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**Constraints on SUSY-GUT unification  
from  $b \rightarrow s\gamma$  decay**

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**Abstract**

The top-quark Yukawa infrared fixed-point solution of the renormalization group equations, with minimal supersymmetry and GUT unification, defines a low-energy spectrum of supersymmetric particle masses in terms of a few GUT-scale parameters assuming universal boundary conditions. We give predictions in this model for the inclusive  $b \rightarrow s\gamma$  branching fraction and investigate the impact of non-universal scalar mass boundary conditions. We find our results do not depend significantly on the value of the GUT-scale trilinear coupling  $A^G$ . The small  $\tan\beta$  region favors predictions for the inclusive  $b \rightarrow s\gamma$  branching fraction close to that of the Standard Model value. Nevertheless forthcoming experimental results can eliminate some regions of the GUT parameter space.

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It is now widely appreciated that the  $b \rightarrow s\gamma$  inclusive branching fraction is potentially sensitive to new physics beyond the Standard Model (SM), such as charged Higgs bosons and supersymmetric particles [1–4]. The inclusion of QCD corrections to the one-loop amplitudes significantly enhances the decay rate and these calculations have been steadily refined [5–7]. The SM amplitude is dominated by the loop with an intermediate weak boson  $W$  and top quark  $t$ , and the branching fraction depends on the mass of  $t$  for which there is now some direct evidence [8]. With supersymmetry (SUSY) there are additional contributions, the most important coming from loops with charged Higgs  $H^\pm$  and  $t$ , charginos  $\chi_i^\pm$  and stop  $\tilde{t}_i$ . The phenomenology can be complex in general, because of the potential for cancellations between different loop contributions, and because of the large number of possible parameters. However, considerations of supersymmetric grand unification [9] (GUT) provide great simplification, with many low-energy parameters determined by just a few GUT-scale parameters through the renormalization group equations. In this Letter we investigate the predictions for  $b \rightarrow s\gamma$  based on the RGE analysis of Ref. [10], and show that measurements of the  $B \rightarrow X_s\gamma$  inclusive decay rate can exclude otherwise allowed regions for the GUT parameter space.

An attractive possibility is the realization of an infrared fixed-point solution [11] for the top-quark Yukawa coupling  $\lambda_t$  in supersymmetric GUT models [10,12–14]. In this scenario the top-quark mass  $m_t$  is related to the ratio of the two Higgs vacuum expectation values  $v_2/v_1 = \tan\beta$  by [12]

$$m_t(pole) \simeq (200 \text{ GeV}) \sin\beta . \quad (1)$$

In the following discussion we use the value  $m_t(pole) = 168 \text{ GeV}$  and  $\alpha_s(M_Z) = 0.120$ , for which the RGE solutions for the sparticle mass spectra were constructed in Ref. [10]; this is consistent with the value  $m_t = 174 \pm 10_{-12}^{+13}$  experimentally observed from the top-quark candidate events [8]. The universal soft SUSY-breaking parameters at the GUT scale  $M_G$  are  $m_0$ ,  $m_{1/2}$ ,  $\mu$ ,  $A$ ,  $B$ ; here  $m_0$  is the scalar mass,  $m_{1/2}$  is the gaugino mass,  $\mu$  is the Higgs mass mixing, and  $A$ ,  $B$  are trilinear, bilinear scalar couplings respectively. In the

ambidextrous approach of Ref. [10] (see also Ref. [15]) used to solve the RGE integration,  $m_{1/2}$ ,  $m_0$ , and  $A$  are input at the GUT scale. Near the fixed point, the value of the trilinear coupling  $A_t$  that enters into squark mixing is drawn to a fixed value (for a given  $\alpha_3(M_Z)$ ) [14,16], so that the dependence of the rate on the initial value  $A^G$  is minimal. These soft SUSY-breaking parameters are evolved down to the electroweak scale  $M_Z$ , and the values  $|\mu(M_Z)|$  and  $B(M_Z)$  are determined from the requirement that the electroweak symmetry be broken radiatively. By this procedure, all sparticle masses and couplings are determined in terms of GUT parameters  $m_0$  and  $m_{1/2}$  for a specified sign of  $\mu$ . Thus we can predict the  $b \rightarrow s\gamma$  decay rate over the region of  $(m_0, m_{1/2})$  parameter space allowed by other constraints, which are experimental lower bounds on sparticle masses and a naturalness criterion taken to be  $|\mu| < 500$  GeV in Ref. [10].

For calculating the  $b \rightarrow s\gamma$  decay rate, we use the latest analysis of QCD corrections by Buras et al. [6], which evaluates the full  $8 \times 8$  anomalous dimension matrix. We do not take into account the small effect of running the scale of the decay process from  $Q = m_t$  down to  $M_W$  (estimated in Ref. [17]), beyond the usual effect of running from  $Q = M_W$  down to  $m_b$ . The QCD effects also receive theoretical corrections from other large mass splittings in the supersymmetric spectrum [18].

The ratio of  $\Gamma(b \rightarrow s\gamma)$  to the inclusive semileptonic decay width is then given by

$$\frac{\Gamma(b \rightarrow s\gamma)}{\Gamma(b \rightarrow c\ell\nu)} = \frac{6\alpha}{\pi\rho\lambda} \frac{|V_{ts}^*V_{tb}|^2}{|V_{cb}|^2} |c_7(m_b)|^2, \quad (2)$$

where  $\alpha$  is the electromagnetic coupling. The phase-space factor  $\rho$  and the QCD correction factor  $\lambda$  for the semileptonic process are given by  $\rho = 1 - 8r^2 + 8r^6 - r^8 - 24r^4 \ln(r)$  with  $r = m_c/m_b = 0.316 \pm 0.013$  and  $\lambda = 1 - \frac{2}{3}f(r, 0, 0)\alpha_s(m_b)/\pi$  with  $f(r, 0, 0) = 2.41$  [19].

The formulas for the renormalization group coefficient in Eq. (2) are [3,20]

$$c_7(m_b) = \left[ \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right]^{16/23} \left\{ c_7(M_W) - \frac{8}{3} c_8(M_W) \left[ 1 - \left( \frac{\alpha_s(m_b)}{\alpha_s(M_W)} \right)^{2/23} \right] \right\} + \sum_{i=1}^8 h_i \left( \frac{\alpha_s(M_W)}{\alpha_s(m_b)} \right)^{\alpha_i}, \quad (3)$$

$$c_7(M_W) = \frac{3}{2} x f_\gamma^{(1)}(x) + \frac{1}{2} y f_\gamma^{(2)}(y) + \frac{1}{2 \tan^2 \beta} y f_\gamma^{(1)}(y) + c_7^{\chi^\pm}, \quad (4)$$

$$c_8(M_W) = \frac{3}{2}x f_g^{(1)}(x) + \frac{1}{2}y f_g^{(2)}(y) + \frac{1}{2 \tan^2 \beta} y f_g^{(1)}(y) + c_8^{\chi^\pm}, \quad (5)$$

with  $x = (m_t/M_W)^2$ ,  $y = (m_t/m_{H^\pm})^2$ , and

$$c_7^{\chi^\pm} = \sum_{j=1}^2 \left\{ \frac{M_W^2}{\tilde{m}_{\chi_j}^2} \left[ |V_{j1}|^2 f_\gamma^{(1)}(z_1) - \sum_{k=1}^2 \left| V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} M_W \sin \beta} \right|^2 f_\gamma^{(1)}(z_2) \right] \right. \\ \left. - \frac{U_{j2}}{\sqrt{2} \cos \beta} \frac{M_W}{\tilde{m}_{\chi_j}} \left[ V_{j1} f_\gamma^{(3)}(z_1) - \sum_{k=1}^2 \left( V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} M_W \sin \beta} \right) T_{k1} f_\gamma^{(3)}(z_2) \right] \right\}, \quad (6)$$

$$c_8^{\chi^\pm} = \sum_{j=1}^2 \left\{ \frac{M_W^2}{\tilde{m}_{\chi_j}^2} \left[ |V_{j1}|^2 f_g^{(1)}(z_1) - \sum_{k=1}^2 \left| V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} M_W \sin \beta} \right|^2 f_g^{(1)}(z_2) \right] \right. \\ \left. - \frac{U_{j2}}{\sqrt{2} \cos \beta} \frac{M_W}{\tilde{m}_{\chi_j}} \left[ V_{j1} f_g^{(3)}(z_1) - \sum_{k=1}^2 \left( V_{j1} T_{k1} - V_{j2} T_{k2} \frac{m_t}{\sqrt{2} M_W \sin \beta} \right) T_{k1} f_g^{(3)}(z_2) \right] \right\}, \quad (7)$$

with  $z_1 = (\tilde{m}/\tilde{m}_{\chi_j})^2$  and  $z_2 = (\tilde{m}_{t_k}/\tilde{m}_{\chi_j})^2$  and the matrices  $T$ ,  $U$  and  $V$  diagonalize the top squark and chargino mass matrices (see Refs. [3,10]). The parameter  $\tilde{m}$  is the mass of the (degenerate) squarks from the first two generations.

The coefficients from the  $8 \times 8$  anomalous dimension matrix are [6]

$$a_i = \left( \frac{14}{23}, \quad \frac{16}{23}, \quad \frac{6}{23}, \quad -\frac{12}{23}, \quad 0.4086, \quad -0.4230, \quad -0.8994, \quad 0.1456 \right) \\ h_i = \left( \frac{626126}{272277}, \quad -\frac{56281}{51730}, \quad -\frac{3}{7}, \quad -\frac{1}{14}, \quad -0.6494, \quad -0.0380, \quad -0.0186, \quad -0.0057 \right) \quad (8)$$

The formulas for the loop contributions  $f_{\gamma,g}$  are given in Ref. [3].

The neutralino and gluino loop contributions have not been included in the calculation of the  $b \rightarrow s\gamma$  inclusive rate. However, these contributions are known to be small, especially in the low  $\tan \beta$  regime.

Our results are displayed in Figs. 1-3. Figure 1 and Figure 2 show the predicted value of the inclusive branching fraction  $B(b \rightarrow s\gamma)$ , that is equal in the spectator approximation to the physical meson branching fraction  $B(B \rightarrow X_s \gamma)$ ; the upper part of these figures gives the branching fraction versus  $m_0$  for fixed  $m_{1/2}$ , while the lower part gives curves versus  $m_{1/2}$  for fixed  $m_0$ . The same information is displayed in Figure 3, in the form of contours of constant  $B(b \rightarrow s\gamma)$  in the  $(m_{1/2}, m_0)$  parameter plane. The region is bounded by experimental limits on the chargino mass  $m_{\chi_1^\pm}$  and the lighter stop mass  $m_{t_1}$  and the

lighter higgs mass  $m_h$  together with the naturalness boundary  $|\mu| = 500$  GeV. The shaded region is allowed by these constraints.

It has come to light in recent months that the leading order QCD calculations are somewhat inadequate for comparison with the current experimental data; hence a brief discussion of the theoretical uncertainties is warranted. The  $(m_b)^5$  dependence of the rate for  $b \rightarrow s\gamma$  is customarily removed by taking the ratio with semileptonic decays. This, however, does not entirely eliminate the theoretical uncertainty due to  $m_b$  because the  $b$ -mass still enters into the phase space factor which depends on the ratio  $m_b/m_c$ . The ratio of CKM matrix entries is taken to be the central value [6]

$$\frac{|V_{ts}^* V_{tb}|}{|V_{cb}|^2} = 0.95 \pm 0.04 . \quad (9)$$

There is even more substantial uncertainty in the higher order (three-loop) contributions. It has been argued [6] that the unknown next-to-leading QCD corrections yield a  $\pm 25\%$  uncertainty in the branching fraction. This number is arrived at by varying the unphysical scale  $\mu$  from  $m_b/2$  to  $2m_b$ , and is particularly large since the process  $b \rightarrow s\gamma$  is dominated by the QCD corrections. The calculation of the next-to-leading corrections is a formidable task, and progress in reducing this theoretical uncertainty is not anticipated in the near future. Furthermore, the value of  $\alpha_s(M_Z)$  is not known very precisely, and for  $\alpha_s(M_Z)$  in the range  $0.12 \pm 0.01$ , the  $b \rightarrow s\gamma$  rate changes by roughly 10%. Finally, the experimental error in the measurement of the semileptonic decay rate must be considered.

Most analyses of the  $b \rightarrow s\gamma$  process have focused on the case where the scalar masses are universal at the GUT scale. It has recently become fashionable to consider situations where the scalar masses take a more general form but still satisfy the stringent bounds on the flavor changing neutral currents of the first two generations. This nonuniversality might be generic [21], or it might arise from the running of the scalar masses from a universal value near the Planck scale to the GUT scale [22]. When the gauge group is broken to one of lesser rank and non-universality in the soft-supersymmetry breaking terms exists, then there are in general additional D-term contributions to the scalar masses [23,24]. One must be careful

to include the often-neglected contributions to the renormalization group equations of the soft-supersymmetry breaking parameters that arise when non-universality is assumed [25]. The combination

$$\mathcal{S} = m_{H_2}^2 - m_{H_1}^2 + \text{Tr}[M_{Q_L}^2 - M_{L_L}^2 - 2M_{U_R}^2 + M_{D_R}^2 + M_{E_R}^2], \quad (10)$$

satisfies the (one-loop) scaling equation [24]

$$\frac{d\mathcal{S}}{dt} = \frac{2b_1 g_1^2}{16\pi^2} \mathcal{S}, \quad (11)$$

so that if it is zero at some scale, for example the GUT scale, then it is zero for all scales. The renormalization group coefficient  $b_1 = 33/5$  in the minimal supersymmetric model. We have used the two-loop renormalization group equations for which three groups are now in complete agreement [26]. The supersymmetric spectrum that results will be different from that obtained with universal masses at the GUT scale. In a particular example based on the minimal  $SU(5)$  GUT investigated recently by Polonsky and Pomarol [22], each of the sfermion (**10** and **5**) and the Higgs masses is separated at the GUT scale due to the evolution from the Planck scale. If one takes the sfermion mass to be twice that of the Higgs scalar mass at the GUT scale, the approach of  $B(b \rightarrow s\gamma)$  to the Standard Model value is faster than in the universal case as one increases the scalar masses: see Figures 4 and 5. This results from the contributions of the comparatively heavier top squarks which enter into the chargino loop diagrams.

One can also consider the nonuniversality of the two Higgs doublet mass parameters [27]. In this case one finds that the value of  $\mu$  needs to be adjusted to achieve the correct electroweak symmetry breaking as described by the (tree-level) expression

$$\frac{1}{2}M_Z^2 = \frac{m_{H_1}^2 - m_{H_2}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. \quad (12)$$

The charged-Higgs mass can be increased or decreased, but since  $\mu$  is large for small  $\tan \beta$ , the mass is guaranteed to be large as well. Consequently the impact on the  $b \rightarrow s\gamma$  rate is small. In some small regions of parameter space, the mixing between the top squarks can



be enhanced. However the overall qualitative behaviour depicted in Figures 1 and 2 is not changed substantially.

The effects of the nonuniversality of the symmetry breaking terms on  $B(b \rightarrow s\gamma)$  in the low  $\tan\beta$  region can be summarized as follows: (1) There is little dependence on the chargino mass and couplings in the low- $\tan\beta$  fixed-point region since  $\mu$  is large and dominates the chargino mass matrix. Therefore the mass of the lightest chargino is approximately  $M_2 = \frac{\alpha_2}{\alpha_G} m_{1/2}$ , and its coupling is predominantly gaugino. (2) Consequently, the dominant effect comes from the top squark masses and mixing. Typically increasing  $m_0^{\tilde{f}}$  relative to  $m_0^H$  increases the top squark mass and pushes the  $b \rightarrow s\gamma$  rate closer to the Standard Model value. (3) The dependence on  $A^G$  is minimal near the top Yukawa coupling fixed point because the low energy value  $A_t$  is driven to a fixed value [14,16]. It has pronounced effects only in regions where there is large top squark mixing. This results in the lightest top squark mass being suppressed (below the experimental limit and one is even in danger of the mass-squared going negative) and an enhancement of the chargino contribution to the  $b \rightarrow s\gamma$  inclusive rate.

To summarize, we have evaluated the inclusive decay branching fraction  $B(b \rightarrow s\gamma)$  through the allowed regions of SUSY-GUT parameters, for solutions with small  $\tan\beta$  near the  $\lambda_t$  fixed point, following Ref. [10]. Our results show that

- (a) In the low  $\tan\beta$  fixed point region, the inclusive rate for  $b \rightarrow s\gamma$  is typically close to that expected in the Standard Model. It is larger (smaller) than the Standard Model value for  $\mu > 0$  ( $\mu < 0$ ) when the difference is substantial.
- (b) An accurate measurement of  $B(b \rightarrow s\gamma)$  must be coupled with reduced theoretical uncertainties in order to constrain the parameters  $m_{1/2}$  and  $m_0$ , or to test the universality of the soft-supersymmetry breaking parameters.
- (c) The contours in Figure 3 illustrate the kind of constraints that may be expected.

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## FIGURES

1. Predicted values of  $B(b \rightarrow s\gamma)$  (a) versus  $m_0$  for fixed values of  $m_{1/2}$  and (b) versus  $m_{1/2}$  for fixed values of  $m_0$ , in the low  $\tan\beta$  SUSY-GUT solutions of Ref. [10] with  $\mu > 0$ . The curves are shown in intervals of 10 GeV in  $m_0$  and  $m_{1/2}$ ; each curve in (a) corresponds to a vertical line in (b).
2. Predicted values of  $B(b \rightarrow s\gamma)$  (a) versus  $m_0$  for fixed values of  $m_{1/2}$  and (b) versus  $m_{1/2}$  for fixed values of  $m_0$ , in the low  $\tan\beta$  SUSY-GUT solutions of Ref. [10] with  $\mu < 0$ . The cross indicates the point at which the lighter top squark mass-squared becomes negative.
3. Contours of fixed  $B(b \rightarrow s\gamma)$  in the  $(m_{1/2}, m_0)$  parameter plane, for (a)  $\mu > 0$  and (b)  $\mu < 0$ . The shaded areas are allowed by direct experimental and naturalness constraints [10].
4. Predicted values of  $B(b \rightarrow s\gamma)$  for  $\mu > 0$  (a) versus  $m_0^{\tilde{f}} = 2m_0^H$  for fixed values of  $m_{1/2}$  and (b) versus  $m_{1/2}$  for fixed values of  $m_0^H$ .
5. Predicted values of  $B(b \rightarrow s\gamma)$  for  $\mu < 0$  (a) versus  $m_0^{\tilde{f}} = 2m_0^H$  for fixed values of  $m_{1/2}$  and (b) versus  $m_{1/2}$  for fixed values of  $m_0^H$ .

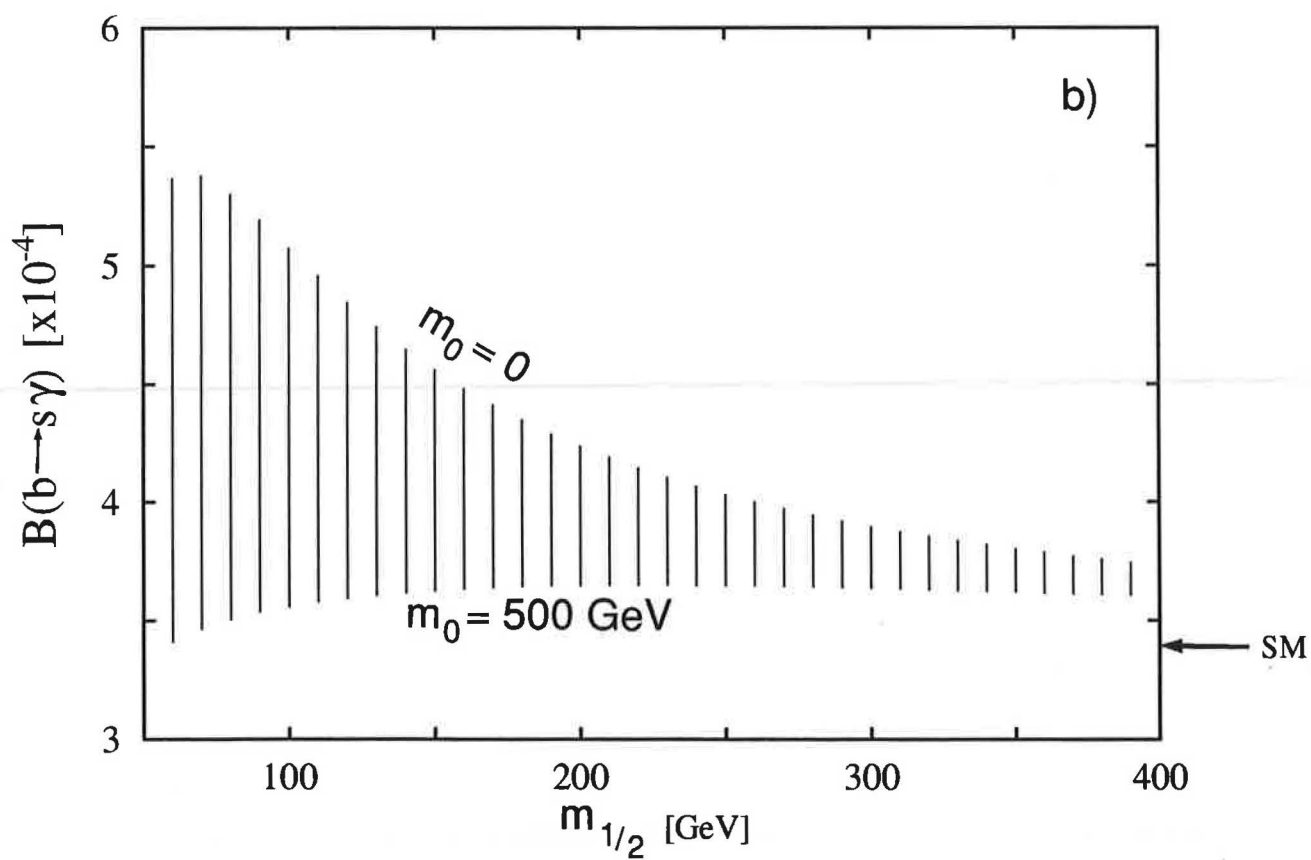
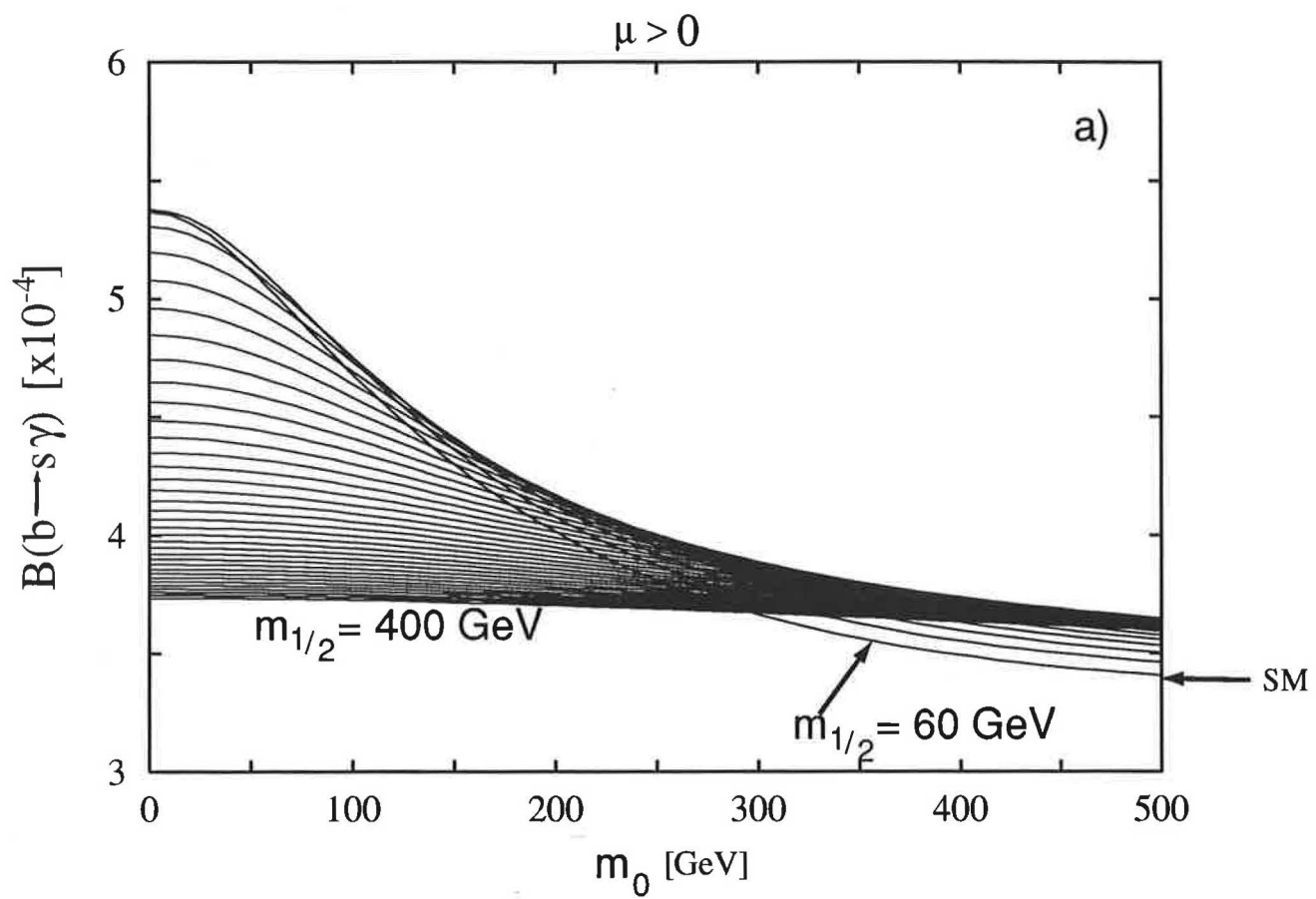


Figure 1

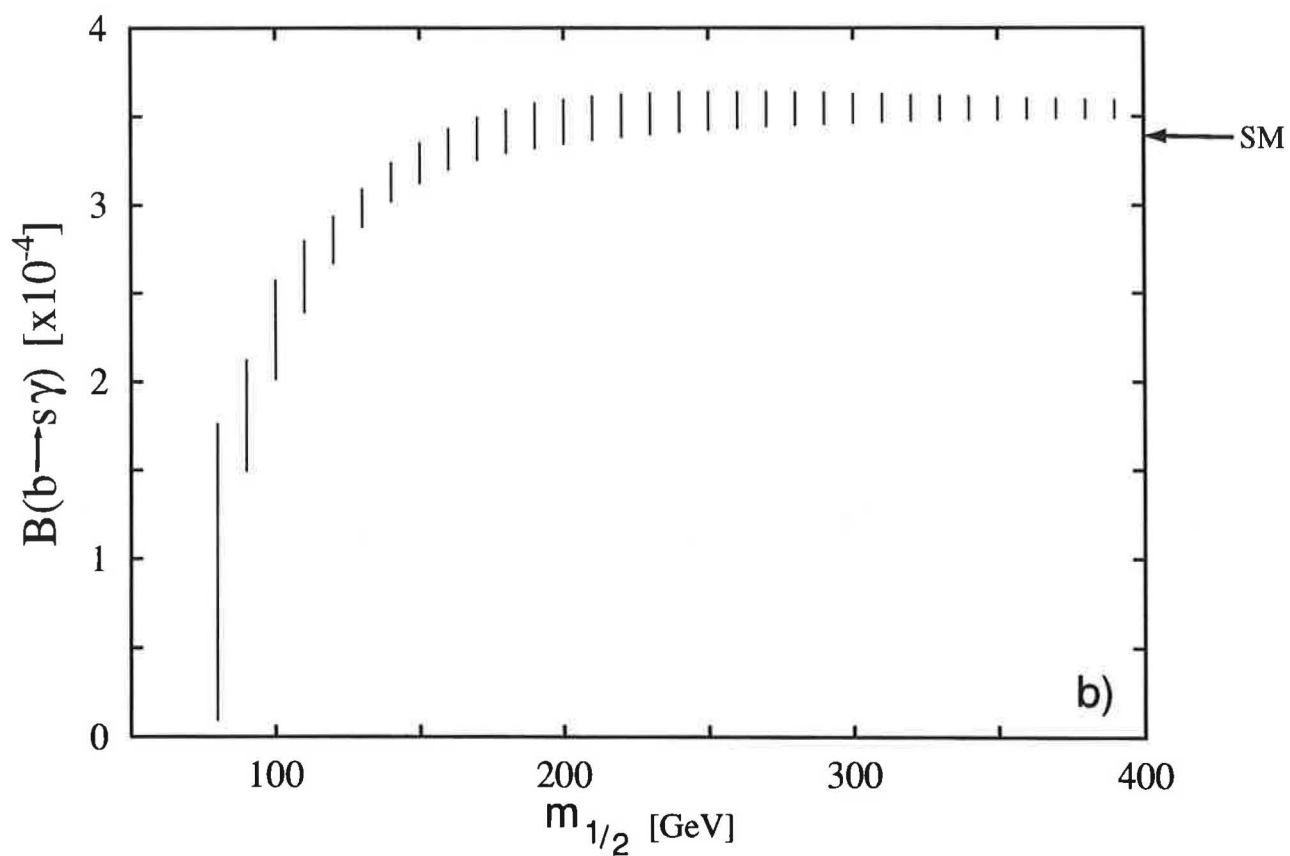
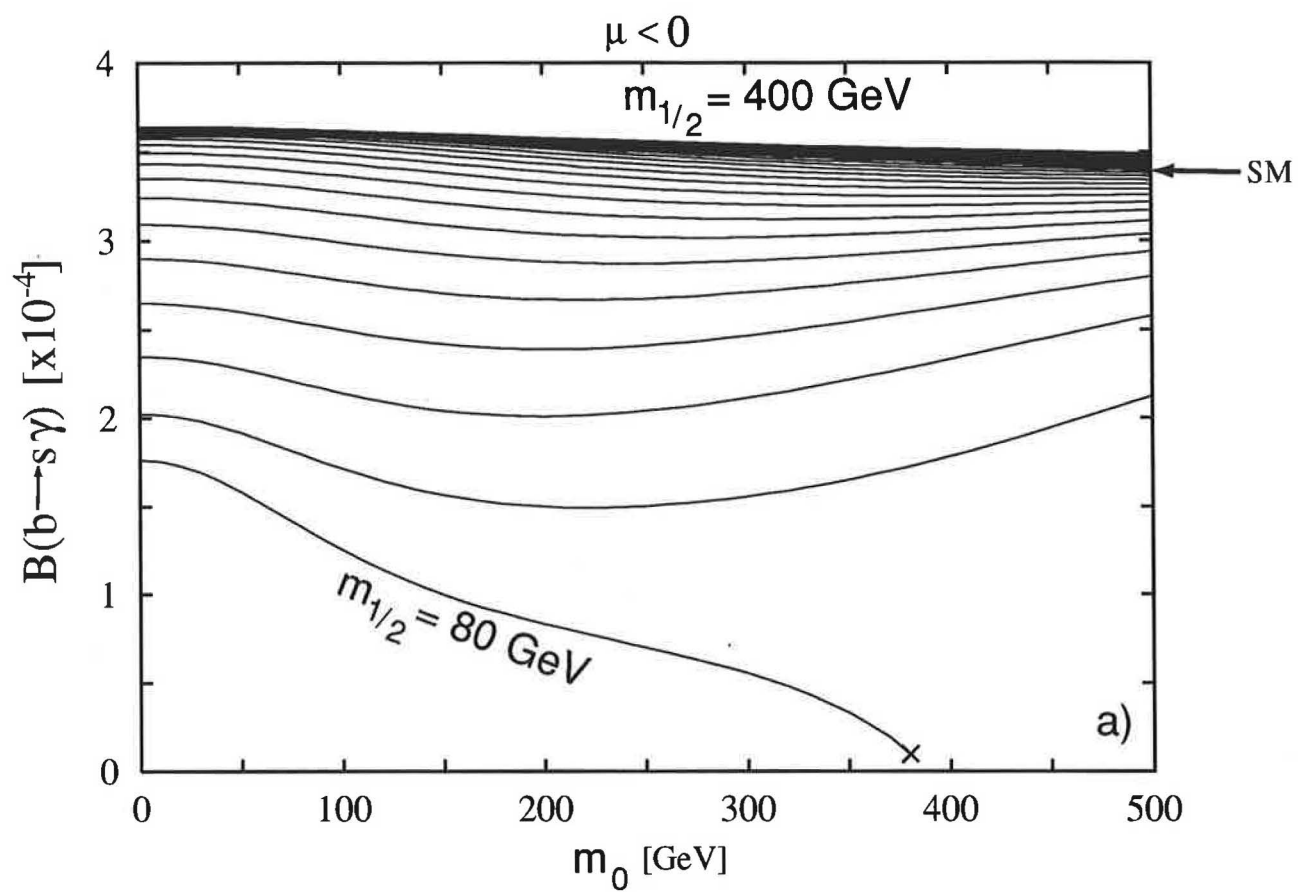


Figure 2



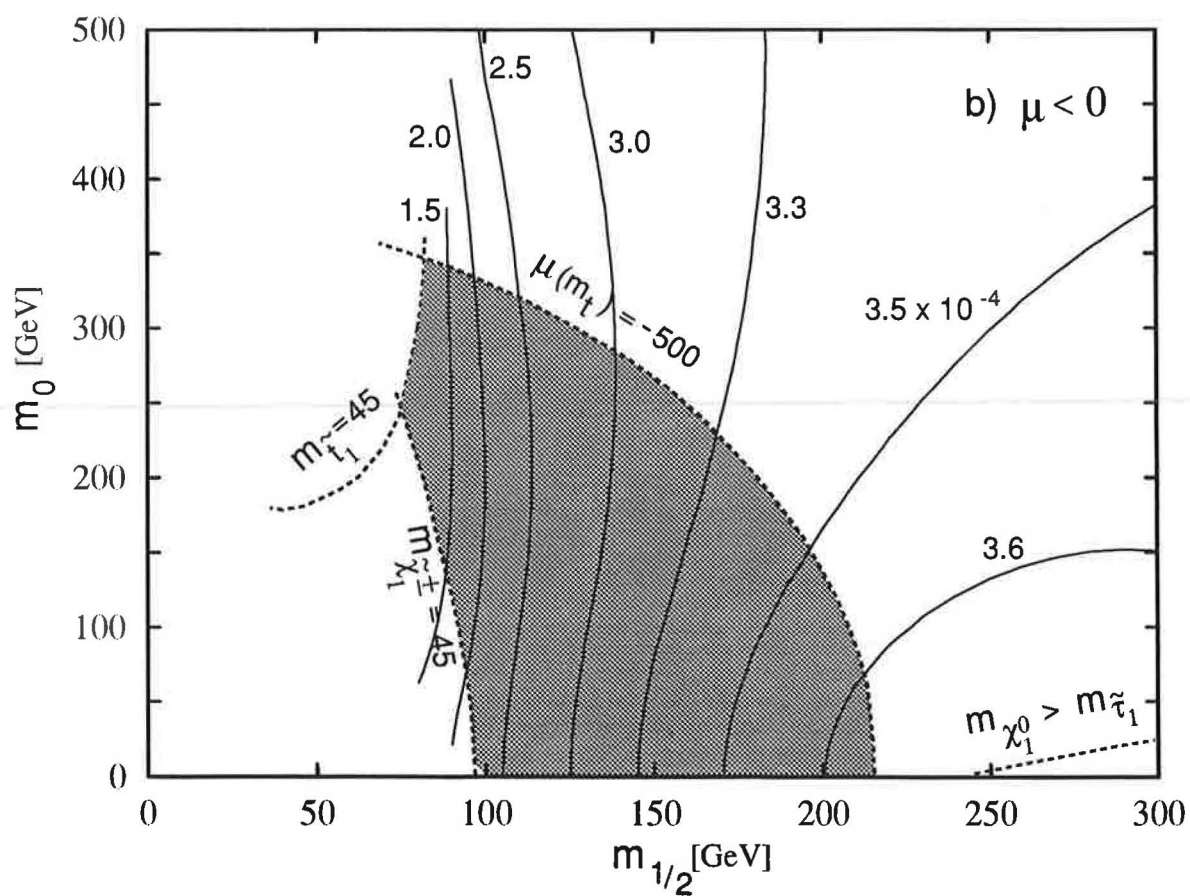
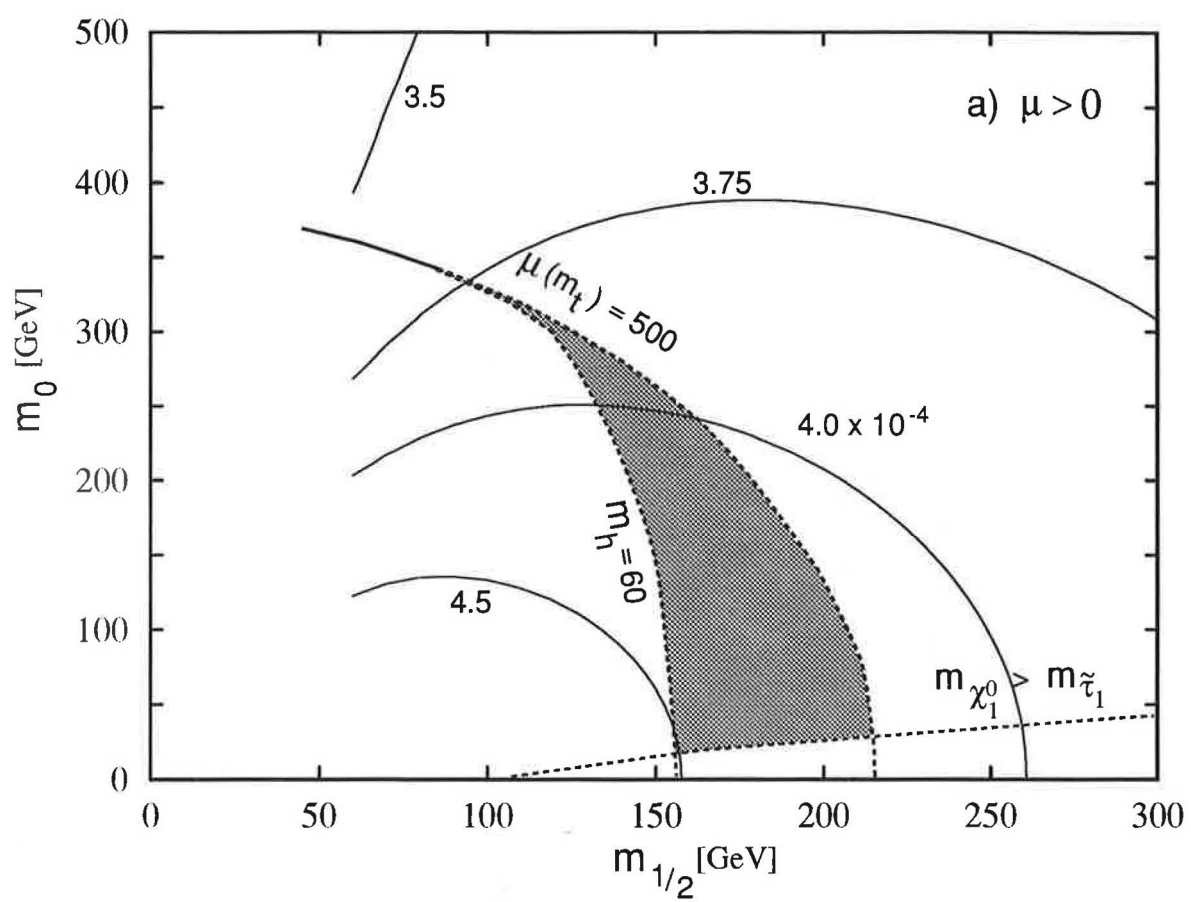


Figure 3

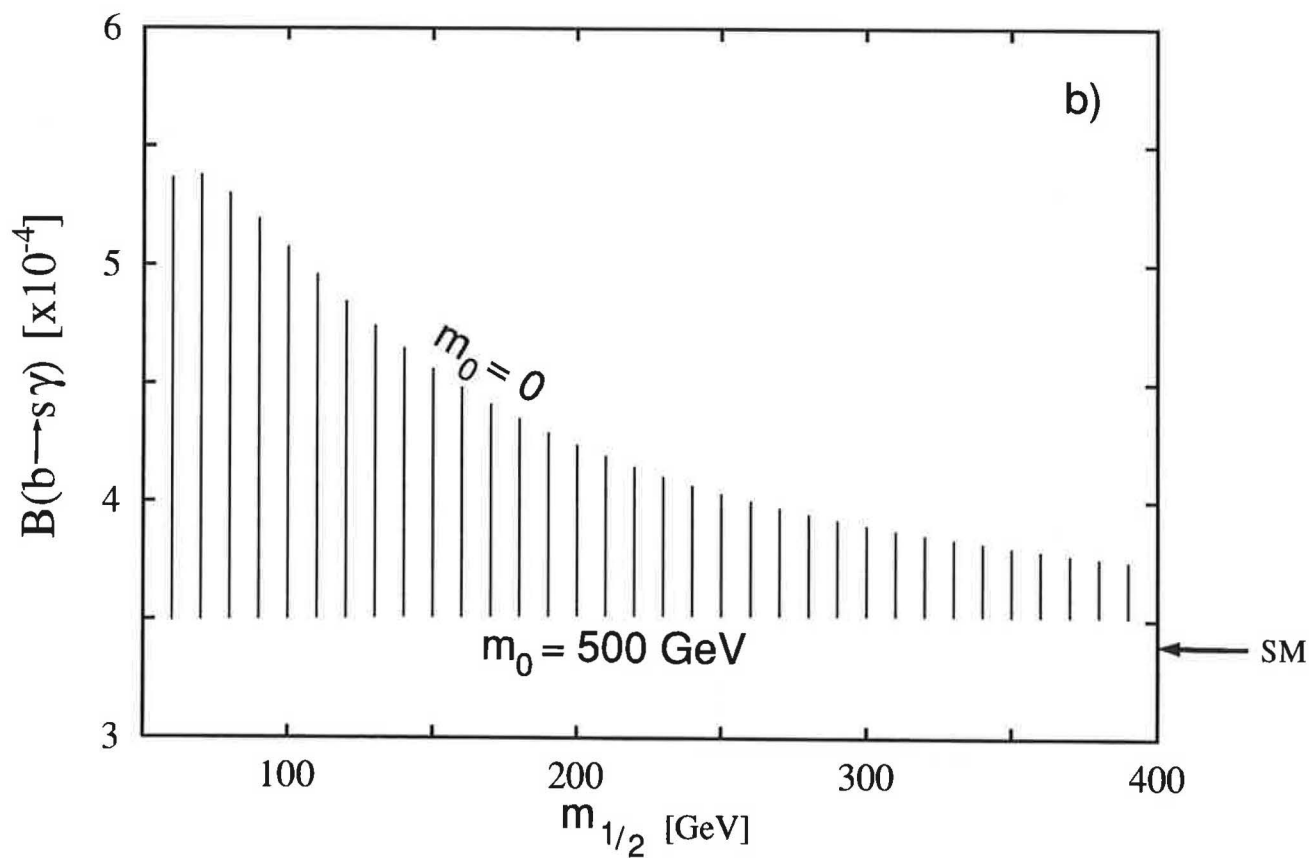
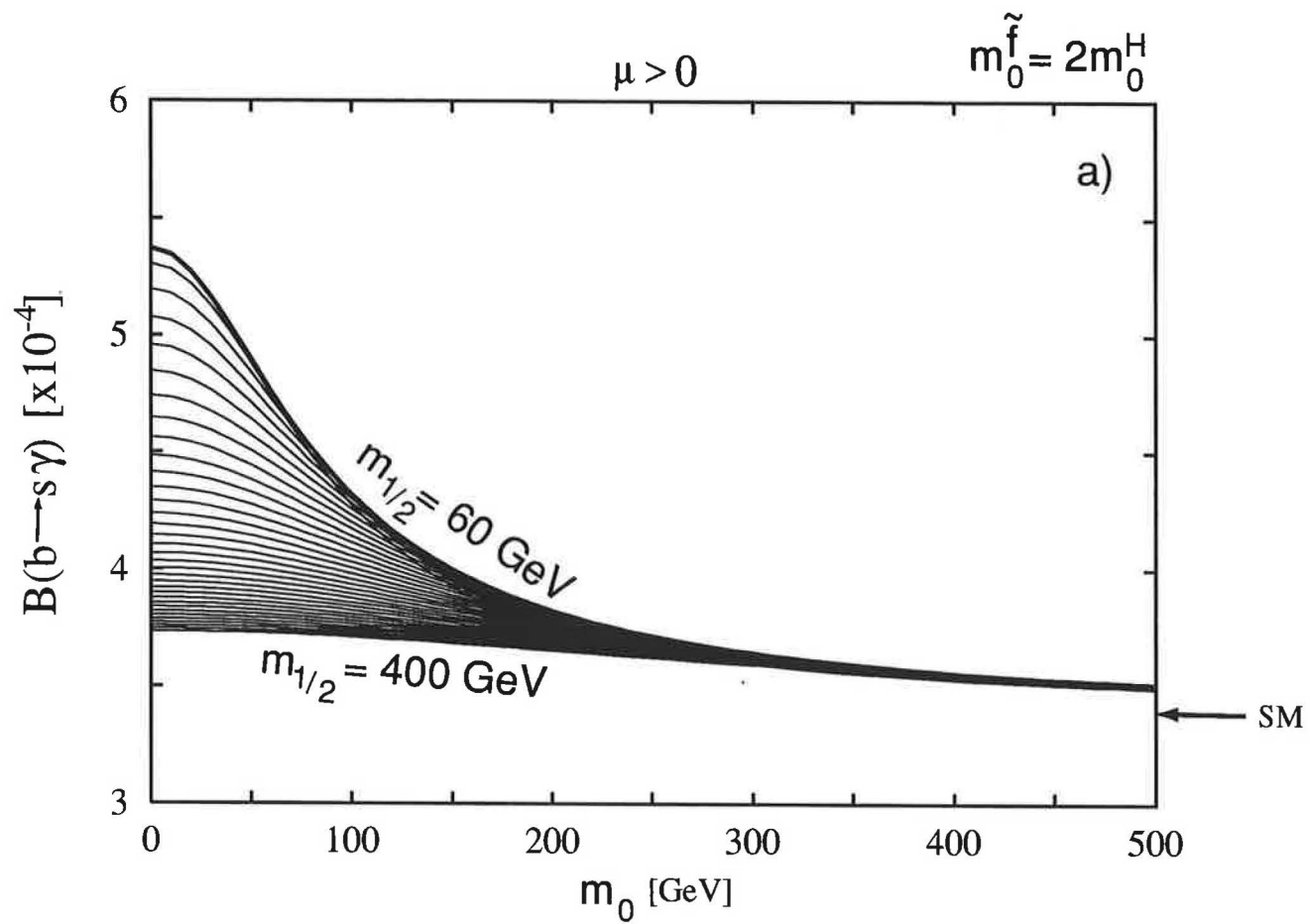


Figure 4

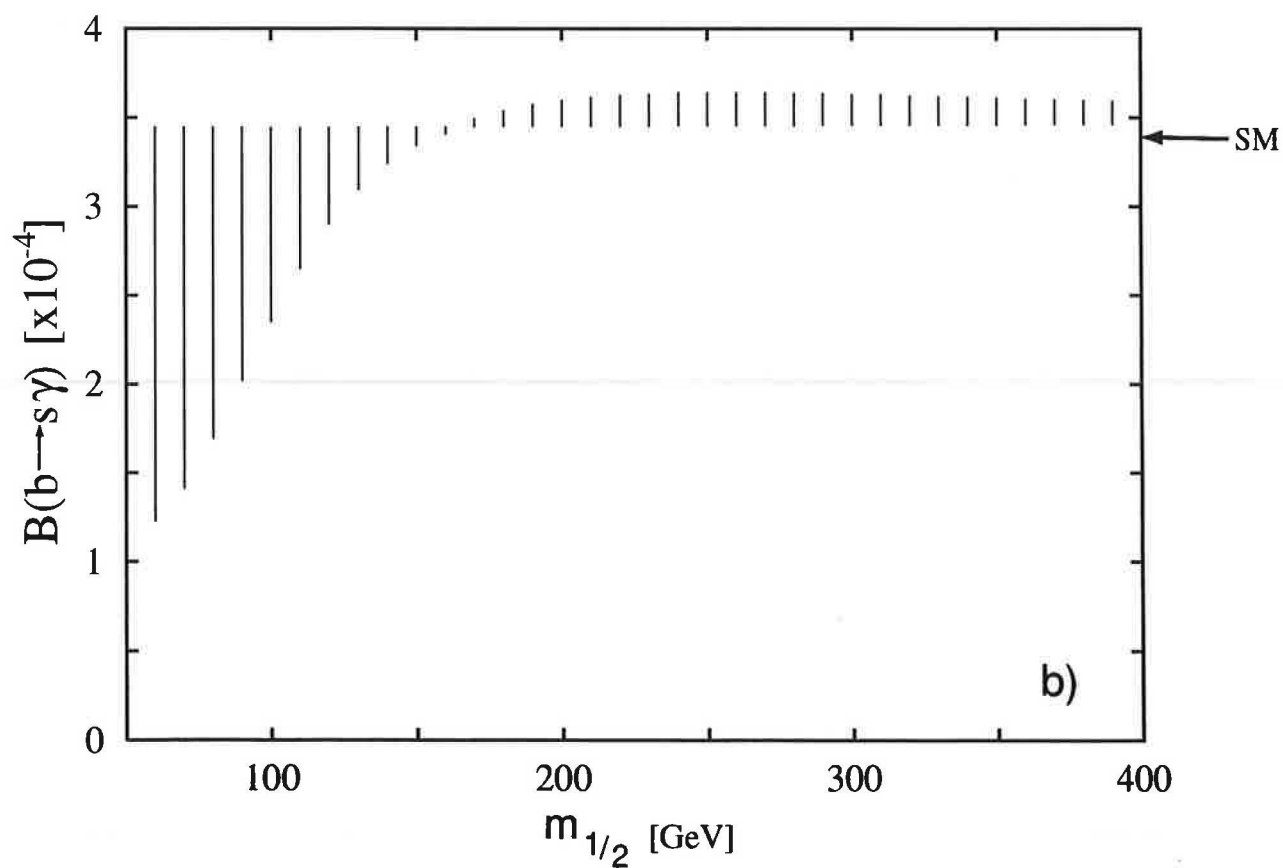
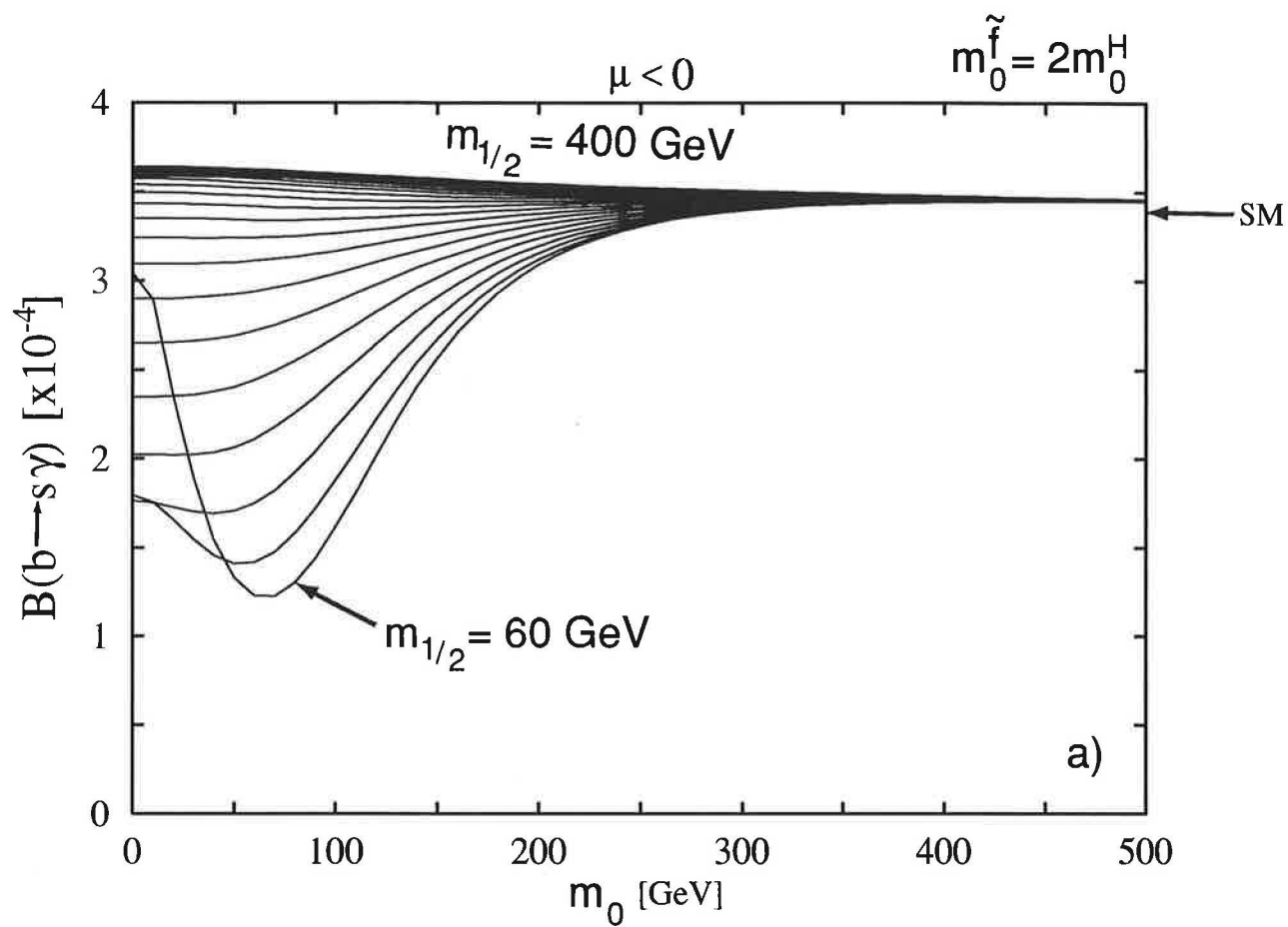


Figure 5





