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Abstract

We present a classification of (2,2) free field compactifications with one twist in which only 95 distinct models (generations and antigenerations) are found. Models with three generations and no antigenerations are given.

Classification of spectra of different string compactifications always serves a twofold aim. On the one hand one is searching for realistic models with three generations and as few antigenerations as possible. On the other hand one would like to get an overview of "what a certain compactification scheme contains", especially in comparison to other schemes. Classification has initiated enormous progress in understanding of the underlying relations between different schemes. In particular, analysis of the $(2,2)$ spectra yielded by minimal $N = 2$ models [1] led to the observation that they realize Calabi-Yau manifolds at specific points in their moduli spaces.

Of the known string compactification schemes there are two of which hardly anything is known concerning their content in terms of vacuum zero modes: Lattice compactifications [2] and compactifications by free fermions, also called fermionic strings [3,4]. Here we will be concerned with the second case.

Classification in this case has been hampered by the huge number of possibilities for boundary conditions for the fermions [5]. In most cases fermions with only periodic and antiperiodic boundary conditions were used, which implied the need to introduce several sets of boundary conditions to create viable models. This strategy was adopted in most of the subsequent literature.

Instead, in a previous paper [6] we proposed the opposite approach, which is to classify models with very general boundary conditions and a minimum number of different sets. Since it is believed that the main features of the vacua are already evident in the possible $(2,2)$ models, we choose to concentrate on them. Furthermore we impose left–right symmetry in anticipation of a possible geometric interpretation. Only with these restrictions is the classification possible.

In this spirit, we gave a prescription for generating all possible left–right symmetric $(2,2)$ models in the fermionic formulation. Our aim in that work was to make some general observations regarding the nature of fermionic string, and its relation to other compactifications. We stressed that the $(2,2)$ structure is realized on the spectrum (e.g. implying space-time supersymmetry, exceptional gauge groups, the existence of moduli), but that the explicit formulation of the algebra in terms of general complex fermions is still unknown. This situation is reminiscent of the one for Calabi-Yau compactifications. Confining ourselves to $D = 6$ and $D = 8$ dimensions, we discovered that there is a considerable overlap with orbifolds and torus compactifications, but that there exist many models in the fermionic formulation which do not belong to any orbifold or known smooth manifold. Specifically, in $D = 8$ there exist only the known tori and no orbifolds. In $D = 6$ dimensions, we found 37 models of which 6 belong to the two-torus T^2 and only 4 had a generation number which could possibly correspond to orbifolds or the Calabi-Yau manifold K3. (In $D = 6$, the generation number is related to the Hodge numbers by $n_+ - n_- = h^{11}$ [7]. All orbifold models and the $K3$ manifold have 10 generations.) Only one of these models could directly be bosonized, namely into the Z_2 orbifold. The other models show very similar spectra, therefore suggesting highly nontrivial identities similar to those proposed in ref.[8] between the partition functions.

In this work, we extend the analysis to the case of $D = 4$. Here we already have the examples of equivalence between the fermionic models and the Z_2, Z_4 and Z_8 orbifolds by the already mentioned types of partition function identities [8,9]. The exact overlap between the two schemes remains contentious however, and it is important to note that the fermionic versions of these orbifolds were established using theta-function identities and not direct bosonisation which indeed does not appear to be possible for the Z_4 and Z_8 cases.

An additional aim here is to refute an assertion which often is made, namely that there exists a unique way of generating three generation models which involves a large set of boundary conditions. In fact, extrapolating from the $D = 6$ case, one would naturally expect there to be

many more than 37 left–right symmetric models in four dimensions. This makes the existence of a unique theory unlikely (being in accordance with the Calabi-Yau and Landau-Ginzburg schemes, in which there are also several three generation models [10,1]). Our method for generating left–right symmetric models was given in ref.[6], but for completeness we shall briefly summarise our choice of vectors of boundary conditions.

In our classification we will make use of the fact that $N = 1$ space-time supersymmetry is equivalent to $N = 2$ world-sheet supersymmetry [11] (and the same is valid after the bosonic string map, turning $N = 1$ space-time supersymmetry into an $E_{4+D/2} \otimes E_8$ gauge group). Furthermore a model with local $N = 1$ space-time supersymmetry at the massless level implies the existence of gravity supermultiplets. Since gravity couples universally to all massless and massive states, it forces them all to appear in supermultiplets. This implies $N = 1$ space-time supersymmetry even at the massive level, which by the above theorem implies $N = 2$ supersymmetry on the world-sheet (as does $E_{4+D/2} \otimes E_8$ after the heterotic string map).

We therefore shall restrict ourselves to left–right symmetric $(1,1)$ models which are promoted into $(2,2)$ models in this manner. Further breaking of the gauge group by embeddings of twists (e.g. Wilson-lines) should then work in the usual way, and will not spoil the relevance of the classification. We use the formulation of ref.[3], and we stress that we are restricting the analysis to only complex fermions. The internal degrees of freedom then have phases associated with them a_r, b_r, c_r; $r = 1, \cdots, 3$ which come in triplets for left and right movers fulfilling the constraint

$$a_r + b_r + c_r \ \in \ 0, \frac{1}{2} \qquad \mathrm{mod}(1) \tag{1}$$

and therefore constituting a product of three $(1,1)$ models to start with.

Without loss of generality (see ref.[6]) we choose the first four vectors to be of the form,

$$\mathbf{W}_0 = \left[\ (\frac{1}{2})(\frac{1}{2}\frac{1}{2}\frac{1}{2})^3 \ \middle| \ (\frac{1}{2}\frac{1}{2}\frac{1}{2})^3(\frac{1}{2})^5(\frac{1}{2})^8 \ \right]$$

$$\mathbf{W}_1 = \left[\ (\frac{1}{2})(a_1^r b_1^r c_1^r) \ \middle| \ (000)^3(0)^5(0)^8 \ \right]$$

$$\mathbf{W}_2 = \left[\ (0)(000)^3 \ \middle| \ (a_1^r b_1^r c_1^r)(\frac{1}{2})^5(0)^8 \ \right]$$

$$\mathbf{W}_3 = \left[\ (0)(000)^3 \ \middle| \ (000)^3(0)^5(\frac{1}{2})^8 \ \right]. \tag{2}$$

The $\mathbf{W}_0$ vector is needed to have a non-trivial modular invariant theory, and to give the gravity multiplet. It implies the existence of Ramond and Neveu-Schwarz sectors as in any string compactification. The $\mathbf{W}_1$ and $\mathbf{W}_2$ vectors respectively implement supersymmetry on the right movers and exceptional gauge groups on the left movers. Finally, in order to give a second seperate E_8' factor we have the $\mathbf{W}_3$ vector. Thus we are able to get copies of $N = 2$ algebras on each side, establishing a $(2,2)$ model [6].

The numerical survey of the spectra generated by the above vectors reveals that, for any choice of (a_1^r, b_1^r, c_1^r), the theory generated has the maximal $N = 4$ supersymmetry (and so $E_8 \otimes E_8'$ gauge group), and therefore corresponds to a torus compactification in the usual sense. Typically one finds tori with enhanced symmetry. For example, consider the choice $a_1^r = b_1^r = 0$ and $c_1^r = \frac{1}{2}$ for all r. Direct bosonisation (of the first two fermions in each triplet) shows that we do not simply obtain a product of three independent tori of radius $R = 1/2$, since the vector $\tilde{\mathbf{W}}_0 = \mathbf{W}_0 - \mathbf{W}_1 - \mathbf{W}_2 - \mathbf{W}_3$ relates them in a non-trivial way. To go beyond torus

compactification, we will need to add more vectors to break down supersymmetry and gauge symmetry. Such additional compactification vectors may be either left–right symmetric,

$$\mathbf{W}_4 = \left[\ (0)(a_4^r b_4^r c_4^r) \ \middle| \ (a_4^r b_4^r c_4^r)(0)^5(0)^8 \ \right], \tag{3}$$

or may occur in left–right symmetric pairs,

$$\begin{aligned}
\mathbf{W}_4 &= \left[\ (0)(a_4^r b_4^r c_4^r) \ \middle| \ (a_5^r b_5^r c_5^r)(0)^5(0)^8 \ \right] \\
\mathbf{W}_5 &= \left[\ (0)(a_5^r b_5^r c_5^r) \ \middle| \ (a_4^r b_4^r c_4^r)(0)^5(0)^8 \ \right],
\end{aligned} \tag{4}$$

and so on. Usually it is assumed that only the first possibility may allow the interpretation of the model as a compactified variety (e.g. in ref.[8]). However we emphasise that one should also consider the second possibility. This is similar to the case of the comparison between Calabi-Yau manifolds and compactifications by products of $N = 2$ models, where the vacua of the latter are not always left–right symmetric.

For $N = 1$, resp. $N = 2$ the theories generated have the gauge group

$$G = g \otimes E_6 \otimes E_8', \tag{5}$$

$$G = g \otimes E_7 \otimes E_8', \tag{6}$$

where the first group, g (which is of rank 8, resp. 7), is some product of low rank subgroups coming from the compactified degrees of freedom. In ref.[6] we found that with such a choice of vectors one should obtain all possible left–right symmetric models, provided that one considers k_{ij} structure constants consistent with the preservation of modular invariance. However this selection of vectors above is not sufficient to guarantee a (2,2) compactification since we still have to choose the structure constants. A poor choice of k_{ij} can spoil an ($N = 2$) algebra by projecting out some of the supersymmetry generators via the modular invariance conditions. This implies the breaking of $N = 1$ space-time supersymmetry and/or the exceptional group. For any $(2, 2)$ model there are always several choices of such k_{ij}. E.g. they are fixing representations and antirepresentations.

In order to guarantee a (2,2) model we need to impose a condition on the structure constants. We usually do this by insisting that, given a gauge group G, the structure constants are such that there are the required number of gravitino degrees of freedom. A sufficient condition for this is [6],

$$k_{ij} + \tilde{k}_{ij} = 0 \qquad \mathrm{mod}(1), \tag{7}$$

where the tilde implies the left–right reflected indices (for example $\tilde{k}_{10} = k_{20}$, $\tilde{s}_1 = s_2$ etc.). This always works because of the chirality degrees of freedom of the gravitino and gaugino[1]. Using this restriction, one only has to ensure that the gauge group has the structure G above by the choice of k_{ij}.

We have examined $\sim 10^7$ possible models with one symmetric twist vectors upto order 20 and with the $\mathbf{W}_1$, $\mathbf{W}_2$ vectors containing fractions of $\frac{1}{2}$ or 0 only, in most of the cases. Not taking the simplest $\mathbf{W}_1$, $\mathbf{W}_2$ gives only a few additional models with low numbers of generations and antigenerations. We shall discuss this point in more detail below. The "uncompactness" of the fermionic string construction prohibits a more complete classification than this, although we find that the number of new models drops off very quickly as the number of twist vectors is

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[1]We confess that this is the correct version of Eq.(7) of ref.[6].

increased as a result of the more and more restrictive constraints for a modular invariant theory. Concerning the increase of the order of the model, we checked a further 10^5 models upto order 40 and no new ones were found. Therefore we believe that almost all possible spectra have been found. We find approximately 10^4 models of which 10^3 have distinct spectra, but most of them differ only in the number of singlets. Many of them are related as in the $N = 2$ minimal models, where for instance in $D = 6$ there exist only two distinct models, namely T^2 and $K3$ [1]. Another example is the case of $D = 6$ compactification by free fermions, where it was found that there are two models corresponding to the Z_2 orbifold with such enhanced symmetries [6]. Beyond that one expects mirror symmetry to be at work.

In this letter we shall only give the models with lowest order and maximal gauge group for each generation number. There are 95 distinct cases. In table 1 we have displayed the internal part of the compactification vectors which can achieve them in conjunction with the choice of vectors specified above.

Let us now discuss the relation to orbifolds. As was pointed out in ref.[8], only Z_N, $Z_N \times Z_M$ orbifolds, where N, M are powers of 2, have any chance to be equivalent to fermionic strings given our current knowledge about partition function identities.

By directly bosonising the Z_2 orbifold, one might expect it to have a compactification vector

$$\mathbf{W}_4 = \left[\; (0) \left(0\frac{1}{2}\frac{1}{2}\right) \left(0\frac{1}{2}\frac{1}{2}\right) (000) \; \middle| \; \left(0\frac{1}{2}\frac{1}{2}\right) \left(0\frac{1}{2}\frac{1}{2}\right) (000)(0)^5(0)^8 \; \right].$$

But here one should be careful, since as discussed above, our starting point was a torus with enhanced symmetries due to the vector $\tilde{\mathbf{W}}_0$ discussed above. Indeed the calculation shows that we have a model with six generations of $\mathbf{56}$ representations of E_7, 96 singlets and 37 additional gauge bosons. Requiring a sectorwise equivalence of the partition functions (as in ref.[8]) one has to introduce the additional vector

$$\mathbf{W}_5 = \left[\; (0) \left(\frac{1}{2}\frac{1}{2}0\right) \left(\frac{1}{2}\frac{1}{2}0\right) (000) \; \middle| \; \left(\frac{1}{2}\frac{1}{2}0\right) \left(\frac{1}{2}\frac{1}{2}0\right) (000)(0)^5(0)^8 \; \right].$$

As expected, with this set of vectors we obtain the complete spectrum of the Z_2 orbifold (10 generations and 80 singlets). This is also apparent from the fact that such a vector is needed to completely decouple one torus from the internal part of the corresponding $D = 6$ model. More specifically, we need to break an initial, enhanced $SO(8)$ symmetry, down to $SO(4) \times SO(4)$. Using this vector we recover, in addition to the $N = 2$ models in the table, all the models of ref.[6] with the obvious changes.

The non-singlet spectrum of the 27-3 version of the Z_4 orbifold (the singlet numbers are not available in the literature, here we find 270 singlets and 20 additional gauge bosons) is generated by the first four vectors plus the vector

$$\mathbf{W}_4 = \left[\; (0) \left(0\frac{1}{4}\frac{3}{4}\right) \left(0\frac{1}{2}\frac{1}{2}\right) \left(\frac{1}{2}\frac{3}{4}\frac{3}{4}\right) \; \middle| \; \left(0\frac{1}{4}\frac{3}{4}\right) \left(0\frac{1}{2}\frac{1}{2}\right) \left(\frac{1}{2}\frac{3}{4}\frac{3}{4}\right) (0)^5(0)^8 \; \right].$$

Adding the $\mathbf{W}_5$ vector to the above gives the 31-7 version of the Z_4 orbifold (with 254 singlets and 12 additional gauge bosons) [12]. This is in accordance with ref.[8], where the authors chose a slightly different form of the superpartner of the stress-energy tensor and slightly different boundary conditions. They found a 29-5 model (similar to a Z_6 or Z_{12} orbifold), which was turned into the 31-7 version of the Z_4 orbifold by adding the vector $\mathbf{W}_5$.

The great majority of work on fermionic strings has been based on the 27-3 left–right symmetric

model above. Traditionally this model is achieved (with exactly the same spectrum) using a pair of symmetric compactification vectors [13]

$$\mathbf{W}_4 = \left[\ (0)\left(0\frac{1}{2}\frac{1}{2}\right)\left(0\frac{1}{2}\frac{1}{2}\right)(000)\ \middle|\ \left(0\frac{1}{2}\frac{1}{2}\right)\left(0\frac{1}{2}\frac{1}{2}\right)(000)(0)^5(0)^8\ \right]$$

$$\mathbf{W}_5 = \left[\ (0)\left(\frac{1}{2}0\frac{1}{2}\right)(000)\left(0\frac{1}{2}\frac{1}{2}\right)\ \middle|\ \left(\frac{1}{2}0\frac{1}{2}\right)(000)\left(0\frac{1}{2}\frac{1}{2}\right)(0)^5(0)^8\ \right].$$

With this choice of vectors, a direct bosonisation exists along the lines of ref.[9]. First we label the nine right moving internal fermions by

$$(\rho_1,\sigma_1,\psi_1)(\rho_2,\sigma_2,\psi_2)(\rho_3,\sigma_3,\psi_3)$$

or in real fermions

$$(\rho_1^{r1},\rho_1^{r2};\sigma_1^{r1},\sigma_1^{r2};\psi_1)(\rho_2^{r1},\rho_2^{r2};\sigma_2^{r1},\sigma_2^{r2};\psi_2)(\rho_3^{r1},\rho_3^{r2};\sigma_3^{r1},\sigma_3^{r2};\psi_3)$$

and similarly the left movers. Then we split the triplets into the fermions which have an odd phase under the supersymmetry vector $\mathbf{W}_1$ (ψ_1, ψ_2, ψ_3) and the rest. The latter we wish correspond to complex bosons (z_1, z_2, z_3). One defines the bosons as

$$\frac{1}{\sqrt{2}}(\rho_i^{r1}+i\sigma_i^{r1})=:e^{iRez}:\ ;\quad \frac{1}{\sqrt{2}}(\rho_i^{r2}+i\sigma_i^{r2})=:e^{iImz}:\ ,$$

thus getting Z_2 twists on the bosonic coordinates. Obviously, for a left-right symmetric model we need to do the same for the left movers. Thus we may write down the action on the new coordinates (ψ_i, z_i) of various combinations of compactification vectors,

$$
\begin{aligned}
\mathbf{W}_4\ &:\ (\psi_1,z_1),(\psi_2,z_2)\rightarrow(-\psi_1,-z_1),(-\psi_2,-z_2)\\
\mathbf{W}_5\ &:\ (\psi_1,z_1),(\psi_3,z_3)\rightarrow(-\psi_1,-z_1+\pi+i\pi),(-\psi_3,-z_3)\\
\mathbf{W}_4+\mathbf{W}_5\ &:\ (\psi_1,z_1),(\psi_2,z_2)\,(\psi_3,z_3)\rightarrow(\psi_1,z_1+\pi+i\pi),(-\psi_2,-z_2),(-\psi_3,-z_3).\quad(8)
\end{aligned}
$$

It is easy to show that this may always be done if we only have phases of $\frac{1}{2}$ or 0. On the other hand one could decide to take both real components of a complex field into a real boson. Then the situation is completely different, since we never get twists - only shifts of the bosonic coordinates are created. This therefore gives us a hint that twists in an orbifold may be reformulated via the fermionic formulation as shifts. So we conclude that in the case of the Z_4 orbifold we are only able to make the action of the Z_2 subroup visible as twists, while the remainder is still hidden as shifts.

From table 1 we see that there are further similarities between spectra. But now the discrete symmetries are sometimes completely different thus making any conclusion difficult. We find models with the same spectra as the (Z_4), Z_8, $Z_3\times Z_3$, or $Z_6\times Z_6$ orbifolds; Z_4, Z_6, Z_{12} orbifolds; Z_7, Z_8 orbifolds; $Z_2\times Z_6$ orbifolds. For a comparison see ref.[14,12].

Also from refs.[14,15] one finds no overlap with Gepner models except the model no.83 in the table with five generations and one antigeneration. This is similar to a Gepner model, namely the well studied 3^5 model [1,16][2]. However the discrete symmetries are completely different for most of the cases and also there is not the usual relation that models are the same up to pairs

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[2]In the Z_5 phase and Z_5 permutationally modded 3^5 model one finds 5 generations, 1 antigeneration, one additional gauge boson and 42 singlets.

of additional gauge bosons and singlets [1].

Comparing the spectra to that of ref.[17], one gets the impression that the models studied here must be related to some varieties with torsion.

Since there is a great deal of interest in three generation models and the question of why we have just three generations, we should discuss a certain peculiarity of our survey in detail. If one chooses the simplest set of $\mathbf{W}_i$ vectors (here having in mind a possible bosonisation as discussed above), three generations occur quite naturally as the lowest possible number of generations. Consider the $\mathbf{W}_0$ sector in such a model where $\mathbf{10}$ representations of $SO(10)$ always arise from

$$b^i_{-\frac{1}{2}}|0\rangle \otimes \tilde{b}^i_{-\frac{1}{2}}\tilde{b}_{-\frac{1}{2}}|0\rangle \ ; \ d^i_{-\frac{1}{2}}|0\rangle \otimes \tilde{d}^i_{-\frac{1}{2}}\tilde{b}_{-\frac{1}{2}}|0\rangle \tag{9}$$

excitations, where $i = (3, 6, 9)$. It is simple to show that these states always satisfy the modular invariance conditions since they are symmetric in left and right excitations. We still need to show that the $\mathbf{27}$ representations have the same chirality, which we can do by examining the corresponding space-time fermionic $\mathbf{16}$ states, which occur in the $\overline{\mathbf{W}_0 + \mathbf{W}_1 + \mathbf{W}_2}$ sector. The modular invariance conditions constrain their chirality;

$$
\begin{array}{llll}
\mathbf{W}_0 & : & \Gamma_5\gamma_3\gamma_6\gamma_9 & = & \tilde{\Gamma}_5\tilde{\gamma}_3\tilde{\gamma}_6\tilde{\gamma}_9(-1)^{2(k_{01}+k_{02}+\frac{1}{2})} \\
\mathbf{W}_1 & : & \Gamma_5\gamma_3\gamma_6\gamma_9 & = & (-1)^{2(k_{11}+k_{12}+\frac{1}{2})} \\
\mathbf{W}_2 & : & 1 & = & \tilde{\Gamma}_5\tilde{\gamma}_3\tilde{\gamma}_6\tilde{\gamma}_9(-1)^{2(k_{21}+k_{22})} \\
\mathbf{W}_4 & : & \gamma_3^{2W_4^3}\gamma_6^{2W_4^6}\gamma_9^{2W_4^9} & = & \tilde{\gamma}_3^{2W_4^3}\tilde{\gamma}_6^{2W_4^6}\tilde{\gamma}_9^{2W_4^9}(-1)^{2(k_{41}+k_{42})}
\end{array}
\tag{10}
$$

where we have labelled the internal degrees of freedom $1, \cdots, 9$. The first condition is given by the $\mathbf{W}_1$ and $\mathbf{W}_2$ conditions via the structure constant relations. Without loss of generality we can choose the structure constants to be zero. Generically, the only solution to the $\mathbf{W}_4$ constraint (which corresponds to the $\mathbf{10}$) is $\gamma_i = \tilde{\gamma}_i = \pm 1$, which gives the three fermionic $\mathbf{16}$ with spin structure $(\gamma_3\gamma_6\gamma_9) = (+ + -), (+ - +), (- + +)$ and their antiparticles with $(\gamma_3\gamma_6\gamma_9) = (- - +), (- + -), (+ - -)$ and in addition two chiralities of gaugino with $(\gamma_3\gamma_6\gamma_9) = (- - -), (+ + +)$. All three matter multiplets have $\Gamma_5 = -\tilde{\Gamma}_5$ and thus the chirality of the $\mathbf{16}$ is the same in each $\mathbf{27}$. (Alternatively we could have established this by examining their charges.) Thus proving that at least three generations appear for the simplest choice of $\mathbf{W}_1$ and $\mathbf{W}_2$ vectors.

Whilst the three generation models are of immediate interest for possible phenomenological considerations[3], the other ones seem to be not so attractive at first glance. However studying specific examples gives the impression that it might also be possible to promote those into models with three net generations (now with antigenerations[4]) by adding additional boundary vectors, naturally leading to $(2, 0)$ models.

We shall demonstrate this by showing two examples. The first model is initially a left–right symmetric 7-1 model with the compactification vector

$$\mathbf{W}_4 = \left[\ (0) \left(0\frac{1}{2}\frac{1}{2}\right) \left(\frac{1}{3}\frac{3}{4}\frac{11}{12}\right) \left(0\frac{5}{12}\frac{7}{12}\right) \ \middle| \ \left(0\frac{1}{2}\frac{1}{2}\right) \left(\frac{1}{3}\frac{3}{4}\frac{11}{12}\right) \left(0\frac{5}{12}\frac{7}{12}\right) (0)^5(0)^8 \ \right], \tag{11}$$

and will lead to a 4-1 model. Initially fermionic $\mathbf{16}$ representations come from the $\overline{\mathbf{W}_0 + \mathbf{W}_1 + \mathbf{W}_2}$ and $\overline{\mathbf{W}_0 + \mathbf{W}_1 + \mathbf{W}_2 + 6\mathbf{W}_4}$ sectors. For the first sector the modular invariance projections are

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[3]Not discussing here the question of Wilson line breakings, which are still to be done to implement reasonable gauge groups.

[4]A pattern which is prefered by certain potentially viable schemes for a realistic phenomenology [18].

as above, and give 3-1 generations, with the following chiralities

$$\mathbf{16} \quad : \quad (+++--++-),(++-+-+-+),(+-++--++)$$
$$\overline{\mathbf{16}} \quad : \quad (+-++++--) \tag{12}$$

and their antiparticles, defined for the product of gamma matrices $(\Gamma_5\gamma_3\gamma_6\gamma_9\tilde{\Gamma}_5\tilde{\gamma}_3\tilde{\gamma}_6\tilde{\gamma}_9)$. The second sector gives 4-0 generations with the chiralities

$$\mathbf{16} \quad : \quad (++------+),(+++--++-),$$
$$(++-+-+-+),(++++-+++) \tag{13}$$

defined for the product of gamma matrices $(\Gamma_5\gamma_3\gamma_5\gamma_8\tilde{\Gamma}_5\tilde{\gamma}_3\tilde{\gamma}_5\tilde{\gamma}_8)$. In order to give such a 4-1 model, we wish to construct an additional $\mathbf{W}_5$ which overlaps in such a way that some of the old generations are projected out, and no new generations are created. One way to do this is for $\mathbf{W}_5$ to give constraints that impose $\gamma_3\gamma_6 = -1$ in the first sector and $\gamma_3\gamma_8 = -1$ in the second. A suitable vector is

$$\mathbf{W}_5 = \left[\; (0) \left(\frac{1}{2}\frac{1}{2}0\right) \left(\frac{1}{2}\frac{1}{2}0\right) (000) \; \middle| \; \left(00\frac{1}{2}\right) \left(0\frac{1}{2}\frac{1}{2}\right) \left(0\frac{1}{2}0\right) (0)^5 \left(\frac{1}{2}\right)^4 (0)^4 \; \right], \tag{14}$$

with the new structure constants chosen to be all zero except $k_{25} = \frac{1}{2}$. This vector projects out the first generation of Eq.(12) and the last two generations of Eq.(13). In addition, the overlap with the E'_8 degrees of freedom ensures that there are no new sectors which could contain more generations. The gauge symmetry of the visible sector is broken down to $SO(10)$, and with further vectors we could clearly arrange to end up with smaller groups still.

The second model is initially a left–right symmetric 9-2 model and gives a 5-2 model. The compactification vector is

$$\mathbf{W}_4 = \left[\; (0) \left(0\frac{1}{5}\frac{4}{5}\right) \left(\frac{1}{2}\frac{3}{5}\frac{9}{10}\right) \left(\frac{3}{10}\frac{4}{5}\frac{9}{10}\right) \; \middle| \; \left(0\frac{1}{5}\frac{4}{5}\right) \left(\frac{1}{2}\frac{3}{5}\frac{9}{10}\right) \left(\frac{3}{10}\frac{4}{5}\frac{9}{10}\right) (0)^5 (0)^8 \; \right]. \tag{15}$$

Fermionic $\mathbf{16}$ representations come from the $\overline{\mathbf{W}_0 + \mathbf{W}_1 + \mathbf{W}_2}$ and $\overline{\mathbf{W}_0 + \mathbf{W}_1 + \mathbf{W}_2 + 5\mathbf{W}_4}$ sectors. The first sector has 5-0 generations, with the chiralities

$$\mathbf{16} \quad : \quad (+--+---+),(+--+--+-),(+-+----+)$$
$$(+-+---+-),(++++-+++) \tag{16}$$

defined for the product of gamma matrices $(\Gamma_5\gamma_3\gamma_6\gamma_9\tilde{\Gamma}_5\tilde{\gamma}_3\tilde{\gamma}_6\tilde{\gamma}_9)$ and the second sector has 4-2 generations with the chiralities

$$\mathbf{16} \quad : \quad (+-------),(+--+---+),$$
$$(+-+---+-),(+-++--++)$$
$$\overline{\mathbf{16}} \quad : \quad (+--+-++-),(+-++--++) \tag{17}$$

defined for the product of gamma matrices $(\Gamma_5\gamma_3\gamma_4\gamma_7\tilde{\Gamma}_5\tilde{\gamma}_3\tilde{\gamma}_4\tilde{\gamma}_7)$. A 5-2 model is obtained by adding the vector $\mathbf{W}_5$

$$\mathbf{W}_5 = \left[\; (0)(000)(000)(000) \; \middle| \; (000) \left(00\frac{1}{2}\right) \left(00\frac{1}{2}\right) (0)^5 \left(\frac{1}{2}\right)^2 (0)^6 \; \right], \tag{18}$$

8

which projects out all the states in Eq.(16) which have $\tilde{\gamma}_6\tilde{\gamma}_9 = -1$, and does not affect any of the states in Eq.(17). Here we have to set all the new structure constants to zero except $k_{54} = \frac{9}{10}$ and $k_{52} = \frac{1}{2}$.

We should add that this second model may still allow the hope for a bosonisation into a manifold (like discussed above), since $\mathbf{W_5}$ has nonzero entries in at most one position of the left triplets.

Finally we address another aspect of the underlying $(2,2)$ models. Since we have constructed $N = 1$ supersymmetric space-time compactifications with maximal exceptional gauge groups, implying $N = 2$ algebras on the world-sheet, there must be the moduli fields associated to this structure.

The first set is obtained by acting with $G^+(\bar{z})$ on the left-handed (chiral) $\mathbf{27}$ superfields, while the second set is obtained by acting with $G^-(\bar{z})$ on the right-handed (antichiral) $\overline{\mathbf{27}}$ superfields. The explicit form of the algebra is only known for Z_2 boundary conditions[5], but nevertheless one is able to construct the moduli by using the superpartner of the stress energy-tensor in the $N = 1$ subalgebra of the $N = 2$ algebra, that is known explicitly.

$$T_F(\bar{z}) = \frac{1}{\sqrt{2}}(G^+(\bar{z}) + G^-(\bar{z})) = i\sum_{i=1}^{3} \tilde{\rho}^i\tilde{\sigma}^i\tilde{\psi}^i + \text{ h.c.} \tag{22}$$

Using the fact that $G^-(\bar{z})$ vanishes on the left-handed superfields and $G^+(\bar{z})$ vanishes on the rigth-handed ones, we may simply use $T_F(\bar{z})$ to construct the moduli.

To demonstrate this explicitly let us give an example. Suppose that a generation exists with a $\mathbf{10}$ of the form,

$$b^i_{-v^i}|0\rangle \otimes \tilde{b}^j_{-\tilde{v}^j}\tilde{b}^{10}_{-\frac{1}{2}}|0\rangle \tag{23}$$

in a sector

$$\overline{\alpha\mathbf{W}} = \left[\ \left(\frac{1}{2}\right)\left(v^1v^2v^3\right)\left(v^4v^5v^6\right)\left(v^7v^8v^9\right) \ \middle| \ \left(\tilde{v}^1\tilde{v}^2\tilde{v}^3\right)\left(\tilde{v}^4\tilde{v}^5\tilde{v}^6\right)\left(\tilde{v}^7\tilde{v}^8\tilde{v}^9\right)(\tfrac{1}{2})^5(\tfrac{1}{2})^8 \ \right],$$

and consider the singlet state which is generated from it by acting with the $T_F(\bar{z})$, together with the removal of the $SO(10)$ excitation,

$$b^i_{-v^i}|0\rangle \otimes \tilde{d}^k_{-\tilde{v}^k}\tilde{d}^l_{-\tilde{v}^l}|0\rangle \tag{24}$$

where the indices k and l are in the same triplet as but not equal to j. Clearly the $\mathbf{W_1}$ modular invariance condition is unchanged (see ref.[6]), but what about the conditions from the vectors overlapping? Taking the constraint associated with $\mathbf{W}_n$

$$\mathbf{W}_n \cdot N_{\alpha\mathbf{W}} = P_n \qquad \text{mod}(1) \tag{25}$$

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[5]In this case the supercurrents are simply the linear combination

$$G^+ = -\sqrt{2}\sum_{j=1}^{3} \psi_j\partial z_j, \tag{19}$$

$$G^- = -\sqrt{2}\sum_{j=1}^{3} \bar{\psi}_j\partial\bar{z}_j, \tag{20}$$

The bosonisation procedure we described above may be used to give

$$\sqrt{2}\partial z_j \equiv i(:\bar{\rho}_j\rho_j: -i\bar{\sigma}_j\sigma_j :) \tag{21}$$

which is precisely the prescription given in ref.[6] for an $N = 2$ algebra on the world-sheet.

where P_n depends only on the sector and is the same for each state, we see that for both the **10** and singlet to exist, we require

$$\mathbf{W}_n^j + \mathbf{W}_n^{10} - \mathbf{W}_n^i = -\mathbf{W}_n^k - \mathbf{W}_n^l - \mathbf{W}_n^i \qquad \mathrm{mod}(1) \qquad (26)$$

But this is simply the triplet constraint and is therefore trivially satisfied. Finally we have to show that the singlet is massless which also follows from the triplet constraint since the vacuum energies for both states are the same. In this way one can construct the moduli for all the models considered here by acting with $T_F(\bar{z})$.

To conclude, we have given a classification of $(2,2)$ free field compactifications that is expected to be exhaustive at the one twist level. The fermionic and orbifold compactifications overlap at least in the way predicted by ref.[8]. Further conclusions about the models in which the numbers of generations and antigenerations coincide with other orbifolds have to be postponed at the current state of knowledge. In particular, a conjecture such as fermionic strings overlap with Z_N orbifolds and Gepner models overlap with $Z_N \times Z_M$ orbifolds, cannot be made unless the appearance of, for example, the model with five generations and one antigenerations is explained.

Three generation models with no antigenerations have been found. For the case of $(2,0)$ models, additional three generation models with antigenerations have been given.

Table Captions

Table 1 Supersymmetric (2,2) models in $D = 4$. n_{gen}, n_{agen} are the numbers of fundamental representations of E_{6+X}. n_g is the number of bosons in g, and n_s is the number of singlets. The gauge group is $g \otimes E_{6+X} \otimes E_8'$ where $2^X = N$. The models marked with a star may only be generated with more complicated $\mathbf{W}_1$, $\mathbf{W}_2$ vectors. Here they have an internal structure, $\left(0\frac{1}{6}\frac{2}{6}\right)\left(0\frac{1}{6}\frac{2}{6}\right)\left(0\frac{1}{6}\frac{2}{6}\right)$.

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Table 1:

number	vector	N-SUSY	n_{gen}	n_{agen}	n_s	n_g	$|\chi/2|$
1	$(000)(000)(000)$	4	1	0	-	66	0
2	$(000)\left(\frac{8}{20}\frac{13}{20}\frac{19}{20}\right)\left(\frac{7}{20}\frac{14}{20}\frac{19}{20}\right)$	2	10	0	36	13	0
4	$(000)\left(0\frac{1}{16}\frac{15}{16}\right)\left(\frac{6}{16}\frac{11}{16}\frac{15}{16}\right)$	2	9	0	63	19	0
3	$(000)\left(\frac{12}{20}\frac{13}{20}\frac{15}{20}\right)\left(\frac{11}{20}\frac{14}{20}\frac{15}{20}\right)$	2	8	0	16	11	0
5	$(000)\left(0\frac{1}{8}\frac{7}{8}\right)\left(\frac{3}{8}\frac{6}{8}\frac{7}{8}\right)$	2	7	0	56	19	0
6	$(000)\left(0\frac{1}{2}\frac{1}{2}\right)\left(0\frac{1}{2}\frac{1}{2}\right)$	2	6	0	96	37	0
7	$(000)\left(\frac{3}{6}\frac{4}{6}\frac{5}{6}\right)\left(\frac{3}{6}\frac{4}{6}\frac{5}{6}\right)$	2	5	0	24	17	0
8	$(000)\left(0\frac{1}{4}\frac{3}{4}\right)\left(0\frac{1}{4}\frac{3}{4}\right)$	2	4	0	48	33	0
9	$(000)\left(0\frac{1}{6}\frac{5}{6}\right)\left(\frac{3}{6}\frac{4}{6}\frac{5}{6}\right)$	2	3	0	27	19	0
10	$(000)\left(0\frac{2}{4}\frac{2}{4}\right)\left(\frac{1}{4}\frac{1}{4}\frac{2}{4}\right)$	2	2	0	14	23	0
11	$(000)\left(0\frac{2}{6}\frac{4}{6}\right)\left(0\frac{2}{6}\frac{4}{6}\right)$	2	1	0	18	33	0
12	$\left(0\frac{2}{16}\frac{14}{16}\right)\left(0\frac{3}{16}\frac{13}{16}\right)\left(\frac{2}{16}\frac{3}{16}\frac{11}{16}\right)$	1	51	0	212	16	51
13	$\left(0\frac{2}{20}\frac{18}{20}\right)\left(0\frac{3}{20}\frac{17}{20}\right)\left(\frac{3}{20}\frac{18}{20}\frac{19}{20}\right)$	1	46	0	204	16	46
14	$\left(0\frac{2}{8}\frac{6}{8}\right)\left(0\frac{1}{8}\frac{7}{8}\right)\left(\frac{2}{8}\frac{7}{8}\frac{7}{8}\right)$	1	41	1	264	18	40
15	$\left(0\frac{4}{16}\frac{12}{16}\right)\left(0\frac{3}{16}\frac{13}{16}\right)\left(\frac{3}{16}\frac{14}{16}\frac{15}{16}\right)$	1	39	0	154	14	39
16	$\left(0\frac{2}{16}\frac{14}{16}\right)\left(0\frac{3}{16}\frac{13}{16}\right)\left(\frac{2}{16}\frac{15}{16}\frac{15}{16}\right)$	1	37	1	214	14	36
17	$\left(0\frac{2}{16}\frac{14}{16}\right)\left(\frac{8}{16}\frac{11}{16}\frac{13}{16}\right)\left(\frac{10}{16}\frac{11}{16}\frac{11}{16}\right)$	1	35	0	174	10	35
18	$\left(0\frac{2}{12}\frac{10}{12}\right)\left(0\frac{1}{12}\frac{11}{12}\right)\left(\frac{3}{12}\frac{10}{12}\frac{11}{12}\right)$	1	35	1	208	16	34
19	$\left(0\frac{4}{16}\frac{12}{16}\right)\left(\frac{6}{16}\frac{13}{16}\frac{13}{16}\right)\left(\frac{3}{16}\frac{4}{16}\frac{9}{16}\right)$	1	33	0	152	12	33
20	$\left(0\frac{4}{12}\frac{8}{12}\right)\left(0\frac{1}{12}\frac{11}{12}\right)\left(\frac{1}{12}\frac{2}{12}\frac{9}{12}\right)$	1	31	0	147	14	31
21	$\left(0\frac{2}{16}\frac{14}{16}\right)\left(0\frac{3}{16}\frac{13}{16}\right)\left(\frac{10}{16}\frac{11}{16}\frac{11}{16}\right)$	1	31	1	134	12	30
22	$\left(0\frac{4}{20}\frac{16}{20}\right)\left(\frac{2}{20}\frac{19}{20}\frac{19}{20}\right)\left(\frac{1}{20}\frac{4}{20}\frac{15}{20}\right)$	1	30	0	118	12	30
23	$\left(0\frac{4}{16}\frac{12}{16}\right)\left(\frac{7}{16}\frac{12}{16}\frac{13}{16}\right)\left(\frac{1}{16}\frac{6}{16}\frac{9}{16}\right)$	1	29	0	118	10	29
24	$\left(0\frac{2}{16}\frac{14}{16}\right)\left(\frac{4}{16}\frac{5}{16}\frac{7}{16}\right)\left(\frac{3}{16}\frac{6}{16}\frac{7}{16}\right)$	1	29	1	124	10	28
25	$\left(0\frac{4}{16}\frac{12}{16}\right)\left(0\frac{1}{16}\frac{15}{16}\right)\left(\frac{9}{16}\frac{10}{16}\frac{13}{16}\right)$	1	27	0	96	12	27
26	$\left(0\frac{8}{16}\frac{8}{16}\right)\left(\frac{1}{16}\frac{2}{16}\frac{13}{16}\right)\left(0\frac{5}{16}\frac{11}{16}\right)$	1	27	1	128	12	26
27	$\left(0\frac{1}{10}\frac{9}{10}\right)\left(0\frac{1}{10}\frac{9}{10}\right)\left(\frac{1}{10}\frac{1}{10}\frac{8}{10}\right)$	1	26	0	204	26	26
28	$\left(0\frac{5}{20}\frac{15}{20}\right)\left(\frac{4}{20}\frac{6}{20}\frac{10}{20}\right)\left(\frac{2}{20}\frac{3}{20}\frac{15}{20}\right)$	1	26	1	77	10	25
29	$\left(0\frac{4}{16}\frac{12}{16}\right)\left(0\frac{1}{16}\frac{15}{16}\right)\left(\frac{1}{16}\frac{2}{16}\frac{13}{16}\right)$	1	25	0	120	14	25
30	$\left(0\frac{2}{4}\frac{2}{4}\right)\left(0\frac{1}{4}\frac{3}{4}\right)\left(\frac{2}{4}\frac{3}{4}\frac{3}{4}\right)$	1	27	3	270	20	24
31	$\left(0\frac{4}{8}\frac{4}{8}\right)\left(0\frac{1}{8}\frac{7}{8}\right)\left(\frac{1}{8}\frac{2}{8}\frac{5}{8}\right)$	1	25	1	168	14	24
32	$\left(0\frac{4}{12}\frac{8}{12}\right)\left(\frac{6}{12}\frac{7}{12}\frac{11}{12}\right)\left(\frac{7}{12}\frac{8}{12}\frac{9}{12}\right)$	1	24	0	122	12	24

number	vector	N-SUSY	n_{gen}	n_{agen}	n_s	n_g	$|\chi/2|$
33	$\left(0\,\frac{4}{12}\,\frac{8}{12}\right)\left(\frac{6}{12}\,\frac{7}{12}\,\frac{11}{12}\right)\left(\frac{3}{12}\,\frac{4}{12}\,\frac{5}{12}\right)$	1	24	1	124	10	23
34	$\left(0\,\frac{3}{12}\,\frac{9}{12}\right)\left(0\,\frac{2}{12}\,\frac{10}{12}\right)\left(\frac{3}{12}\,\frac{10}{12}\,\frac{11}{12}\right)$	1	23	0	102	16	23
35	$\left(0\,\frac{2}{8}\,\frac{6}{8}\right)\left(0\,\frac{1}{8}\,\frac{7}{8}\right)\left(\frac{1}{8}\,\frac{2}{8}\,\frac{5}{8}\right)$	1	23	1	134	16	22
36	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)\left(\frac{3}{6}\,\frac{4}{6}\,\frac{5}{6}\right)\left(\frac{3}{6}\,\frac{4}{6}\,\frac{5}{6}\right)$	1	22	0	114	20	22
37	$\left(0\,\frac{4}{16}\,\frac{12}{16}\right)\left(\frac{8}{16}\,\frac{11}{16}\,\frac{13}{16}\right)\left(\frac{2}{16}\,\frac{5}{16}\,\frac{9}{16}\right)$	1	21	0	92	10	21
38	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)\left(0\,\frac{1}{6}\,\frac{5}{6}\right)\left(0\,\frac{1}{6}\,\frac{5}{6}\right)$	1	21	1	162	24	20
39	$\left(0\,\frac{4}{12}\,\frac{8}{12}\right)\left(\frac{1}{12}\,\frac{3}{12}\,\frac{8}{12}\right)\left(\frac{1}{12}\,\frac{3}{12}\,\frac{8}{12}\right)$	1	20	0	60	18	20
40	$\left(0\,\frac{3}{12}\,\frac{9}{12}\right)\left(\frac{4}{12}\,\frac{10}{12}\,\frac{10}{12}\right)\left(\frac{3}{12}\,\frac{10}{12}\,\frac{11}{12}\right)$	1	19	0	84	12	19
41	$\left(0\,\frac{2}{8}\,\frac{6}{8}\right)\left(\frac{4}{8}\,\frac{5}{8}\,\frac{7}{8}\right)\left(\frac{2}{8}\,\frac{3}{8}\,\frac{3}{8}\right)$	1	19	1	130	12	18
42	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)\left(0\,\frac{2}{6}\,\frac{4}{6}\right)\left(0\,\frac{2}{6}\,\frac{4}{6}\right)$	1	18	0	126	32	18
43	$\left(0\,\frac{2}{18}\,\frac{16}{18}\right)\left(\frac{3}{18}\,\frac{4}{18}\,\frac{11}{18}\right)\left(\frac{2}{18}\,\frac{7}{18}\,\frac{9}{18}\right)$	1	18	1	84	10	17
44	$\left(0\,\frac{3}{12}\,\frac{9}{12}\right)\left(\frac{6}{12}\,\frac{8}{12}\,\frac{10}{12}\right)\left(\frac{3}{12}\,\frac{10}{12}\,\frac{11}{12}\right)$	1	17	0	90	10	17
45	$\left(0\,\frac{6}{12}\,\frac{6}{12}\right)\left(\frac{1}{12}\,\frac{4}{12}\,\frac{7}{12}\right)\left(\frac{6}{12}\,\frac{7}{12}\,\frac{11}{12}\right)$	1	19	3	132	12	16
46	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)\left(\frac{2}{10}\,\frac{3}{10}\,\frac{5}{10}\right)\left(\frac{6}{10}\,\frac{7}{10}\,\frac{7}{10}\right)$	1	17	1	94	12	16
47	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)\left(\frac{1}{10}\,\frac{3}{10}\,\frac{6}{10}\right)\left(\frac{1}{10}\,\frac{3}{10}\,\frac{6}{10}\right)$	1	16	0	61	14	16
48	$\left(0\,\frac{2}{12}\,\frac{10}{12}\right)\left(\frac{6}{12}\,\frac{7}{12}\,\frac{11}{12}\right)\left(\frac{3}{12}\,\frac{10}{12}\,\frac{11}{12}\right)$	1	16	1	58	12	15
49	$\left(0\,\frac{1}{16}\,\frac{15}{16}\right)\left(\frac{4}{16}\,\frac{14}{16}\,\frac{14}{16}\right)\left(\frac{9}{16}\,\frac{10}{16}\,\frac{13}{16}\right)$	1	15	0	62	8	15
50	$\left(0\,\frac{6}{12}\,\frac{6}{12}\right)\left(\frac{1}{12}\,\frac{2}{12}\,\frac{9}{12}\right)\left(\frac{7}{12}\,\frac{8}{12}\,\frac{9}{12}\right)$	1	17	3	90	10	14
51	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)\left(0\,\frac{1}{6}\,\frac{5}{6}\right)\left(\frac{3}{6}\,\frac{4}{6}\,\frac{5}{6}\right)$	1	16	2	134	16	14
52	$\left(0\,\frac{6}{12}\,\frac{6}{12}\right)\left(\frac{1}{12}\,\frac{4}{12}\,\frac{7}{12}\right)\left(\frac{3}{12}\,\frac{10}{12}\,\frac{11}{12}\right)$	1	15	1	90	8	14
53	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)\left(\frac{5}{10}\,\frac{7}{10}\,\frac{8}{10}\right)\left(\frac{2}{10}\,\frac{2}{10}\,\frac{6}{10}\right)$	1	14	0	74	16	14
54	$\left(0\,\frac{4}{12}\,\frac{8}{12}\right)\left(0\,\frac{1}{12}\,\frac{11}{12}\right)\left(\frac{3}{12}\,\frac{4}{12}\,\frac{5}{12}\right)$	1	14	1	84	14	13
55	$\left(0\,\frac{1}{6}\,\frac{5}{6}\right)\left(0\,\frac{1}{6}\,\frac{5}{6}\right)\left(\frac{1}{6}\,\frac{1}{6}\,\frac{4}{6}\right)$	1	13	0	102	26	13
56	$\left(0\,\frac{1}{18}\,\frac{17}{18}\right)\left(\frac{3}{18}\,\frac{7}{18}\,\frac{8}{18}\right)\left(\frac{1}{18}\,\frac{8}{18}\,\frac{9}{18}\right)$	1	15	3	89	10	12
57	$\left(0\,\frac{1}{12}\,\frac{11}{12}\right)\left(\frac{3}{12}\,\frac{4}{12}\,\frac{5}{12}\right)\left(\frac{1}{12}\,\frac{3}{12}\,\frac{8}{12}\right)$	1	14	2	70	12	12
58	$\left(0\,\frac{1}{4}\,\frac{3}{4}\right)\left(0\,\frac{1}{4}\,\frac{3}{4}\right)\left(\frac{1}{4}\,\frac{1}{4}\,\frac{2}{4}\right)$	1	13	1	120	26	12
59	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)\left(\frac{1}{10}\,\frac{3}{10}\,\frac{6}{10}\right)\left(\frac{6}{10}\,\frac{6}{10}\,\frac{8}{10}\right)$	1	12	0	71	12	12
60	$\left(0\,\frac{1}{18}\,\frac{17}{18}\right)\left(\frac{1}{18}\,\frac{2}{18}\,\frac{15}{18}\right)\left(\frac{5}{18}\,\frac{15}{18}\,\frac{16}{18}\right)$	1	13	2	83	10	11
61	$\left(0\,\frac{4}{12}\,\frac{8}{12}\right)\left(\frac{6}{12}\,\frac{7}{12}\,\frac{11}{12}\right)\left(\frac{2}{12}\,\frac{3}{12}\,\frac{7}{12}\right)$	1	12	1	66	8	11
62	$\left(0\,\frac{1}{8}\,\frac{7}{8}\right)\left(0\,\frac{3}{8}\,\frac{5}{8}\right)\left(\frac{1}{8}\,\frac{1}{8}\,\frac{6}{8}\right)$	1	11	0	56	18	11
63	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)\left(\frac{2}{10}\,\frac{3}{10}\,\frac{5}{10}\right)\left(\frac{1}{10}\,\frac{2}{10}\,\frac{7}{10}\right)$	1	12	2	76	14	10
64	$\left(0\,\frac{3}{6}\,\frac{3}{6}\right)\left(0\,\frac{2}{6}\,\frac{4}{6}\right)\left(\frac{3}{6}\,\frac{4}{6}\,\frac{5}{6}\right)$	1	11	1	84	18	10

number	vector			N-SUSY	n_{gen}	n_{agen}	n_s	n_g	$|\chi/2|$
65	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)$	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)$	$\left(\frac{2}{10}\,\frac{2}{10}\,\frac{6}{10}\right)$	1	10	0	80	26	10
66	$\left(0\,\frac{3}{12}\,\frac{9}{12}\right)$	$\left(\frac{2}{12}\,\frac{5}{12}\,\frac{5}{12}\right)$	$\left(\frac{1}{12}\,\frac{1}{12}\,\frac{10}{12}\right)$	1	11	2	59	10	9
67	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)$	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)$	$\left(\frac{2}{6}\,\frac{2}{6}\,\frac{2}{6}\right)$	1	9	0	66	30	9
68	$\left(0\,\frac{1}{16}\,\frac{15}{16}\right)$	$\left(\frac{4}{16}\,\frac{4}{16}\,\frac{8}{16}\right)$	$\left(\frac{3}{16}\,\frac{6}{16}\,\frac{7}{16}\right)$	1	15	7	108	10	8
69	$\left(0\,\frac{2}{4}\,\frac{2}{4}\right)$	$\left(0\,\frac{1}{4}\,\frac{3}{4}\right)$	$\left(0\,\frac{1}{4}\,\frac{3}{4}\right)$	1	11	3	122	24	8
70	$\left(0\,\frac{4}{8}\,\frac{4}{8}\right)$	$\left(0\,\frac{3}{8}\,\frac{5}{8}\right)$	$\left(\frac{4}{8}\,\frac{5}{8}\,\frac{7}{8}\right)$	1	9	1	72	18	8
71	$\left(0\,\frac{2}{14}\,\frac{12}{14}\right)$	$\left(\frac{2}{14}\,\frac{4}{14}\,\frac{8}{14}\right)$	$\left(\frac{2}{14}\,\frac{6}{14}\,\frac{6}{14}\right)$	1	8	0	55	16	8
72	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)$	$\left(\frac{3}{6}\,\frac{4}{6}\,\frac{5}{6}\right)$	$\left(\frac{2}{6}\,\frac{5}{6}\,\frac{5}{6}\right)$	1	9	2	81	16	7
73	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)$	$\left(\frac{5}{10}\,\frac{6}{10}\,\frac{9}{10}\right)$	$\left(\frac{1}{10}\,\frac{2}{10}\,\frac{7}{10}\right)$	1	7	0	48	10	7
74	$\left(0\,\frac{2}{16}\,\frac{14}{16}\right)$	$\left(\frac{1}{16}\,\frac{7}{16}\,\frac{8}{16}\right)$	$\left(\frac{5}{16}\,\frac{5}{16}\,\frac{6}{16}\right)$	1	11	5	72	10	6
75	$\left(0\,\frac{10}{20}\,\frac{10}{20}\right)$	$\left(\frac{6}{20}\,\frac{15}{20}\,\frac{19}{20}\right)$	$\left(\frac{5}{20}\,\frac{6}{20}\,\frac{9}{20}\right)$	1	9	3	40	12	6
76	$\left(0\,\frac{2}{8}\,\frac{6}{8}\right)$	$\left(0\,\frac{1}{8}\,\frac{7}{8}\right)$	$\left(0\,\frac{3}{8}\,\frac{5}{8}\right)$	1	7	1	60	20	6
77	$\left(0\,\frac{2}{14}\,\frac{12}{14}\right)$	$\left(0\,\frac{4}{14}\,\frac{10}{14}\right)$	$\left(0\,\frac{6}{14}\,\frac{8}{14}\right)$	1	6	0	54	20	6
78	$\left(0\,\frac{1}{8}\,\frac{7}{8}\right)$	$\left(\frac{2}{8}\,\frac{3}{8}\,\frac{3}{8}\right)$	$\left(\frac{1}{8}\,\frac{1}{8}\,\frac{6}{8}\right)$	1	7	2	60	14	5
79	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)$	$\left(0\,\frac{2}{10}\,\frac{8}{10}\right)$	$\left(0\,\frac{4}{10}\,\frac{6}{10}\right)$	1	5	0	39	24	5
80	$\left(0\,\frac{1}{4}\,\frac{3}{4}\right)$	$\left(\frac{2}{4}\,\frac{3}{4}\,\frac{3}{4}\right)$	$\left(\frac{1}{4}\,\frac{1}{4}\,\frac{2}{4}\right)$	1	9	5	124	22	4
81	$\left(0\,\frac{3}{6}\,\frac{3}{6}\right)$	$\left(0\,\frac{2}{6}\,\frac{4}{6}\right)$	$\left(0\,\frac{1}{6}\,\frac{5}{6}\right)$	1	7	3	84	20	4
82	$\left(0\,\frac{4}{8}\,\frac{4}{8}\right)$	$\left(0\,\frac{2}{8}\,\frac{6}{8}\right)$	$\left(\frac{1}{8}\,\frac{1}{8}\,\frac{6}{8}\right)$	1	5	1	26	16	4
83	$\left(0\,\frac{2}{14}\,\frac{12}{14}\right)$	$\left(0\,\frac{4}{14}\,\frac{10}{14}\right)$	$\left(\frac{2}{14}\,\frac{4}{14}\,\frac{8}{14}\right)$	1	3	0	29	16	3
84	$\left(0\,\frac{2}{18}\,\frac{16}{18}\right)$	$\left(\frac{2}{18}\,\frac{7}{18}\,\frac{9}{18}\right)$	$\left(\frac{5}{18}\,\frac{6}{18}\,\frac{7}{18}\right)$	1	9	7	108	10	2
85	$\left(0\,\frac{1}{18}\,\frac{17}{18}\right)$	$\left(\frac{1}{18}\,\frac{8}{18}\,\frac{9}{18}\right)$	$\left(\frac{4}{18}\,\frac{6}{18}\,\frac{8}{18}\right)$	1	7	5	61	10	2
⋆ 86	(000)	$\left(0\,\frac{3}{12}\,\frac{9}{12}\right)$	$\left(\frac{2}{12}\,\frac{11}{12}\,\frac{11}{12}\right)$	1	6	4	144	14	2
87	$\left(0\,\frac{1}{6}\,\frac{5}{6}\right)$	$\left(\frac{2}{6}\,\frac{2}{6}\,\frac{2}{6}\right)$	$\left(\frac{1}{6}\,\frac{2}{6}\,\frac{3}{6}\right)$	1	5	3	84	16	2
88	$\left(0\,\frac{6}{12}\,\frac{6}{12}\right)$	$\left(0\,\frac{4}{12}\,\frac{8}{12}\right)$	$\left(\frac{1}{12}\,\frac{1}{12}\,\frac{10}{12}\right)$	1	3	1	18	14	2
⋆ 89	(000)	$\left(\frac{4}{18}\,\frac{15}{18}\,\frac{17}{18}\right)$	$\left(\frac{2}{18}\,\frac{3}{18}\,\frac{13}{18}\right)$	1	2	0	31	10	2
⋆ 90	(000)	$\left(\frac{1}{18}\,\frac{7}{18}\,\frac{10}{18}\right)$	$\left(\frac{1}{18}\,\frac{1}{18}\,\frac{16}{18}\right)$	1	2	1	21	10	1
⋆ 91	(000)	$\left(0\,\frac{5}{18}\,\frac{13}{18}\right)$	$\left(\frac{5}{18}\,\frac{15}{18}\,\frac{16}{18}\right)$	1	1	0	12	20	1
92	$\left(0\,\frac{1}{8}\,\frac{7}{8}\right)$	$\left(\frac{2}{8}\,\frac{2}{8}\,\frac{4}{8}\right)$	$\left(\frac{2}{8}\,\frac{3}{8}\,\frac{3}{8}\right)$	1	11	11	172	14	0
93	$\left(0\,\frac{1}{12}\,\frac{11}{12}\right)$	$\left(\frac{3}{12}\,\frac{4}{12}\,\frac{5}{12}\right)$	$\left(\frac{1}{12}\,\frac{5}{12}\,\frac{6}{12}\right)$	1	5	5	57	10	0
94	$\left(0\,\frac{1}{10}\,\frac{9}{10}\right)$	$\left(\frac{2}{10}\,\frac{4}{10}\,\frac{4}{10}\right)$	$\left(\frac{2}{10}\,\frac{3}{10}\,\frac{5}{10}\right)$	1	3	3	50	10	0
⋆ 95	(000)	$\left(0\,\frac{2}{18}\,\frac{16}{18}\right)$	$\left(\frac{2}{18}\,\frac{6}{18}\,\frac{10}{18}\right)$	1	1	1	23	14	0