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# One-Loop $QCD$ Mass Effects in the Production of Polarized Bottom and Top Quarks

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## ABSTRACT

The analytic expressions for the production cross sections of polarized bottom and top quarks in  $e^+e^-$  annihilation are explicitly derived at the one-loop order of strong interactions. Chirality-violating mass effects will reduce the longitudinal spin polarization for the light quark pairs by an amount of 3%, when one properly considers the massless limit for the final quarks. Numerical estimates of longitudinal spin polarization effects in the processes  $e^+e^- \rightarrow b\bar{b}(g)$  and  $e^+e^- \rightarrow t\bar{t}(g)$  are presented.

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Precision tests at  $LEP$  have so far shown great agreement with electroweak theory giving strong experimental support for the Standard Model ( $SM$ ) [1]. However, quantum chromodynamics ( $QCD$ ), though being non-convergent or weakly convergent in the perturbative expansion at low energies, provides an interesting testing ground at the  $Z$  peak to probe many field-theoretical aspects of this asymptotically free theory. In this note, we will study the longitudinal polarization asymmetry  $P_L$  of the bottom quark ( $b$ ) and top quark ( $t$ ) produced through the  $e^+e^-$  annihilation reaction at  $LEP$  and future colliders. A suprising outcome of our calculations is that the  $O(\alpha_s)$  correction of  $P_L$  in the limit  $m_q \rightarrow 0$  differs substancially from the corresponding result of a theory where  $m_q$  was originally set to zero. In the following, we will call such a theory a *naive massless theory*, since it leads to the wrong result for the  $O(\alpha_s)$  contributions to  $P_L$  for the process  $e^+e^- \rightarrow Z, \gamma \rightarrow b\bar{b}(g)$ .

At the one-loop  $QCD$  level, the  $\gamma(q) \rightarrow q(p_1) - \bar{q}(p_2)$  and  $Z(q) \rightarrow q(p_1) - \bar{q}(p_2)$  vertex functions relevant for the production of massive quarks can be written down as follows:

$$\Gamma_\mu^\gamma = -ieQ_q \left[ (1+A)\gamma_\mu + B \frac{(p_2 - p_1)_\mu}{2m_q} \right], \quad (1)$$

$$\begin{aligned} \Gamma_\mu^Z = & -i \frac{e}{4s_w c_w} \left[ (1 - 4s_w^2 Q_q)(1+A)\gamma_\mu + (1 - 4s_w^2 Q_q)B \frac{(p_2 - p_1)_\mu}{2m_q} \right. \\ & \left. - (1+C)\gamma_\mu \gamma_5 - D \frac{(p_1 + p_2)_\mu}{2m_q} \gamma_5 \right], \end{aligned} \quad (2)$$

where  $c_w = M_W/M_Z$ ,  $s_w = \sqrt{1 - c_w^2}$  and the form factors  $A$ ,  $B$ ,  $C$  and  $D$  have been calculated by using dimensional regularization. These form factors are given by

$$A = \frac{\alpha_s}{4\pi} C_F \left[ \left( \frac{1+v^2}{v} \ln \left( \frac{1+v}{1-v} \right) - 2 \right) \left( \frac{2}{\epsilon} - \gamma_E + \ln \left( \frac{4\pi\mu^2}{m_q^2} \right) \right) + F(v) \right], \quad (3)$$

$$B = -\frac{\alpha_s}{4\pi} C_F \frac{1-v^2}{v} \ln \left( \frac{1+v}{1-v} \right), \quad (4)$$

$$\begin{aligned} C = & \frac{\alpha_s}{4\pi} C_F \left[ \left( \frac{1+v^2}{v} \ln \left( \frac{1+v}{1-v} \right) - 2 \right) \left( \frac{2}{\epsilon} - \gamma_E + \ln \frac{4\pi\mu^2}{m_q^2} \right) \right. \\ & \left. + F(v) + 2 \frac{1-v^2}{v} \ln \left( \frac{1+v}{1-v} \right) \right], \end{aligned} \quad (5)$$

$$D = -\frac{\alpha_s}{4\pi} C_F \left[ (2+v^2) \frac{1-v^2}{v} \ln \left( \frac{1+v}{1-v} \right) - 2(1-v^2) \right]. \quad (6)$$

The function  $F(v)$  given in Eqs. (3) and (5) is defined as

$$F(v) = \left[ 3v - \frac{1+v^2}{2v} \ln \frac{4v^2}{1-v^2} \right] \ln \left( \frac{1+v}{1-v} \right) + \frac{1+v^2}{v} \left[ \text{Li}_2 \frac{v+1}{2v} - \text{Li}_2 \frac{v-1}{2v} \right] - 4 + \frac{\pi^2}{2} \frac{1+v^2}{v}, \quad (7)$$

where  $v = \sqrt{1-\xi}$  with  $\xi = 4m_q^2/q^2$ . To remove the  $UV$  divergences in the vertex functions Eqs. (1) and (2), we have considered the wave-function renormalization constant of the final quarks

$$Z_q = -\frac{\partial}{\partial \not{p}} \Sigma_q(\not{p}) \Big|_{\not{p}=m_q} = \frac{\alpha_s}{4\pi} C_F \left[ -\frac{2}{\varepsilon_{UV}} + \gamma_E - \ln \frac{4\pi\mu^2}{m_q^2} - 4 \right] \quad (8)$$

and renormalized the form factors  $A$  and  $C$  according the prescription

$$A = A_0 + \delta Z_q, \quad C = C_0 + \delta Z_q \quad (9)$$

Our final result is also valid when employing dimensional reduction methods for the calculation of  $QCD$  quantum corrections [2]. In addition to the  $UV$  divergences, one encounters in Eqs. (3)–(6) infrared ( $IR$ ) singularities due to the soft-gluon part of the one-loop contributions, which will exactly cancel with those of the real-gluon emission graphs at the same order of strong interaction.

Decomposing the hadronic tensors in terms of their Lorentz structure and considering the one-loop  $QCD$  corrections, one gets

$$H^{VV}(virtual) = -q^2(3-v^2) - 2A q^2(3-v^2) - 2B q^2 v^2, \quad (10)$$

$$H^{AA}(virtual) = -2q^2 - 4C q^2 v^2, \quad (11)$$

$$H_{\pm}^{VA}(virtual) = H_{\pm}^{AV}(virtual) = \mp(1+A+C)q^2 v, \quad (12)$$

where the superscripts refer to the parity-parity combination of the corresponding squared amplitudes. A straightforward computation of the hadronic tensors involving real-gluon emission gives

$$H^{VV}(real) = -4 \frac{(1-y)^2 + (1-z)^2}{yz} + 2\xi \left( \frac{1}{y^2} + \frac{1}{z^2} + \frac{2}{y} + \frac{2}{z} \right) + \xi^2 \left( \frac{1}{y} + \frac{1}{z} \right)^2 \quad (13)$$

$$\begin{aligned}
H^{AA}(real) &= -4 \frac{(1-y)^2 + (1-z)^2}{yz} - 2\xi^2 \left( \frac{1}{y} + \frac{1}{z} \right)^2 \\
&\quad + 2\xi \left( -2 + \frac{1}{y^2} + \frac{1}{z^2} - \frac{4}{y} - \frac{4}{z} + \frac{6}{yz} - \frac{y}{z} - \frac{z}{y} \right), \tag{14}
\end{aligned}$$

$$\begin{aligned}
\pm H_{\pm}^{VA}(real) &= \pm H_{\pm}^{AV}(real) \\
&= -4(1-y) \frac{(1-y)^2 + (1-z)^2}{yz \sqrt{(1-y)^2 - \xi}} + \frac{2\xi^2}{\sqrt{(1-y)^2 - \xi}} \left[ \left( \frac{1}{y} + \frac{1}{z} \right)^2 - \frac{2}{y^2} \right] \\
&\quad + \frac{2\xi}{\sqrt{(1-y)^2 - \xi}} \left( 1 + \frac{1}{y^2} + \frac{1}{z^2} - \frac{5}{y} - \frac{6}{z} + \frac{6}{yz} + \frac{y}{z} + \frac{z}{y} - \frac{y}{z^2} \right. \\
&\quad \left. + \frac{y^2}{z^2} \right), \tag{15}
\end{aligned}$$

where  $y = 1 - 2p_1 \cdot q/q^2$  and  $z = 1 - 2p_2 \cdot q/q^2$  are kinematical variables. Note that gluon-mass effects can safely be neglected in the computation of the squared amplitudes, so that Eqs. (13)–(15) only display a quark-mass dependence through  $\xi$ .

The integration of Eqs. (13)–(15) over the three-body phase-space has been performed analytically. The various types of integrals contained in the hadronic tensors are identified and listed in Appendix A. For compactness, it will be useful to re-express the hadronic tensors in a different basis [3]

$$H^1 = \frac{1}{2} (H^{VV} + H^{AA}), \tag{16}$$

$$H^2 = \frac{1}{2} (H^{VV} - H^{AA}), \tag{17}$$

$$H^3 = \frac{i}{2} (H^{VA} - H^{AV}) = 0, \tag{18}$$

$$H_{\pm}^4 = \frac{1}{2} (H_{\pm}^{VA} + H_{\pm}^{AV}) = H_{\pm}^{VA}. \tag{19}$$

In this basis only  $H^4$  depends on the spin orientation of the final quark. The production cross section of a polarized bottom or top quark can formally be written as

$$d\sigma_{tot} = \sum_i g_i [H^i(virtual) dPS_2 + H^i(real) dPS_3], \tag{20}$$

where the electroweak couplings in the above basis, Eqs. (16)–(19), read

$$g_1 = Q_q^2 - 2Q_q v_e v_q \operatorname{Re} \chi_z + (v_e^2 + a_e^2)(v_q^2 + a_q^2) |\chi_z|^2, \tag{21}$$

$$g_2 = Q_q^2 - 2Q_q v_e v_q \operatorname{Re} \chi_z + (v_e^2 + a_e^2)(v_q^2 - a_q^2) |\chi_z|^2, \tag{22}$$

$$g_3 = -2Q_q v_e a_q \text{Im} \chi_z, \quad (23)$$

$$g_4 = 2Q_q v_e a_q \text{Re} \chi_z - (v_e^2 + a_e^2) 2v_q a_q |\chi_z|^2. \quad (24)$$

In Eqs. (21)–(24)  $Q_q$  denotes as usual the fractional charge of the final-state quarks whereas  $v_f = 2T_z^f - 4Q_f s_W^2$  and  $a_f = 2T_z^f$  are the electroweak vector- and axial-vector coupling constants for fermions ( $f$ ), respectively. The couplings containing  $\text{Im} \chi_z$  and  $\text{Re} \chi_z$  stem from the  $\gamma$ - $Z$  interference. Here,  $\chi_z(s) = g_W M_Z^2 s (s - M_Z^2 + i M_Z \Gamma_Z)^{-1}$  characterizes the Breit-Wigner ( $BW$ ) form of the  $Z$  propagator, where  $M_Z$  and  $\Gamma_Z$  are the mass and the total decay width of the  $Z$  boson. Note that we use a  $BW$  function with a constant decay width as obtained by a Laurent series expansion in terms of the complex pole mass of the  $Z$  boson [4]. This approach ensures the gauge invariance in the vicinity of the  $Z$ -boson resonance [4,5]. Denoting the two helicity states of the final quark  $q$  by  $\lambda_\pm = \pm 1/2$ , one defines the following longitudinal polarization asymmetry:

$$\langle P_L \rangle = \frac{\sigma_{tot}(e^+ e^- \rightarrow q(\lambda_+) \bar{q}(g)) - \sigma_{tot}(e^+ e^- \rightarrow q(\lambda_-) \bar{q}(g))}{\sigma_{tot}(e^+ e^- \rightarrow q \bar{q}(g))}. \quad (25)$$

At this point it is important to comment on the fact that in  $O(\alpha_s)$  the mass-zero limit of the longitudinal polarization does not coincide with the  $O(\alpha_s)$  result of the naive massless theory  $\langle P_L \rangle_{m_q=0}$ . In fact, one finds in the correct limit

$$\langle P_L \rangle_{m_q \rightarrow 0} = \frac{1 + \alpha_s/3\pi}{1 + \alpha_s/\pi} \langle P_L \rangle_{m_q=0} \simeq 0.975 \langle P_L \rangle_{m_q=0}, \quad (26)$$

where it is  $\langle P_L \rangle_{m_q=0} = -93.9\%$  for down-type quarks (i.e.  $d, s, b$ ) and  $\langle P_L \rangle_{m_q=0} = -68.5\%$  for the up and charm quark at  $q^2 = M_Z^2$ . This surprising effect arises from the fact that there are chirality-violating or helicity-flip contributions proportional to  $m_q^2$  which are multiplied with a would-be singularity proportional to  $1/m_q^2$ . Thus one has to include a finite spin-flip contribution to  $\langle P_L \rangle$  in the limit  $m_q \rightarrow 0$ , which evidently cannot be seen in the naive massless theory. Similar observations have been reported by the authors of Ref. [6]. In their analysis, this chirality-violating effect originating solely from quantum electromagnetic corrections was found to be small. Here, this chirality-violating mass effect is of the order of 3% (i.e.  $2\alpha_s/3\pi$ ) and can, in principle, be observed at  $LEP$  by analyzing  $\langle P_L \rangle$  via the decay products of the final quarks.

In Figs. 1 and 2 we have plotted the numerical values of  $\langle P_L \rangle$  versus the c.m.s. energy of the  $b$ - and  $t$ -pair production, respectively. The solid line is the Born approximation whereas the dashed line refers to the  $O(\alpha_s)$  corrections. In particular, for the case of  $b$ -quark production the solid line corresponds approximately to the line obtained in the naive massless theory, while the dashed one represents the analytic result in the limit  $m_b \rightarrow 0$ . A similar effect occurs in the top production which is smaller of the order of 0.5%. For practical purposes, we present the corresponding production cross sections for the  $b$  and  $t$  quark, which have found to be in excellent agreement with Ref. [7–9]. For our numerical estimates we have used running masses and the running coupling constant  $\alpha_s$  with  $\Lambda_{\overline{MS}} = 0.238$  GeV [10].

In conclusion, we have analytically calculated the longitudinal polarization asymmetry of the final  $b$  and  $t$  quark for the processes  $e^+e^- \rightarrow b\bar{b}(g)$  and  $e^+e^- \rightarrow t\bar{t}(g)$ . Especially, we have found that chirality-violating mass terms reduce the spin-polarization asymmetry of the naive massless theory by 3% which is, in principle, quite sizable to be detected in the *LEP* data. Since the top quark will predominantly decay via electroweak interactions, spin polarization asymmetry tests by analyzing the angular distribution of the charged leptons arising from semileptonic top decays will furnish very attractive and feasible experiments at future TeV  $e^+e^-$  colliders.

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# A Three-Body Phase Space

## I. Spin-Independent Phase-Space Integrals

The three body phase-space boundaries for the processes at hand are given by

$$y_- = \Lambda^{\frac{1}{2}}\sqrt{\xi} + \Lambda, \quad y_+ = 1 - \sqrt{\xi} \quad (\text{A1})$$

$$z_{\pm} = \frac{2y}{\xi + 4y} \left( 1 - y - \frac{1}{2}\xi + \Lambda + \frac{\Lambda}{y} \pm \frac{1}{y}\sqrt{(1-y)^2 - \xi}\sqrt{(y-\Lambda)^2 - \Lambda\xi} \right) \quad (\text{A2})$$

In Eqs. (A1) and (A2) it is  $\Lambda = \lambda^2/q^2$ , where  $\lambda$  denotes the gluon mass that has been introduced to regulate the soft *IR* singularities. After identifying the different types of integrals in Eqs. (13) and (14) and integrating over the above phase-space boundaries, one has to the leading order of  $\Lambda$  [7]

$$\begin{aligned} I_1 &= \int dy dz \\ &= \frac{1}{2}v \left( 1 + \frac{1}{2}\xi \right) - \frac{1}{2}\xi \left( 1 - \frac{1}{4}\xi \right) \ln \left( \frac{1+v}{1-v} \right), \end{aligned} \quad (\text{A3})$$

$$\begin{aligned} I_2 &= \int dy dz \frac{1}{y} = \int dy dz \frac{1}{z} \\ &= -v + \left( 1 - \frac{1}{2}\xi \right) \ln \left( \frac{1+v}{1-v} \right), \end{aligned} \quad (\text{A4})$$

$$\begin{aligned} I_3 &= \int dy dz \frac{1}{y^2} = \int dy dz \frac{1}{z^2} \\ &= -\frac{4v}{\xi} \left( \ln \Lambda^{\frac{1}{2}} + \ln \xi - 2 \ln v - 2 \ln 2 + 1 \right) + 2 \left( 1 - \frac{3}{\xi} \right) \ln \left( \frac{1+v}{1-v} \right), \end{aligned} \quad (\text{A5})$$

$$\begin{aligned} I_4 &= \int dy dz \frac{y}{z} = \int dy dz \frac{z}{y} \\ &= -\frac{1}{4}v \left( 5 - \frac{1}{2}\xi \right) + \frac{1}{2} \left( 1 + \frac{1}{8}\xi^2 \right) \ln \left( \frac{1+v}{1-v} \right), \end{aligned} \quad (\text{A6})$$

$$\begin{aligned} I_5 &= \int dy dz \frac{1}{yz} \\ &= \left( -2 \ln \Lambda^{\frac{1}{2}} - \ln \xi + 4 \ln v + 2 \ln 2 \right) \ln \left( \frac{1+v}{1-v} \right) \\ &\quad + 2 \left[ \text{Li}_2 \left( \frac{1+v}{2} \right) - \text{Li}_2 \left( \frac{1-v}{2} \right) \right] + 3 \left[ \text{Li}_2 \left( -\frac{2v}{1-v} \right) - \text{Li}_2 \left( \frac{2v}{1+v} \right) \right], \end{aligned} \quad (\text{A7})$$

$$\begin{aligned} I_6 &= \int dy dz \frac{y}{z^2} = \int dy dz \frac{z}{y^2} \\ &= \frac{2}{\xi} v + \ln \left( \frac{1-v}{1+v} \right). \end{aligned} \quad (\text{A8})$$

Note that the expressions  $I_1 - I_5$  agree with the ones given in [7] whereas we get a different result for  $I_6$ .

## II. Spin-Dependent Phase-Space Integrals

To the best of our knowledge, the following analytic expressions for the basic set of phase-space integrals involving spin-dependent integrand functions have not been presented in the literature before. Due to the presence of the additional square root in the integrand and the rather involved phase-space boundaries a number of sophisticated manipulations had to be applied to arrive at analytic expressions. After straightforward but tedious algebra and neglecting all terms that vanish in the limit  $\Lambda \rightarrow 0$ , we obtain the following expressions:

$$\begin{aligned} S_1 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \\ &= 1 - \sqrt{\xi} - \frac{1}{2}\xi \ln \left( \frac{2 - \sqrt{\xi}}{\sqrt{\xi}} \right), \end{aligned} \quad (\text{A9})$$

$$\begin{aligned} S_2 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{1}{y} \\ &= 2 \ln \left( \frac{2 - \sqrt{\xi}}{\sqrt{\xi}} \right), \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} S_3 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{1}{y^2} \\ &= \frac{4}{\xi} \left[ -\ln \Lambda^{\frac{1}{2}} + \frac{1}{2} \ln \xi + \ln(1 - \sqrt{\xi}) - 2 \ln(2 - \sqrt{\xi}) + \ln 2 - 1 \right], \end{aligned} \quad (\text{A11})$$

$$\begin{aligned} S_4 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{1}{z} \\ &= \text{Li}_2 \left( \frac{1+v}{2} \right) + \text{Li}_2 \left( \frac{1-v}{2} \right) + 2 \text{Li}_2 \left( -\frac{\sqrt{\xi}}{2 - \sqrt{\xi}} \right) + \frac{1}{4} \ln^2 \xi \\ &\quad + \ln^2 \left( \frac{2 - \sqrt{\xi}}{2} \right) - \ln(1+v) \ln(1-v), \end{aligned} \quad (\text{A12})$$

$$\begin{aligned}
S_5 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{1}{z^2} \\
&= \frac{4}{\xi} \left[ -\ln \Lambda^{\frac{1}{2}} - \frac{1}{2} \ln \xi + \ln(1 - \sqrt{\xi}) - \frac{1+v^2}{2v} \ln \left( \frac{1+v}{1-v} \right) + \ln 2 \right], \quad (\text{A13})
\end{aligned}$$

$$\begin{aligned}
S_6 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{y}{z^2} \\
&= \frac{4}{\xi} (1 - \sqrt{\xi}), \quad (\text{A14})
\end{aligned}$$

$$\begin{aligned}
S_7 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{y^2}{z^2} \\
&= \frac{2}{\xi} (1 - \sqrt{\xi})^2, \quad (\text{A15})
\end{aligned}$$

$$\begin{aligned}
S_8 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} z \\
&= \frac{1}{32} \left[ 12 - (2 + \xi)^2 - \frac{2 + \sqrt{\xi}}{2 - \sqrt{\xi}} \xi^2 + 2(8 - \xi) \xi \ln \frac{\sqrt{\xi}}{2 - \sqrt{\xi}} \right], \quad (\text{A16})
\end{aligned}$$

$$\begin{aligned}
S_9 &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{z}{y} \\
&= -\frac{1}{2} \ln \xi + \ln(2 - \sqrt{\xi}) + \frac{2}{2 - \sqrt{\xi}} - 2, \quad (\text{A17})
\end{aligned}$$

$$\begin{aligned}
S_{10} &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{y}{z} \\
&= \text{Li}_2 \left( \frac{1+v}{2} \right) + \text{Li}_2 \left( \frac{1-v}{2} \right) - 2 \text{Li}_2 \left( \frac{\sqrt{\xi}}{2} \right) + \frac{1}{4} \ln^2 \left( \frac{1}{4} \xi \right) \\
&\quad + (2 - \frac{1}{2} \xi) \ln \left( \frac{2 - \sqrt{\xi}}{\sqrt{\xi}} \right) - \sqrt{\xi} + 2v \ln \left( \frac{1-v}{1+v} \right) \\
&\quad - \ln \left( \frac{1+v}{2} \right) \ln \left( \frac{1-v}{2} \right) + 1, \quad (\text{A18})
\end{aligned}$$

$$\begin{aligned}
S_{11} &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{y^2}{z} \\
&= (1 + \frac{1}{2} \xi) \left[ \text{Li}_2 \left( \frac{1+v}{2} \right) + \text{Li}_2 \left( \frac{1-v}{2} \right) - 2 \text{Li}_2 \left( \frac{1}{2} \sqrt{\xi} \right) + \frac{1}{4} \ln^2 \left( \frac{1}{4} \xi \right) \right. \\
&\quad \left. - \ln \left( \frac{1+v}{2} \right) \ln \left( \frac{1-v}{2} \right) \right] + 3v \ln \left( \frac{1-v}{1+v} \right) + \frac{1}{8} (18 + \xi) - \frac{1}{8} (20 - \xi) \sqrt{\xi} \\
&\quad + (3 - \xi + \frac{1}{16} \xi^2) \ln \left( \frac{2 - \sqrt{\xi}}{\sqrt{\xi}} \right), \quad (\text{A19})
\end{aligned}$$

$$\begin{aligned}
S_{12} &= \int \frac{dy dz}{\sqrt{(1-y)^2 - \xi}} \frac{1}{yz} \\
&= \frac{1}{v} \ln \left( \frac{1-v}{1+v} \right) \left[ 2 \ln \Lambda^{\frac{1}{2}} + \frac{1}{2} \ln \xi + 4 \ln(2 - \sqrt{\xi}) - 4 \ln v - 4 \ln 2 - 2 \ln \left( \frac{1-v}{1+v} \right) \right] \\
&\quad + \frac{1}{v} \ln^2 \left( \frac{(1-v)^2}{\sqrt{\xi}(2 - \sqrt{\xi})} \right) + \frac{2}{v} \ln \left( \frac{\sqrt{\xi}(2 - \sqrt{\xi})}{2} \right) \ln \left( \frac{2\sqrt{\xi}(1 - \sqrt{\xi})}{(1 - \sqrt{\xi} - v)^2} \right) \\
&\quad + \frac{2}{v} \left[ \text{Li}_2 \left( \frac{\sqrt{\xi}(2 - \sqrt{\xi})}{(1+v)^2} \right) - \text{Li}_2 \left( \left( \frac{1-v}{1+v} \right)^2 \right) + \text{Li}_2 \left( \frac{(1-v)^2}{\sqrt{\xi}(2 - \sqrt{\xi})} \right) \right] \\
&\quad + \frac{1}{v} \left[ \text{Li}_2 \left( \frac{1+v}{2} \right) - \text{Li}_2 \left( \frac{1-v}{2} \right) + \text{Li}_2 \left( -\frac{2v}{1-v} \right) - \text{Li}_2 \left( \frac{2v}{1+v} \right) - \frac{\pi^2}{3} \right]. \quad (\text{A20})
\end{aligned}$$

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## Figure Captions

- Fig. 1:** Longitudinal polarization asymmetry for bottom quark as a function of the c.m.s. energy. The solid line corresponds to the Born approximation (or the  $O(\alpha_s)$  result of the naive massless theory) while the dashed one is calculated at the one-loop  $QCD$  order. We have used  $\alpha_s(M_Z^2) = 0.12$ .
- Fig. 2:** Longitudinal polarization asymmetries for three different masses of top quark as a function of the c.m.s. energy. The dashed and dotted lines are indicated similar to Fig. 1.
- Fig. 3:** (a). Production cross section for the bottom quark as a function of the c.m.s. energy. (b). Production cross section of the  $b$  quark in the vicinity of the  $Z$ -boson mass, where the solid line represents the born approximation, the dotted one denotes the one-loop  $QCD$  result for  $m_b \rightarrow 0$  and the dashed line gives the massive one-loop  $O(\alpha_s)$  corrections.
- Fig. 4:** Production cross sections for three different masses of top quark.

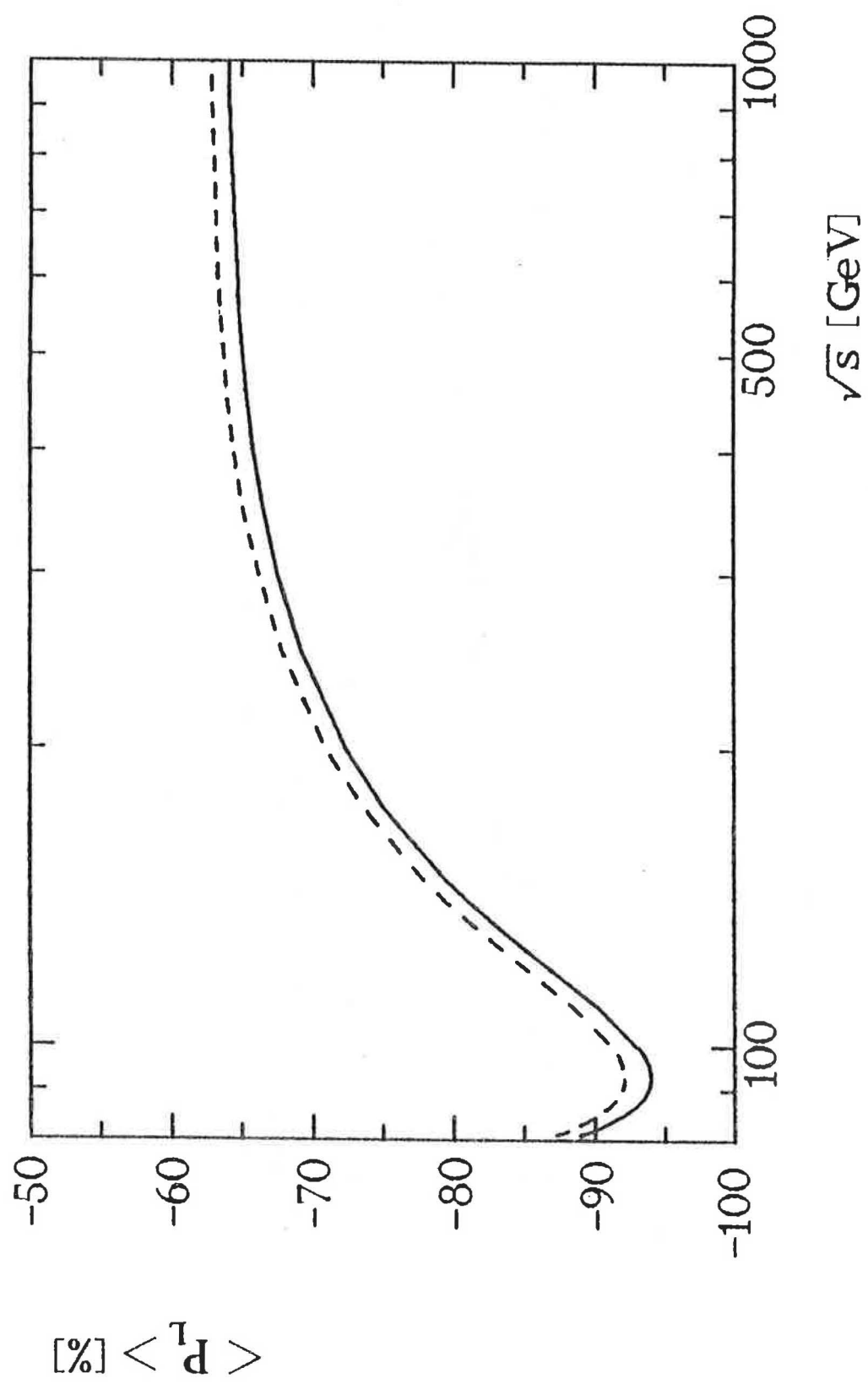


Fig. 1

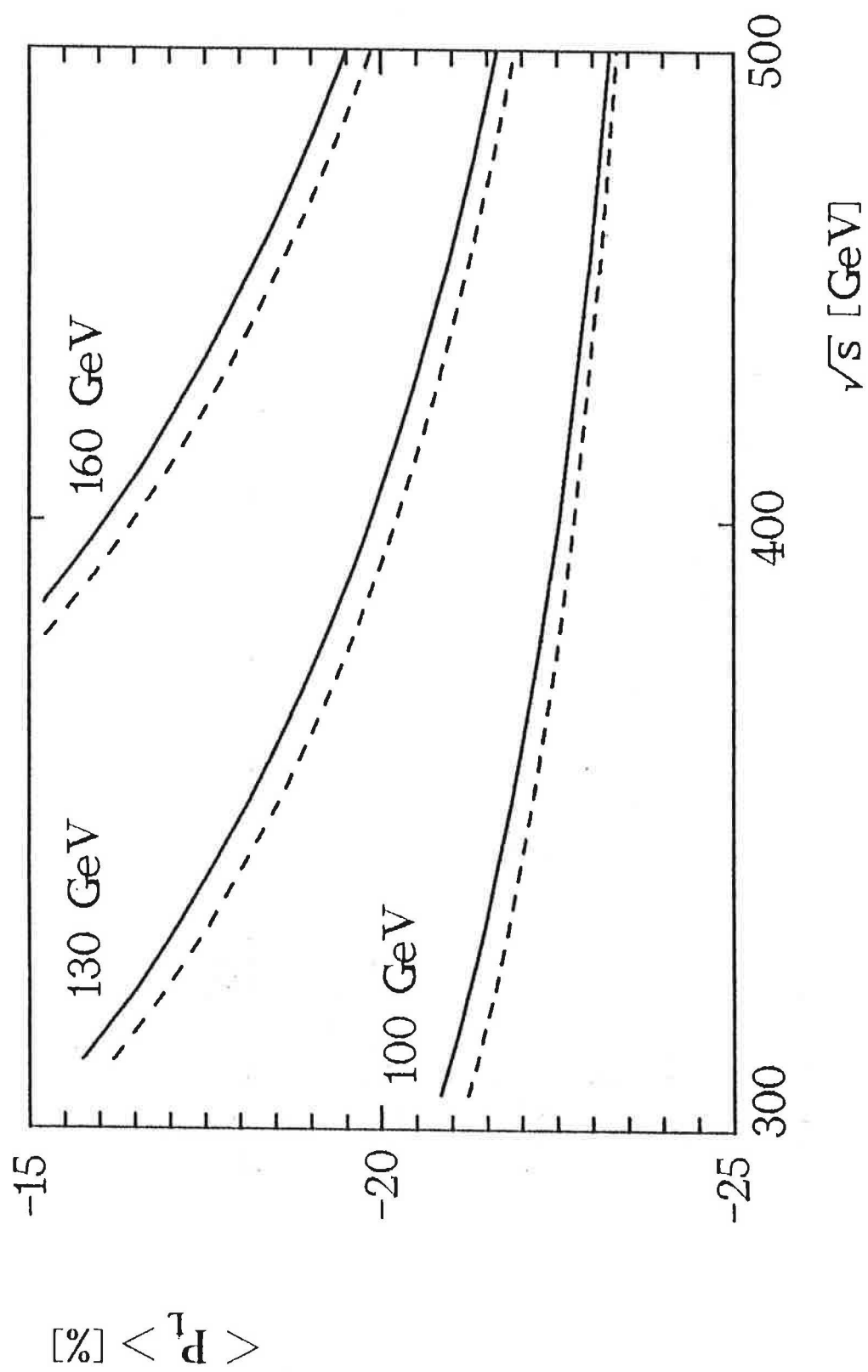


Fig. 2



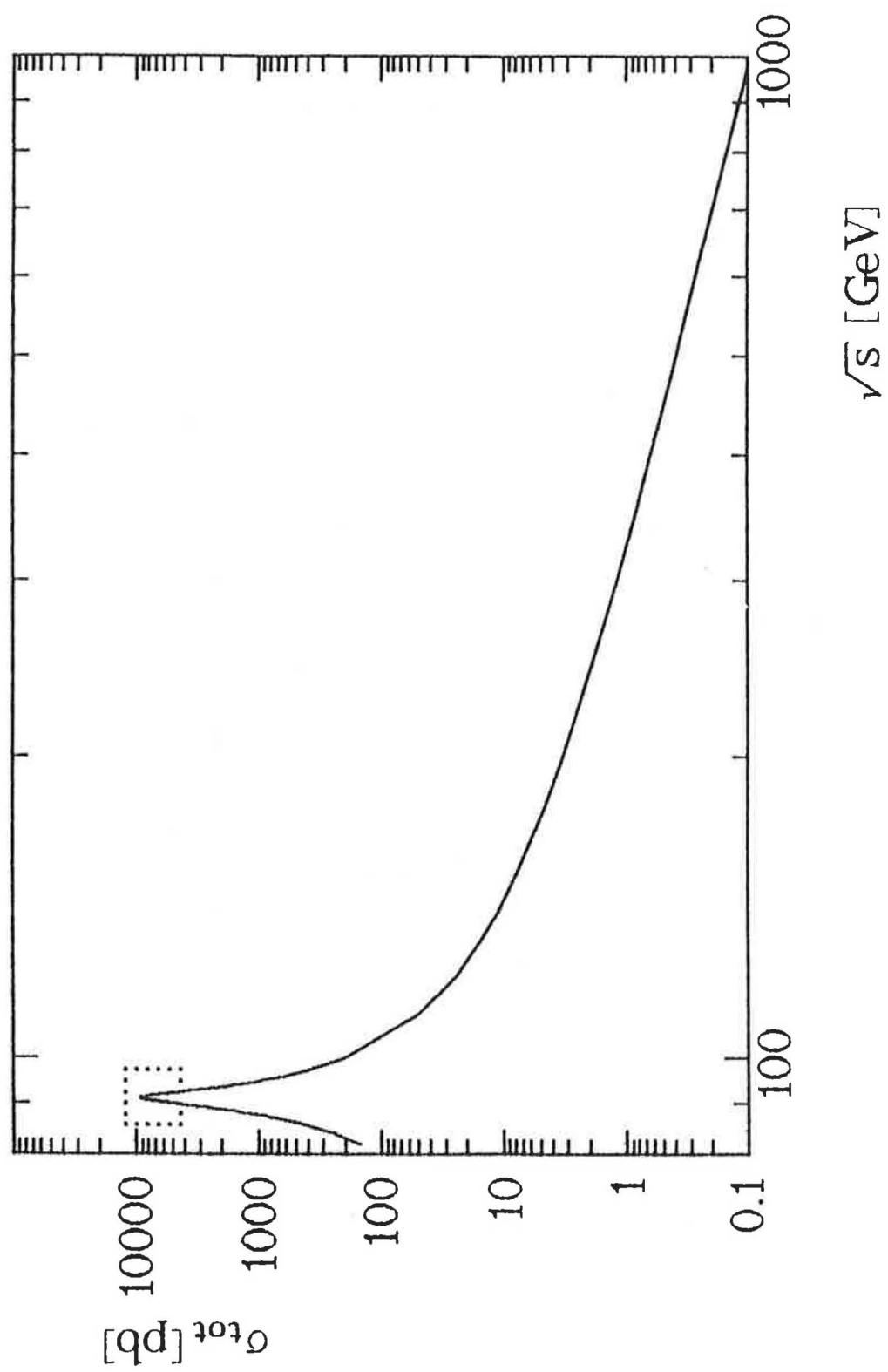


Fig. 3 (a)

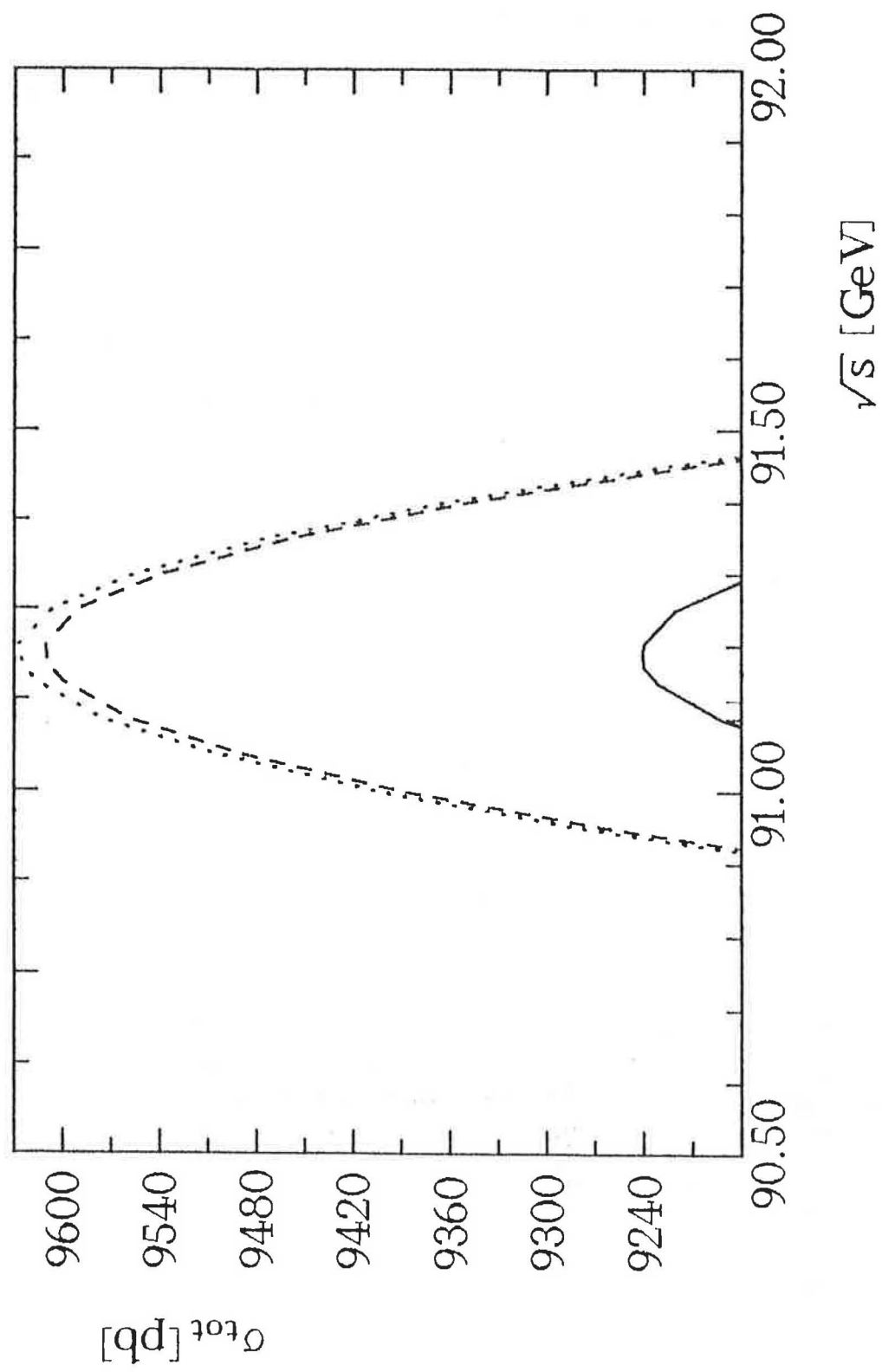


Fig. 3(b)

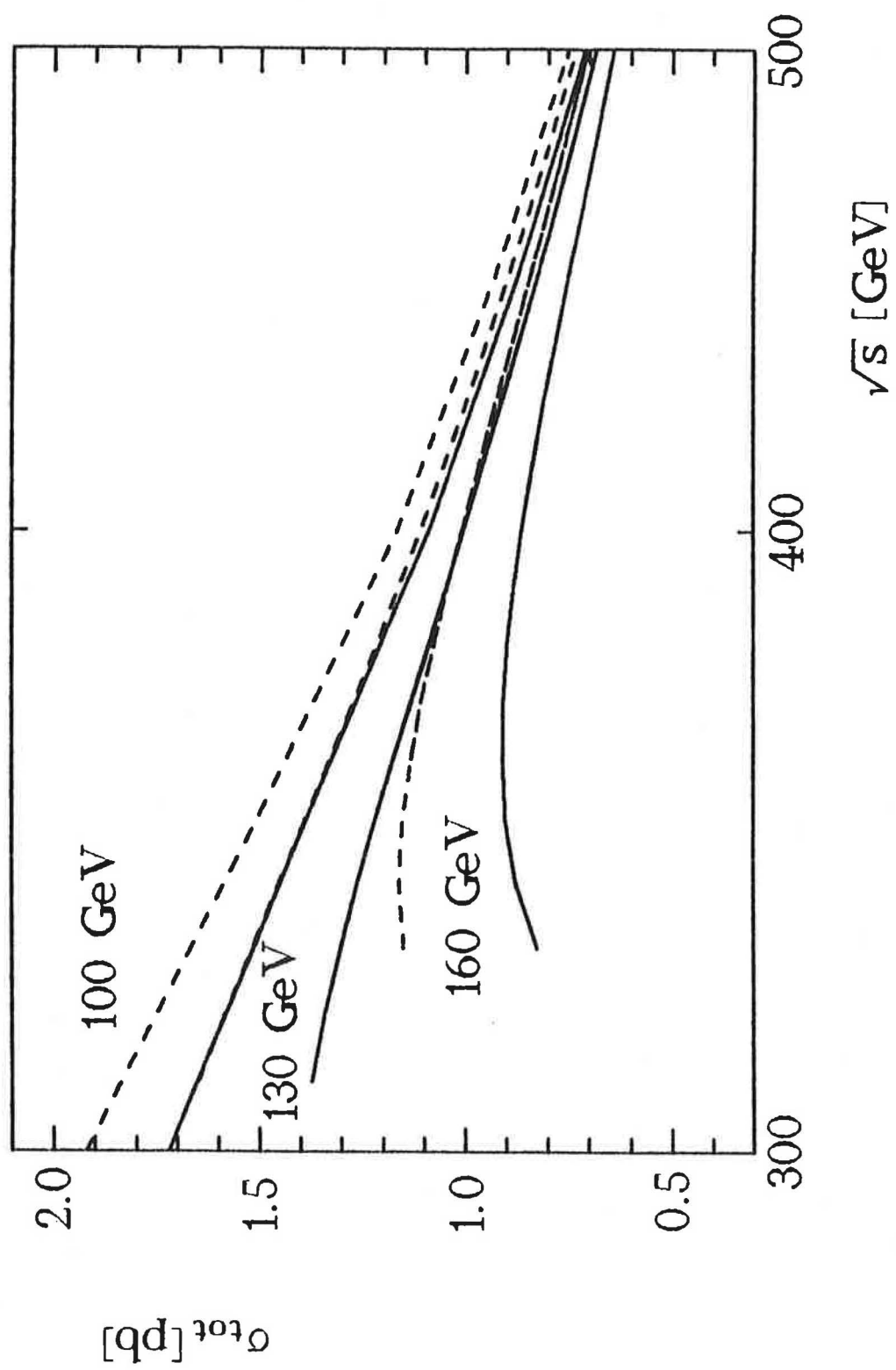


Fig. 4





