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An Approximate Next-to-Leading Order Relation Between the Low-x F_2 Scaling Violations and the Gluon Density

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An approximate next-to-leading order relation between the low-x F_2 scaling violations and the gluon density.

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Abstract. Due to the large α_s^2 corrections to the F_2 scaling violations in the kinematical region of HERA, the previously determined approximate leading order relation between the F_2 scaling violations and the gluon distribution at low-x need to be corrected. A new relation is presented in next-to-leading order and found to give reasonable agreement with the exact calculation.

On the basis of the DGLAP equations it is known that the dominant source for the F_2 scaling violations at low-x ($x < 10^{-2}$) is the conversion of gluons into quark-antiquark pairs. This fact makes it possible to relate the gluon density directly to the Q^2-dependence of the structure function F_2 as has been suggested previously [1], i.e.

$$\frac{dF_2}{dlnQ^2}(x) = 2\sum_f e_i^2 \frac{\alpha_s}{4\pi} \int_x^1 G(x/z) K_g^{(1)}(z) dz \approx 2\sum_f e_i^2 \frac{\alpha_s}{4\pi}\frac{2}{3} G(2x) \tag{1}$$

where e_i is the quark electric charge, $K_g^{(1)}$ is the lowest order gluon to quark splitting function and G is the gluon momentum distribution. The approximation of the integral in the final step of (1) was found to be better than 10%, depending slightly on x. However, the first step in (1) already ignores the contribution from quarks and from α_s^2 corrections and the validity of this approximation has been discussed before [2, 3, 4]. We therefore make an estimate of these contributions, utilizing the MRS D_0 and D_- parton distributions [5]. At low-x, these distributions differ markedly so an indicative of the real contribution can be obtained.

The DGLAP prediction for the F_2 scaling violations is:

$$\frac{dF_2}{dlnQ^2}(x) = \int_x^1 F_2(x/z) K_q(z) dz + \sum_{f+\bar{f}} e_i^2 \int_x^1 G(x/z) K_g(z) dz \tag{2}$$

where the sum runs over flavours and antiflavours and e_i is the electric charge of quark with flavour i.

The splitting functions, K_q and K_g, are expanded to next-to-leading order (NLO)

$$K_g = \frac{\alpha_s}{4\pi} K_g^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 K_g^{(2)} \tag{3}$$

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and equivalently for K_q. The formal expressions for these functions are fully known to NLO. Here we use the formulae derived by Floratos et al. [6] which agree with the independent calculation by Furmanski and Petronzio [7]. The renormalisation scheme is $\overline{MS}$ with $n_f = 4$ and $\Lambda^{(4)}_{\overline{MS}}$ was set to 215 MeV as obtained by MRS.

The relative importance of the F_2 term in (2) to the full equation is shown in fig. 1 in NLO at $Q^2 = 20\ GeV^2$ for $x < 10^{-2}$. The contribution from the F_2 term to $dF_2/dlnQ^2$ is around $5-8\%$ in this kinematical region and can be neglected in a first approximation. On the other hand, as shown in fig. 2, the α_s^2 correction (F_2 term neglected) to the leading order result is large, up to 35% in the kinematical region of HERA for $Q^2 = 20\ GeV^2$. In view of the accurate data which will soon be available, α_s^2 corrections must be taken into account and we do this as follows.

The second order contribution is

$$dF_2^{(2)}/dlnQ^2 = 2\sum_f e_i^2 (\frac{\alpha_s}{4\pi})^2 \int_x^1 G(x/z) K_g^{(2)}(z) dz \tag{4}$$

The integrand of (4) is shown in fig. 3 for $Q^2 = 20\ GeV^2$ and $x = 10^{-3}$. The main contribution to (4) comes from $z < 0.1$ and $z > 0.9$, independent of gluon distribution. For the region $z < 0.1$, the gluon distribution derived from existing data can be used since at HERA $x > 5 \cdot 10^{-4}$ so the argument of G, x/z, is greater than $5 \cdot 10^{-3}$, close to the minimum value of existing data. Indeed, the different parametrisations obtained from a QCD fit to data agree fairly well in this region.

For the region $z > 0.9$, $1.0x < x/z < 1.1x$. Here we fix the gluon distribution at $G(2x)$ and keep it outside the integral. Strictly, it would have been more appropriate to use $G(x)$ instead but when merging the result with the approximate leading order expression (1) it is convenient to use $G(2x)$. This choice will give a wrong value with maximum 30% in case of D_-. But this is a correction to the correction and the resulting error is small, however included in the estimate of the uncertainty below. We can then extend this region down to $z = 0.5$ or so to include an additional small contribution.

The region $0.1 < z < 0.5$ must be excluded but is expected to give a small contribution since the positive and negative parts of the integrand tend to cancel out each other, as is seen in fig. 3.

We can then write

$$\begin{aligned} \frac{dF_2/dlnQ^2(x)}{2\sum_f e_i^2} &\approx \frac{\alpha_s}{4\pi}\int_x^1 G(x/z)K_g^{(1)}(z)dz \\ &+ (\frac{\alpha_s}{4\pi})^2\left[\int_x^{0.1} G^{exp}(x/z)K_g^{(2)}(z)dz + G(2x)\int_{0.5}^1 K_g^{(2)}(z)dz\right] \end{aligned} \tag{5}$$

where G^{exp} is the gluon distribution found from the complete QCD analysis of existing data. The accuracy of this formula relative to the exact NLO calculation (F_2 term excluded) is shown in fig. 4, using D_- gluon distribution for G^{exp}. It is seen that the approximate form of the α_s^2 contribution is good to the level of 5%.

Replacing the first integral in (5) according to (1), introducing the function $N(x, Q^2)$ for the second integral and evaluating the third integral we obtain for four flavours

$$dF_2/dlnQ^2(x) \approx G(2x)\frac{20}{9}\frac{\alpha_s}{4\pi}\left[\frac{2}{3} + \frac{\alpha_s}{4\pi}\cdot 3.58\right] + (\frac{\alpha_s}{4\pi})^2\frac{20}{9}N(x, Q^2) \tag{6}$$

where $N(x, Q^2)$ is given in the appendix. As seen in fig. 5 the different gluon distributions give just a small uncertainty for $N(x, Q^2)$.

Finally, a comparison between (6) and the complete NLO calculation (F_2 term included) is shown in fig. 6 and in fig. 7. When calculating the function $N(x, Q^2)$, D_- was used in all cases. The worse accuracy at high Q^2 for both D_0 and D_- is due to the increasing importance of the F_2 term with increasing Q^2.
The resulting accuracy includes effects of neglecting the F_2 term as well as the numerical errors obtained in approximating both the α_s and the α_s^2 terms.

In summary, α_s^2 corrections to the Q^2 dependence of F_2 are large in the kinematical region of HERA. An approximate NLO formula relating $dF_2/dlnQ^2$ to the gluon distribution was derived and found to give satisfactory agreement with the exact NLO calculation. The simplicity of the LO formula (1) is in principle not lost, retaining the experimental attraction.
In using the relation (6) one must also include the error on α_s and the uncertainty coming from so called low-x effects which involves gluon recombination and transverse momentum of the partons, neglected in the DGLAP equations.

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Appendix: $N(x, Q^2)$

The complete expression for the function $N(x, Q^2)$ is

$$N(x, Q^2) = \int_x^{0.1} G^{exp}(x/z, Q^2) P_g^{(2)}(z) dz \tag{7}$$

where

$$\begin{aligned} P_g^{(2)}(z) &= 1/3(52/z - 157 - 304z + 423z^2 + 2(2 - 208z + 201z^2)ln(z) \\ &- (41 + 134z + 2z^2)ln^2(z) + 2(12/z - 13 + 156z - 177z^2)ln(1-z) \\ &- 4(13 + 28z + 8z^2)(Li_2(z) - \pi^2/6) - 2(1 - 2z + 2z^2)(26ln(z)ln(1-z) \\ &- 10ln^2(1-z) + 17\pi^2/6 + ln^2(1-z)) + 18(1 + 2z + 2z^2)F(z) \end{aligned} \tag{8}$$

where

$$Li_2(z) = -\int_0^z \frac{ln|1-y|}{y} dy \tag{9}$$

and

$$F(z) = \frac{ln^2(z)}{2} - 2Li_2(-z) - 2ln(z)ln(1+z) - \pi^2/6 \tag{10}$$

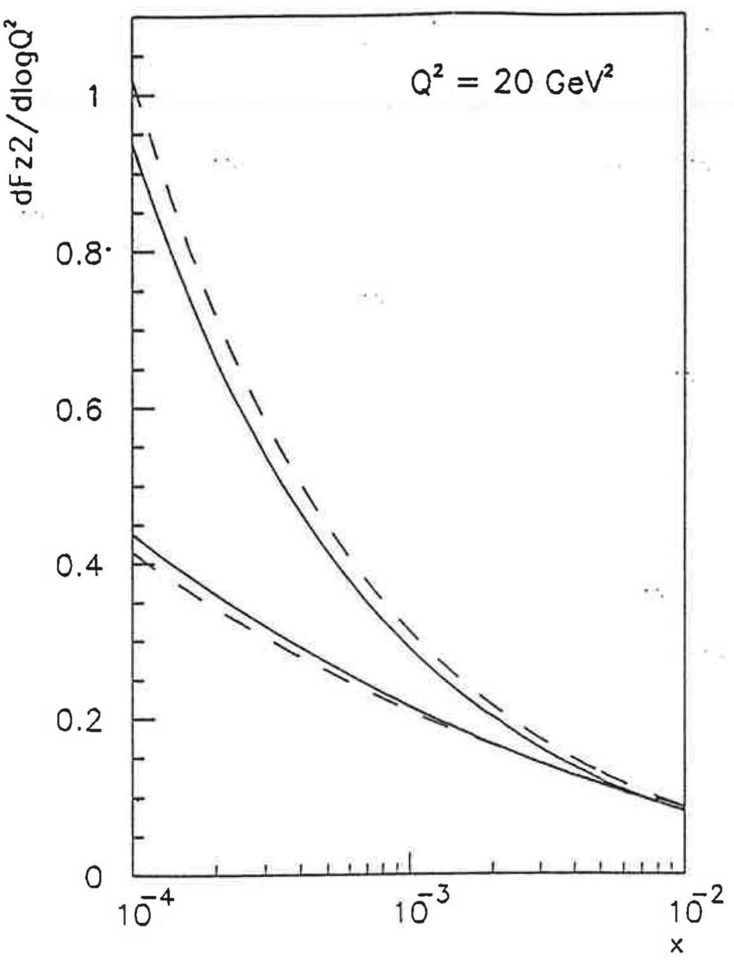

Figure 1: *Effect of quarks.*
a) Solid line is the exact NLO DGLAP prediction for $dF_2/dlnQ^2$. The dashed line was obtained neglecting the F_2 term. Upper (lower) lines correspond to D_- (D_0).

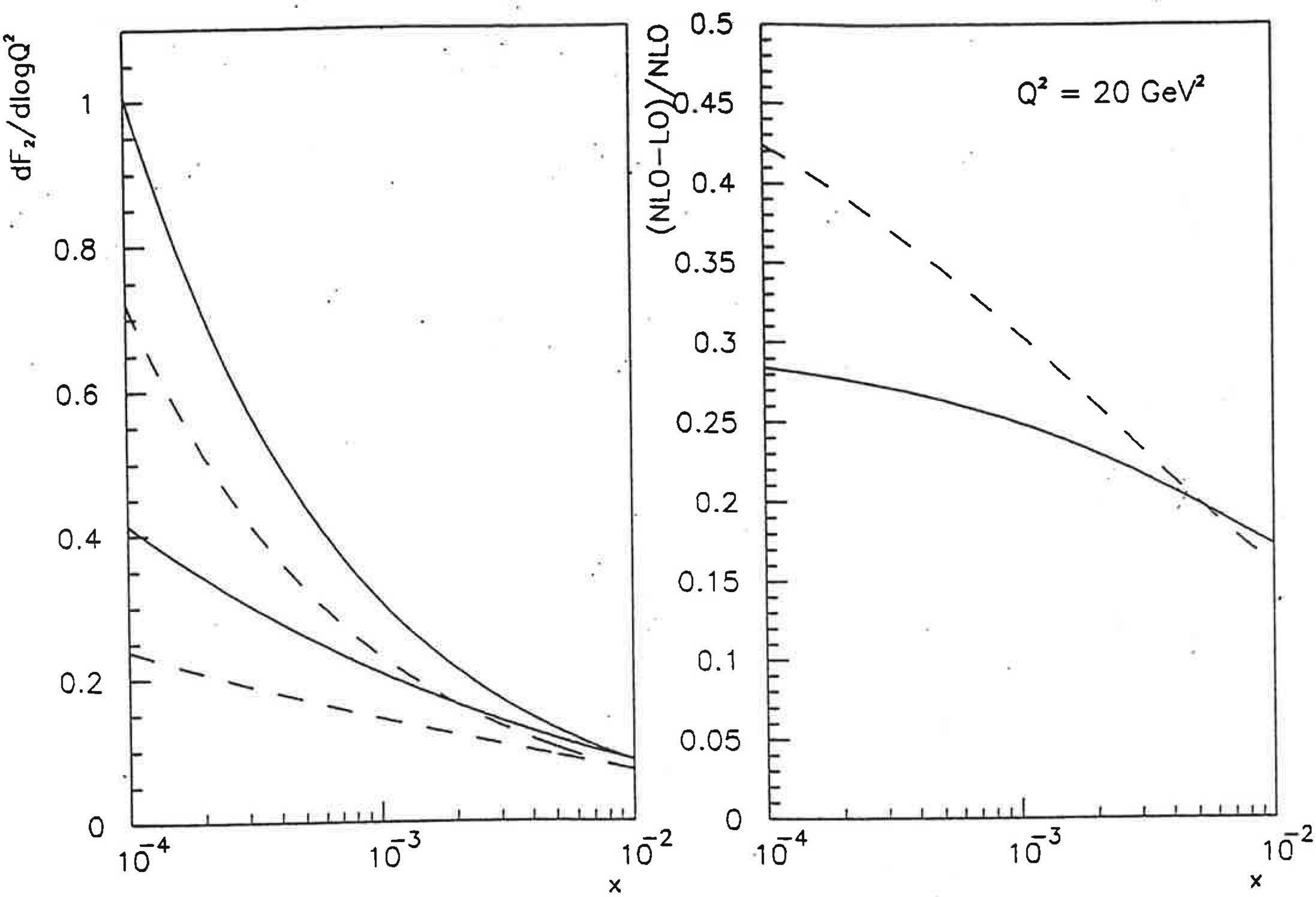

Figure 2: *Effect of α_s^2 correction.*
a) Solid (dashed) line is the NLO (LO) prediction for $dF_2/dlnQ^2$ (F_2 term neglected). The two upper (lower) lines are for D_- (D_0) gluon distribution. b) Fractional difference of the above. Dashed (solid) line corresponds to D_0 (D_-).

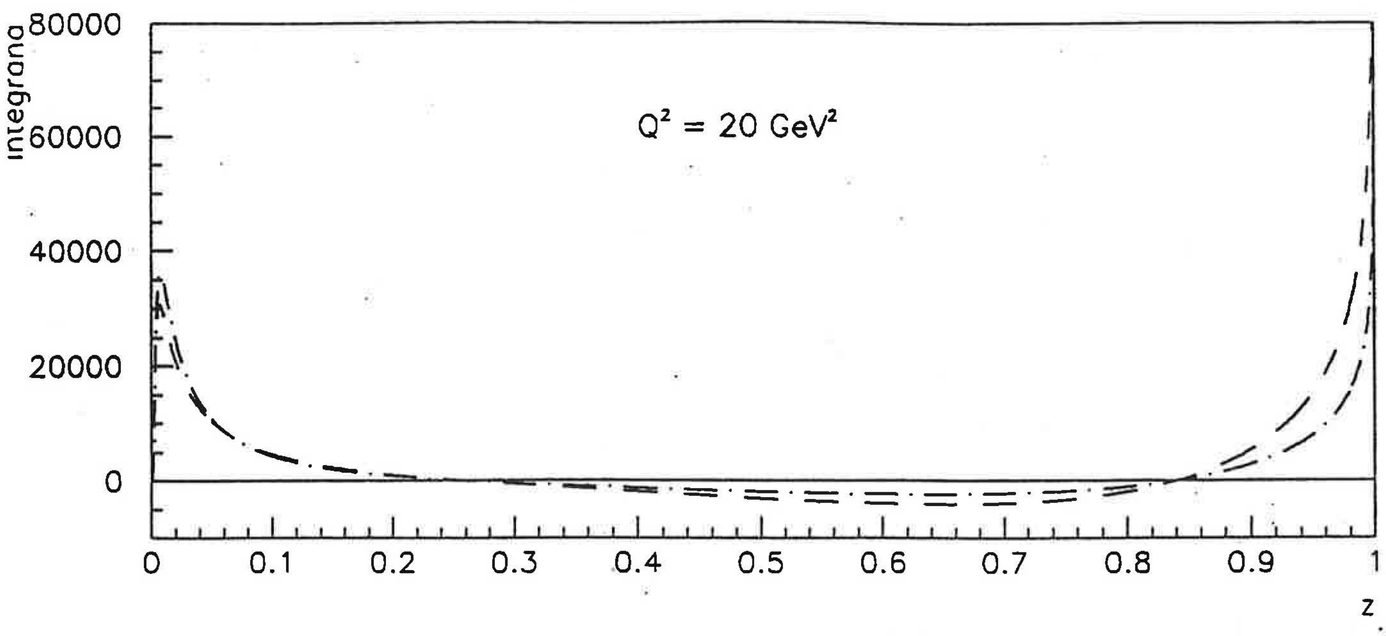

Figure 3: *Integrand $K_g^{(2)}(z)G(x/z)$ vs z for D_- (dashed line) and D_0 (dashed-dotted line) input gluon distribution for $x = 10^{-3}$.*

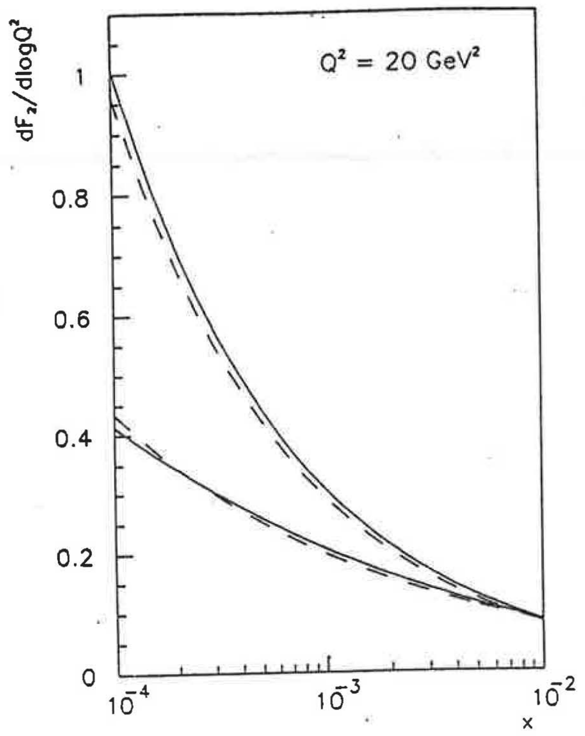

Figure 4: ***Effect of approximate α_s^2 correction.***
The NLO prediction (F_2 term neglected) is shown with a solid line and the dashed line is the sum of the exact LO and the approximate higher order correction, i.e. eq. (5). The D_0 (lower lines) and the D_- (upper lines) were used as input. For G^{exp} in (5) D_- was used in both cases.

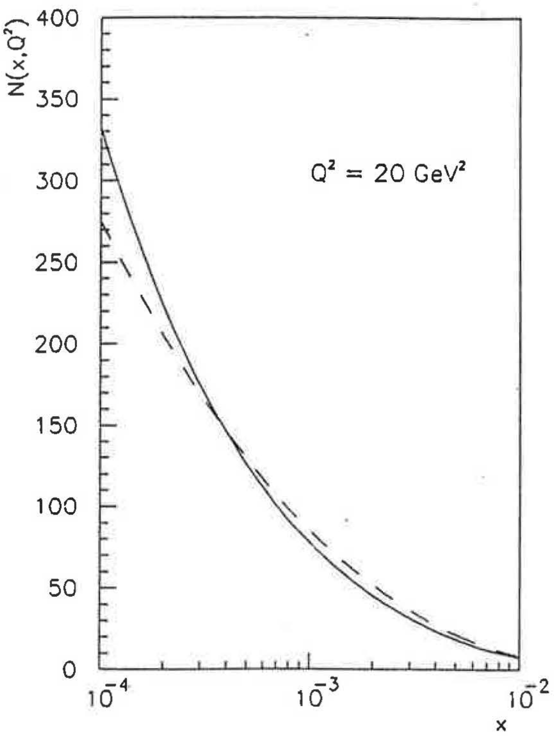

Figure 5: *The function $N(x, Q^2)$ at $Q^2 = 20$ GeV^2 for D_- (solid line) and D_0 (dashed line).*

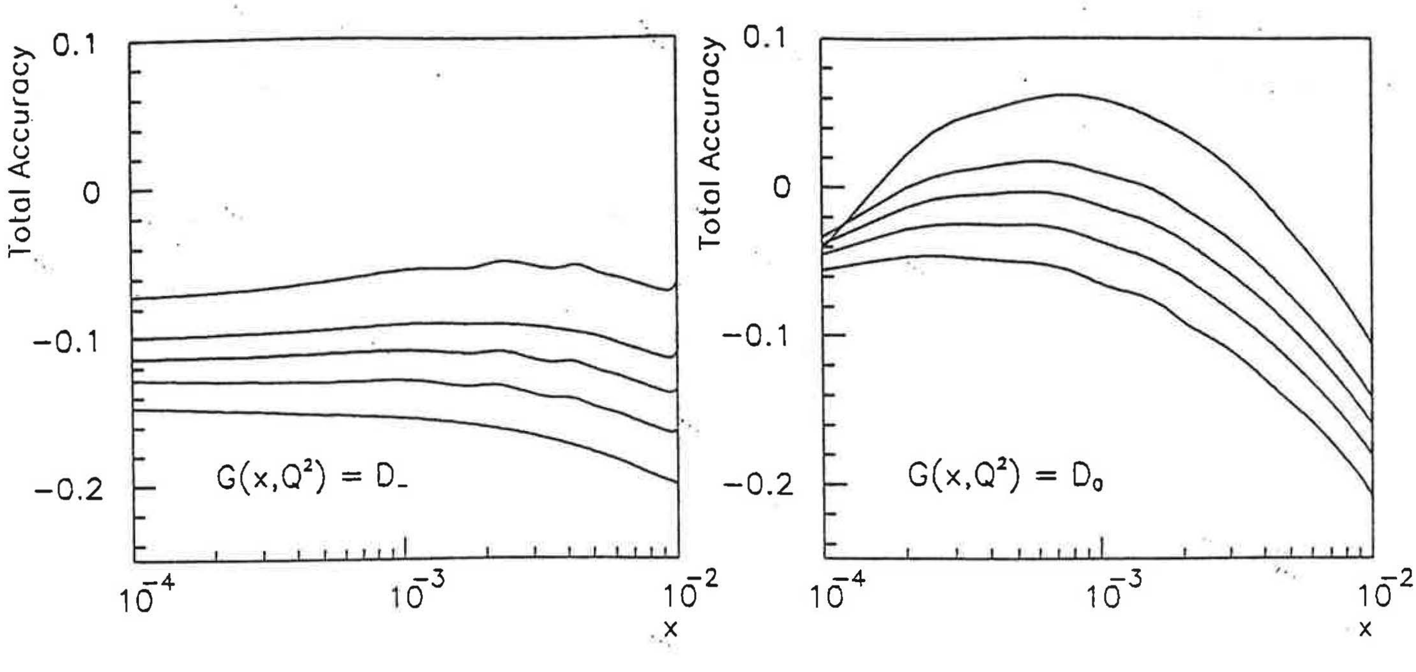

Figure 6: *The total accuracy of (6) (Q^2 dependence).*
Plotted is the fractional difference between the exact NLO result and the approximate formula (6) for $Q^2 = 10, 20, 30, 50$ *and* 100 GeV^2 *(top to bottom).* $N(x, Q^2)$ *was calculated using the* D_- *gluon distribution.*

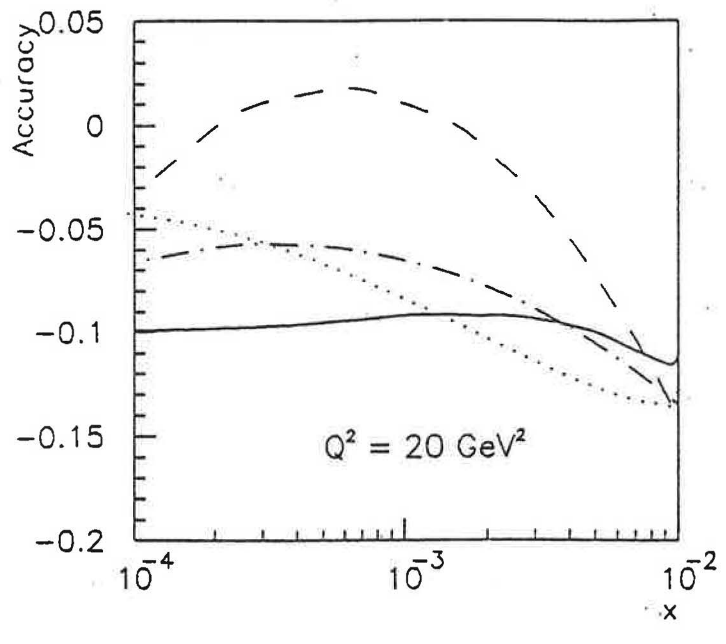

Figure 7: *The total accuracy of (6) (dependence on parametrisations).*
At $Q^2 = 20$ GeV^2 *the total accuracy is plotted for different existing parton parametrisations:*
Solid: D_-
Dashed: D_0
Dashed-dotted: $CTEQM$
Dotted: GRV
$N(x, Q^2)$ *was calculated using the* D_- *gluon distribution.*