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RAL Report
RAL-94-074

Where Are We Going With Bose-Einstein - a Mini Review

S Haywood

July 1994

Rutherford Appleton Laboratory Chilton DIDCOT Oxfordshire OX11 0QX

June 17, 1994

Where are we going with Bose-Einstein - a Mini Review

Stephen Haywood - RAL

Abstract

In this paper, I consider the progress (or otherwise) in understanding the Bose-Einstein effect in the context of high energy e^+e^- interactions. The agreement between published experimental results is poor and there are difficulties in explaining the observations theoretically, especially in the light of resonance narrowing. Systems of more than two pions are investigated using event-weights, and the expected distortion of the ρ^0 is considered.

This paper is written from the perspective of an experimentalist. It seeks to highlight the experimental problems and draw attention to relevant theoretical literature.

1 Introduction

The Bose-Einstein effect corresponds to an enhancement in the production rate of identical bosons, emitted from similar regions of space and time, arising from the imposition of Bose symmetry. The effect has been studied in High Energy Physics for thirty years [1], with the original aim being to understand the space-time distribution of the sources of particles produced in high energy collisions. This is still relevant when studying regimes where collective effects are important and a semi-classical picture of particle sources is appropriate, for example in nucleus-nucleus collisions. However, in the aftermath of the string model, this is of less interest for high energy collisions of electrons or protons, as will be seen in section 2.

Enhancements in the mass spectrum of same-sign pion pairs are readily seen in e^+e^- data (for example, [2, 3, 4, 5, 6, 7]) and easily described by very simple models. Further, these enhancements can be explained qualitatively in the context of the string model. However, to describe the observations quantitatively is non-trivial, and the more sophistication used, the more difficult it seems to become to understand the effect. Further there are discrepancies between different experimental measurements which make the analysis even more unsure.

Experiments at PEP and PETRA made basic measurements associated with the Bose-Einstein effect. These measurements were followed by various theoretical papers. However, the LEP experiments, rather than building on this theoretical insight returned to making more basic measurements, albeit with improved statistics.

One may well ask "Why continue to study the effect ?" For many studies it is important to understand the distributions of particles in jets, and these are influenced by the symmetry imposed at production. Secondly, the composition of jets needs to be understood, and as will be discussed, the Bose-Einstein effect can distort the line-shape of broad resonances making the measurement of their production rates difficult.

This paper is organised as follows: in section 2, a short review of theoretical ideas behind the Bose-Einstein effect is given. Reference is made to papers which I have found useful. In section 3, attention is drawn to some of the most important considerations, both experimental and theoretical, which have gone into the derivation of the published results. The results from several experiments are contrasted in section 4. The remaining sections correspond to Monte-Carlo studies which I have performed. In section 5, the use of the JETSET program to simulate the Bose-Einstein effect is discussed. The effect of resonances on the observable Bose-Einstein enhancement is considered in section 6. In section 7, the use of event-weights is investigated in order to examine the effect of many pions on the observable enhancement. These methods are subsequently used to look at the effect of Bose symmetry on the ρ^0 resonance in section 8.

Having read this paper, the reader should appreciate that all is not well with our studies of the Bose-Einstein effect in e^+e^-. In 1990, Goldhaber [8] stated:

> "What is clear is that we have been working on this effect for thirty years. What is not as clear is that we have come that much closer to a precise understanding of the effect."

1.1 The Scope of this Paper

Traditionally the Bose-Einstein effect has been studied by looking at charged pions or charged kaons. Neutral pions are rather more difficult to study due to the difficulty of reconstructing the π^0's from the daughter photons, while the photons themselves in hadronic environments do not yield measurable enhancements due to the relatively long lifetime of the pseudoscalar mesons (see section 3.3).

In what follows in this paper, all discussion will be limited to the studies of charged pions seen in hadronic processes in high energy e^+e^- collisions, as seen at PEP, PETRA or LEP.

Throughout, the JETSET 7.3 Monte-Carlo generator [9] has been used. Effects of final-state interactions are not explicitly included and all the studies which I have performed use only the charged pions which are created, but not the other stable particles. Therefore the distributions which are created correspond to the purity corrected[1] data, as plotted by many experiments. The preferred JETSET parameters which I have used are given in appendix A.

While JETSET on the whole does quite well at describing many of the features of e^+e^- data, such as single particle spectra, it cannot be taken for granted that it will provide an adequate description of more complex distributions corresponding to many particles. Therefore any distributions which rely on the correct relationships between the momenta of several particles will have some inevitable uncertainties.

2 The Physics of the Bose-Einstein effect

2.1 The Semi-classical model

In a semi-classical model of particle production, particles are emitted from a set of sources distributed in space-time with a distribution $\rho(x)$. If the particles are **identical**, then it is necessary to impose Bose symmetry on the production amplitude. This leads to an interference term in the amplitude-squared which is observable only if the sources are **incoherent**. For more details, the reader is referred to the literature, for example [10].

If two identical particles with 4-momenta p_1 and p_2 are observed in the pair rest-frame, then the enhancement due to the interference relative to the rate with no interference is

$$B(q) = 1 + \lambda|\tilde{\rho}(q)|^2 \tag{1}$$

where $\tilde{\rho}$ is the Fourier transform of $\rho(x)$ and λ is a measure of the **incoherence** of the source: $\lambda = 1$ for complete incoherence and $\lambda = 0$ for complete coherence. ρ is normalised such that $\tilde{\rho}(0) = \int \rho(x) d^4x = 1$. For this formulation to be valid, it is assumed that there is no correlation between the momentum distribution of the pions and the positions of their sources.

One of the first models for a source which was used was that of a sphere of emitters with a Gaussian density, described by a radius parameter σ:

$$\rho(x) = \rho(0)e^{-\frac{x^2}{2\sigma^2}} \tag{2}$$

While this simple picture is quite naive when applied to a high energy e^+e^- collision, it works surprisingly well and provides a useful parameterisation of the measurements. This is helpful when it comes to comparing results from different experiments. Such a source leads to an enhancement:

$$B(q) = 1 + \lambda e^{-\sigma^2 Q^2} \tag{3}$$

where $Q^2 = -q^2 = m^2 - 4m_\pi^2$. m is the pair mass, and so the Q distribution is closely related to the pair mass spectrum. σ measures the source size in fm; alternatively σ^{-1} measures the range of the enhancement in Q and is given in GeV.

[1] *Purity corrected* pertains to correcting the measured distributions to correspond directly with what would be expected if one species of particle could be examined alone - see section 3.2.

2.2 The String Model

The string model of fragmentation [11] has proved very successful in describing the distributions of particles seen in high energy e^+e^- collisions. Bowler [10] used the space-time structure inherent in the Artru-Mennessier model [12] to obtain an expression for the enhancement of identical particles. Andersson and Hofmann [13] showed how this enhancement could be described in terms of string amplitudes. These formulations describe the measured data as well as any semi-classical models in the variable Q. However, due to the implicit $1+1$ dimensional nature of the string,[2] the complete description of the Bose-Einstein enhancement as a function of the 4-momentum difference q is less satisfactory [10, 4, 6].

In the string model, there are no explicit spatial dimensions of the flux-tube which appear. Instead the length scales which can be measured, corresponding to the extent of the Bose-Einstein correlations seen as a function of Q, are related to the string parameters, in particular, the *string tension*. The amplitude for a set of particles of given momentum and produced in a given order along the string is related to the area of the *world sheet* spanned by the string in space-time. Hence the Bose-Einstein enhancement between a pair of identical particles with the same momentum arises because the configuration in space-time is unchanged when the two particles are exchanged. Incoherence of the "sources" arises naturally from the sum over many different configurations, each with a slightly different phase. The reader can learn more of this from the earlier references and [14].

Fragmentation of the string occurs after a time corresponding to about 1 fm. In this time, the quarks at either end have separated by 10's of fermi's, owing to the substantial Lorentz boost which they acquire in high energy e^+e^- collisions. Therefore one might naively think that the source represented by the string would be substantially elongated along the jet axis. This is not the case as Andersson and Hofmann pointed out [13] and Hofmann subsequently amplified [15]. In the string model, configurations which have small areas spanned by the world sheet are enhanced. This leads to a longitudinal-momentum ordering of particles as a function of distance along the string: particles produced at the ends tending to have higher momentum than those coming from the centre. This violates the assumption given in the previous subsection that there is no correlation between the positions of sources and the momenta of particles coming from them. Since the Bose-Einstein effect is only apparent for identical particles with *similar* momenta, particles separated by many fermi at production are incapable of exhibiting a significant enhancement because their momenta are so different.[3] It transpires that measurements are sensitive to local volumes within the string flux-tube whose dimensions are of the order of a fermi both parallel and transverse to the jet axis. This agrees well with the experimental observations that the "sources" appear to be approximately spherical [2, 3, 4, 6].

While the string model provides an appropriate framework in which to consider Bose-Einstein enhancements, the associated calculations are quite complex and most auxiliary calculations to allow for other effects have been performed in the context of the simpler semi-classical model with source densities. For these reasons, little reference will be made to the string model in what follows and Gaussian sources will be used for Monte-Carlo calculations.

3 Important Considerations when Making Measurements

Before looking at some experimental measurements, it is important to review some of the considerations which affect the results and their interpretation. The first aspect considered is a matter of experimental method, while those following concern corrections to the data and rely on a good description of particle distributions in the Monte-Carlo used.

[2] The string is described in the spatial dimension along the jet axis and in time.

[3] This argument is semi-classical and corresponds to the notion that phase information is related to the product of the momentum and position 4-vectors: $p \cdot x$.

3.1 Reference Sets

Obviously to demonstrate an *enhancement* in Q amongst identical particles, it is necessary to have a reference with which to compare the data. Ideally, a reference set should contain all the features which are present in the distributions for the identical pairs, **except** of course the Bose symmetry. Further, it should not contain additional features. Since pions are the particles most copiously produced in hadronic interactions, it is natural to look for the effects of Bose symmetry amongst **same-sign** pion pairs (see figure 4). Therefore the obvious reference set is provided by **opposite-sign** pion pairs. Unfortunately the Q distribution of the opposite-sign pairs includes peaks due to neutral meson resonances; exhibits a slow rise with Q relative to the same-sign pairs arising from simple counting of pairs and charge conservation; and suffers from other dynamical effects, including local charge ordering.

An alternative reference set can be derived using the method of **event-mixing**, whereby particles observed in one event can be combined with particles from another event with similar topology and orientation. For events with high centre-of-mass energies, where two-jet topologies tend to dominate, particles from opposite hemispheres can be combined. This is done by reflecting particle momenta: $\boldsymbol{p} \rightarrow -\boldsymbol{p}$. The second approach is valid provided the pair masses are typically much greater than those influenced by the Bose-Einstein effect. It has been used for the work presented here since it is simpler to implement. Using this reference set, some correlations are lost because of the additional gluon radiation causing jets no longer to be collinear.

A third type of reference set can be derived from Monte-Carlo generators such as JETSET. However this relies on a correct simulation of the physics **in the complete absence of any Bose symmetry** and a correct simulation of apparatus effects. In practice, it is more useful to use simulated data to correct both the same-sign and the reference data for the imperfections mentioned above and the effects of acceptance.

The usual method to measure the Bose-Einstein enhancement is thus to measure the cross-section as a function of Q for same-sign pairs $\pi^+\pi^+$ and $\pi^-\pi^-$ (denoted $++$) and compare with the opposite-sign pairs $\pi^+\pi^-$ (denoted $+-$) or mixed pairs (denoted mix) (see figure 4). This is done both for the data and Monte-Carlo and the double ratios are formed:

$$r(data)_{+-} = \frac{d\sigma(data)_{++}}{dQ} / \frac{d\sigma(data)_{+-}}{dQ} \qquad r(data)_{mix} = \frac{d\sigma(data)_{++}}{dQ} / \frac{d\sigma(data)_{mix}}{dQ} \tag{4}$$

and

$$R_{+-} = r(data)_{+-}/r(MC)_{+-} \qquad R_{mix} = r(data)_{mix}/r(MC)_{mix} \tag{5}$$

3.2 Purity

The Bose-Einstein effect is manifested only for identical particles. Some experimental analyses have explicitly attempted to identify specific types of particles by the use of ionisation, for example [17], while others have simply used all charged particles with the understanding that pions dominate the samples. Even when particle identification is made, it results in some contamination, and in both cases, it is necessary to correct for impurities amongst the pairs, since pairs consisting of distinguishable particles (for example, π^+K^+) will not exhibit an enhancement. When no form of identification is performed, the enhancement $(R-1)$ for pions is reduced by the order of 25%, dependent on experimental cuts. Most of the experimental data is corrected as a function of Q for purity.

3.3 The Origin of Pions

In principle, all pairs of identical pions can exhibit the Bose-Einstein effect. However, since the measurements in Q are inversely related to the spatial dimensions of sources, measurements corresponding to large dimensions are at low Q. As Q tends to zero, the phase space of particle pairs vanishes, causing the statistical errors on R to become worse at small Q (see figure 4). Further, in this region, corrections due to final-state interactions (see next subsection) become large, as do the corrections for the efficiency, which vanishes due to cuts to remove overlapping of tracks in detectors. These effects are more significant than the resolution on Q which can be obtained with the LEP detectors and place an upper limit of a few fermi on the spatial dimensions which can be measured.

Many of the pions produced in e^+e^- collisions have their origins in other particles which have decay lengths significantly greater than a few fermi. The parents of pions can be classified as follows:

- **Stable particles** (such as Λ and K^0_s) which travel sufficiently far that their daughters may be removed by track cuts.
- **Weakly decaying particles** (such as charm or beauty mesons)
- **Resonances** (such as $\eta, \eta', \omega, \phi$) which decay strongly but have decay lengths of many fermi.

Finally there are the pions coming from **short-lived** sources ($c\tau < 10$ fm) which can contribute measurably to the Bose-Einstein effect. These include pions coming from the string itself as well as pions coming from ρ's, Δ's and K^*'s.

The fractions of pions ($fract$) coming from these different sources, *as determined with* JETSET, are shown in table 1. The stable particles have not been decayed, and therefore do not represent a source of pions. The cumulative sum squared ($(\Sigma fract)^2$) gives a measure of the number of pairs of pions where both pions come from that source or one which is shorter-lived. Similar information is presented as a function of Q in figure 5. No momentum thresholds are used.

origin	$c\tau$ (fm)	fract	Σfract	$(\Sigma\text{fract})^2$
string	0.0	0.18	0.18	0.03
ρ	1.3	0.34	0.52	0.28
Δ	1.7	0.03	0.56	0.31
K^*	4.0	0.09	0.65	0.42
strong	>10.0	0.27	0.92	0.85
weak		0.08	1.00	1.00
Effective radius measured for string: 0.5-1.0 fm				

Table 1: Sources of pions.

Pions originating from long-lived sources do not exhibit a **measurable** Bose-Einstein enhancement.[4] On the face of it, this would appear to be because such pions effectively come from extended sources whose dimensions exceed a few fermi's. However, since the Bose-Einstein effect corresponds to an interference between the phases of the pion wave-functions, and the decay of particles is a coherent process, preserving the phase from the point of production at the original source, this cannot be the explanation [19]. Rather, the suppression of the Bose-Einstein effect arises from the effect of symmetrising the Breit-Wigner function - this is examined more in section 6. Another way of looking at this is as follows. The Bose-Einstein effect is exhibited by **indistinguishable** particles. However, if one considers the production of particles by a resonance, and one combination π^+x (where x is the other daughter) is compatible with the resonance

[4] Strictly speaking, it is pion pairs where the production points of the pions are separated by many fermi which cannot exhibit a measurable Bose-Einstein enhancement. In principle, identical pions originating from the same long-lived mother without going through intermediate states could interfere measurably with each other, but not with other pions. However, the number of pairs of this type is much less than those coming from short-lived sources.

parameters ($\Delta m \leq \Gamma_0$), while a second combination is not, then it is possible to distinguish between the different π^+'s, and it is no longer appropriate to consider their exchange.

Because a large fraction of pions are unable to exhibit the Bose-Einstein interference, the size of the enhancement which is measured is reduced. Generally, experiments have not corrected their results to allow for this *dilution* and it is necessary to compare the measured values of λ with the maximum values λ_{max} which could be observed if all pions from short-lived sources interfered with $\lambda = 1$. λ_{max} corresponds to the fraction of pairs which can be formed where both pions come from short-lived sources.

3.4 Final-state Interactions

Charged pions exhibit both the **Coulomb** and the **Strong Interaction**. The strength and sign of the interactions are not the same for the three types of dataset: $++, +-$ and *mix*. All experiments have tried to correct for the Coulomb interaction, although this has been done incorrectly as was pointed out by Bowler [18]. In principle the Coulomb correction should be applied only to particles originating from similar regions of space-time. These particles are essentially the same as the ones originating from the *short-lived* sources discussed in the previous subsection. Since such particles cannot be identified explicitly in the data, a correction which is correlated to the *origin* of particles should be derived from Monte-Carlo. This turns out to be far from trivial.

The exact form of the distortion to the Q distribution for same-sign pairs caused by the Strong Interaction is unknown, however earlier estimates of its magnitude by Suzuki [20] were around 10-20%. More recent work by Bowler [19] has suggested that the effect of repulsive interactions between same-sign pions may be cancelled out by the effect of attractive interactions between opposite-sign pions. The Strong Interaction is not incorporated explicitly in Monte-Carlo generators.

4 Measurements at e^+e^- Colliders

In table 2, comparison is made between the measured values of λ and σ obtained by: TPC (PEP) [2], MARK II (PEP) [3], TASSO (PETRA) [4], OPAL (LEP) [5], ALEPH (LEP) [6] and DELPHI (LEP) [7]. For the most part, these values correspond to fits made to the distributions $R_{+-}(Q)$ and $R_{mix}(Q)$ - corrected for purity and the Coulomb interaction, and normalised by Monte-Carlo data. All experiments have made cuts to select good hadronic events using well measured charged tracks. A selection of additional cuts made by some of the experiments which have some bearing on the measurements (especially before acceptance corrections) are indicated in summary. The maximum values of λ, λ_{max}, which could be expected have been determined from JETSET by the individual experiments (not by myself), and these are also shown in the table. However, these values depend on the assumptions as to what will or will not lead to observable enhancements, and some of these assumptions are questionable. The same measurements are also shown in figure 6.

Initial observations which can be made from reading the published analyses are:

- Measured source sizes are of the order of 0.5-1 fm. At least, looking at the measurements using opposite-sign pairs, there is a reasonable agreement between the various parameters. The various systematic corrections have a smaller effect on σ than λ.
- Sources do appear to be roughly spherically symmetric.[5]

[5]This conclusion arises from studies made as a function of $q = (q_0, q_{\parallel}, q_{\perp})$ relative to the jet axis rather than simply $Q = \sqrt{-q^2}$.

Experiment	$\sqrt{s}$ (GeV)	Opposite-sign normalisation σ (fm)	λ	Mixed-event normalisation σ (fm)	λ	λ_{max} (JETSET)
TPC	29			0.65 ± 0.04	0.61 ± 0.05	0.90
Mark II	29	0.84 ± 0.06	0.50 ± 0.03	1.01 ± 0.09	0.45 ± 0.03	0.63
Tasso	34	0.80 ± 0.06	0.35 ± 0.03			0.50
Opal	91	0.90 ± 0.02	0.93 ± 0.05			0.60
Aleph	91	0.80 ± 0.04	0.62 ± 0.04	0.50 ± 0.02	0.40 ± 0.02	0.24
Delphi	91	0.82 ± 0.03	0.45 ± 0.02	0.42 ± 0.04	0.35 ± 0.04	

Notes:	
General	Correct for Purity, Coulomb and normalise by MC. Only statistical errors shown.
TPC	No MC normalisation. λ_{max} only allows for V^0s.
Mark II	No MC normalisation. λ_{max} allows for weak decays, but not long-lived hadrons; could reduce by 35%.
Tasso	No purity or Coulomb corrections. λ_{max} does not allow for long-lived hadrons.
Opal	I have corrected for purity and normalisation by MC from numbers quoted in corresponding reference.
Aleph	Purity corrections removes 25% of pairs. Of the remainder, 27% contain a weak decay pion, 49% contain a long-lived ($c\tau > 10$ fm) hadronic decay pion. If replace π's from η' with π's from string, λ_{max} becomes 0.40. If ignore Coulomb corrections, λ's become 0.53 and 0.37 respectively.

Interesting cuts:	
General	Select good hadronic events.
TPC	sphericity$< 0.25, p < 1.45, \theta_{open} > 3$ to 15^o.
Opal	$p < 10, \theta_{open} > 6^o$.
Aleph	sphericity$< 0.03, p_t^{\perp jet} < 1.2, p < 4.5$, kill close tracks.
Delphi	thrust$> 0.95, p < 5, \theta_{open} > 2^o$.

Table 2: Experimental measurements.

- The distributions in Q are *reasonably* well described (or parameterised) by the Gaussian form (equation 3, corresponding to a Gaussian source density).

These observations justify the use of the simple Gaussian description of the Bose-Einstein enhancement to make comparisons between different measurements.

4.1 Problems of Comparisons

It is not easy to compare the measurements made by different experiments for some of the following reasons:

- The experiments have not used the same corrections in all cases. Further, some corrections are difficulty to apply, while others have been applied incorrectly.
- The observed enhancements depend to some extent on cuts made for the analyses. In particular, daughters of particles such as η' can lead to fake enhancements [2].
- While statistical errors are not a limitation at LEP, systematics are difficult to control or estimate, and can have a very significant effect.

Because the determination of the parameter λ, which measures the enhancement, depends critically on the analysis, it is only meaningful to consider it with respect to the maximum possible value λ_{max}. The ratio of the two can be considered to represent a *fully corrected* measurement of the incoherence parameter. However, this comparison is only meaningful to the extent to which reconstructed Monte-Carlo data describes the measurements.

Even taking into account the above remarks, the discrepancies between the various values of both λ and λ_{max} are alarmingly large. The errors given in table 2 are statistical only. Most experiments which have used both types of reference set have estimated systematic errors from the range of the fitted parameters. However, there remain significant uncertainties from physical processes which are very difficult to estimate.

4.2 Problems Indicated by the Measurements

Apart from the fact that there is little consistency between the measurements made by different experiments, there are some common problems:

- Measurements made by a single experiment have significant differences between the parameters derived using the two types of reference distribution.
- The measurements of λ tend to exceed the maximum expected values λ_{max}. While this is not immediately apparent for the measurements from PEP and PETRA, it should be recognised that the published estimates of λ_{max} do not allow for some significant classes of long-lived sources. As Bowler [21] put it, the problem is not to explain why the value of λ is so small, but rather, why it is so large.
- The measured enhancements (refer to published distributions) tend to be more peaked at low Q than a Gaussian.

Possible explanations for the discrepancies arising from use of different reference sets:

- Overestimation of Coulomb correction - it affects the two methods differently.
- Similarly the Strong Interaction may have different effects on the datasets used in the two methods.
- Effects which arise from particle correlations related to dynamical processes, in particular decays of hadrons, are different for the two reference distributions and may not be corrected or may be over-corrected by Monte-Carlo.

Possible explanations for the surprisingly large values of λ:

- Overestimation of the Coulomb correction. (A correction made for the Strong Interaction, if anything, may reduce the measurement.)
- Overestimation by JETSET of the number of long-lived resonances, in particular the η'. This leads to an estimate of λ_{max} which is too small.
- Fake correlations which arise from particle decays, and which are not described and hence corrected adequately by Monte-Carlo.

Improved estimates of the Coulomb correction and η' rate by ALEPH led to estimates of λ which were close to the maximum possible value predicted by JETSET. Confirmation of this comes from a recent Delphi analysis [22], which decreased the η' and ρ^0 production rates, and ignored the Coulomb correction. This resulted in a fully corrected measurement of $\lambda = 1.06 \pm 0.05 \pm 0.16$ using mixed-event normalisation. However, it becomes more difficult to explain the measurements when one considers the effect of resonances with $c\tau < 10$ fm. This is discussed further in section 6.

Possible explanations for the shape of the observed enhancements:

- The string model would suggest a distribution which is sharper at $Q = 0$ [10, 13].
- The superposition of contributions from different sources at low Q also causes the enhancement to pile up at $Q = 0$ - see section 6.

5 Monte-Carlo Simulation

5.1 Implementation within Jetset

Within JETSET, a model of the Bose-Einstein effect has been implemented and may be used optionally by setting flags. The use of the 5 parameters (MSTJ(51), MSTJ(52), PARJ(91), PARJ(92) and PARJ(93)) is described in the JETSET writeup [9], as is some of the methodology. The JETSET implementation is contained entirely in the subroutine LUBOEI. In this section, only those pions which can be influenced by the Bose-Einstein effect are considered: by my choice of PARJ(91) (see Appendix A), this means pions coming from sources with $c\tau < 10$ fm.

The basic idea is that to generate a given distribution in some variable, a simpler distribution may be generated and for each occurrence of the variable, a transformation can be made to give the desired distribution. With the Bose-Einstein option of JETSET, particles are generated with their usual distributions which do not include the Bose-Einstein effect. Then for each pair, the corresponding value of Q is shifted to a new value Q', such that if the new distribution were plotted as a function of Q' and were compared with the original distribution of Q, the result ought to be the desired Bose-Einstein enhancement. Precisely how this shift is made is described below. Since it is particle momenta which are generated, the shift in Q must be made by shifting the momenta. There is no *a priori* way in which this should be done. In the scheme used in LUBOEI, the shift in Q for each pair is calculated and the corresponding shift in the vector momenta of each particle is evaluated. The momentum shifts are made along the line connecting the endpoints of the two momentum vectors and are equal and opposite so as to conserve the overall momentum. For each particle influenced by the Bose-Einstein effect, there are shifts coming from each combination with identical particles. The vector sum of all shifts is made and is applied at the end. This does not quite conserve the overall event energy, and a small rescaling is made. The whole process is carried out in the event rest-frame, so as to provide a well defined procedure (this is the same as the lab-frame in e^+e^-).

This can be summarised schematically:

1. From the 4-momenta p of each pair, Q is formed.
2. A shift is made to Q: $Q \to Q' \equiv Q + \Delta Q$.
3. Corresponding shifts are made to the 3-momenta of the pair: $\boldsymbol{p} \to \boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{\Delta p}$.
4. After all shifts are made, the effective Q can be determined.

As Sjostrand points out, the above is simply a method for obtaining some plausible description of the Bose-Einstein effect in so far as it affects individual particles. As such, it will describe reasonably well certain features of the effect, but some of the consequences of the implementation have to be regarded with care - in particular, secondary effects. Some comments and criticisms follow.

Basic assumption

The basic assumption in the technique of shifting variables is that the the feature being added by the transformation is not already included in any way in the initial distribution, and that this feature is independent of all other features. This is clearly only an approximation for the Bose-Einstein effect, since the symmetrisation of the wave-functions should be imposed right from the start of particle creation and will have consequences which transcend the complete process. This is seen in the way in which some *ad hoc* scheme has to be developed to shift the momenta.

Of particular concern is the fact that the Bose-Einstein effect undeniably tends to bring particles closer together, as is seen with the LUBOEI implementation. However, in practice, the tendency is to determine parameters for Bose-Einstein and then impose the Bose-Einstein effect in the Monte-Carlo on top of the standard event generation. However, this event generation is tuned to reproduce the measured topology of events, which naturally contain the consequences of the symmetrisation already. Therefore there is a tendency to double count the effect with respect to some quantities. A better procedure would be to include Bose-Einstein in the simulation and then retune the Monte-Carlo parameters.

Phase space

While all that is required by transforming Q is to produce a new distribution such that the ratio with the old distribution is the Bose-Einstein enhancement (as parameterised by equation 1), it is actually necessary to have a description of the distribution of Q, namely dN/dQ (see figure 4). This is a non-trivial function and cannot be parameterised in a general way. It is a function of phase-space, kinematic cut-offs and the detailed dynamics of the particle production, all of which will depend on the collision process. Since the Bose-Einstein effect is predominantly in the region of Q where dN/dQ is described by phase-space, the choice is made in LUBOEI to replace the true dN/dQ by phase-space[6] (see figure 4) over the whole Q range. This leads to a dN/dQ which is independent of $\sqrt{s}$. Firstly this approximation is not perfect (the peak of dN/dQ at LEP energies is at $Q = 0.4$ GeV, while the Bose-Einstein enhancement has an inverse length scale around $Q = 0.3$ GeV). Secondly, in principle, the distribution dN/dQ should be normalised so that the integral corresponds to the number of pairs of particles produced. However, with the phase-space description, dN/dQ tends to infinity as Q tends to infinity, and the integral is therefore infinite. So with the adoption of this choice for dN/dQ, it is not possible to normalise dN/dQ'.

Improved formulation

The assumptions concerning phase-space, and consequently the absence of renormalisation, mean that the ratio of the distribution in Q' to that in Q will not be described perfectly by the desired parameterisation of the Bose-Einstein effect. A more complete derivation is as follows.

Let the distribution of pairs be $f(Q) \equiv dN/dQ$, and the transformation is $Q \to Q'$, then

$$dN = f(Q)dQ \propto (1 + b(Q'))f(Q')dQ' \tag{7}$$

where $b(Q')$ is the Bose-Einstein enhancement, for example $\lambda e^{-\sigma^2 Q'^2}$. Integrating the differential equations gives

$$\int_0^Q f(q)dq = \frac{1}{1+\beta}\int_0^{Q'} (1 + b(q))f(q)dq. \tag{8}$$

[6]If two pions in their joint rest-frame have momentum p, then $Q = 2p$ and the Lorentz invariant phase space is

$$d\text{Lips} \propto \frac{d^3p}{E} \propto \frac{Q^2 dQ}{\sqrt{Q^2 + 4m_\pi^2}} \tag{6}$$

The normalisation $1+\beta$ is given by

$$1+\beta \equiv 1+\frac{\int_0^\infty b(q)f(q)dq}{\int_0^\infty f(q)dq}. \tag{9}$$

For example, with $\sigma^{-1} = 0.3$ GeV and $\lambda = 1.0$, $\beta = 8\%$.

Multiplying both sides of equation 8 by $1+\beta$ and expanding the integral into two intervals from 0 to Q and from Q to Q' gives

$$(1+\beta)\int_0^Q f(q)dq = \int_0^Q (1+b(q))f(q)dq + \int_Q^{Q'} (1+b(q))f(q)dq. \tag{10}$$

Subtracting $\int_0^Q f(q)dq$ from both side leads to an exact equation for Q'

$$\int_Q^{Q'} (1+b(q))f(q)dq = \beta\int_0^Q f(q)dq - \int_0^Q b(q)f(q)dq. \tag{11}$$

If the distribution in Q' from this equation is compared with that in Q, the ratio is found to be exactly as expected.

Expanding the left hand side about Q leads to an approximation for ΔQ which is good provided $|Q'-Q| \ll Q$

$$\Delta Q \equiv Q' - Q = \frac{\beta\int_0^Q f(q)dq - \int_0^Q b(q)f(q)dq}{(1+b(Q))f(Q)} \tag{12}$$

By using $Q+\Delta Q/2$ in the denominator, rather than Q, this form can be used iteratively to obtain an improved approximation. It should be noted that for $\lambda \geq 1$, the shifts ΔQ are quite significant for small Q. LUBOEI avoids iteration by providing an approximation to Q' based on the assumption of phase-space.

The form derived here should be contrasted with the form used in LUBOEI

$$\Delta Q \equiv Q' - Q = \frac{-\int_0^Q b(q)f(q)dq}{f(Q)} \tag{13}$$

The assumptions used in LUBOEI lead to $\beta = 0$ and ignore the term $b(Q)$ in the denominator - which is reasonable for $\sigma Q \gg 1$.

In figure 7a, the ratio of the distributions of Q' to Q is shown as solid points. This is obtained by examining the quantities **inside** LUBOEI. The histogram is a plot of the function of equation 3. It can be seen that the measured ratio exceeds the expectation for $Q < 0.1$ GeV and is lower for 0.2 GeV $< Q <$ 0.4 GeV. When the distribution is examined **after** the shifts to the particle momenta, it is found that the enhancement is greatly increased below 0.2 GeV (hollow points in figure 7).

In figure 7b, the approximation derived above has been used along with an approximate parameterisation of dN/dQ. It can be seen that the distribution in Q' is closer to that which is expected. The deviations for $Q > 1$ GeV arise from inadequacies in the parameterisation of dN/dQ. Again, it can be seen that the effect of the momentum shifts is to increase the enhancement. The fact that the enhancement which can be examined as the output of LUBOEI does not equal the input parameterisation may cause concern amongst the unsuspecting. This will be considered more in section 7.

The use of equation 12 is not without problems. Firstly, it is necessary to use a complete parameterisation of the whole of the Q distribution. This is process dependent, and loses the generality of the phase-space description. Secondly, to renormalise the distribution, it is necessary to obtain β by an integral up to infinity. In practice, the integral is performed up to some cutoff, and at this point, there may be small but undesirable discontinuities. Despite these problems, the above formulation has been used in the proceeding work.

Resonances

The fact that pions coming from resonances or long-lived parents only exhibit the Bose-Einstein effect at very low Q, so low that the effect may be unobservable, is handled in LUBOEI by restricting the pions which are subjected to momentum shifts to those whose parents have widths greater than a parameter PARJ(91). This excludes pions which come from particles with long flight lengths (or small widths).

However, for particles which are allowed to participate in the simulation of the Bose-Einstein effect, no allowance is made for the fact that they may come from parents whose flight lengths are not small compared with the effective string dimensions (see table 1).

5.2 Estimation and Use of Parameters

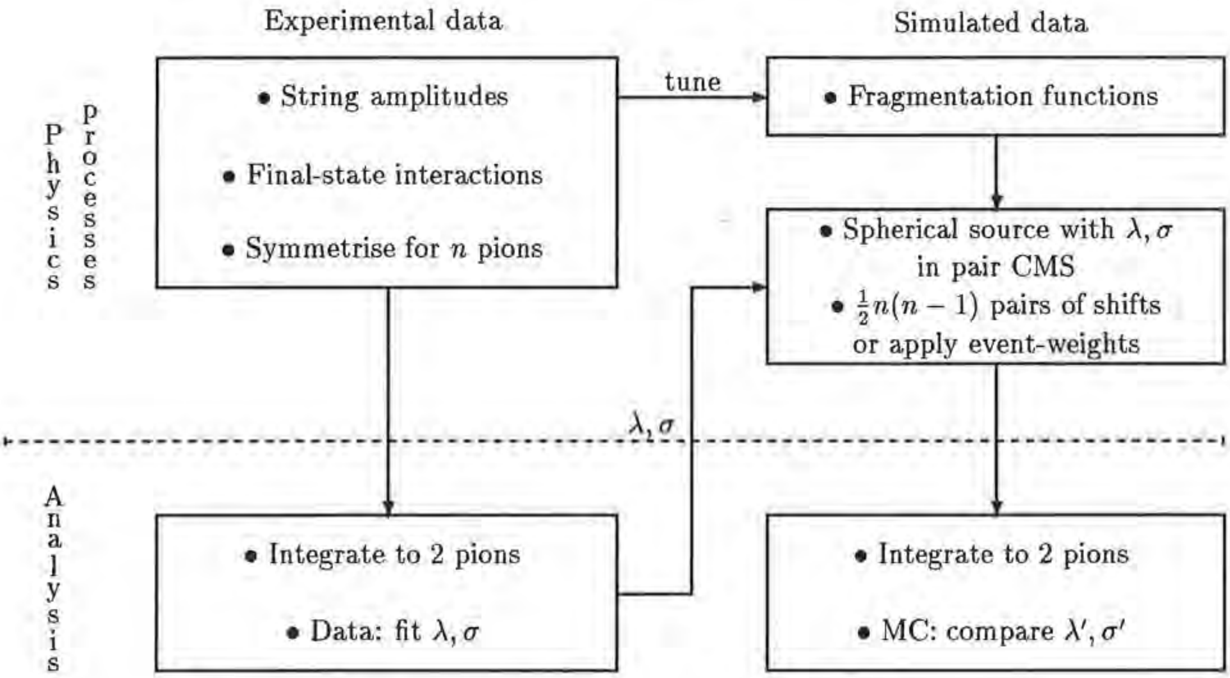

Figure 1: Treatment of experimental vs simulated data.

In figure 1, a schematic comparison is made between what happens with the experimental data and the Monte-Carlo simulation (ignoring details of the detector and analysis cuts). Several observations can be made:

1. The Monte-Carlo parameters are tuned to reproduce the data, which already includes the effects of Bose symmetry.

2. The Bose-Einstein effect which exists in the data is expressed in the Monte-Carlo in a very different manner (for practical reasons).

3. The analysis tends to reduce the data to consideration of pion pairs since theoretical descriptions of the Bose-Einstein effect are indeed functions of the momentum difference between pairs. However, such treatments miss the complications caused by multipion effects.

4. The parameters (λ, σ) fitted from the data as *output* are used as *input* for the models considered in Monte-Carlo. In turn, the *output* of the Monte-Carlo is not necessarily the same as its *input*. In

principle, consistency between the output from the real data and Monte-Carlo can be obtained by iteration.

5.3 Conclusions on the Use of Luboei

1. To first order, LUBOEI provides a reasonable description of the Bose-Einstein effect, bearing in mind that some of the enhancement seen in the same-sign distribution at low Q arises from multipion effects.

2. Although there are some deficiencies, it is not trivial to improve it in a general way.

3. The description of the Bose-Einstein effect in LUBOEI is intended to describe the enhancement seen in the same-sign data. It is not reasonable to assume that it will automatically describe all other effects resulting from the imposition of Bose symmetry.

As a final comment, it is worth remarking that although the fragmentation within JETSET is based on the Lund string model [11], this is only at the level of the fragmentation function which give the momentum distributions of particles. No space-time structure within the string is accessible in the program.

6 The Effect of Resonances on Bose-Einstein.

6.1 Resonance Narrowing of the Bose-Einstein effect

In 1976, Grassberger [23] pointed out that same-sign pions with 3-momentum difference $\boldsymbol{q}$, where one of the pions comes from a resonance (mass m_0, width $\Gamma_0 \sim \tau^{-1}$) of momentum $\boldsymbol{p}$, would not contribute to the Bose-Einstein effect if $|\boldsymbol{q} \cdot \boldsymbol{p}| \gg m_0\Gamma_0$. Loosely, this corresponds to pions from resonances being unable to contribute to the Bose-Einstein effect for $Q \gg (c\tau)^{-1}$. It is as if the resonances represent a source whose spatial dimensions are of the order of $c\tau$. Consequently such pions only give rise to enhancements for $Q \leq O((c\tau)^{-1})$, which is less than the scale σ^{-1} coming from pion pairs originating from the string (see table 1). Therefore the Q distribution is *narrowed* by the resonances. Furthermore, since some of the pion pairs in the range $(c\tau)^{-1} \leq Q \leq \sigma^{-1}$ come from resonances and therefore are unable to contribute to the Bose-Einstein enhancement, the effect is *diluted* in this Q range.[7] When Andersson and Hofmann [13] showed how the Bose-Einstein effect could be understood in the context of string amplitudes, they observed that Grassberger's suppression would dilute the measurable Bose-Einstein enhancement so drastically as to make it impossible to explain the current observations. Bowler [24] subsequently pointed out that if the production of η''s in JETSET (used by Andersson and Hofmann) was very wrong, and there were reasons to suspect this, then the dilution would be less, and the theory could once again be consistent with the data.

6.2 Resonance Weights

Bowler [19] has suggested an expression for the amplitude for the three particle configuration $\pi^+ x \pi^+$ (where x is the other daughter), illustrated in figure 2, where one pion comes from a resonance. If the resonance comes from point $\boldsymbol{a}$ and the other pion from $\boldsymbol{b}$ then the amplitude, symmetric in 1 and 2, is

$$\mathcal{A} = (bw_{13}e^{-i\boldsymbol{p_1} \cdot \boldsymbol{a}}e^{-i\boldsymbol{p_2} \cdot \boldsymbol{b}} + bw_{23}e^{-i\boldsymbol{p_2} \cdot \boldsymbol{a}}e^{-i\boldsymbol{p_1} \cdot \boldsymbol{b}})\xi(\boldsymbol{a})\xi(\boldsymbol{b}) \tag{14}$$

[7] DELPHI [22] have shown that in a b quark enriched sample, the measured enhancement is reduced compared to that seen in a light quark sample.

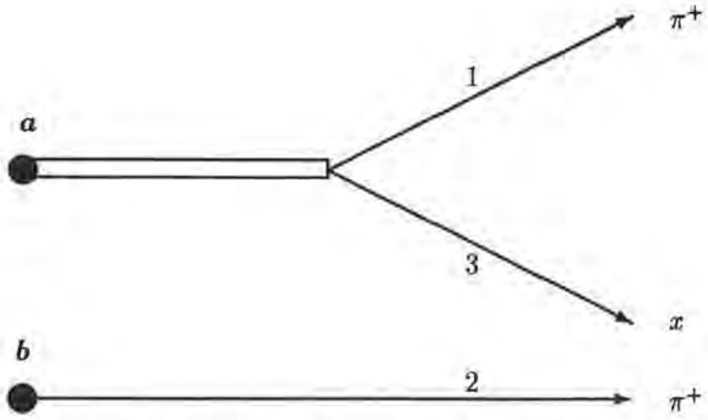

Figure 2: $\pi^+ x \pi^+$ production with one resonance.

where the ξ's represent the source oscillator strengths and bw_{ij} is the Breit-Wigner amplitude[8] to produce particles i and j via the resonance. Bowler [24] has indicated that the alternative approach of applying Grassberger's criterion to string amplitudes does not yield significant differences. If this is integrated over space, the corresponding distribution is

$$BW_{13} + BW_{23} + 2\mathrm{Re}(bw_{13} bw_{23}^{*}) \lambda |\tilde{\rho}(q)|^2 \tag{16}$$

where BW is the modulus squared of bw, and $q = p_1 - p_2$.

Clearly this expression is not the complete picture in the sense that it only pertains to the resonant part of the three-particle production cross-section. Further, if used in conjunction with a Monte-Carlo generator where resonances are explicitly created, allowance must be made for this. Therefore, when applied as a weight to a pair of identical pions where one of the pions is identified as coming from a resonance, it is necessary to renormalise. This can be done by dividing by the sum of the first two terms. Hence a suitable weight for Monte-Carlo is

$$1 + \frac{2\mathrm{Re}(bw_{13} bw_{23}^{*})}{BW_{13} + BW_{23}} \lambda |\tilde{\rho}(q)|^2 \tag{17}$$

It can be seen in the limit that there is no resonance (Γ_0 becomes very large) that this expression reduces to equation 1. In the presence of a resonance, the enhancement in the production rate is only achieved if *both* opposite-sign combinations have masses close to the resonance mass *as well as* the same-sign pions having similar momentum.

When considering two identical pions, it is frequently the case that they *both* come from resonances. This means that it is not clear which of the two resonances to consider when using equation 17. Therefore, I have extended the above to describe the situation illustrated in figure 3. Symmetry is imposed between the two π^+'s, but no possible symmetries are considered between x and x'. Corresponding to equation 14, the amplitude is

$$\mathcal{A} = (bw_{13}^{A} bw_{24}^{B} e^{-i\boldsymbol{p_1} \cdot \boldsymbol{a}} e^{-i\boldsymbol{p_2} \cdot \boldsymbol{b}} + bw_{23}^{A} bw_{14}^{B} e^{-i\boldsymbol{p_2} \cdot \boldsymbol{a}} e^{-i\boldsymbol{p_1} \cdot \boldsymbol{b}}) \xi(\boldsymbol{a}) \xi(\boldsymbol{b}) \tag{18}$$

This yields a normalised weight:

$$1 + \frac{2\mathrm{Re}(bw_{13}^{A} bw_{24}^{B} bw_{23}^{A*} bw_{14}^{B*})}{BW_{13}^{A} BW_{24}^{B} + BW_{23}^{A} BW_{14}^{B}} \lambda |\tilde{\rho}(q)|^2 \tag{19}$$

[8] I have used a relativistic formulation:

$$bw_{ij} \propto \frac{1}{m_{ij}^2 - m_0^2 + i m_0 \Gamma_0} \tag{15}$$

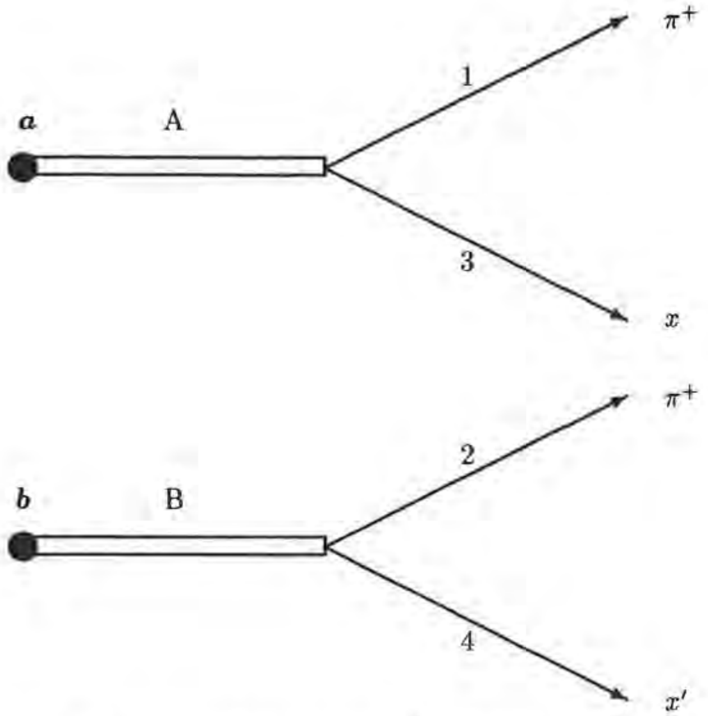

Figure 3: $\pi^+ x \pi^+ x'$ production with two resonances.

6.3 Application of Resonance Weights

I have applied the weight of equation 19 (with $\lambda = 1$, $\sigma^{-1} = 0.3$ GeV) to pions whose mothers have $c\tau < 10$ fm; pions from long-lived sources have not been enhanced; and pions from the string are described as coming from a source of zero lifetime (therefore no Breit-Wigner term) but which is extended in space-time as described in section 2. Comparison has been made with the unweighted distributions - these constitute perfect reference sets.

The enhancements expected for pion pairs where (a) both pions come from the string, (b) at least one pion comes from a rho ($\rho^{0,+,-}$), (c) at least one pion comes from some other source but with $c\tau < 10$ fm are shown in figure 8a.

The enhancements, which are built up from the different components allowing for their relative rates, are shown in figure 8b. They can be compared with the enhancement which would result if there were no resonance narrowing, as described by equation 3. One can see that (with the rates predicted by JETSET), pions from ρ's dominate the observable Bose-Einstein enhancement.

What is shown by experiments is the ratio of the same-sign pion spectrum to the reference sample. This is usually corrected for purity (namely, with non-pion pairs removed). However, this includes the dilution from pions arising from long-lived decays. In a simplified form, if the distribution of pairs which can exhibit the Bose-Einstein enhancement is denoted as $f_{short}(Q)$ and the distribution of pairs which cannot is $f_{long}(Q)$ and the reference samples perfectly describe the distributions in the absence of enhancements (at least after all ratios have been taken), then the measured ratio is

$$\frac{B(Q) f_{short}(Q) + f_{long}(Q)}{f_{short}(Q) + f_{long}(Q)} = 1 + \frac{f_{short}(Q)}{f_{short}(Q) + f_{long}(Q)}(B(Q) - 1) \tag{20}$$

The enhancement will only have the form expected, namely $(B(Q) - 1)$, if the scale factor multiplying it is a constant, which implies that $f_{short}(Q)$ and $f_{long}(Q)$ have the same shape. From figure 5, it can be seen that in the context of JETSET, this is not true. In figure 8c, the observable ratio is shown when all pions coming from sources with lifetimes corresponding to $c\tau < 10$ fm are treated as being capable of the full Bose-Einstein enhancement of equation 3. The dilution caused by pions coming from sources with $c\tau > 10$ fm is clear, resulting in a maximum enhancement of about 1.3. The fit (for $0.1 < Q < 2$) yields $\lambda = 0.29$ and $\sigma^{-1} = 0.32$ GeV - to be compared with an input value of 0.3 GeV. The fitted value of λ is a measure of the dilution and corresponds to the value of the curve for $c\tau < 10$ fm of figure 5 for small Q. When resonance narrowing is included, figure 8d, the fitted values are $\lambda = 0.14$ and $\sigma^{-1} = 0.24$ GeV (corresponding to 0.81 fm). Because the observable distribution is manifestly non-Gaussian, the fitted values depend on the range over which the fit is made. Consequently, the fitted values of λ and σ are correlated.

It can be seen that the observable enhancements which can be measured, even with a perfect reference distribution, are potentially complicated because they combine the different shapes of the underlying distributions of pairs (before the effects of symmetry) with different enhancements described by equation 19. One might try to *correct* measured distributions for the effect of pairs which cannot lead to measurable enhancements, but even then, the interpretation of the corrected distributions is not trivial. Alternatively, one can attempt to tune the Monte-Carlo input so as to reproduce the experimental data. All of these procedures depend critically on the Monte-Carlo describing the underlying distributions in Q.

6.4 Comparison with Experimental Measurements

While there are plenty of uncertainties in the results shown in figure 8d, it is not clear that the measurements made at LEP can be explained readily. (Although it should be recalled that the correction for the Coulomb effect has generally been over estimated.) The discrepancies between different LEP collaborations only add to the confusion. The enhancement displayed in figure 8d is less pronounced than those found in references [5, 6, 7]. It is difficult to reconcile simultaneously both the size of the enhancement (λ) and its width (σ^{-1}) with the measured values in table 2. If the value of σ^{-1} input to the Monte-Carlo is increased, then the contribution from prompt pions from the string extends further in Q, but it is a small fraction of the total enhancement, and there is little change in the contribution from the resonances.[9]

The above conclusion differs from Bowler's in reference [24] (corresponding to $\sqrt{s} = 35$ GeV), where he suggested that the measurements could just about be explained in the presence of resonance narrowing, provided the true η' rate was negligible compared to the value coming from JETSET (and indeed, this was shown to be the case [16]). However, there were two significant differences between that work and this: (a) Bowler treated all pions from η''s as if they came from the string; (b) he generated only light quarks *uds*. The result of these two steps is to increases the fraction of pion pairs coming from the string at low Q. Bowler's treatment of the η''s contrasts with what has been done in the work reported here. By setting PARJ(26)=0.2 in JETSET, 80% of the η''s which are created during the string fragmentation on a first attempt are discarded and a second attempt to generate a suitable meson is made. This results in an appropriate η' rate (compatible with measurements [16]) and a few more of all the other mesons (*not necessarily* prompt pions from the string) in the prescribed ratios. On the face of it, this might seem to be a more appropriate way of handling the excess of η''s found in the default version of JETSET (but see the conclusions for this section). With respect to the second difference (b), by generating only light quarks, the number of daughters originating from weak decays of heavy flavour particles is reduced to close to zero, causing the fraction of prompt pions to be increased. Both of these assumption (a) and (b) tend to increase the fraction of pions from short-lived sources (especially from the string) and to do so

[9] Increasing σ^{-1}, reduces the source size, but this has little effect for pions originating from resonances, whose effective source size is dominated by $c\tau$.

even more at low Q. The consequence is to increase the observable Bose-Einstein enhancement. Using assumptions (a) and (b), I generated the distribution corresponding to figure 8d. From this, the fitted values are $\lambda = 0.34$ and $\sigma^{-1} = 0.23$ GeV (corresponding to 0.86 fm). As before, the shape differs from the Gaussian form, and reaches a maximum of 1.52 at $Q \approx 0$.

The E665 Collaboration [25] claim to have seen two distinct components in their measurement of $R(Q)$ in μ-N interactions. The hypothesis is that the shorter-range component comes from pions originating from ρ^0's. Evidence for this is suggested by the absence of the shorter-range component in the distribution with ρ^0's removed. I do not believe these observations could be so clear at LEP since, firstly the sum of different components described by equation 19 is quite smooth, with no significant discontinuities. Secondly, due to the large multiplicities at LEP and hence combinatorial backgrounds, it is not trivial to remove the ρ^0's and further, the pions from $\rho^\pm$'s would be left in the sample and would show the same resonance narrowing as those from the ρ^0's.

6.5 Other Effects of Resonances

The distributions presented in figure 8 were derived with perfect reference sets. However, resonances can introduce kinematic correlations other than the trivial ones such as the ρ^0 and K* peaks in the $\pi^+\pi^-$ spectrum. This is particularly apparent for the η' which can decay (via an η) giving four charged pions. In turn, these can show correlations at low Q in precisely the region of interest for the Bose-Einstein effect [2]. In principle, these effects should be removed when (implicitly) correcting with Monte-Carlo, but again, this assumes that the Monte-Carlo provides a good description of the resonance rates. Further, such effects depend on the experimental cuts which are made, especially those aimed at avoiding overlapping or broken tracks. Kulka and Lorstad [26] indicated how sensitivity to the η' might be reduced.

6.6 Conclusions for Resonance Narrowing

If one accepts the model of resonance narrowing in which the decay of resonances is a coherent process whereby the phase information from production is transmitted to the daughters, then it is hard to explain the measured Bose-Einstein effect with JETSET. Therefore, either the model is incorrect, or there are other effects at work (eg. final-state interactions), or the distribution of pions from various sources is different from that which is suggested by JETSET. Recently, Andersson, Gustafson and Samuelsson [27] have suggested that due to the approximation of the pion to a Goldstone boson, there may be correlations between pion momenta measured perpendicular to the jet axis. This could increase the fraction of pion pairs from the string at low Q, which would improve the agreement with the data. Within the framework provided by the current version of JETSET, to have some sort of reasonable description of the measured data (despite all the inconsistencies), it seems to be necessary to treat all of the pions coming from sources with $c\tau < 10$ fm as if they were coming from the string.

7 Beyond Interactions between Pairs of Identical Pions

7.1 Multipion Effects

Frequently, studies of the Bose-Einstein effect are limited to the study of pairs of pions, and sometimes triplets. However, as has already been stated, nature does not operate on pairs of particles, rather the amplitude for the *complete* event is symmetrised between *all* identical particles. It was seen when studying the behaviour of LUBOEI that the effect of multipion interactions is significant compared to the simple Bose-Einstein enhancement.

Further, because the symmetrisation occurs at the event level, rather than simply for pairs of particles, it will produce correlations between *all* pairs (including opposite-sign pairs) and to the extent that global event parameters (eg. jet cone size) will be changed, it will introduce correlations between particles in different hemispheres or events (mixed pairs).

7.1.1 Different schemes for investigating multipion effects

It is important to compare different schemes in order to cross-check the predictions of one scheme in predicting more subtle effects, for example distortions of the ρ^0 resonance line-shape. In this section, two schemes are considered.

(a) Shifting particle momenta

Firstly, in LUBOEI, each pion has its momentum vector shifted corresponding to an effective interaction with each other identical pion. The multipion effects are explicitly included and show up in the deviation of the ratio from the input function, as seen in figure 7b. With n identical pions, there are $n(n-1)/2$ pairs which can be formed and this results in a total of $n(n-1)$ shifts which are made to the momentum vectors.

(b) Event-weights

The second scheme corresponds to forming event-weights W. One way in which this may be done is by taking the product of the enhancements from all pairs:

$$W = \prod_{all\ pairs} w(q_{pair}) = \prod_{all\ pairs} (1 + \lambda|\tilde{\rho}(q_{pair})|^2) \tag{21}$$

This will be referred to as the *product* event-weight. For $\lambda = 1$, this has a maximum value of $2^{n(n-1)/2}$. (This weight can only be attained by a somewhat bizarre configuration of particle momenta. n is more usefully interpreted as the number of identical pions in a given event hemisphere.)

Alternatively, one can create a symmetric amplitude, which has $n!$ terms. The corresponding weight is given by

$$W = \sum_{all\ permutations} \lambda^{\frac{p}{2}} \tilde{\rho}(q_{1i_1})\tilde{\rho}(q_{2i_2})....\tilde{\rho}(q_{ni_n}) \tag{22}$$

where the indices $i_1, i_2, ...i_n$ are taken from the set $1, 2, ...n$ and p is the number of times the first and second indices differ. This will be referred to as the *symmetric-amplitude* event-weight, and represents the most correct way to obtain a weight for n pions. For two particles, this yields

$$W = \tilde{\rho}(q_{11})\tilde{\rho}(q_{22}) + \lambda\tilde{\rho}(q_{12})\tilde{\rho}(q_{21}) = 1 + \lambda|\tilde{\rho}(q_{12})|^2 \tag{23}$$

since $\tilde{\rho}(0) = 1$ by normalisation. For $\lambda = 1$, equation 22 has a maximum value of $n!$. There is some overlap between the expressions of equations 21 and 22 and to first order in λ they are the same.

The weight for a complete event is the product of the weights for the system of π^+'s and the system of π^-'s. In what follows, only pions originating from parents with $c\tau < 10$ fm are considered, and pions from opposite hemispheres are treated separately.[10] Since it is complicated and time consuming to calculate weights corresponding to the symmetrised amplitudes for large numbers of particles, the maximum number of pions of the same sign in the same hemisphere which may be considered is restricted to six - events which violate this condition are not used. Distributions of the event-weights of equations

[10] This is a good approximation since the masses of pairs where the pions are taken from opposite hemispheres are generally much greater than the reciprocal length scale σ^{-1} characterising the observed Bose-Einstein effect.

21 and 22 found for events generated by JETSET are shown in figure 9.[11] It can be seen that the two sets of event-weights are quite similar, although there are significant overflows corresponding to very large weights. Comparison between the distributions derived with these weights are made in section 7.4.

7.2 Additional Enhancements Arising from Event-weights

The weighted Q spectrum is given by

$$\frac{d\sigma}{dQ} = \int W(Q_{12}, Q_{13}, ..) \frac{d^n\sigma}{dp_1 dp_2...} \delta(Q - Q_{12}) dp_1 dp_2... \tag{24}$$

where Q_{ij} is the function $\sqrt{-(p_i - p_j)^2}$ and p_i are the 4-momenta of *like-sign pions*. The enhancement is obtained by taking the ratio of this with the same expression but with W set to 1. If the product of weights (equation 21) is used, then the factor $w(Q)$ can be extracted from the integral and the ratio of the residual integral to the normalisation consists of a geometric average of the weights for all other pairs, averaged over the distribution of particles in an event. If this ratio were just a numerical constant, then the simple two-particle Bose-Einstein enhancement would be recovered, namely $w(Q) = B(Q) = 1+\lambda|\tilde{\rho}(Q)|^2$. However, the ratio does contain terms correlated with Q.

If we consider a pair of pions i and j, then in the limit where the pion mass is ignored for illustration, $Q_{ij} = 2|\boldsymbol{p_i}||\boldsymbol{p_j}|(1 - cos\theta_{ij})$. Clearly, if $|\boldsymbol{p_i}|$ is small, then Q_{ij} will be small, but so will $Q_{ij'}$, where $j' \neq j$. Hence, equation 24 contains a set of terms like $w(Q_{1j'})$ and $w(Q_{i'2})$ which will be correlated with $Q = Q_{12}$ and which will cause extra additional enhancement, on top of the simple function $w(Q)$. The amount of the correlations can be seen in figure 10 where the mean values of (a) Q_{ij}, (b) $Q_{ij'}$ and $Q_{i'j}$ and (c) $Q_{i'j'}$ ($i' \neq i$, $j' \neq j$), averaged over single hemispheres, are plotted as a function of $Q = Q_{ij}$. (a) is trivial while (b) indicates a clear correlation (although it is actually the effects of the smaller values of $Q_{ij'}$, rather than the mean, which are the most important). (c) is flat, indicating that in JETSET there are no correlations in Q between independent pairs of particles. Whether this observation truly reflects actual particle production remains to be seen. One would expect that final-state interactions would induce some global correlations, as would the Bose-Einstein effect itself; the latter making it difficult to establish from measured data the extent of correlations in the absence of Bose symmetry.

If we consider a system of $n\pi^+$'s and $n\pi^-$'s in a hemisphere (the effect of pions in the other hemisphere is small), then from the integral of equation 24, $\frac{d\sigma_{++}}{dQ}$ at $Q = Q_{ij}$ will contain a primary term $w(Q)$ and $2(n-2)$ secondary terms of the form $w(Q_{ij'})$ or $w(Q_{i'j})$ (since there are $(n-2)$ other pions of the same sign not including i or j). For the reference distribution formed from $\pi^+\pi^-$ pairs, $\frac{d\sigma_{+-}}{dQ}$ will be the product of two independent integrals with the form of equation 24 (one performed over the π^+'s, the other over the π^-'s). Each of these integrals will result in $(n-1)$ secondary terms, giving a total of $2(n-1)$ terms. So we would expect that $\frac{d\sigma_{++}}{dQ}$ would be enhanced above $w(Q)$, but the enhancement of $\frac{d\sigma_{+-}}{dQ}$ would more than compensate for this.

7.3 Problems with Event-weights

When event-weights are constructed, either as simple products of the weights for pairs, or as sums of terms in the case of symmetrised amplitudes, they are greater than or equal to unity. The number of terms is a strongly increasing function of the event multiplicity and for large multiplicity events, the weight can be very large. This causes numerical problems since a few high multiplicity events dominate the weighted distributions, reducing statistical precision. While the weight derived from the Bose-Einstein enhancement is correct, there is the implicit assumption that such a weight is not already included in

[11] The discontinuity which occurs at $w = 2$ is a result of a pole in the Jacobean $(\frac{dw_{pair}}{dQ})^{-1}$ at $Q = 0$ or $w_{pair} = 2$.

the Monte-Carlo data. This is not true in so far as in measured data, the intrinsic production rate of high multiplicity events must be lower, but has actually been enhanced by the Bose-Einstein effect, and the Monte-Carlo's are tuned to reproduce the multiplicities observed in measured data. This is true not only when events are viewed as a function of multiplicity, but also when looking at distributions such as those in Q. There is a tendency to strongly weight bins which have already been implicitly enhanced by tuning the event shape parameters, and this can lead to substantial distortions.

Therefore, if event-weights are used, it is necessary to *control* them. This can be done in several ways:

- *Truncate event-weights:* this is not very satisfactory since it reduces the enhancement which should be seen for some pairs.
- *Limit studies to lower multiplicities:* not completely satisfactory.
- *Renormalise weights using opposite-sign pairs.* This can be done by dividing the weights for all same-sign pions by weights obtained by applying the same procedure to pairs of *opposite-sign* pions. This is fine when looking at same-sign distributions, but does not allow investigation of opposite-sign pairs.
- *Renormalise so as to preserve multiplicity.* This is my preferred scheme, and is discussed below.

It is easier to consider event-weights which are constructed from products of the weights for pairs of pions. This is roughly equivalent to using symmetric amplitudes, as was shown above. The number of terms in the product is equal to the number of pairs of the same sign, and the product can be expected to rise roughly as some constant $\bar{w}$ to the power of the number of pairs n_{pair}. So an event-weight can be constructed:

$$W = \frac{\prod_{++pairs} w_{++pair} \prod_{--pairs} w_{--pair}}{\bar{w}^{n_{pair}}} \tag{25}$$

With values of $\lambda = 1$ and $\sigma^{-1} = 0.3$ GeV, it is found that choosing $\bar{w} = 1.055$, the mean event-weight as a function of multiplicity stays roughly at unity. Even with this scheme, it is necessary to truncate high weight events ($W > 20$), but this can be done with the loss of just a few per mille of events. Nevertheless, if the incoherence parameter λ is increased beyond the value of 5, the weights become uncontrollable. This procedure can also be performed by explicitly renormalising distributions for each multiplicity class separately (as in references [28, 29]), and then combining them.

7.4 Comparing Multipion Effects in Different Models

Taking ratios

When the size of the Bose-Einstein effect is studied with experimental data, the double ratio $R = r(data)/r(MC)$ is formed, where $r(data)$ is the ratio of the same-sign spectrum to some reference. The same-sign spectrum will not only contain contributions from single pairs, but also multipion effects. The double ratio R will only have the interpretation commonly used, corresponding to the two-particle correlation function (equation 1), if the multipion effects cancel. The extent to which this is true can be investigated by replacing *measured data* by Monte-Carlo data with the Bose-Einstein effect simulated, and denoted as MC^*. The double ratio becomes $R = r(MC^*)/r(MC)$, which is

$$R = \frac{\frac{d\sigma(MC^*)_{++}}{dQ} / \frac{d\sigma(MC^*)_{ref}}{dQ}}{\frac{d\sigma(MC)_{++}}{dQ} / \frac{d\sigma(MC)_{ref}}{dQ}} = \epsilon_{++}/\epsilon_{ref} \tag{26}$$

where

$$\epsilon_{++} \equiv \frac{\frac{d\sigma(MC^*)_{++}}{dQ}}{\frac{d\sigma(MC)_{++}}{dQ}} \qquad \text{and} \qquad \epsilon_{ref} \equiv \frac{\frac{d\sigma(MC^*)_{ref}}{dQ}}{\frac{d\sigma(MC)_{ref}}{dQ}} \tag{27}$$

This corresponds to the ratios of the enhancements seen by the same-sign (ϵ_{++}) and reference samples (ϵ_{ref}). Using the double ratio also provides a natural way of renormalising when event-weights are used. This would not be achieved if the numerator ϵ_{++} were used alone.

Some comparisons

In figures 11a, b, c and d, the enhancements obtained from (a) *symmetric-amplitude* event-weights[12] (equation 22), (b) *product* event-weights (equation 21), (c) *normalised-product* event-weights (equation 25) and (d) LUBOEI are shown. Only pions with $c\tau < 10$ fm have been considered, and no resonance narrowing has been included. The inputs to the distributions were $\lambda = 1$ and $\sigma^{-1} = 0.3$ GeV. These enhancements have been fitted with a form commonly used in the analysis of the experimental data, $\alpha(1 + \beta Q)(1 + \lambda e^{-\sigma^2 Q^2})$, and the results are presented in table 3. Lafferty and Edwards [29] have performed similar studies with the GENLON generator [33].

Weighting scheme	ϵ_{++}		ϵ_{+-}		ϵ_{mix}		R_{+-}		R_{mix}	
	λ	σ^{-1}	λ	σ^{-1}	λ	σ^{-1}	λ	σ^{-1}	λ	σ^{-1}
(a) *Symmetric-amplitude*	1.12	0.35	0.34	0.66			0.80	0.31		
(b) *Product*	1.48	0.33	0.43	0.66			1.05	0.29		
(c) *Normalised-product*	1.16	0.35	0.35	0.65	0.23	0.59	0.85	0.30	0.86	0.33
(d) LUBOEI	1.51	0.29	0.39	0.78	0.10	0.72	1.22	0.25	1.40	0.28

Table 3: Fitted parameters resulting from multipion effects. Errors are about 0.02. σ^{-1} is measured in GeV.

Several observations can be made:

- Multipion effects do cause enhancements for same-sign pairs in excess of the factor of two expected for just simple pair interactions (ie. $\lambda_{meas} > 1$).
- There are enhancements in opposite-sign pairs and mixed pairs. These have a Q scale which is about twice the input value.
- Although the ++ enhancements are greater than the input values, when normalised by the reference samples, the fitted values of λ may fall below 1. If anything, this makes it harder to explain the measurements.
- The spread of values indicates uncertainties of the order of 10's of percent.
- If we believe that *symmetric-amplitude* weights are the most correct, event-weights derived from the products of weights seem to overestimate the enhancement for same-sign pairs.
- With LUBOEI, a significant bump is seen in the +− ratio resulting from a displacement of the ρ^0 (see section 8).

[12] As explained earlier, the event-weights derived from symmetric amplitudes and products of pair weights have been derived for individual hemispheres, where the number of identical pions in the hemisphere participating in the Bose-Einstein effect is no more than six.

- The enhancement of the *mixed* distribution is less than that of $+-$.
- The effects which are seen are not small, and will probably be different for different source models. Therefore there is probably little value in trying to be too quantitative.

While the scheme employed by LUBOEI is arranged to reproduce the enhancement expected for a single same-sign pair and manifestly contains some form of multipion effects in so far as each pion is simultaneously perturbed by all other pions of the same sign, it is not obvious that LUBOEI reproduces the enhancements described by multipion weights. It can be readily seen that the two schemes are structurally different: for (b) and (c), weights corresponding to *pairs* are *multiplied* and the momentum differences enter as the *scalers* represented by the variable Q. For LUBOEI, momentum shifts are evaluated for *individual* pions and *added* in a *vector* like manner.

Because the enhancement of the mixed-event reference set is less than in the opposite-sign data,[13] the ratio R_{mix} tends to be more greatly enhanced than R_{+-} (this is more apparent for entry (d) than (c)). This contradicts what seems to be seen in the LEP data and raises questions about the role of final-state interactions, especially the Strong Interaction.

7.5 $\pi^+\pi^-$ Interactions

It was seen above that the reference samples are distorted by the Bose-Einstein effect. Juricic [30] has tried to correct the mixed-event distribution of pairs by an iterative procedure. He found that the corrections to R_{mix} were of the order of a few percent, in contrast to the values of table 3. However, the form used in [30] was derived from the application of the two-particle correlation to all other identical pions in an event. This attributes an enhancement to the measured single-particle spectrum $\frac{d\sigma}{dp}$ corresponding to the **average** value of the two-particle function of equation 1, averaged over all identical particles in an event. This weight, for which a correction is made, has the form

$$w(p_1) = \frac{1}{n}\sum_i^n B(Q_{1i}) = 1 + \frac{1}{n}\sum_i^n b(Q_{1i}) = 1 + \bar{b} \tag{28}$$

where $B(Q)$ has been written as $1 + b(Q)$. By contrast, if an event-weight formed from the **product** of two-particle functions is used, then the effective weight for a single particle is dominated by the terms containing its momentum:

$$w(p_1) = \prod_i^n B(Q_{1i}) \approx 1 + \sum_i^n b(Q_{1i}) = 1 + n\bar{b} \tag{29}$$

where it is assumed $b(Q)$ can be treated as a perturbation. Thus the Bose-Einstein enhancement of the mixed-event reference sample $\frac{d^2\sigma_{mix}}{dp_1 dp_2} \propto \frac{d\sigma}{dp_1}\frac{d\sigma}{dp_2}$ is actually expected to be greater than found in [30], when multipion effects are correctly allowed for.

It was found that with the scheme used in [30], the iteration converged rapidly. However, if the formulae are rewritten in the form probably intended, it can be seen that the convergence is immediate. Let the measured two-particle density for same-sign pairs be $D(p_1,p_2) = \frac{d^2\sigma}{dp_1 dp_2}$ and the single-particle density be $s(p_1) = \frac{d\sigma}{dp_1}$, which is effectively obtained in event mixing by integrating $D(p_1,p_2)$. Since, by construction, $s(p_1)$ is obtained from $D(p_1,p_2)$, which is distorted by the Bose-Einstein effect, by a function (to be determined) $R(p_1,p_2)$, $s(p_1)$ is obtained by correcting with the reciprocal of the weight from equation 28. If the superscripts label successive approximations, then from [30]

[13] Gluon radiation causes the jets of the two hemispheres no longer to be collinear, and since the amount of correlation which will be present is reduced, particles in the the two hemispheres will be pulled in towards jet axes which are not back-to-back. Thus, on reflecting the particle momenta, the additional correlation which is produced by the Bose-Einstein effect is not as great as it would have been in the absence of radiation.

$$s^n(p_1) = \frac{\int D(p_1,p_2)dp_2}{\int R^{n-1}(p_1,p_2)s^{n-1}(p_2)dp_2} \tag{30}$$

and

$$R^n(p_1,p_2) = \frac{D(p_1,p_2)}{s^n(p_1)s^n(p_2)} \tag{31}$$

The indices in the denominator of equation 31 have been modified from [30] to use the best estimates of $s(p)$. With this modification, when equation 31 is substituted back in to equation 30 on a subsequent iteration, it is found that there is no change to the function $s(p)$.

In addition to the trivial *knock-on* effects discussed already, Andreev, Plumer and Weiner [31] have proposed that correlations can exist between π^+ and π^-. These correlations are on the same footing as the effect between like sign pions and arise from symmetry at the Quantum Mechanical level. Crudely, this arises because the π^+ and π^- are described by the same creation-annihilation operators, albeit with different coefficients. Applying the formula of [31] to a $\pi^+\pi^-$ pair in their centre-of-mass frame, under the assumption that the source density is purely a function of proper time, as in equation 2, gives an enhancement which has the same shape as that for the same-sign pions, but scaled by a factor $e^{-4\sigma^2 m_\pi^2}$, which is 0.4 for $\sigma^{-1} = 0.3$ GeV. If this is true, it would make it even more difficult to reconcile Monte-Carlo predictions with the measured data.

7.6 Conclusions for the Effects of Many Pions

- While all methods for implementing the Bose-Einstein effect at the pair level, by arrangement, give the expected answers, there is disagreement when the higher order effects due to many pions are included.
- Only one form of the Bose-Einstein enhancement has been studied, but other forms corresponding to different source densities could be considered, and would undoubtedly lead to slightly different results.
- Investigating the Bose-Einstein effect with Monte-Carlo tuned to measured data is not entirely satisfactory. Ideally, Monte-Carlo with the Bose-Einstein effect included should be tuned to reproduce measurements. However, in the light of all the uncertainties associated with the Bose-Einstein effect this is a daunting task.
- It is not clear whether JETSET will necessarily describe all correlations between particles, and this will certainly have bearing on higher order effects.
- It seems far from obvious that one can understand parameters pertaining to the Bose-Einstein effect with precisions which are more accurate than O(10%). Consequently, their interpretation, especially in terms of the string model seems of questionable interest.

8 Consequences of the Bose-Einstein effect on the ρ^0 Resonance

8.1 Introduction

The fact that resonances modify the observation of the Bose-Einstein effect, implies the converse, namely that the Bose-Einstein effect will modify the line-shapes of resonances. This can be seen explicitly when equation 19 is perceived as describing the production of $\pi^+\pi^-$ pairs associated with a resonance, as opposed to a same-sign pair. This suggestion was made by OPAL [32], who showed that their $\pi^+\pi^-$ data, especially in the vicinity of the ρ^0 resonance, could be better described when the Bose-Einstein correlations modeled by LUBOEI in JETSET were turned on. In particular, they showed that the slope of the background and the approximate shape of the ρ^0 resonance itself could be better described (at least when looking at the difference between the opposite and same-sign mass distributions). This work was amplified by Lafferty [28], who also emphasised the importance of other contributions to the ρ^0 line-shape. With respect to reference [32], one of the concerns was the use of an incoherence parameter λ in excess of the theoretical maximum of 1. In this section, the results from using LUBOEI will be contrasted with those using weighting techniques.

In general, the Bose-Einstein effect tends to pull particles together, and to this extent will tend to pull the momentum vectors of pions from resonance decays towards a central direction. In the LUBOEI model of the Bose-Einstein effect, identical particles are pulled together by an effective force, and to the extent that all the π^+'s and π^-'s in a jet have a common direction, then opposite-sign pions will tend to be drawn together and this will smear the masses of resonances, with a tendency to lower the position of the peak. There are no limits to the shift which can be made to the mass of a resonant pair within kinematic bounds.

In the context of weighting, pairs whose pions lie closer to the centre of a jet will tend to pick up larger weights, and such pairs will tend to correspond to those pairs whose pion momentum vectors lie closer together and which tend to have a lower invariant mass. The result of the Bose-Einstein effect is thus to reweight the resonance spectrum, with a preference for lower masses. Since the effect is limited to a reweighting, the distortion will be apparent only over the width of the resonance. In both cases, there should be changes to the non-resonant spectrum in the vicinity of the resonance; and this should be strongest for $\pi^+\pi^-$ pairs where one of the pions comes from a ρ^0. In practice, this is not significant unless weights are used which explicitly incorporate the Breit-Wigner terms of equation 19.

In weighting schemes where explicit Breit-Wigner terms are incorporated (equation 19), the mass dependence of the interference term is

$$\text{interference} \quad \propto \quad 2\frac{(m_{12}^2 - m_0^2)(m_{13}^2 - m_0^2) - m_0^2\Gamma_0^2}{((m_{12}^2 - m_0^2)^2 + m_0^2\Gamma_0^2)((m_{13}^2 - m_0^2)^2 + m_0^2\Gamma_0^2)} \tag{32}$$

The Dalitz plot for the two opposite-sign pion pairs is divided into four quadrants by the ρ^0 bands; and the interference enhances the production rate in the on-diagonal quadrants, while suppressing it in the off-diagonal quadrants.

It can be seen in the context of specific examples that the process of shifting momenta does not behave in the same way as reweighting, which I believe provides a more appropriate description of the true production rates.

An example

In the case of a very narrow resonance, it is evident that the daughter pions will not play an observable part in the Bose-Einstein effect. This is communicated to LUBOEI or simple weighting schemes through *ad hoc* parameters which indicate the minimum width of a parent of participating pions. This is warning in itself. If we ignore this for the moment, then the effect of LUBOEI will be such as to smear the narrow resonance. However, a weighting scheme will recognise that certain configurations are favoured, yet the entries in the mass plot close to the resonance from such favoured configurations will be enhanced, but not shifted. So the resonance will remain very narrow.

8.2 Comparisons of Luboei and Weighting

The choice of parameters

One of the concerns about the work in [32] was the fact that they needed to use a large value for λ to describe their same-sign data. (This confirms the problem illustrated in figure 6a). This may provide a satisfactory way of describing the same-sign data, however it cannot be expected that this will be appropriate for investigating other distributions. In particular, the large value of λ which corresponds to a fully corrected value can be explained at some level by the unrealistically high rate of η''s produced by JETSET and the incorrect application of the Coulomb correction. ALEPH implicitly showed that by taking these things into account, a reasonable description[14] could be obtained by a fully corrected value of λ (which is what is needed as input to JETSET) of 1 to 1.5. Similarly, DELPHI obtained good agreement with their data using a value close to 1. Using a large value of λ is dangerous since distortions caused by the Bose-Einstein effect grow very rapidly as λ increases beyond 1. Also, the enhancement observed in the same-sign spectrum is proportional to the strength λ and the number of *pairs* of pions which can participate in the effect (proportional to the square of the number of pions originating from short-lived sources). However, the distortion of the ρ^0 line shape is proportional to the strength λ and the number of *individual* pions which can participate.

Some comparisons

To examine the consequences of the Bose-Einstein effect on the ρ^0 line-shape, the mass distribution of pion pairs originating from ρ^0's was examined directly. This is to be contrasted with examining the complete opposite-sign mass spectrum. By doing this, the statistical effects from the background from all other pairs were avoided along with complications arising from the effects caused by other resonances near to the ρ^0 mass. With Bose-Einstein correlations turned on, the opposite-sign mass spectra with the ρ^0's removed changed smoothly, and except for the case where weights explicitly contained Breit-Wigner terms, did not show additional enhancements in the vicinity of the ρ^0 mass.

The masses of pairs from ρ^0 decays produced in JETSET were examined under different conditions:

(a) No Bose-Einstein effect.

(b) Using a weight for a $\pi^+\pi^-$ pair equal to the product of weights derived for the π^+ and π^- separately. These were taken as the products of weights formed from the simple Gaussian expression (equation 3) for all pairs including the corresponding pion from the ρ^0.

$$W(\pi_1^+\pi_2^-) = \prod_{++pairs} B(Q_{1i}) \prod_{--pairs} B(Q_{2j}) \tag{33}$$

(c) As for (b), but with the Gaussian expression for $B(Q)$ replaced by the formulation in the presence of resonances, equation 19.

(d) Using the event-weight of equation 25.

(e) Using LUBOEI from JETSET.

JETSET generates ρ^0's according to a simple Breit-Wigner

$$BW(m) \propto \frac{1}{(m-m_0)^2 + \Gamma_0^2/4} \tag{34}$$

[14] Ignoring resonance narrowing coming from parents with $c\tau < 10$ fm.

with $m_0 = 772$ MeV and $\Gamma_0 = 153$ MeV. The same formula has been used to fit the mass distributions, some of which are presented in figure 12, and the results are presented in table 4. One set of measurements corresponds to a *narrow* resonance: Γ_0 was reduced to 21 MeV, just above the minimum width requested by my choice of PARJ(91) (so that the LUBOEI mechanism will operate on the daughter pions).

Weighting scheme	Input parameters ($\sigma^{-1} = 0.3$ GeV)					
	$\lambda = 1$		$\lambda = 2$		$\lambda = 1$	
	$\Gamma_0 = 153$ MeV		$\Gamma_0 = 153$ MeV		$\Gamma_0 = 21$ MeV	
	Fitted quantities (MeV)					
	m_0	Γ_0	m_0	Γ_0	m_0	Γ_0
(a) No B-E	770.3	154.5	770.3	154.5	772.1	25.4
(b) *Product*	767.5	156.3	764.2	158.7	772.0	25.4
(c) *Product* with B-W	768.6	155.5	766.8	156.8	772.0	25.5
(d) *Normalised-product*	767.7	155.2	766.2	155.8	771.9	25.3
(e) LUBOEI	738.9	197.7	717.4	250.0	749.4	83.3

Table 4: Fitted masses and widths for ρ^0 distorted by Bose-Einstein correlations.

The differences between the input parameters and the fitted ones for no Bose-Einstein enhancement seem to arise from a bias towards lower masses coming from the phase-space available during fragmentation. This causes the line-shape to be multiplied by a function which decreases slowly with mass. The width of the 21 MeV resonance is overestimated due to binning effects.

The numbers presented in the table undoubtedly have some errors associated with them and the fits made in the range 600 to 900 MeV do not describe what is happening in the tails of the distribution. In particular, there can be significant enhancements in the tails of the line-shape at low mass. Nevertheless, the trend is clear, namely that the distortions of the line-shape predicted by weighting techniques are substantially less than those predicted by LUBOEI. If the former can be taken as a reasonable representation of the Bose-Einstein effect, then it would seem that serious distortions of the line-shape should not be expected in the data as a result of the Bose-Einstein effect. In addition, there are distortions of the non-resonant background and LUBOEI makes predictions for the magnitude of this which are compatible with those from weighting methods (see table 3).

In reference [28], the ρ^0 line-shape *appears* to be substantially distorted after weights are applied to the pions created by the GENLON generator [33]. However the distortion to the background under the ρ^0 makes it difficult to see what is the actual effect on the ρ^0 line-shape itself. Fits which I have made to the plot shown in [28] indicate a shift of the fitted mass of -10 MeV, which corresponds to about one standard-deviation.

8.3 Three Pion Tau Decays

The distortions of the ρ^0 occur in the context of the production of a ρ^0 and a charged pion. In principle, the three-prong decays of the tau as measured by Argus [34], for example, should provide a good test of what is happening:

$$\tau^\pm \to \nu_\tau a_1^\pm \qquad \text{with} \qquad a_1^\pm \to \rho^0\pi^\pm \qquad \text{with} \qquad \rho^0 \to \pi^+\pi^- \tag{35}$$

While the magnitude of the distortion from the Bose-Einstein effect depends on the number and momentum distribution of other charged pions, in principle it should be possible to observe comparable effects. In the Dalitz plot, the effects discussed in association with equation 32 should be apparent. In practice, it is not so easy to study the effect in tau decays:

- Interpreting the results is complicated by the uncertainties associated with the a_1 line shape [35].
- The rho resonance is not particularly obvious due to the limits of phase-space: the mass of the tau limits the a_1, the mass of the a_1 limits the ρ^0.
- People working on this are not really interested in testing basic Quantum Mechanics, but rather, since it is a simple exclusive decay, the decay amplitudes are written down explicitly and the Bose symmetry is included as a matter of course. The main interest of these people is the extraction of the electroweak asymmetry parameters (see for example [36]).
- The effect of other final-state interactions (in particular, those highlighted in [28]) will be different compared with that found in the hadronic decays of the Z.

8.4 Other Problems

In all of the above, reference has been made to *pions originating from* ρ^0's as opposed to those from other sources. While this has a clear meaning in terms of a Monte-Carlo generator, in the context of symmetries between identical particles, it is not possible to say which pions did or did not come from a ρ^0, as is made explicit in equation 18. Further, there is a large irreducible background to the ρ^0, as can be seen in figure 4. This makes it more difficult to study the effect of the pions from the ρ^0 alone.

There is some uncertainty as to the correct description of the line-shape of a broad resonance (see [28]) and care needs to be taken to fit appropriate parameterisations of the line-shape.[15] By being broad, there is greater overlap with other resonances (see [32]) and reflections such as that coming from the K^*.

Various factors can distort the simple line shape in addition to the effects of Bose symmetry: constraints from the production mechanism, which therefore depend on the interactions being studied (see discussion in [9]); the phase-space available in fragmentation processes; ρ^0-ω interference and non-resonant backgrounds [28].

An investigation of the effect of the Bose-Einstein effect on the ρ^0 is further complicated if one believes that pions from ρ's should be treated differently from prompt pions from the string. In which case, the distortion and measurement of the ρ^0 rate will depend on the assumed rate, requiring an iterative approach.

8.5 Conclusions for the Distortion of the ρ^0 Line-shape

It seems that the LUBOEI implementation of the Bose-Einstein effect in JETSET does not describe the distortions which one might expect the effect to cause to the ρ^0 line-shape. Rather, it overestimates the shift of the peak. Part of the problem in describing the ρ^0 in the inclusive $\pi^+\pi^-$ mass spectrum arises from the shape (especially the slope) of the combinatorial background under the resonance. While the implementation in LUBOEI may provide an improved description of this[16] - or more correctly, the difference between the opposite-sign and same-sign mass spectra - it is less clear exactly how well it describes the ρ^0 itself (see figures of [32]). Indeed, the Bose-Einstein effect should shift the ρ^0 peak, but only by a few MeV. Should careful measurements[17] show that the shift is much greater, then it is more likely that it arises from other effects, such as those highlighted in [28].

[15] If the simple line-shape parameterisation for the ρ^0 (equation 34) is fitted to the central part of the distribution suggested in [28], the resonance mass is overestimated by 10 MeV and the width is underestimated by 6 MeV.

[16] See also [27].

[17] DELPHI [37] considered the $\pi^+\pi^-$ mass spectrum in the light of reflections but not the Bose-Einstein effect and measured a ρ^0 mass 11 ± 2 MeV below the Particle Data Group value [38].

It does not seem appropriate to measure the parameters of the ρ^0 resonance in these complicated multiparticle environments for their own sake, but only as a measure of the distortion of the resonance. It is not a matter of the resonance parameters changing, but rather that the line-shape is modified in non-parametric ways. The best one can do is to verify that the observed line-shape is consistent with all known effects and to ensure that the description provided by Monte-Carlo's is consistent with the data.

9 Conclusions

- There remain many uncertainties associated with the observation of the Bose-Einstein effect in the context of hadronic decays of the Z.
- These uncertainties are not helped by the various treatments of the experimental data. However, even bearing these variations in mind, the results obtained by different experiments do not seem consistent.
- The results using different reference datasets are not consistent, indicating that more complicated interactions are probably present. This contrasts with the conclusions of [21] expressed before the publication of results from LEP.
- While the observed Bose-Einstein enhancements in same-sign mass spectra can readily be described at a simple level, it is still difficult to accommodate the effects of resonance narrowing. Either the model used in this work is wrong, or the role of the strong interaction is still not understood, or distributions of pions coming from different sources are not well described in JETSET.
- The treatment of many-pion effects does not help resolve any of the problems.
- LUBOEI in JETSET provides a reasonable description of the Bose-Einstein enhancement for the same-sign mass spectrum, and to a lesser extent the opposite-sign spectrum. However, it cannot be expected to reproduce all effects associated with the Bose-Einstein effect.
- It does not seem appropriate to pursue this work in order to determine a description of the space-time structure of particle creation. At best, it may be possible to obtain consistency.
- Nevertheless, it is important to have a good Monte-Carlo description of the data in order to understand other processes.
- LUBOEI in JETSET does not seem to provide a good description of what might be expected for the Bose-Einstein distortion of the ρ^0 line-shape, and it is likely that there are other phenomena which are distorting the measured line-shape in hadronic decays of the Z.
- It is difficult to extract conclusions or to make definite predictions for the various measurements presented in this paper without them being based on a large number of assumptions (nature of source, origin of pions, nature of final-state interactions) coupled with a several complications (treatment of string amplitudes, Coulomb interaction, multipion effects, resonances). Were one to attempt this, then it is unclear with which data it should be compared.
- There does not seem the interest theoretically or experimentally to come to definite conclusions on the subject for e^+e^- physics.

In summary, it seems very difficult to make progress in studying the Bose-Einstein effect in the context of e^+e^- physics, and it is not clear to what extent it can be considered a useful and interesting activity.

Acknowledgements

I would like to thank: Armin Bohrer for his thoughts on the ρ^0, Mike Bowler for various helpful remarks and his many inspiring papers in the field, Gosta Gustafson for providing me with [27], George Lafferty for explaining the work of [28] and [32] and for his comments on this paper, Torbjorn Sjostrand for helping me to understand relevant parts of JETSET and John Thompson for his encouragement with this work.

Appendix A: Jetset Parameters used in this Paper

When JETSET has been used to simulate the Bose-Einstein effect, the following parameters have been used by default, unless stated otherwise:

- MSTJ(51) = 2 - Use the Gaussian parameterisation of source shape, described in equation 3.
- MSTJ(92) = 3 - Only consider the Bose-Einstein effect applied to pions.
- PARJ(26) = 0.2 - Reduce the rate of η' production by a factor of four, as suggested in [16].
- PARJ(91) = 20 MeV - The minimum width of parents whose daughter pions can contribute to the Bose-Einstein enhancement, equivalent to 10 fm.
- PARJ(92) = 1 - The strength of the Bose-Einstein effect- a measure of the incoherence of the source. Denoted by λ.
- PARJ(93) = 0.3 GeV - The inverse radius of the pion source, equivalent to 0.66 fm and compatible with the range of experimental measurements. Denoted by σ^{-1}.

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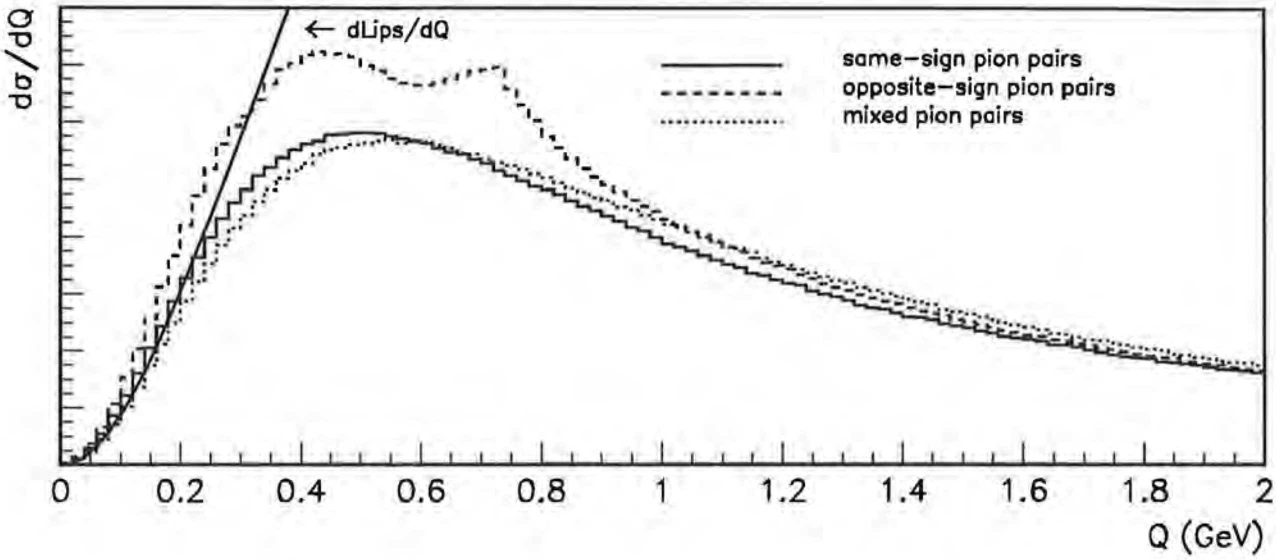

Figure 4: $d\sigma/dQ$ derived from JETSET for different types of pion pairs. No V^0 decays are included (hence no K_s^0 peak) and there is no resolution smearing. The normalisation of $dLips/dQ$ is chosen to cause the curve to approximate the differential cross-sections at low Q.

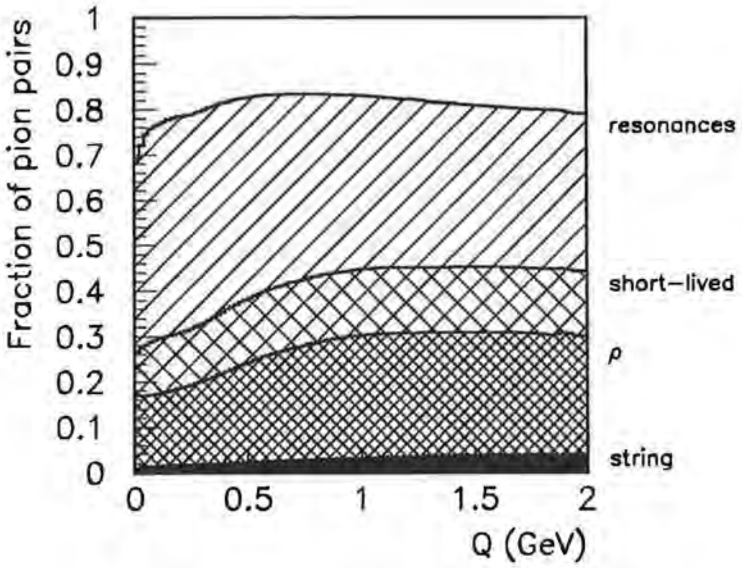

Figure 5: The origins of pion pairs as a function of Q, determined with JETSET. Pairs where both pions come from the string are shown as dark region. Pairs where neither pion comes from a source longer lived than: a ρ are shown as densely hatched; a decay with $c\tau < 10$ fm are shown as hatched; a strongly decaying resonance with $c\tau > 10$ fm are shown as striped. The remainder include one pion from a weak decay. Pions from *stable* particles are not included.

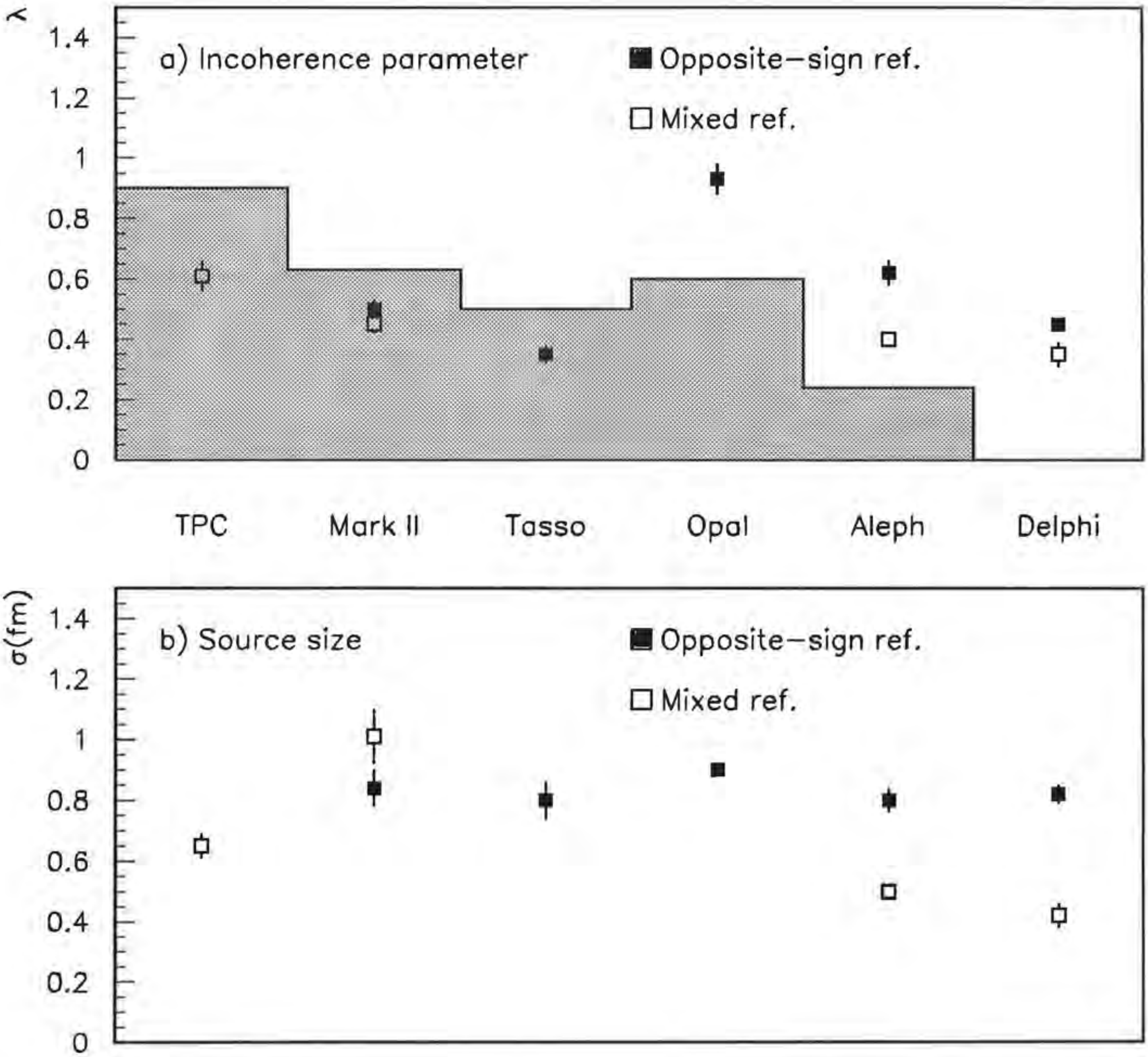

Figure 6: a) The *measured* incoherence parameter λ measured by different experiments, using opposite-sign or mixed reference sets - see table 2. The shaded region indicates (where determined) the "allowed" range for the measured value of λ, namely between 0 and λ_{max}, as predicted with JETSET. It is important to consider i) the notes of table 2, which would tend to lower λ_{max} for TPC, MARKII and TASSO and ii) the role of final-state interactions and the η' rate. b) The measured source sizes for the same experiments, assuming the Gaussian parameterisation of equation 3.

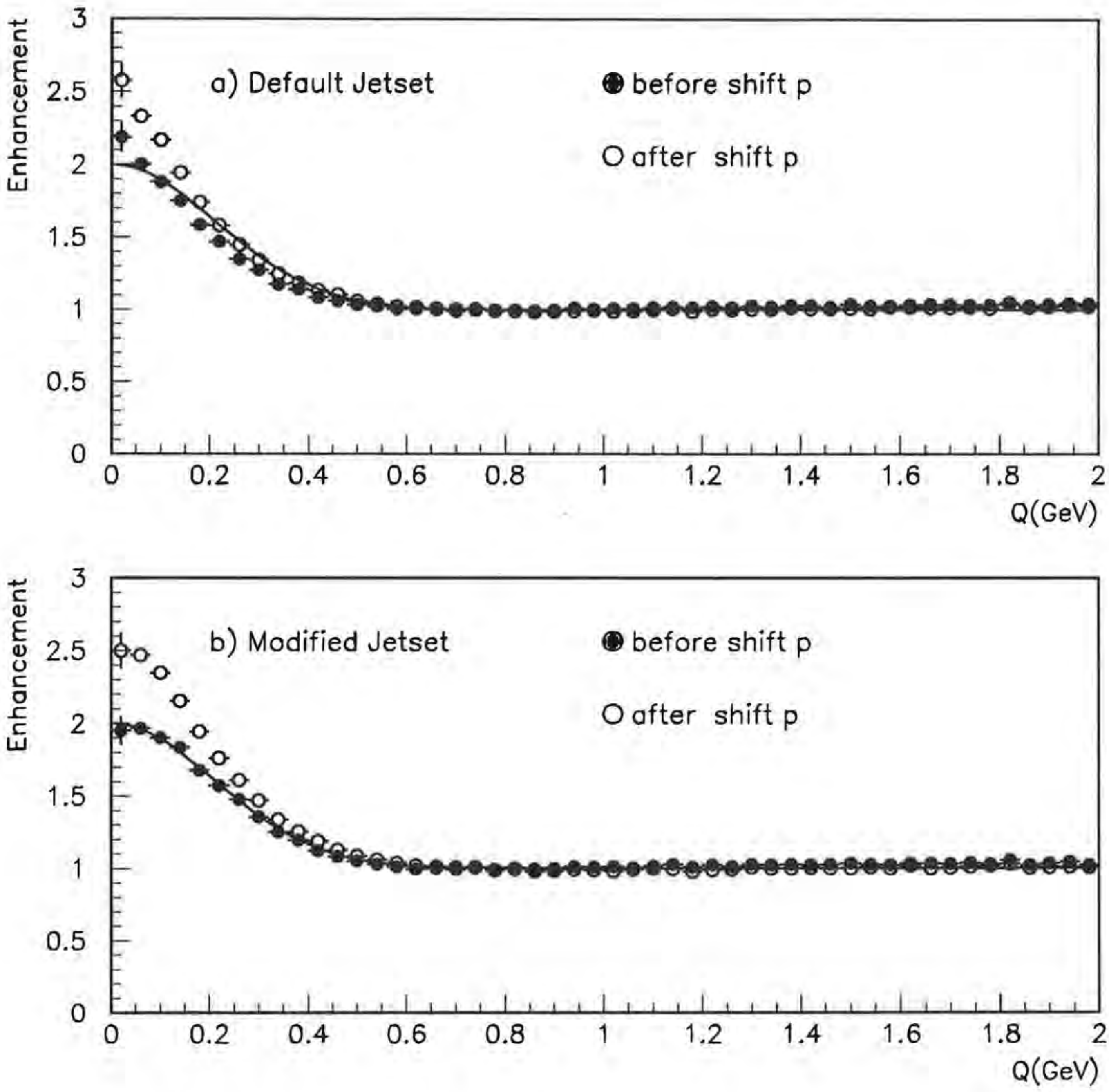

Figure 7: a) The enhancement of the same-sign pion distribution obtained from the routine LUBOEI in JETSET. Obtained from the ratio of the Q distributions for pions from sources with $c\tau < 10$ fm with and without the Bose-Einstein effect. The ratios are normalised to 1 in the region $0.6 < Q < 1$ and the input distribution (equation 3, $\lambda = 1$, $\sigma^{-1} = 0.3$ GeV) is shown as a curve. The solid points show the enhancement which would be obtained as an intermediate step (before shifting all the momentum vectors); the hollow points are the result of the multipion effects, after the momentum shifts, and correspond to what is seen as the output of JETSET. b) As a), but with the modifications described in section 5.1.

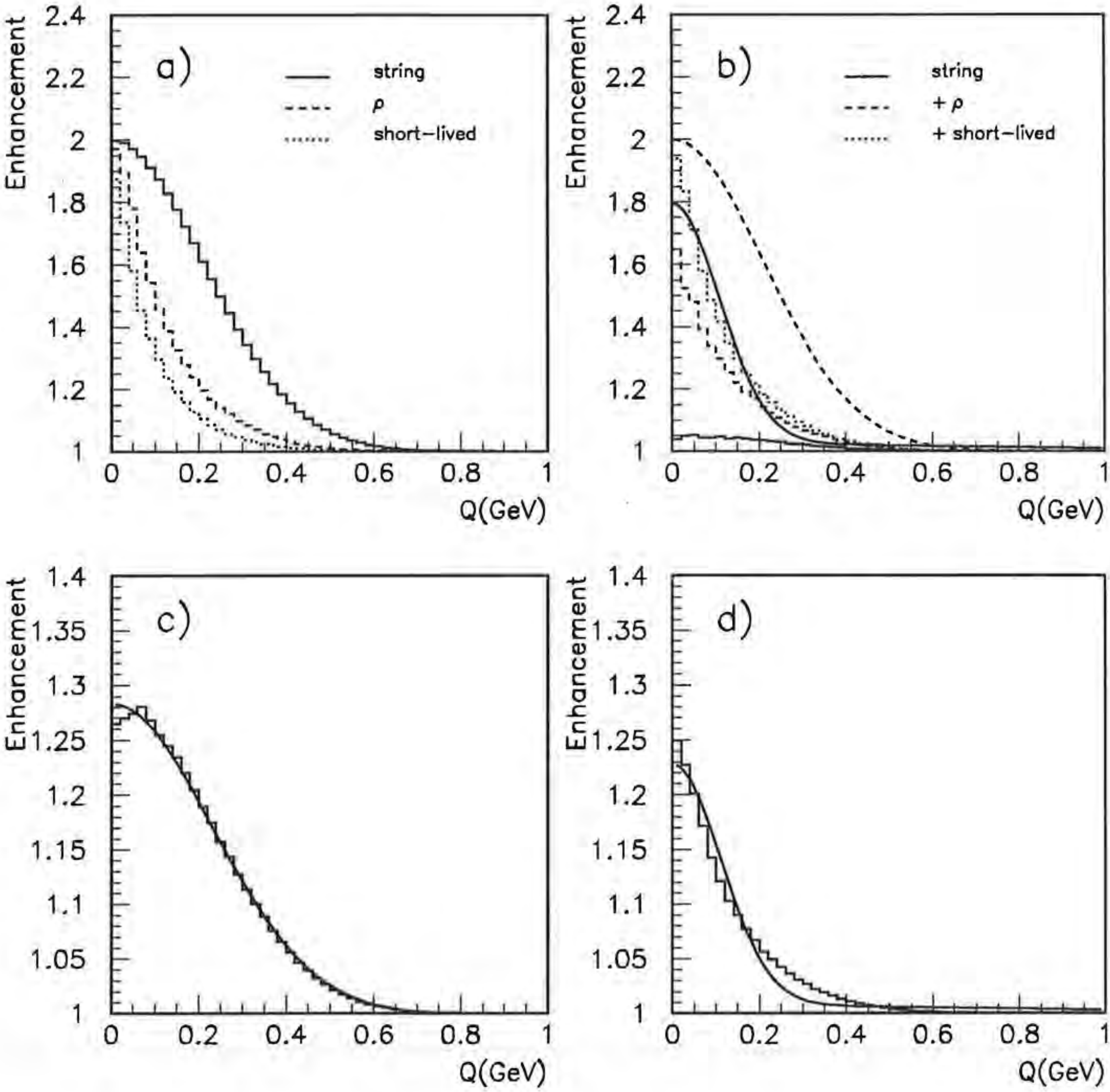

Figure 8: a) The effect of resonance narrowing on the enhancement of different components of the Q distribution. Pions from the string (no narrowing) (line), pions where at least one comes from a ρ (dash) and pions where at least one comes from a short-lived mother ($ct < 10$ fm) (dot) are shown. Note the suppressed zero. b) The different components from figure a) are added in the correct proportions - the resulting distributions are cumulative. For comparison, a fit to the total enhancement is shown (curve), and the input is also shown (dashed curve). c) The enhancement which would be observed in the absence of resonance narrowing, allowing for the correct proportions of **all** pion pairs. d) As for c), but with resonance narrowing. The fitted curve is sensitive to the range over which the fit is made.

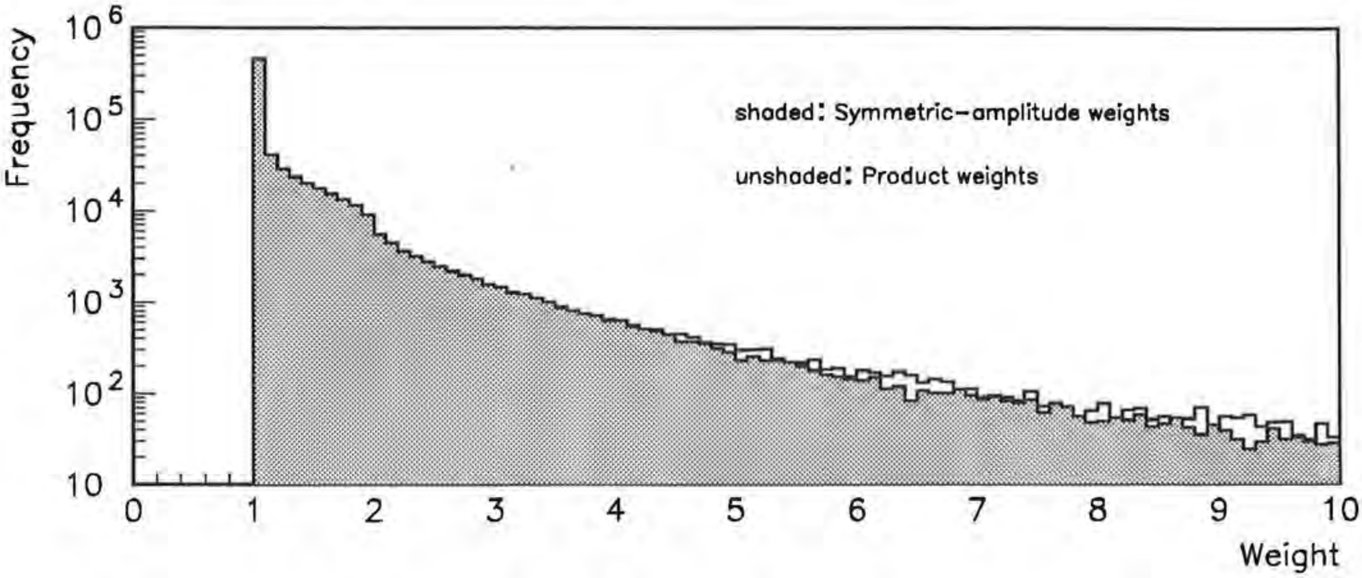

Figure 9: Distributions of *symmetric-amplitude* (equation 22) and *product event-weights* (equation 21). There are significant numbers of overflows with very large weights.

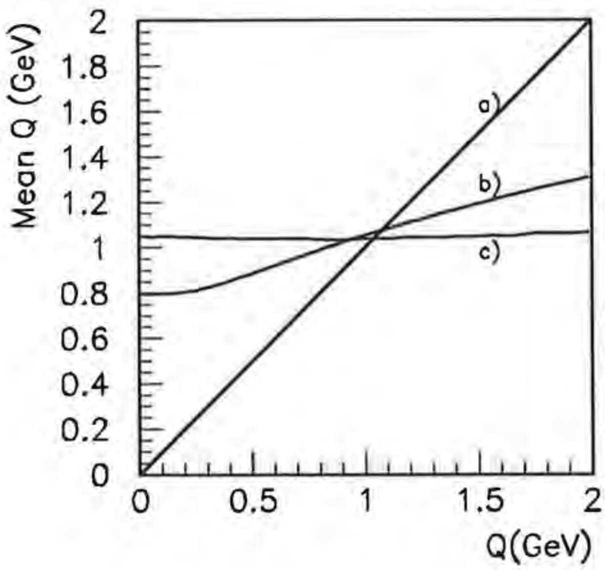

Figure 10: Mean Q of different pion pairs as a function of the Q of a given pair: a) the same pair (trivial), b) one and only one pion in common, c) no pions in common. See section 7.2.

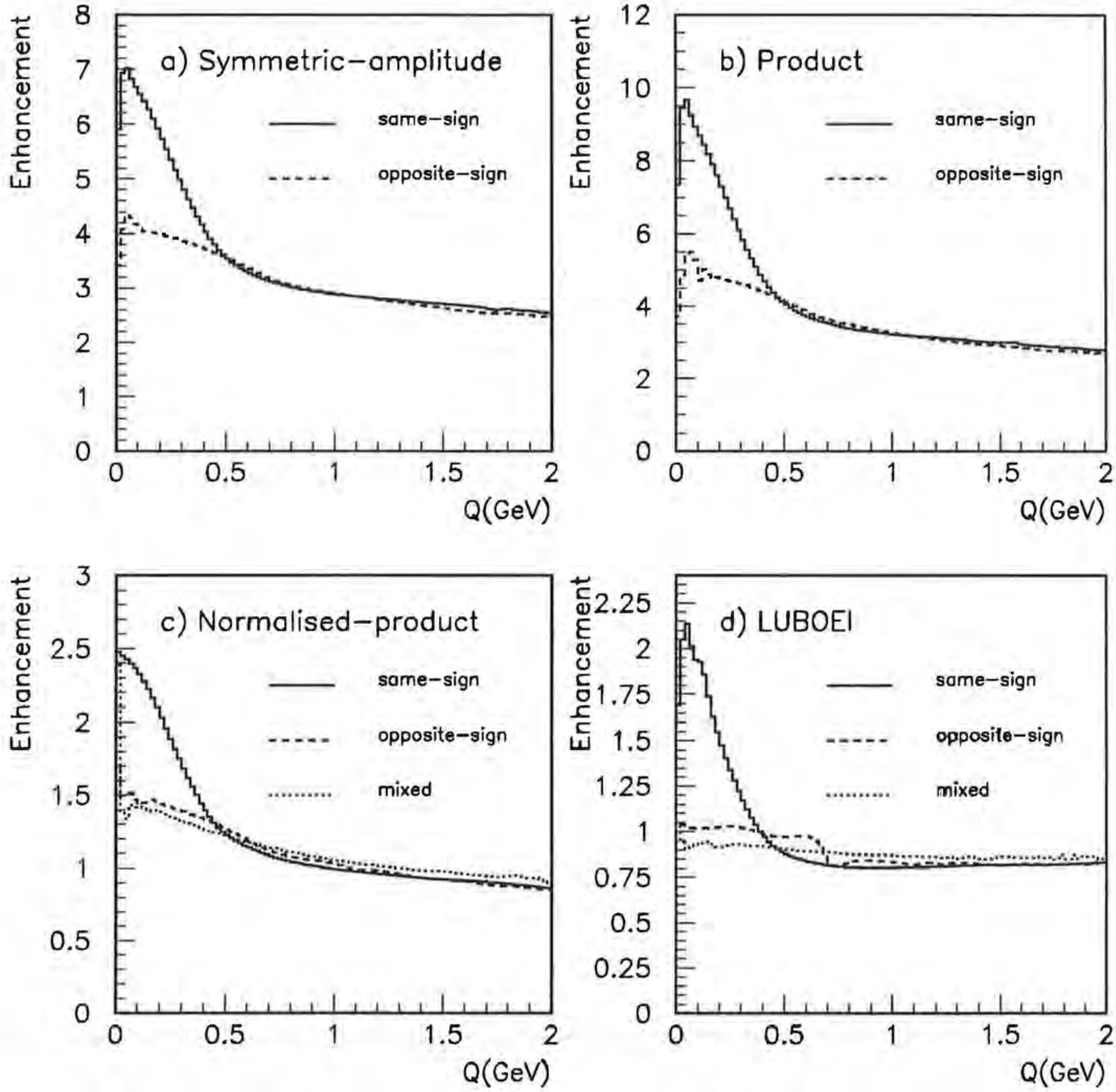

Figure 11: Demonstration of multipion effects. The enhancements seen by same-sign, opposite-sign and mixed pion pairs with different event-weight schemes. Only pions from sources with $c\tau < 10$ fm are considered, and all of these are considered capable of exhibiting the full Bose-Einstein enhancement, with no resonance narrowing. While the distributions are smoothed to some extent, statistical effects are still visible.

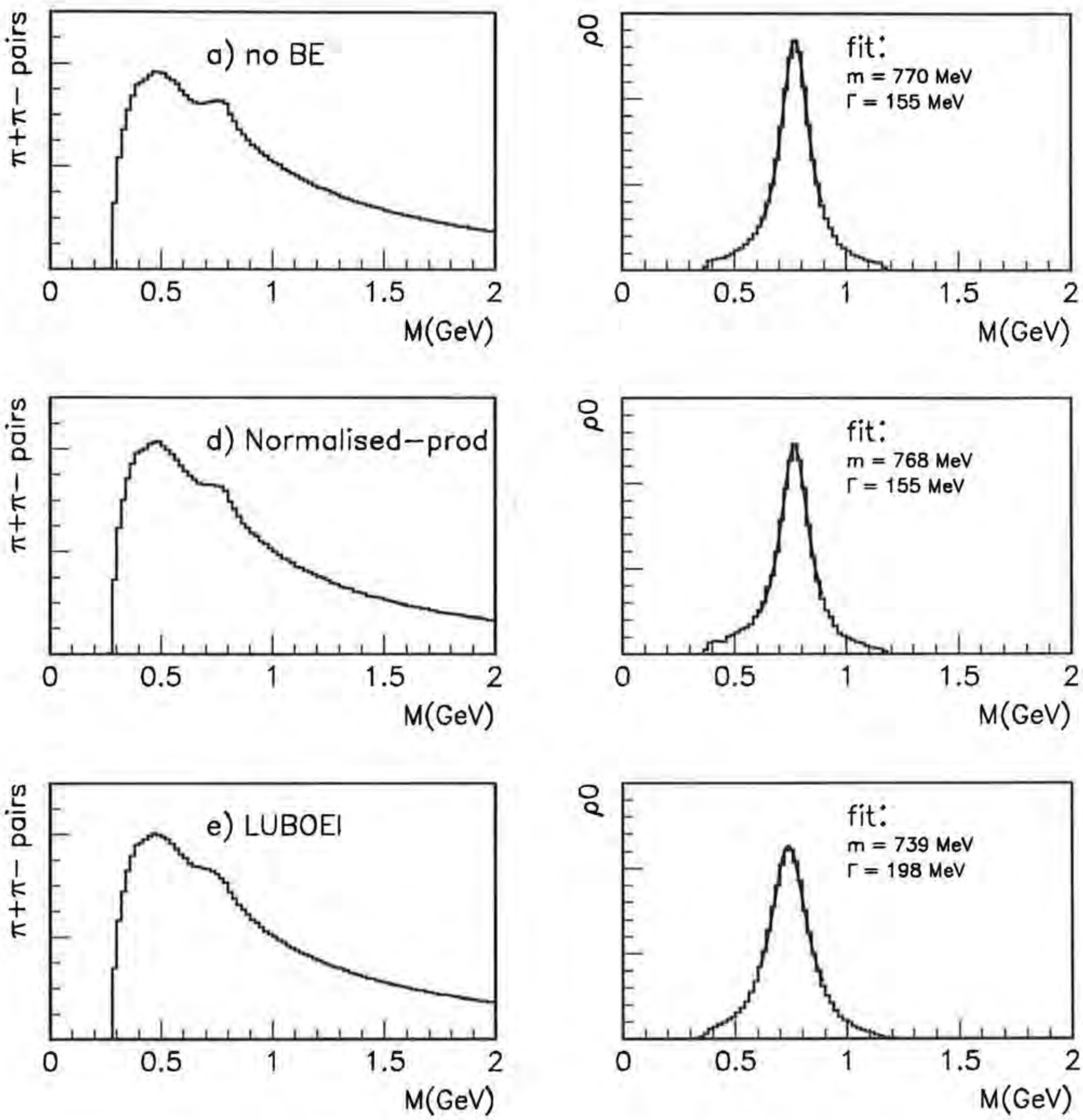

Figure 12: The $\pi^+\pi^-$ mass spectrum for all pion pairs (left) and just those flagged as coming from a ρ^0 (right) for different event-weight schemes.