Statistics for Linguists with R – a SIGIL course

Unit 2: Corpus Frequency Data & Statistical Inference

Marco Baroni¹ & Stefan Evert² http://SIGIL.R-Forge.R-Project.org/

¹Center for Mind/Brain Sciences, University of Trento ²Corpus Linguistics Group, FAU Erlangen-Nürnberg

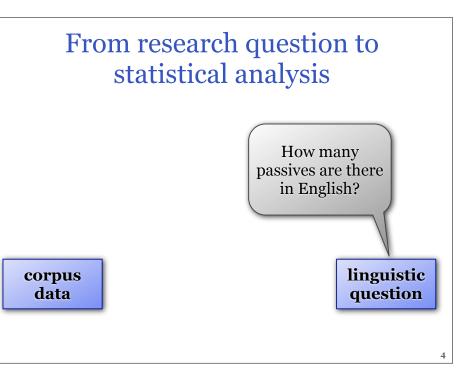
A simple toy problem

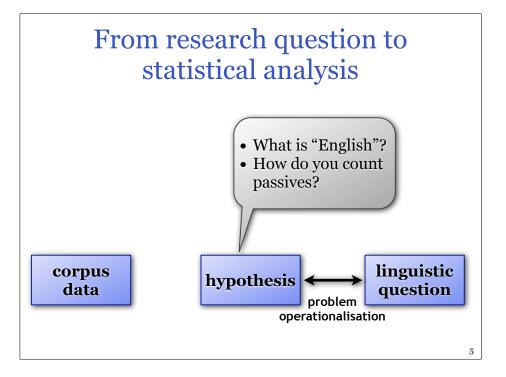
How many passives are there in English?

- American English style guide claims that
 - "In an average English text, no more than 15% of the sentences are in passive voice. So use the passive sparingly, prefer sentences in active voice."
 - http://www.ego4u.com/en/business-english/grammar/passive actually states that only 10% of English sentences are passives (as of January 2009)!
- We have doubts and want to verify this claim

Frequency estimates & comparison

- How often is *kick the bucket* really used?
- What are the characteristics of "translationese"?
- Do Americans use more split infinitives than Britons? What about British teenagers?
- What are the typical collocates of *cat*?
- Can the next word in a sentence be predicted?
- Do native speakers prefer constructions that are grammatical according to some linguistic theory?
- ➡ evidence from frequency comparisons / estimates





How do you count passives?

- ◆ Types vs. tokens
 - **type** count: How many *different* passives are there?
 - token count: How many *instances* are there?
- How many passive tokens are there in English?
 - infinitely many, of course!
- Absolute frequency is not meaningful here



- Sensible definition: group of speakers
 - e.g. American English as language spoken by native speakers raised and living in the U.S.
 - may be restricted to certain communicative situation
- ♦ Also applies to definition of sublanguage
 - dialect (Bostonian, Cockney), social group (teenagers), genre (advertising), domain (statistics), ...
- Here: professional writing by native speakers of AmE (i > target audience of style guide)

Against "absolute" frequency

- ◆ Are there **20,000** passives?
 - Brown (1M words)



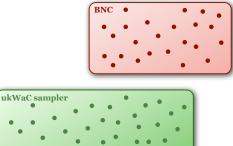
6

8

- ♦ Or 1 million?
 - BNC (90M words)
- ♦ Or 5.1 million?

7

• ukWaC sampler (450M words)



How do you count passives?

- Only **relative frequency** can be meaningful!
- What is a sensible unit of measurement?
 - ... 20,300 per million words?
 - ... 390 per thousand sentences?
 - ... 28 per hour of recorded speech?
 - ... **4,000** per **book**?
- How many passives could there be at most?

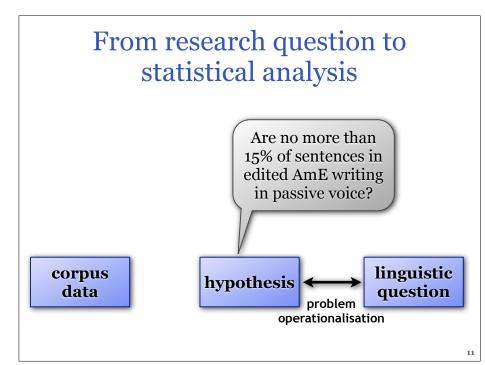
9

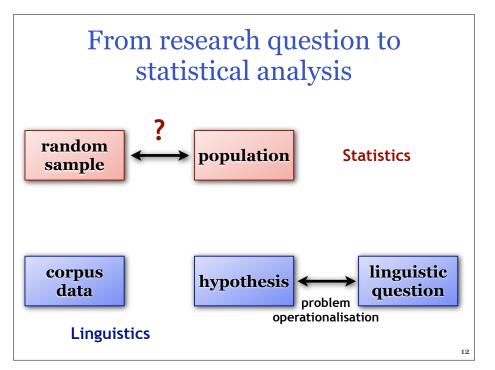
How do you count passives?

- How many passives could there be at most?
 - every VP can be in active or passive voice
 - frequency of passives has a meaningful interpretation by comparison with frequency of potential passives
- What proportion of VPs are in passive voice?
 - easier: proportion of sentences that contain a passive
 - in general, proportion wrt. some **unit of measurement**

10

Relative frequency = proportion π



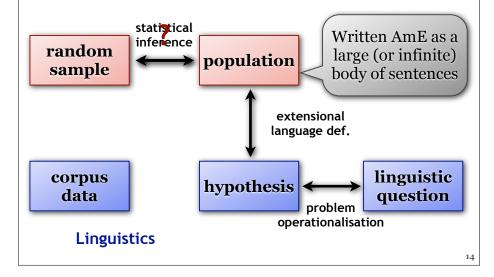


Using inferential statistics

- Statistics deals with similar problems:
 - goal: determine properties of **large population** (human populace, objects produced in factory, ...)
 - method: take (completely) **random sample** of objects, then extrapolate from sample to population
 - this works only because of **random** sampling!
- Many statistical methods are readily available

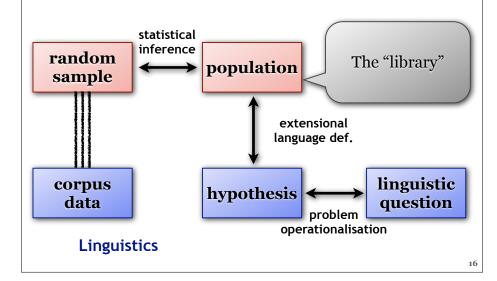
13

From research question to statistical analysis



The library metaphor Extensional definition of a language: "All utterances made by speakers of the language under appropriate conditions, plus all utterances they *could* have made" Imagine a huge library with all the books written in a language, as well as all the hypothetical books that have never been written Ibrary metaphor (Evert 2006)

From research question to statistical analysis



A random sample of a language

- ◆ Apply statistical procedure to linguistic problem
 ⇒ need random sample of objects from population
- Quiz: What are the objects in our population?
 - words? sentences? texts? ...
- Objects = whatever unit of measurement the proportions of interest are based on

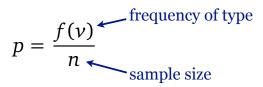
17

19

• we need to take a random sample of such units

Types, tokens and proportions

- Proportions and relative sample frequencies are defined formally in terms of types & tokens
- Relative frequency of type *v* in sample {*t₁, ..., t_n*}
 = proportion of tokens *t_i* that belong to this type



Compare relative sample frequency *p* against (hypothesised) population proportion π



Types, tokens and proportions

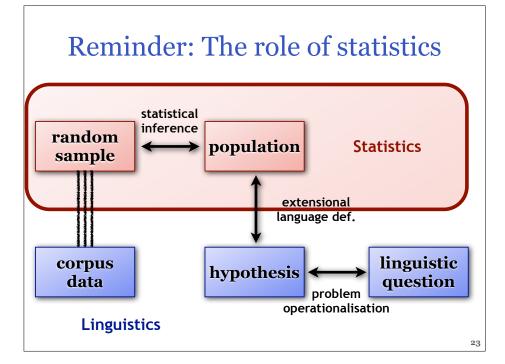
- Example: word frequencies
 - word type = dictionary entry (distinct word)
 - word token = instance of a word in library texts
- Example: passive VPs
 - relevant VP types = active or passive (\rightarrow abstraction)
 - VP token = instance of VP in library texts
- Example: verb sucategorisation
 - relevant types = itr., tr., ditr., PP-comp., X-comp, ...
 - verb token = occurrence of selected verb in text

Inference from a sample

- Principle of inferential statistics
 - if a sample is picked at random, proportions should be roughly the same in sample and population

21

- ◆ Take a sample of 100 sentences
 - observe 19 passives $\rightarrow p = 19\% = .19$
 - style guide \rightarrow population proportion π = 15%
 - $p > \pi \rightarrow$ reject claim of style guide?
- Take another sample, just to be sure
 - observe 13 passives $\rightarrow p = 13\% = .13$
 - $p < \pi \rightarrow$ claim of style guide confirmed?



Sampling variation

- Random choice of sample ensures proportions are the same on average in sample & population
- ◆ But it also means that for every sample we will get a different value because of chance effects
 → sampling variation
 - problem: erroneous rejection of style guide's claim results in publication of a false result
- The main purpose of statistical methods is to estimate & correct for sampling variation
 - that's all there is to inferential statistics, really



22

The null hypothesis

• Our "goal" is to refute the style guide's claim, which we call the **null hypothesis** *H*₀

 $H_0: \pi = .15$

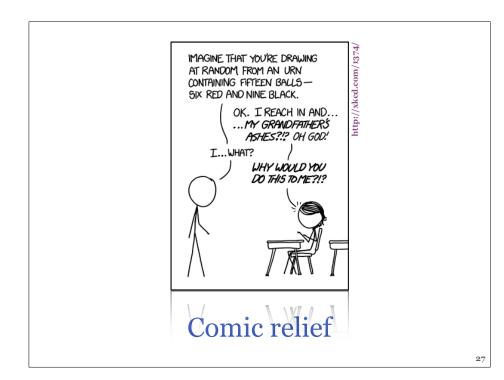
- we also refer to π_0 = .15 as the **null proportion**
- Erroneous rejection of H_0 is problematic
 - leads to embarrassing publication of false result
 - known as a **type I error** in statistics
- Need to control risk of a type I error

Estimating sampling variation

- Assume that style guide's claim H_0 is correct
 - i.e. rejection of H_0 is always a type I error
- Many corpus linguists set out to test H_0
 - each one draws a random sample of size n = 100
 - how many of the samples have the expected *k* = 15 passives, how many have *k* = 19, etc.?
 - if we are willing to reject H_0 for k = 19 passives in a sample, all corpus linguists with such a sample will publish a false result

25

• risk of type I error = percentage of such cases

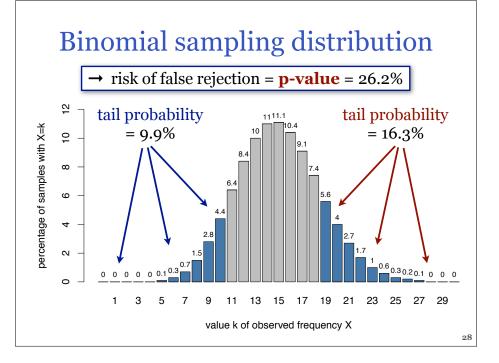


Estimating sampling variation

- We don't need an infinite number of monkeys (or corpus linguists) to answer these questions
 - randomly picking sentences from our metaphorical library is like drawing balls from an infinite urn
 - red ball = passive sent. / white ball = active sent.
 - H_0 : assume proportion of red balls in urn is 15%
- This leads to a **binomial distribution**

$$\Pr(k) = \binom{n}{k} (\pi_0)^k (1 - \pi_0)^{n-k}$$

percentage of samples = probability 26



Statistical hypothesis testing

- Statistical hypothesis tests
 - define a **rejection criterion** for refuting H_0
 - control the risk of false rejection (type I error) to a "socially acceptable level" (significance level α)
 - **p-value** = risk of type I error given observation, interpreted as amount of evidence against *H*₀
- ◆ Two-sided vs. one-sided tests
 - in general, two-sided tests are recommended (safer)
 - one-sided test is plausible in our example

Binomial hypothesis test in R

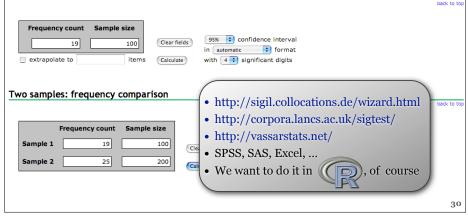
- ◆ Relevant R function: binom.test()
- We need to specify
 - observed data: 19 passives out of 100 sentences
 - null hypothesis: H_0 : $\pi = 15\%$
- Using the binom.test() function:
 - > binom.test(19, 100, p=.15) # two-sided

Hypothesis tests in practice

SIGIL: Corpus Frequency Test Wizard

This site provides some online utilities for the project Statistical Inference: A Gentle Introduction for Linguists (SIGIL) by Marco Baroni @ and Stefan Evert @. The main SIGIL homepage can be found at purl.org/stefan.evert/SIGIL @.

One sample: frequency estimate (confidence interval)



binomission by pothesis test in A . binomission by pothesis test binomial test data: f y and for under of successes = 19 number of trials = 100, p-value = 0.2623 lternative hypothesis: true probability of success is not equal to 0.15 . f percent confidence interval: 0.184432 0.2806980 sample estimates: probability of success 0.19

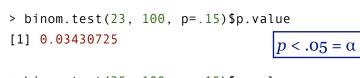
Rejection criterion & significance level > binom.test(19, 100, p=.15)\$p.value [1] 0.2622728 p > .05

n.s.

*

**

33

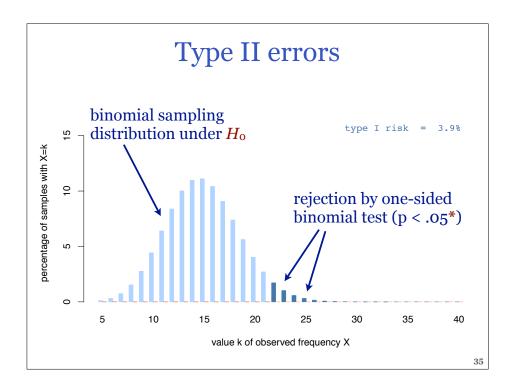


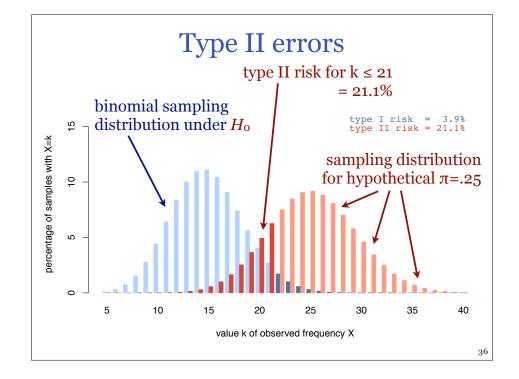
> binom.test(25, 100, p=.15)\$p.value [1] 0.007633061 $p < .01 = \alpha$

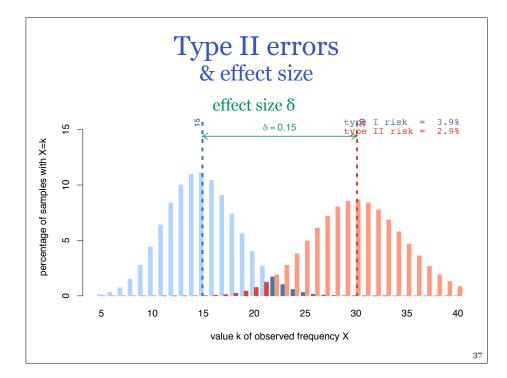
> binom.test(29, 100, p=.15)\$p.value [1] 0.0003529264 $p < .001 = \alpha$

Type II errors

- Rejection criterion controls risk of type I error
 - only for situation in which H_0 is true
- Type II error = failure to reject incorrect H_0
 - for situation in which *H*₀ is not true
 → rejection correct, non-rejection is an error
- What is the risk of a type II error?
 - depends on unknown true population proportion $\boldsymbol{\pi}$
 - intuitively, risk of type II error will be low if the difference $\delta = \pi \pi_0$ (the **effect size**) is large enough

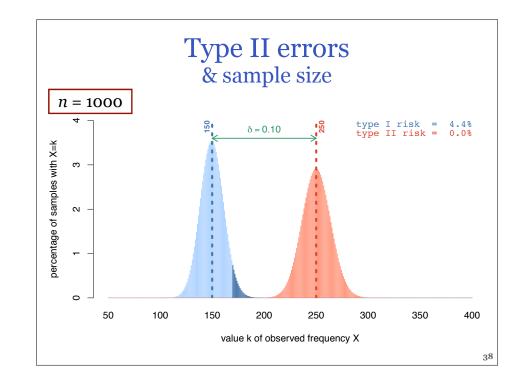




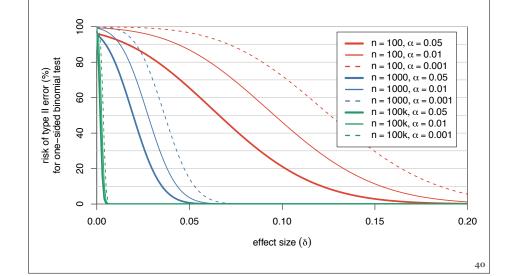


Power

- Type II error = failure to reject incorrect H_o
 - the larger the difference between *H*₀ and the true population proportion, the more likely it is that *H*₀ can be rejected based on a given sample
 - a **powerful** test has a low **type II error**
 - power analysis explores the relationship between effect size and risk of type II error
- Key insight: larger sample = more power
 - relative sampling variation becomes smaller
 - power also depends on significance level



Power analysis for binomial test



Power analysis for binomial test

- Key factors determining the power of a test
 - **sample size** → more evidence = greater power
 - **significance level** \rightarrow trade-off btw. type I / II errors
- Influence of hypothesis test procedure
 - one-sided test more powerful than two-sided test
 - parametric tests more powerful than non-parametric
 - statisticians look for "uniformly most powerful" test
- Tests can become too powerful!
 - reject H_0 for 15.1% passives with n = 1,000,000

Trade-offs in statistics

- Inferential statistics is a trade-off between type I errors and type II errors
 - i.e. between **significance** and **power**
- ◆ Significance level
 - determines trade-off point
 - low significance level $\alpha \rightarrow$ low type I risk, but low power
- ♦ Conservative tests
 - put more weight on avoiding type I errors \rightarrow weaker
 - most non-parametric methods are conservative

Parametric vs. non-parametric

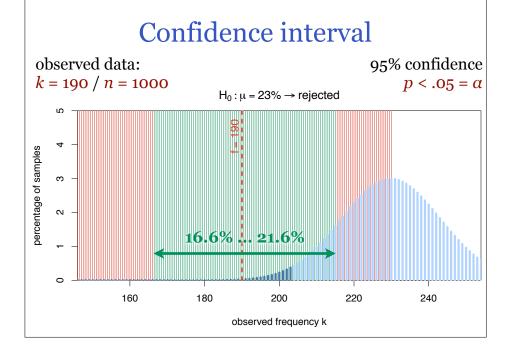
- People often talk about parametric and nonparametric tests without precise definition
- Parametric tests make stronger assumptions
 - not just normality assuming (= Gaussian distribution)
 - binomial test: strong random sampling assumption
 → might be considered a parametric test in this sense!
- Parametric tests are usually more powerful
 - strong assumptions allow less conservative estimate of sampling variation → less evidence needed against H₀

Confidence interval

- We now know how to test a null hypothesis *H*₀, rejecting it only if there is sufficient evidence
- But what if we do not have an obvious null hypothesis to start with?
 - this is typically the case in (computational) linguistics
- We can estimate the true population proportion from the sample data (relative frequency)
 - sampling variation \rightarrow range of plausible values
 - such a **confidence interval** can be constructed by inverting hypothesis tests (e.g. binomial test)

41

43



Confidence intervals in R

- Most hypothesis tests in R also compute a confidence interval (including binom.test())
 - omit H_0 if only interested in confidence interval
- Significance level of underlying hypothesis test is controlled by conf.level parameter
 - expressed as confidence, e.g. conf.level=.95 for significance level $\alpha = .05$, i.e. 95% confidence
- Can also compute one-sided confidence interval
 - controlled by alternative parameter
 - two-sided confidence intervals strongly recommended

47

I'm cheating here a tiny little bit (not always an interval)

- Confidence interval = range of plausible values for true population proportion
 - H_0 rejected by test iff π_0 is outside confidence interval
- Size of confidence interval depends on power of the test (i.e. sample size and significance level)

	n = 100 $k = 19$	n = 1,000 k = 190	n = 10,000 k = 1,900
$\alpha = .05$ $\alpha = .01$ $\alpha = .001$	$10.1\% \dots 31.0\%$	16.6% 21.6% 15.9% 22.4% 15.1% 23.4%	$18.0\% \dots 20.0\%$

Confidence intervals in R

> binom.test(190, 1000, conf.level=.99)

Exact binomial test

data: 190 and 1000

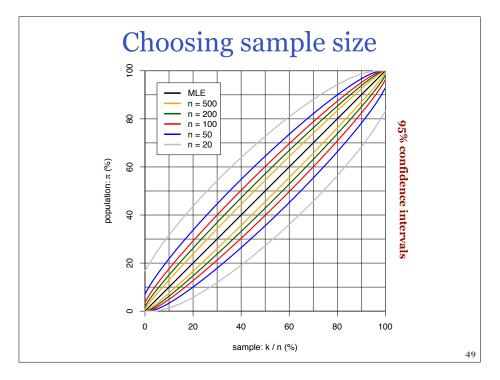
```
number of successes = 190, number of
trials = 1000, p-value < 2.2e-16</pre>
```

alternative hypothesis: true probability of success is not equal to $0.5\,$

99 percent confidence interval: 0.1590920 0.2239133

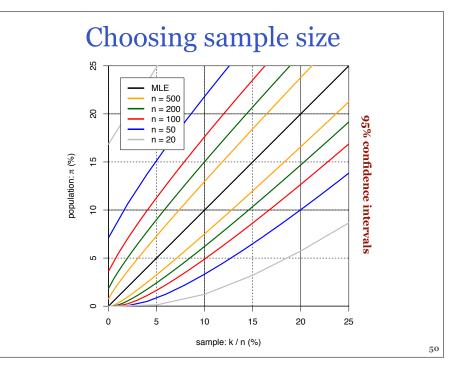
```
sample estimates:
probability of success
0.19
```

48



Using R to choose sample size

- Call binom.test() with hypothetical values
- Plots on previous slides also created with R
 - requires calculation of large number of hypothetical confidence intervals
 - binom.test() is both inconvenient and inefficient
- The corpora package has a vectorised function
 - > library(corpora)
 - > prop.cint(190, 1000, conf.level=.99)
 - > ?prop.cint # "conf. intervals for proportions"



Frequency comparison

- Many linguistic research questions can be operationalised as a frequency comparison
 - Are split infinitives more frequent in AmE than BrE?
 - Are there more definite articles in texts written by Chinese learners of English than native speakers?
 - Does *meow* occur more often in the vicinity of *cat* than elsewhere in the text?
 - Do speakers prefer *I couldn't agree more* over alternative realisations such as *I agree completely*?
- Compare observed frequencies in two samples

Frequency comparison

• Null hypothesis for frequency comparison

 $H_0:\pi_1=\pi_2$

- no assumptions about the precise value $\pi_1 = \pi_2 = \pi$
- Observed data
 - target count k_i and sample size n_i for each sample i
 - e.g. $k_1 = 19 / n_1 = 100$ passives vs. $k_2 = 25 / n_2 = 200$
- Effect size: difference of proportions
 - effect size $\delta = \pi_1 \pi_2$ (and thus $H_0: \delta = 0$)

53

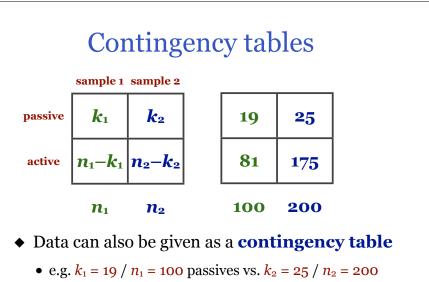
Frequency comparison in R

- Frequency comparison test: prop.test()
 - observed data: counts k_i and sample sizes n_i
 - also computes confidence interval for effect size
- E.g. for 19 passives out of 100 / 25 out of 200
 - parameters conf.level and alternative can be used in the familiar way
 - > prop.test(c(19,25), c(100,200))

54

56

Frequency comparison in R
> prop.test(c(19,25), c(100,200))
2-sample test for equality of proportions with
continuity correction
data: c(19, 25) out of c(100, 200)
X-squared = 1.7611, df = 1, p-value = 0.1845
alternative hypothesis: two.sided
95 percent confidence interval:
-0.03201426 0.16201426
sample estimates:
prop 1 prop 2
0.190 0.125



- represents a cross-classification of n = 300 items
- generalization to larger tables possible

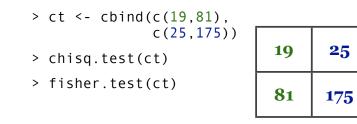
Tests for contingency tables

- Fisher's exact test = generalization of binomial test to contingency tables
 - computationally expensive, mostly for small samples
- Pearson's chi-squared test = asymptotic test based on test statistic X²
 - larger value of $X^2 \rightarrow$ less likely under H_0
 - X^2 can be translated into corresponding p-value
 - suitable for large samples and small balanced samples
- ◆ Likelihood-ratio test based on statistic *G*²
 - popular in collocation and keyword identification
 - suitable for highly skewed data

57

Tests for contingency tables

- Can easily carry out chi-squared (chisq.test) and Fisher's exact test (fisher.test) in R
 - likelihood ratio test not included in R standard library
- ◆ Table for 19 / 100 vs. 25 / 200





60

Significance vs. relevance

- Much focus on significant p-value, but ...
 - large differences may be non-significant if sample size is too small (e.g. 10/80 = 12.5% vs. 20/80 = 25%)
 - increase sample size for more powerful/sensitive test
 - very large samples lead to highly significant p-values for minimal and irrelevant differences (e.g. 1M tokens with 150,000 = 15% vs. 151,000 = 15.1% occurrences)
- It is important to assess both significance and relevance (= effect size) of frequency data!
 - confidence intervals combine both aspects

Effect size in contingency tables

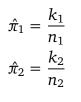
 Simple effect size measure: difference of proportions

$$\delta = \pi_1 - \pi_2$$

- $H_0: \delta = 0$
- ♦ Issues
 - depends on scale of π_1 and π_2
 - small effects for lexical freq's

π_1	π_2	
1 -π ₁	1 -π ₂	

population equivalent of a contingency table, which determines the multinomial sampling distribution



Effect size in contingency tables

◆ Effect size measure: (log) relative risk

 $r = \frac{\pi_1}{\pi_2}$

• $H_0: r = 1$

 $1 - \pi_1$ $1 - \pi_2$ population equivalent of a contingency table, which determines the multinomial sampling distribution

 π_1

 π_2

- ♦ Issues
 - can be inflated for small π_2
 - mathematically inconvenient

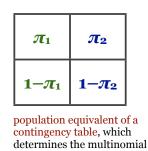


61

63

Effect size in contingency tables

- ◆ Effect size measure: φ coefficient / Cramér V $\phi = \sqrt{\frac{X^2}{r}}$ ◆ H₀: ??? $n = n_1 + n_2$
- ♦ Issues
 - this is a property of the sample rather than the population!



sampling distribution



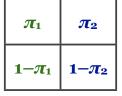
Effect size in contingency tables

◆ Effect size measure: (log) odds ratio π_1 (1

$$\theta = \frac{\overline{1-\pi_1}}{\frac{\pi_2}{1-\pi_2}} = \frac{\pi_1(1-\pi_2)}{\pi_2(1-\pi_1)}$$

• H₀: $\theta = 1$





population equivalent of a contingency table, which determines the multinomial sampling distribution

- ♦ Issues
 - can be inflated for small π_2
 - interpretation not very intuitive

 $\hat{\pi}_1 = \frac{k_1}{n_1}$ $\hat{\pi}_2 = \frac{k_2}{n_2}$

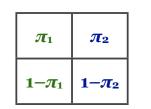
62

Effect size in contingency tables

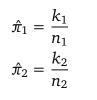
- ◆ Effect size measure: π_1 π_2 φ coefficient / Cramér V $\phi = \frac{\pi_1(1 - \pi_2) - \pi_2(1 - \pi_1)}{\sqrt{(r_1\pi_1 + r_2\pi_2)(1 - r_1\pi_1 - r_2\pi_2)/r_1r_2}}$ $1 - \pi_1$ $1 - \pi_2$ population equivalent of a • H₀: $\varphi = 0$ $n = n_1 + n_2$ contingency table, which determines the multinomial sampling distribution $r_1 = n_1 / n$ $r_2 = n_2 / n$ $\hat{\pi}_1 = \frac{k_1}{n_1}$ ♦ Issues • depends on relative sample sizes $\hat{\pi}_2 = \frac{k_2}{n_2}$ • interpretation entirely unclear
 - 64

Effect size in contingency tables

- We can estimate effect sizes by inserting sample values *k*_{*i*}/*n*_{*i*}
- But such point estimates are meaningless!
- Confidence intervals available only for some effect measures
 - approximate interval for δ from proportions test
 - exact interval for odds ratio θ from Fisher's test
 - *φ* computed from chi-square statistic is still a point estimate!



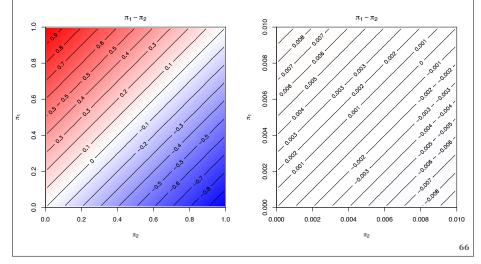
population equivalent of a contingency table, which determines the multinomial sampling distribution

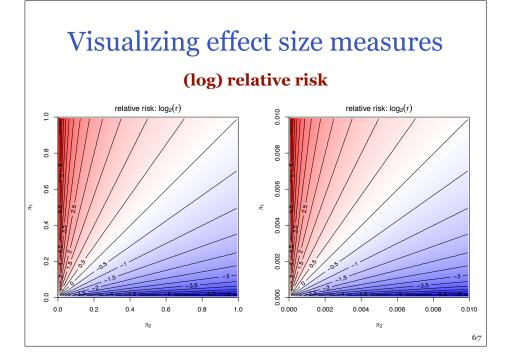


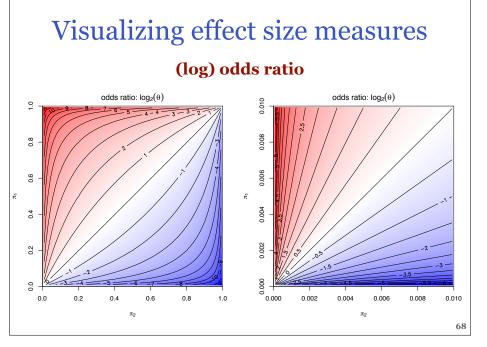
65

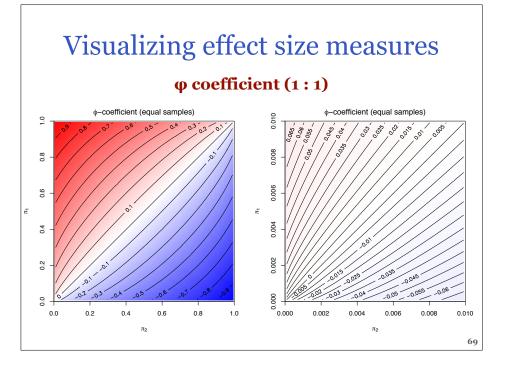
Visualizing effect size measures

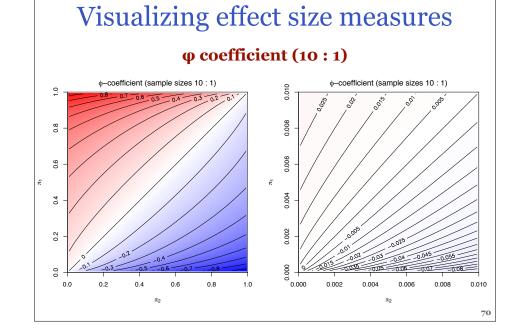
difference of proportions











Visualizing effect size measures φ coefficient (1:10) coefficient (sample sizes 1 : 10) ient (sample sizes 1 : 10 0.8 0.6 4.0 0.0 0.2 0.4 0.6 0.8 0.000 0.002 0.004 0.006 0.008 0.010

 π_2

71

 π_2

A case study: passives

- As a case study, we will compare the frequency of passives in Brown (AmE) and LOB (BrE)
 - pooled data
 - separately for each genre category
- ◆ Data files provided in CSV format
 - passives.brown.csv & passives.lob.csv
 - cat = genre category, passive = number of passives, n_w = number of word, n_s = number of sentences, name = description of genre category

Preparing the data

> Brown <- read.csv("passives.brown.csv")
> LOB <- read.csv("passives.lob.csv")</pre>

```
> library(SIGIL) # or use versions in SIGIL package
```

- > Brown <- BrownPassives</pre>
- > LOB <- LOBPassives

}

now take a look at the two tables: what info do they provide?

```
# pooled data for entire corpus = column sums (col. 2 ... 4)
```

- > Brown.all <- colSums(Brown[, 2:4])</pre>
- > LOB.all <- colSums(LOB[, 2:4])</pre>

Frequency tests for pooled data

proportions test reports p-value is based on chi-squared test # and approximate confidence interval for effect size δ > prop.test(c(10123, 10934), c(49576, 49742))

> ct # contingency table for chi-squared / Fisher

> fisher.test(ct) # exact confidence interval for odds ratio θ

we could in principle do the same for all 15 genres ...

```
Automation: user functions
```

user function do.test() executes proportions test for samples
k₁/n₁ and k₂/n₂, and summarizes relevant results in compact form
> do.test <- function (k1, n1, k2, n2) {
 # res contains results of proportions test (list = data structure)
 res <- prop.test(c(k1, k2), c(n1, n2))</pre>

```
# data frames are a nice way to display summary tables
fmt <- data.frame(p=res$p.value,
    lower=res$conf.int[1], upper=res$conf.int[2])</pre>
```

fmt # return value of function = last expression

> do.test(10123, 49576, 10934, 49742) # pooled data
> do.test(146, 975, 134, 947) # humour genre

A nicer user function

```
# nicer version of user function with genre category labels
> do.test <- function (k1, n1, k2, n2, cat="") {
    res <- prop.test(c(k1, k2), c(n1, n2))
    data.frame(
        p=res$p.value,
        lower=100*res$conf.int[1], # scaled to % points
        upper=100*res$conf.int[2],
        row.names=cat # add genre as row label
    ) # return data frame directly without local variable fmt
}
# extract relevant information directly from data frames
> do.test(Brown$passive[15], Brown$n_s[15],
        LOB$passive[15], LOB$n s[15],
```

cat=Brown\$name[15])

75

73

Ad-hoc functions & loops

```
R wizardry: working with lists
```

our code only works if rows of Brown/LOB are in the same order!
> all(Brown\$cat == LOB\$cat)
it would be nice to collect all these results in a single overview table
for this, we need a little bit of R wizardry ...
apply function quick.test() to each number 1, ..., 15
res.list <- lapply(1:15, quick.test)
pass res.list as individual arguments to rbind()
(think of this as an idiom you just have to remember ...)
res <- do.call(rbind, res.list)
res # data frame with one row for each genre
round(res, 3) # rounded values are easier to read</pre>

78

It's your turn now ...

• Questions:

}

- Which differences are significant?
- Are the effect sizes linguistically relevant?
- A different approach:

print(quick.test(i))

- You can construct a list of contingency tables with the cont.table() function from the corpora package
- Apply fisher.test() or chisq.test() directly to each table in the list using the lapply() function
- Try to extract relevant information with sapply()

79