x_0$ . In fig.(2b), we show the equivalent plots using a  $\Phi$  which has been computed by taking the derivative of the GRV structure function parametrisations for the proton [10]. As we remarked earlier, such an approach is strictly inconsistent with the BFKL formalism – one extracts a parton  $k_T$ -distribution from the  $Q^2$ -factorisation approach (in which the GRV structure functions are derived) – it is not clear that such an extraction is meaningful. Even so, our results again show significant deviation from the standard approach, especially in the backward direction.

From the plots of figs.(1) and (2) it should be clear that the essential BFKL prediction that the incoming partons have non-negligible transverse momenta has important consequences at HERA. It destroys the notion of universal small  $x$  parton distributions in the standard ( $Q^2$ -factorisation) formalism. Consequently, it is essential to measure the small- $x$  parton distri-

bution functions in as many independent processes as is possible, e.g.  $xg(x, Q^2)$  from dijets [1],  $\partial F_2/\partial \ln Q^2$  [11],  $F_L$  [12] and  $J/\Psi$  production [13] – we *do not* expect universal agreement between all processes.

Another consequence of the non-zero parton  $k_T$  is illustrated in fig.(3). The azimuthal distribution of the final state partons is plotted. Instead of back-to-back ‘jets’, we expect a de-correlating effect due to the imbalance of the incoming  $k_T$ . This could show up experimentally, as a broadening of the azimuthal distribution for jets produced in the backward direction compared to those produced in more forward directions. We note that the apparent spikes at the peaks of the distributions are, of course, not physical and are simply due to numerical limitations.

In summary, we have performed a quantitative study of the ‘ $k_T$ -factorisation’ prescription when applied to a process that is currently being measured at HERA. Comparison with the traditional ‘ $Q^2$ -factorisation’ approach shows that, in the region where small- $x$  gluons within the proton are being probed, the discrepancy can be sizeable, i.e.  $\simeq 20\%$ . This deviation arises essentially due to the relaxation of the assumption that the partons which enter the hard scatter are on-shell and has the important consequence that we expect  $xg(x, Q^2)$  (for small  $x$ ) to be process dependent. Another signal for such corrections would be a deviation from the ideal ‘back-to-back’ configuration in the azimuthal angle, as shown in fig.(3).

## References

- [1] J.R. Forshaw and R.G. Roberts, Phys. Lett. **B319** (1993) 539.
- [2] M. Derrick et al (ZEUS Collaboration), Phys. Lett. **322** (1994) 287.
- [3] V.N. Gribov and L.N. Lipatov, Sov. J. Nucl. Phys. **15** (1972) 438.  
L.N. Lipatov, Sov. J. Nucl. Phys. **20** (1974) 94; G. Altarelli and G. Parisi, Nucl. Phys. **B126** (1977) 298.  
Yu.L. Dokshitzer, Sov. Phys. JETP **46** (1977) 641.
- [4] D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. **B146** (1978) 29.  
R.K. Ellis *et al*, Phys.Lett. **78B** (1978) 281; Nucl.Phys. **B152** (1979) 285.  
A.H. Mueller, Phys. Rev. **D18** (1978) 3705.  
S.B. Libby and G. Sterman, Phys. Rev. **D19** (1979) 2468.
- [5] E.A. Kuraev, L.N. Lipatov and V.S. Fadin, Sov. Phys. JETP **45** (1977) 199.  
Ya. Ya. Balitsky and L.N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822.
- [6] S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. **B242** (1990) 97.
- [7] J.C. Collins and R.K. Ellis, Nucl. Phys. **B360** (1991) 3.
- [8] E.M. Levin, M.G. Ryskin, Yu.M. Shabelskii and A.G. Shuvaev, Sov. J. Nucl. Phys. **53** (1991) 657.
- [9] F. Halzen and A.D. Martin, 'An Introductory Course in Modern Particle Physics', John Wiley 1984.
- [10] M. Glück, E. Reya and A. Vogt, Phys. Lett. **B306** (1993) 391.
- [11] R.G. Roberts, 'The structure of the proton' Cambridge Univ. Press (1990).  
J. Blümlein and M. Klein, in Proc. of the workshop 'Physics at HERA' eds. W. Buchmüller and G. Ingelman, vol.1, p.101 (1991).  
K. Prytz, Phys. Lett. **B311** (1993) 286.  
R.K. Ellis, Z. Kunszt and E.M. Levin, preprint Fermilab-pub-93-350T, ETH-93-41, 1994.  
N.N. Nikolaev and B.G. Zakharov, KFA-IKP(Th)-1994-6 (1994).
- [12] A.M. Cooper-Sarkar *et al*, Zeit. Phys. **C39** (1988) 281.

[13] A.D. Martin, C.K. Ng and W.J. Stirling, Phys. Lett. **B191** (1987) 200.

## Figure Captions

- Fig. 1 A comparison of the  $Q^2$ - and  $k_T$ -factorisation calculations of the dijet rapidity distributions. We have chosen  $y_1 = y_2 = y$  and define negative rapidity to be in the proton direction. The normalisation of  $\Phi$  was adjusted to reproduce approximately the value of  $xg(x, Q^2)$  of GRV at  $x=0.01$  and  $Q^2 = 10 \text{ GeV}^2$ .
- Fig. 2 To illustrate the transition from  $k_T$ - to  $Q^2$ -factorisation as the delineating scale,  $\mu^2$ , is increased (strictly speaking we use  $\min(\mu^2, Q^2)$  as the delineating scale). In each case the prediction of eq.(3) is described by the dotted line, the first (second) term in eq.(17) by the long (short) dashed line and their sum by the dot-dashed line.
- Fig. 3 The azimuthal distribution of the final state partons, determined at  $y_1 = y_2 = 1$ . and for two different values of  $k_0$ .

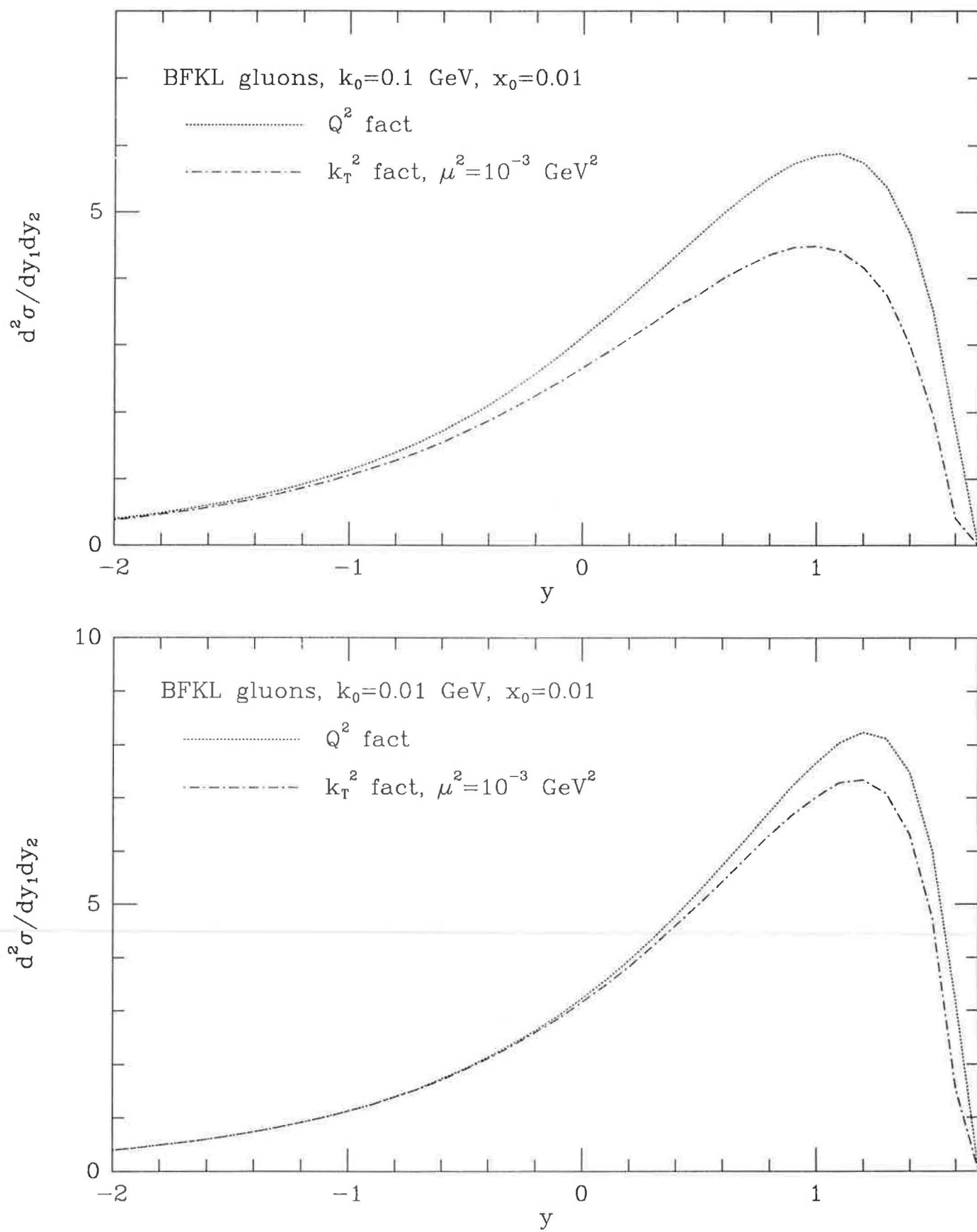


Fig. 1a

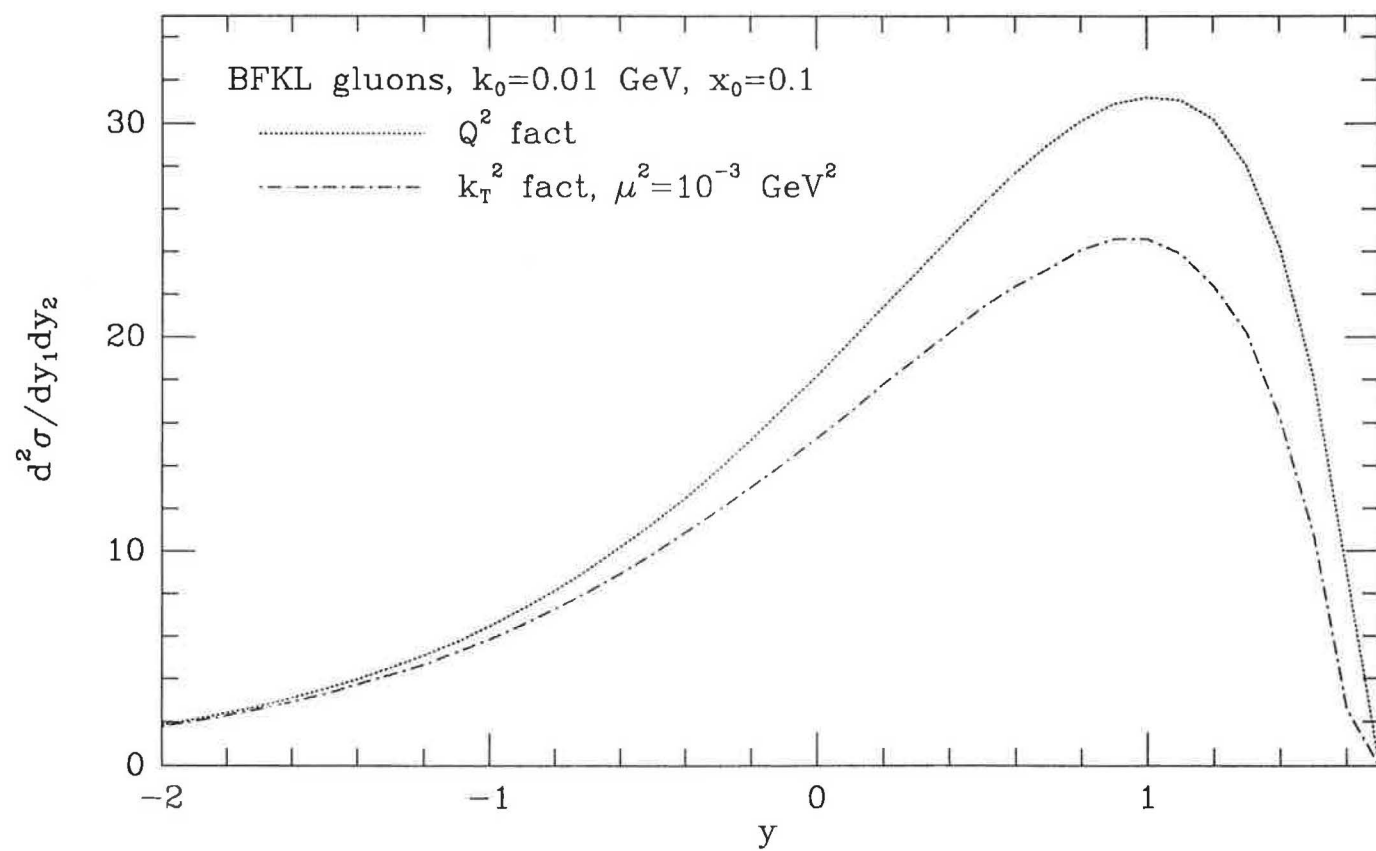
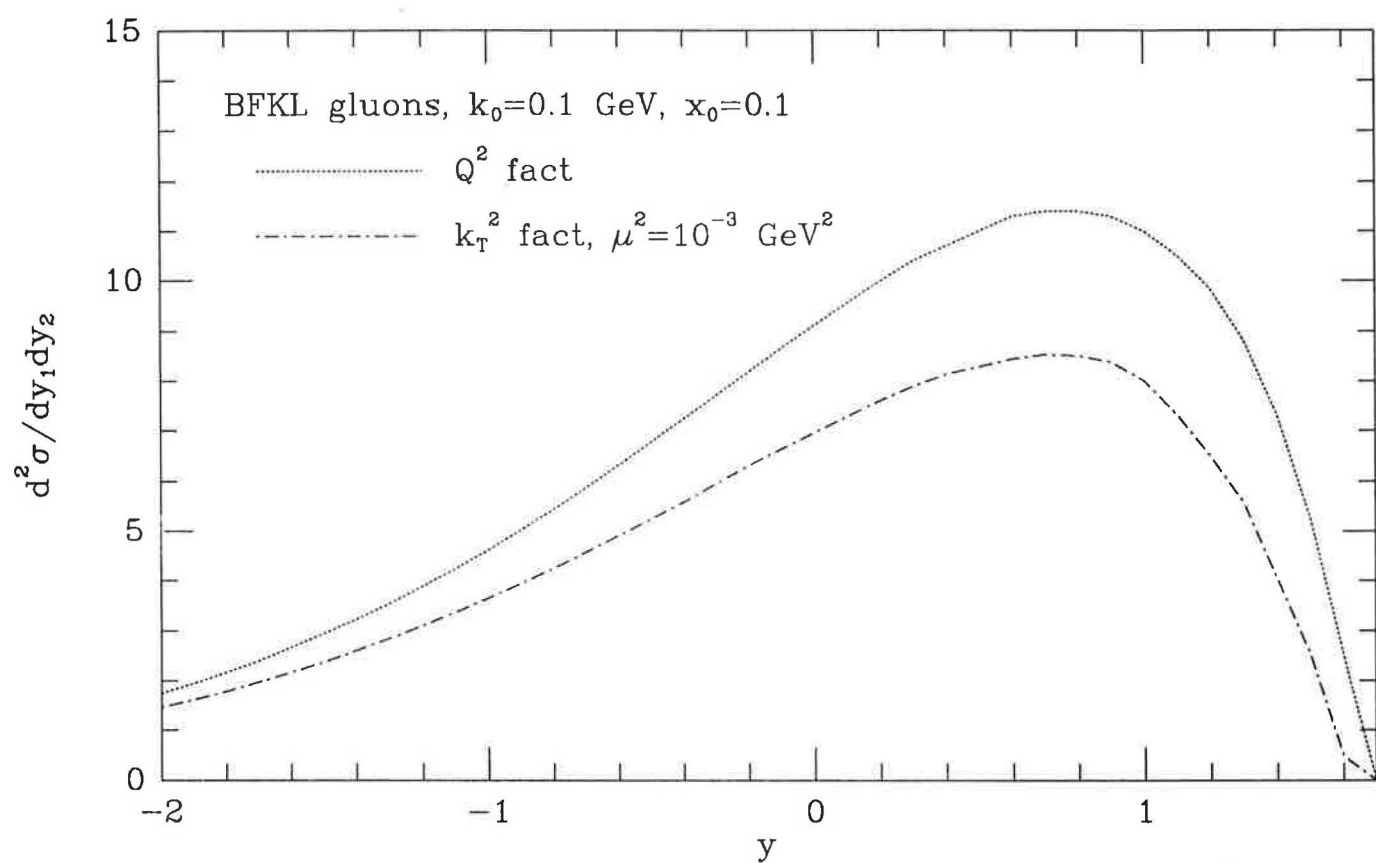


Fig. 1b

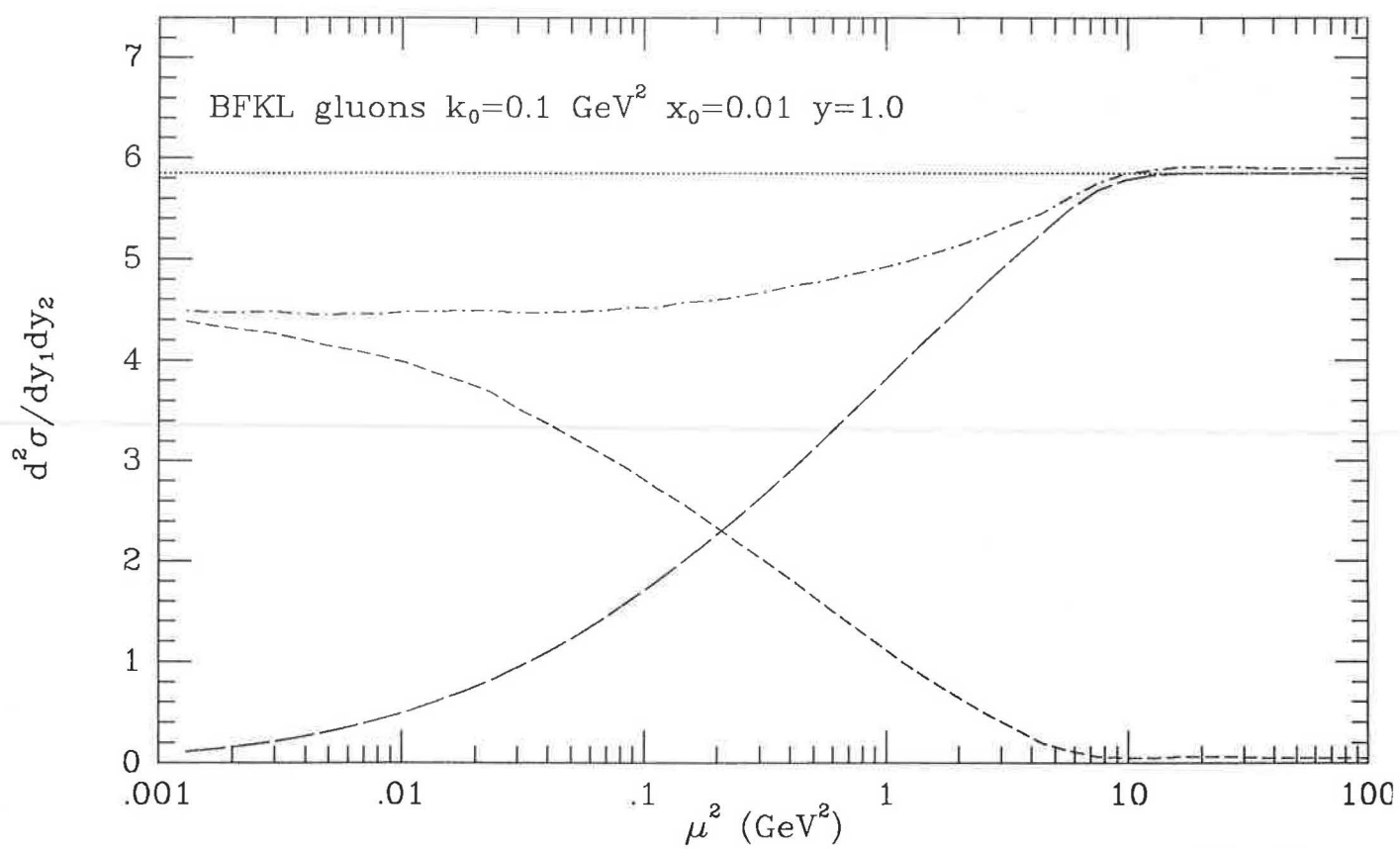
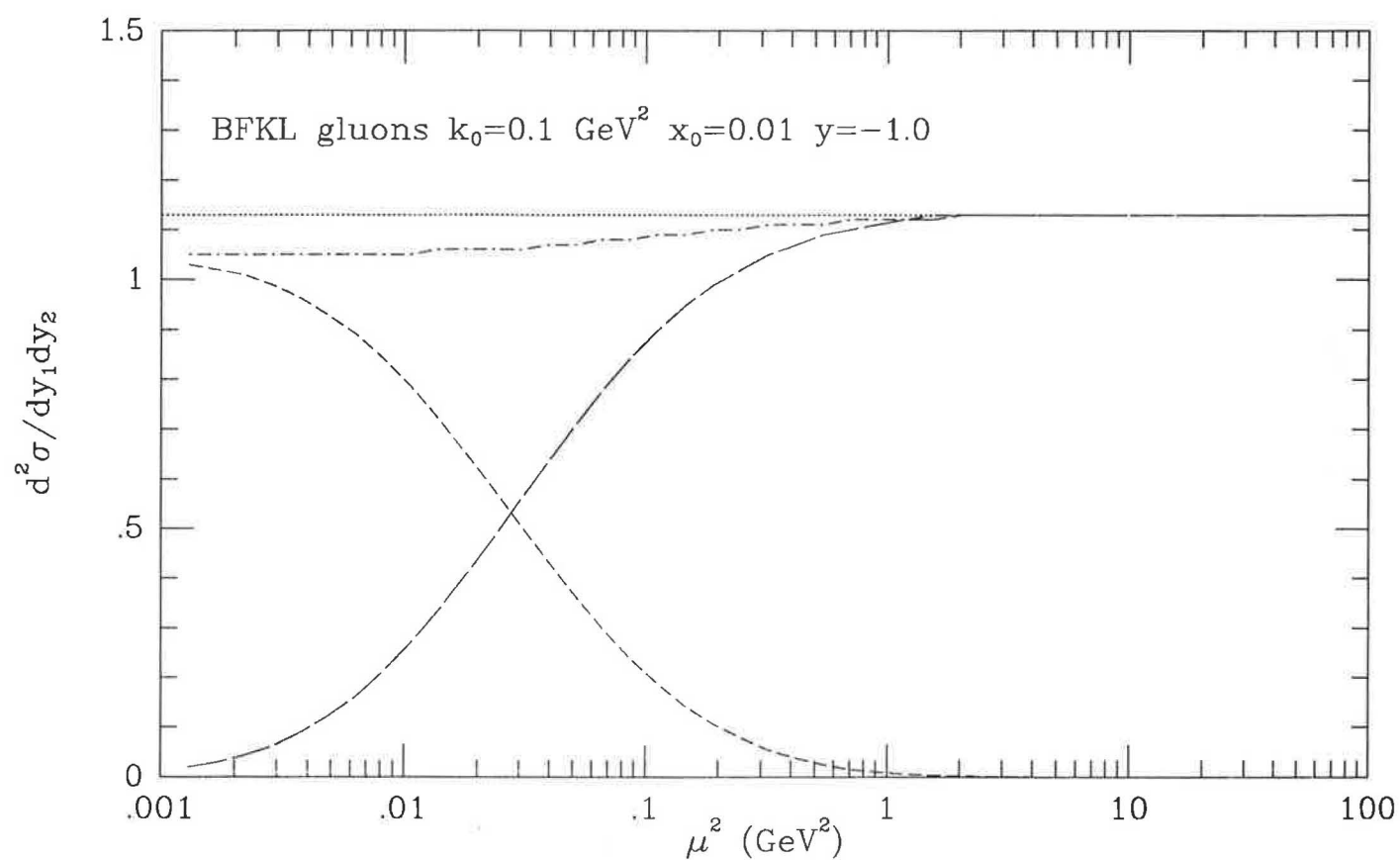


Fig. 2a

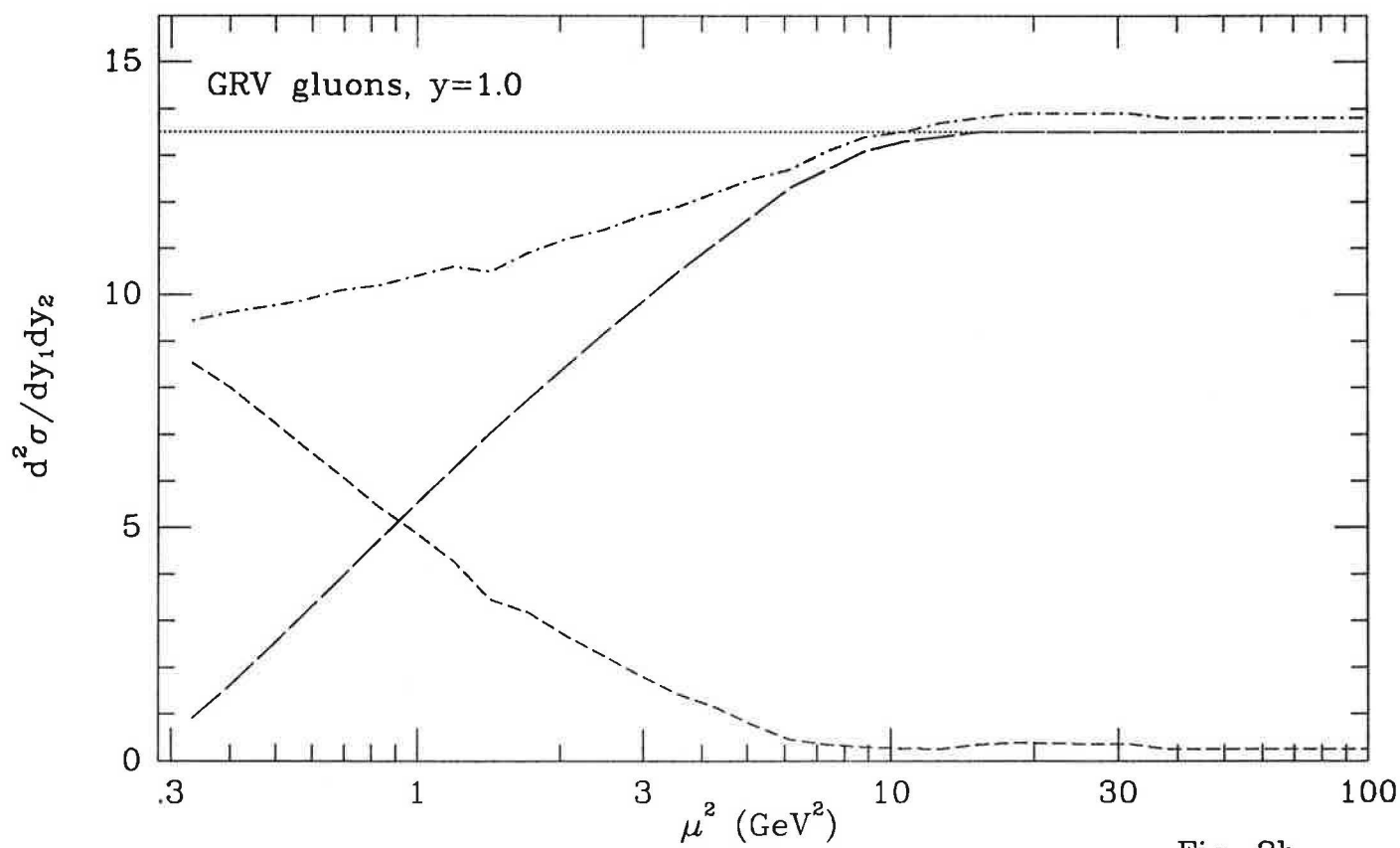
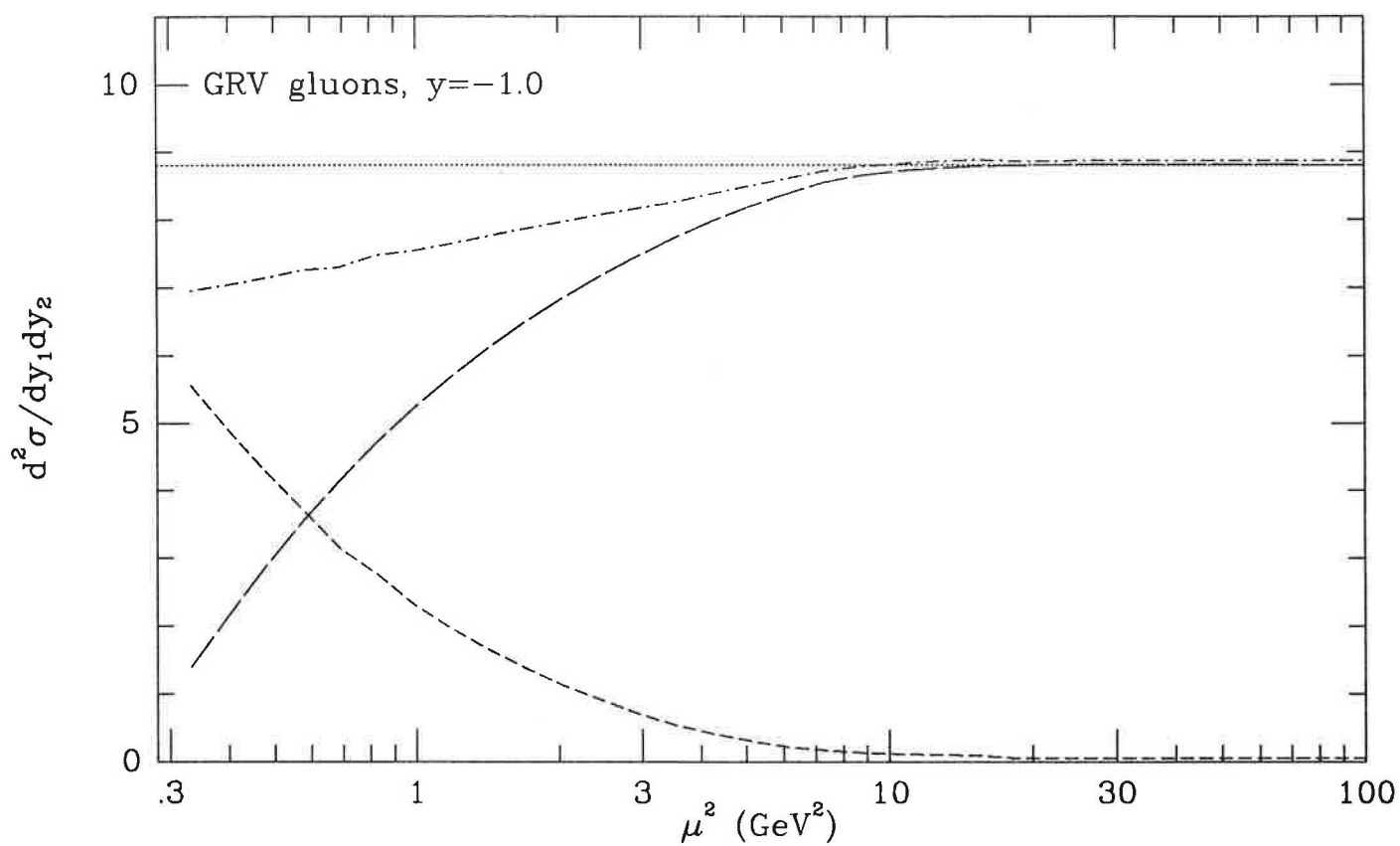


Fig. 2b

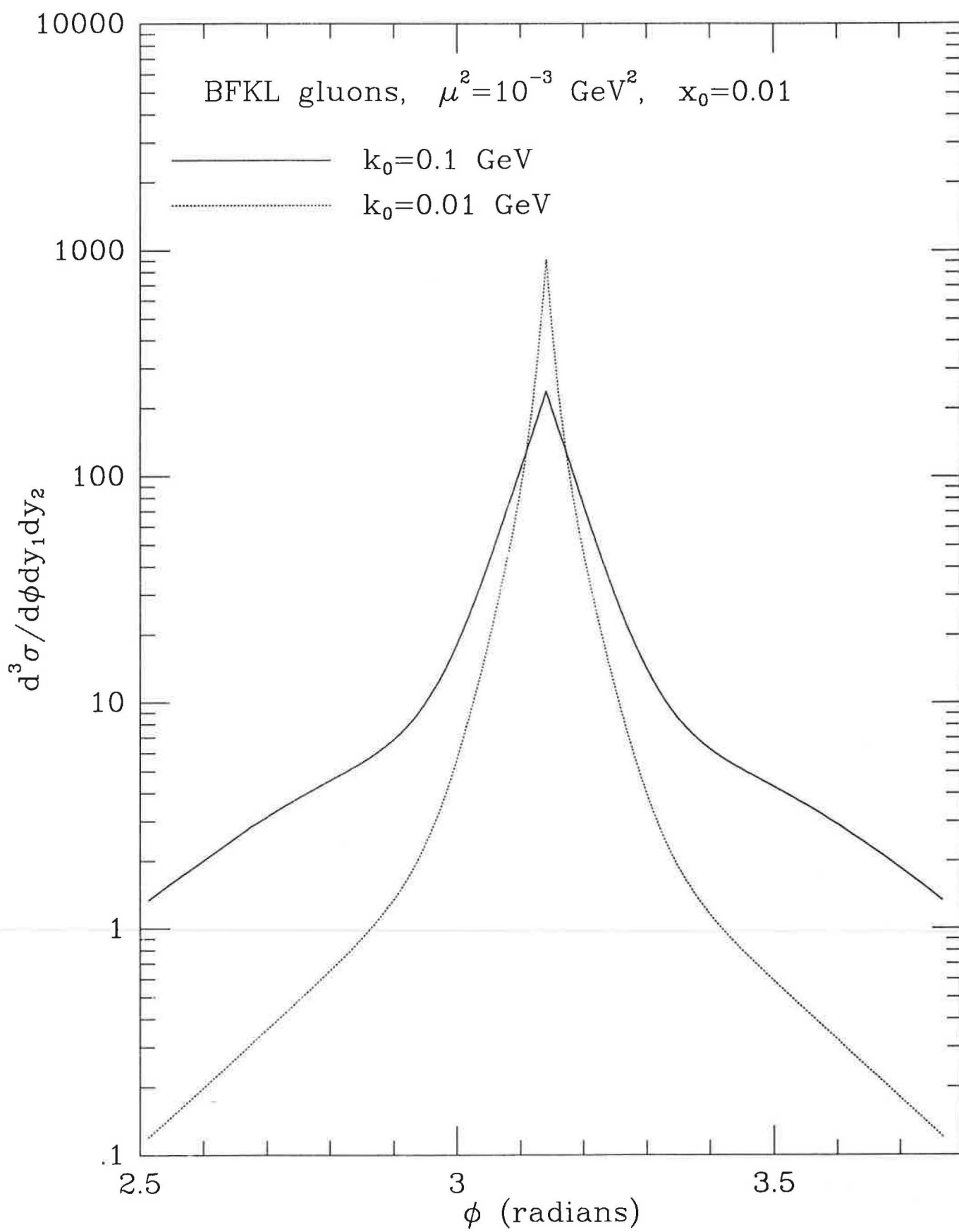


Fig. 3



